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Article *in* Industrial Engineering & Management Systems · December 2023

DOI: 10.7232/iems.2023.22.4.449

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Industrial Engineering
& Management Systems
Vol 22, No 4, December 2023, pp.449-462
ISSN 1598-7248 | EISSN 2234-6473 |
https://doi.org/10.7232/iems.2023.22.4.449

Developing a Two-Stage Supply Chain Model using a Discrete Time Markov Chain Model during Supply Chain Disruptions

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(Received: February 24, 2023 / Revised: August 22, 2023 / Accepted: September 4, 2023)

ABSTRACT

Disruptions in the supply chain have become more frequent in recent years. The unpredictability and pressure of existing problems are also growing. Dealing with unforeseen events necessitates considering the possibility of supply chain disruption. This paper proposes a strategic model to aid supply chain managers and practitioners in system planning in advance of unforeseen events. First, this study develops a Discrete-time Markov Chain model and defines the steady-state probabilities that measure each state's likelihood over time. The expected disruption costs are then considered in the steady-state probabilities. This assists in estimating the costs of each state's and the system's disruption. In addition, a sensitivity analysis is performed to examine the effects of transition rates on the expected disruption cost for each of the considered groups and the expected disruption cost for the system overall.

Keywords: Supply Chain Disruption, Discrete-Time Markov Chain, Supply Chain Management

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1. INTRODUCTION

In recent years, academics and industry practitioners have focused a significant deal of attention on supply chain disruption in light of the increasing complexity and severity of global challenges (Tang and Nurmaya Musa, 2011). This is exemplified by the COVID-19 pandemic, characterized by a protracted period of disruption and has a substantial and unavoidable impact on the supply chain network. Therefore, even though supply chain disruption has been studied for over twenty years, it is necessary to reevaluate how to respond to various types of disruption. A negative impact on revenue, downtime, delivery times, customer loss, stock returns, and brand reputation are just some of the negative outcomes that can occur as a result of disruptions (Hendricks and Singhal, 2005).

Supply chain disruptions can occur for various reasons, including but not limited to natural disasters, terrorism, factory explosions, blackouts, pandemic, and political or economic crisis. For example, the worldwide semiconductor industry was severely impacted by the earthquake that struck Taiwan in 1999. Delays in the delivery of necessary components led to a decrease in production at Ford Motor Company after the 2001 terrorist attacks in the United States. In 2005, oil and gas production platforms in the Gulf of Mexico were in danger from Hurricane Katrina. In addition, Toyota's market leadership was eroded after the 2011 Japanese earthquake and tsunami caused widespread disruptions worldwide to supply chains for automobiles and electronics (Gurnani *et al.*, 2013). The global pandemic known as COVID-19 is another glaring example that severely damaged the supply chain network worldwide and lasted for a very long period. However, the severity of risks can be reduced by rethinking supply chain strategy and using new technical approaches.

Given that most previous research has focused on supply chain disruption, this indicates that the Markov Chain model was designed to improve operational rather than strategic levels. Analyzing a system using Markov Chains requires determining the probability of a future event given the present value of a specific variable. The likelihood of an outcome can be computed after a transition diagram has been produced based on the probabilities of future actions at each state. Consequently, the network behavior of two suppliers and one manufacturer is investigated in this study. First, a Discrete Time Markov Chain model will be constructed to estimate the probability that members of the supply chain will be in each system state over the long term. In addition, a sensitivity analysis will be conducted to determine the impact of each parameter on the expected disruption costs. This could assist decision-makers in preparing their capacities and budgets and mitigating negative consequences before the emergence of risks.

The structure of this study is as follows: section 2 conducts a literature evaluation to identify the research gap. In Section 3, we will construct event graphs to represent the transitions between each state of the system when unexpected occurrences disturb the system from the perspective of two suppliers and one manufacturer. Section 4 then employs a numerical experiment and sensitivity analysis to estimate the impact of each group parameter under consideration on the overall expected disruption costs. Finally, section 5 is the conclusion.

2. LITERATURE REVIEW

Supply chain management is the cross-department and cross-enterprise integration and coordination of material, information, and capital flow to convert and utilize supply chain resources most sensibly across the entire value chain, from upstream to downstream members (Ivanov *et al.*, 2017). Many risks and disruptions can affect supply chains since they are complex coordinated networks functioning in uncertain contexts (Chopra *et al.*, 2007; Simchi-Levi *et al.*, 2015). Many methods exist for assessing the performance of a supply chain network in the face of disruptions. Our research uses Markov Chain analysis to look at how networks behave. There is no single best way to handle a supply chain disruption. However, many strategies can be implemented after reviewing the literature, particularly regarding supply chain disruption using Markov Chain analysis.

Under the threat of disruptions at several levels of the supply chain, Schmitt (2011) developed strategies to maintain service levels to customers. This paper aimed to help managers choose between mitigation measures by demonstrating which backup strategies are most effective for various types and locations of system disruptions. Shao (2012) analyzed and compared several demand-side reactive approaches in light of supply chain disruptions. They consider an assembly-on-demand system where two comparable goods are produced, and customers are price- and disruption-sensitive. Gurning *et al.* (2013) created a Continuous Time Markov Chain model to enable maritime service operators and users to manage a Wheat Supply Chain during disruptive maritime events. The results help in indicating Wheat Supply Chain costs and time functions over a year to optimize mitigation strategies. Lin (2011) created an adaptive production-procurement system using the 3C theory and Markov Chain. The transition probabilities matrix of each uncertain production category can be used to compute its limiting probability due to Markov chain recurrence. The 3C Theory classifies and predicts market demands using capacity and consumption replenishment with market changes. A hybrid Markov process-mathematical programming method for combined location-inventory under supply disruptions was designed by Dehghani *et al.* (2018). The proposed location-inventory model can handle random supply disruptions. Thus, opened facilities alternate between available and unavailable statuses. To examine the optimal inventory control policy facing non-stationary disruption risk and random lead times, Hekimoğlu *et al.* (2018) developed the Markov-modulated analysis of a spare parts network with random lead times and interruption risks. Sato and Takezawa (2018) used a Markov chain Decision to represent supply chain disruption using a state-dependent supply network and demand distribution whose states follow a Markov chain. Instead of improving production capacity, the authors included supply transportation network flexibility to reduce the supply chain interruption likelihood. Hosseini *et al.* (2020) modeled the Ripple impact of supplier disruption using a Markov chain in combination with dynamic Bayesian networks. The authors sought to create a metric that measures the overall expected utility and service level impacts of supplier disruption on manufacturers. Using Discrete-Time Markov Decision Process, Karim and Nakade (2021) modeled a supply chain with one retailer and one manufacturer, considering environmental sustainability and interruption. The authors calculated the optimal work order quantity for the retailer, which maximizes the retailer's total expected revenue, along with the optimal production quantity and green technology investment cost for the manufacturer, which maximizes the manufacturer's total expected profit. Poormoaied

Table 1. Literature review summary of supply chain disruption using the Markov Chain model

No.	Authors	Model technique	Measurements/ Strategies
1	Ross *et al.* (2008)	CTMC	Robustness of a time-dependent ordering policy
2	Wu *et al.* (2010)	DTMC	Lost sales, backorder
3	Gurning and Cahoon (2011)	CTMC	Inventory and sourcing mitigation, contingency rerouting, recovery and business continuity planning
4	Schmitt (2011)	Optimization Model	Service level,partial backordering
5	Lin (2011)	MC and 3C theory	Optimal production quantity, Economic Production quantity
6	Shao (2012)	Optimization Model	Backordering, the upgrading/ downgrading, the compensation, and the mixed strategy
7	Gurning *et al.* (2013)	CTMC and Simulation Model	Inventory and sourcing mitigation, contingency rerouting, recovery and business continuity planning
8	Gao (2015)	MC and Dynamic Programming	Lost sales, fixed transportation cost
9	Saghafian and Van Oyen (2016)	DTMC	Backup flxible capacity
10	Ahmed *et al.* (2017)	CTMC	Optimizing decision values (order quantities, reorder point) to minimize cost per unit time
11	Dehghani *et al.* (2018)	MC, Optimization Model, Fuzzy and Robust Programming	Minimizing total costs of locations, transportaion, and inventory
12	Hekimoğlu *et al.* (2018)	MC	Probability for the service rate of queneing machanism
13	Sato and Takezawa (2018)	MDP	Total expected cost
14	Hosseini *et al.* (2020)	DTMC and DBN	Risk propagation, service level, expected utility cost
15	Karim and Nakade (2021)	Mathematical Model and DTMDP	Defective production quantity, optimal workorder and production quantity
16	Poormoaied and Demirci (2021)	Optimization Model and CTMC	Unit lost sales cost, expected total cost.
17	Anuat *et al.* (2022)	DTMC and DBN	Energy resilience impact metric, risk propagation

Note: DTMC = Discrete-Time Markov Chain, DBN = Dynamic Bayesian Network, CTMC = Continuous-Time Markov Chain, MDP = Markov Decision Process, DTMDP = Discrete-Time Markov Decision Process, WSC = Wheat Supply Chain, MC = Markov Chain.

and Demirci (2021) analyzed a stock-keeping system that allowed for emergency reordering in the context of a supply disruption. By analyzing the workings of the inventory policy, the continuous Time Markov chain model provided a method for calculating steady-state probability, and the optimization model assisted in determining the best values for the policy's parameters.

Most previous research on the Markov Chain model, including disruptions, has focused mainly on the operational level, particularly the inventory model, as shown in Table 1. Therefore, this study aims to construct a Discrete Time Markov Chain (DTMC) model that aids in the strategic planning of supply chain networks prior to disruptions. The model can assess the likelihood that the system will be in each condition over a long period. In addition, the model includes the disruption cost, which assists practitioners in making the appropriate decision.

3. METHODOLOGY

In this section, we depict the model's assumptions and the Markov Chain model of two suppliers and one manufacturer, which facilitates the development of a transition diagram and a set of balance equations. After that, the expected disruption costs and the steady-state probability are combined to generate the performance measure. Table 2 provides a summary of the notations employed in this study.

3.1 Model Assumptions

3.1.1 Supply Chain Model Assumptions

This research considers a two-stage supply chain with two suppliers and one manufacturer. We assume that the first supplier sends the component A to the manufacturer, while the second supplier sends the component B. A

Table 2. Notation used in this study

Notation	Description
π_j	Steady-state probability that the process is in state $j (j = 1, 2, 3, ..., 13)$
λ	The rate at which a system transitions from a less severe to a more severe state
μ	The rate at which a system transitions from a more severe to a less severe state
C_1	The disruption costs for Group 1 (G_1) when both suppliers are partly disrupted
C_2	The disruption costs for Group 2 (G_2) when 1st supplier is completely while 2nd supplier is partly disrupted
C_3	The disruption costs for Group 3 (G_3) when 1st supplier is partly while 2nd supplier is completely disrupted
C_4	The disruption costs for Group 4 (G_4) when both suppliers are completely disrupted
$E[G_1]$	The expected disruption costs for Group 1
$E[G_2]$	The expected disruption costs for Group 2
$E[G_3]$	The expected disruption costs for Group 3
$E[G_4]$	The expected disruption costs for Group 4
$E[C]$	Total expected disruption cost in the system

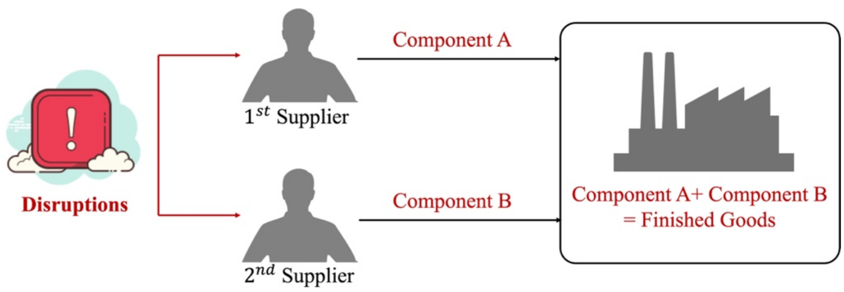

Figure 1. The supply chain model with two suppliers and one manufacturer.

manufacturer will then assemble component A and component B into a finished product. If both suppliers are unable to deliver components to a manufacturer due to disruptions, the manufacturer will be unable to continue production since their safety stock is run out (see Figure 1). The following is a list of the assumptions this study considers. It should be mentioned that the research assumptions comprise the supply chain model and Markov Chain model assumptions.

3.1.2 Disruption Assumptions

- The disruption could randomly affect suppliers.
- Both suppliers experience disruptions simultaneously.
- A manufacturer will recover simultaneously if at least one supplier recovers from the disruption.
- Once the suppliers become conscious of the situation, they will seek to recover it; if they have lost a fraction of their capacity, they will not allow the problem to increase.

3.1.3 Markov Chain Model Assumptions

- It is assumed that the transition rate follows an exponential distribution. λ represents the rate at which the system moves from lower to higher severity. In contrast, the rate at which the severity goes from high to low is μ.
- In this study, we assume that state 0 is a normal functional state, state 1 is a partial loss of capacity (i.e., partly disrupted state), and state 2 is a total loss of capacity (i.e., completely disrupted state).

3.2 Markov Chain Model of a Two-stage Supply Chain

To observe the network behavior of a supply chain, this study considers a two-stage supply chain with two suppliers and one manufacturer considering when they experience the unexpected events. The state number and the description of each state are given in Table 3. The state transition diagram of two suppliers and one manufacturer is shown in Figure 2.

Figure 2 illustrates the Markov chain model's state transition diagram. It is used to analyze the network behavior of a two-stage supply chain model and aids in displaying the transitions between each state when the system is disturbed by random occurrences.

Table 3. A description of a supply chain model's state notation

State No. (X, Y, Z)	π_j	1^{st} Supplier X	2^{nd} Supplier Y	Manufacturer Z
0,0,0	π_1	normal operation	normal operation	normal operation
1,1,0	π_2	partly disrupted	partly disrupted	normal operation
1,1,1	π_6	partly disrupted	partly disrupted	partly disrupted
1,1,2	π_7	partly disrupted	partly disrupted	completely disrupted
1,2,0	π_4	partly disrupted	completely disrupted	normal operation
1,2,1	π_{10}	partly disrupted	completely disrupted	partly disrupted
1,2,2	π_{11}	partly disrupted	completely disrupted	completely disrupted
2,1,0	π_5	completely disrupted	partly disrupted	normal operation
2,1,1	π_8	completely disrupted	partly disrupted	partly disrupted
2,1,2	π_9	completely disrupted	partly disrupted	completely disrupted
2,2,0	π_3	completely disrupted	completely disrupted	normal operation
2,2,1	π_{12}	completely disrupted	completely disrupted	partly disrupted
2,2,2	π_{13}	completely disrupted	completely disrupted	completely disrupted

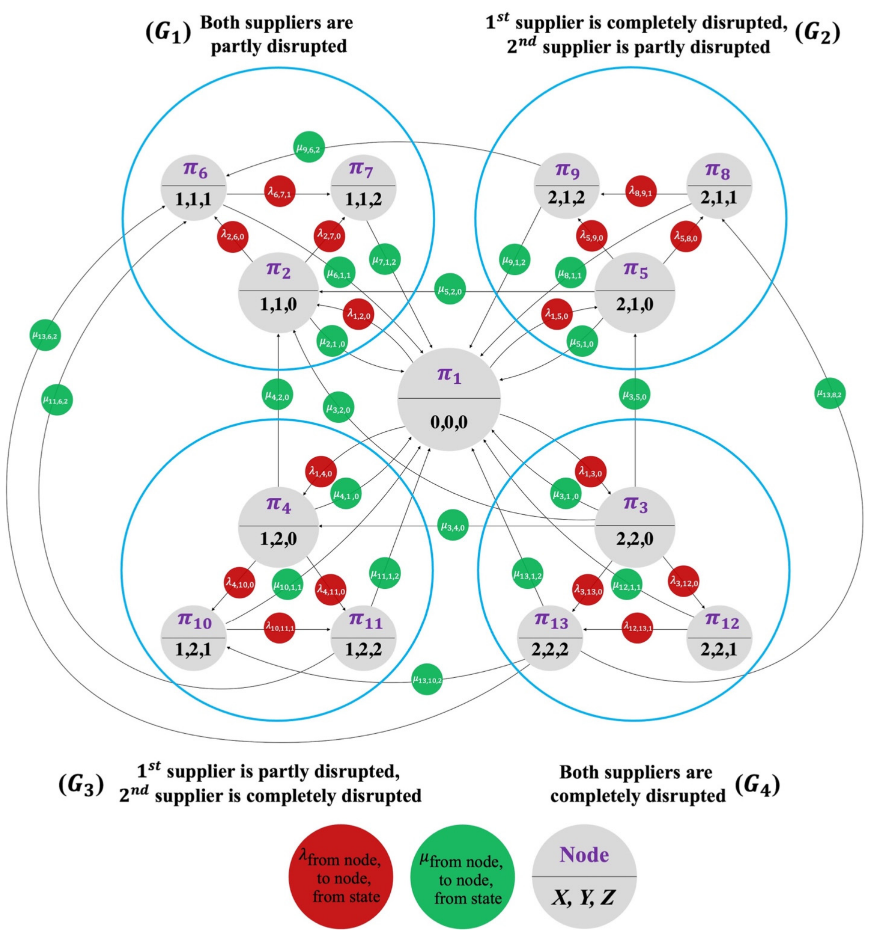

Figure 2. The state transition diagram.

3.3 Balance Equations

The balance equations can be determined from the state transition diagram (see Figure 2). It helps in determining the steady state probability. In other words, the probability that the system will be in each state in the long run (π_j) which is given in Table 4.

Hence, the steady-state probability can be seen as follows.
Let denote

$$A=\left(\frac{1}{\lambda_{2,6,0}+\lambda_{2,7,0}+\mu_{2,1,0}}\right)\left[\lambda_{1,2,0}+\frac{\mu_{5,2,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)+\frac{\mu_{4,2,0}}{\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0}}\left(\lambda_{1,4,0}+\frac{\mu_{3,4,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)+\frac{\mu_{3,2,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right]$$

$$B=\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$C=\left(\frac{1}{\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0}}\right)\left(\lambda_{1,4,0}+\frac{\mu_{3,4,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$D=\left(\frac{1}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}\right)\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$E=\left(\frac{1}{\lambda_{6,7,1}+\mu_{6,1,1}}\right)\left\{\frac{\lambda_{2,6,0}}{\lambda_{2,6,0}+\lambda_{2,7,0}+\mu_{2,1,0}}\left[\lambda_{1,2,0}+\frac{\mu_{5,2,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)+\frac{\mu_{4,2,0}}{\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0}}\left(\lambda_{1,4,0}+\frac{\mu_{3,4,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)\right.$$

Table 4. The balance equations of two suppliers and one manufacturer

State No. (X, Y, Z)	Outgoing rate	=	Incoming rate
0,0,0	$(\lambda_{1,2,0}+\lambda_{1,3,0}+\lambda_{1,4,0}+\lambda_{1,5,0})\pi_1$	=	$\mu_{2,1,0}\pi_2+\mu_{3,1,0}\pi_3+\mu_{4,1,0}\pi_4+\mu_{5,1,0}\pi_5$ $+\mu_{6,1,1}\pi_6+\mu_{7,1,2}\pi_7+\mu_{8,1,1}\pi_8+\mu_{9,1,2}\pi_9$ $+\mu_{10,1,1}\pi_{10}+\mu_{11,1,2}\pi_{11}+\mu_{12,1,1}\pi_{12}+\mu_{13,1,2}\pi_{13}$
1,1,0	$(\lambda_{2,6,0}+\lambda_{2,7,0}+\mu_{2,1,0})\pi_2$	=	$\lambda_{1,2,0}\pi_1+\mu_{3,2,0}\pi_3+\mu_{4,2,0}\pi_4+\mu_{5,2,0}\pi_5$
2,2,0	$(\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0})\pi_3$	=	$\lambda_{1,3,0}\pi_1$
1,2,0	$(\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0})\pi_4$	=	$\lambda_{1,4,0}\pi_1+\mu_{3,4,0}\pi_3$
2,1,0	$(\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0})\pi_5$	=	$\lambda_{1,5,0}\pi_1+\mu_{3,5,0}\pi_3$
1,1,1	$(\lambda_{6,7,1}+\mu_{6,1,1})\pi_6$	=	$\lambda_{2,6,0}\pi_2+\mu_{9,6,2}\pi_9+\mu_{11,6,2}\pi_{11}+\mu_{13,6,2}\pi_{13}$
1,1,2	$\mu_{7,1,2}\pi_7$	=	$\lambda_{2,7,0}\pi_2+\lambda_{6,7,1}\pi_6$
2,1,1	$(\lambda_{8,9,1}+\mu_{8,1,1})\pi_8$	=	$\lambda_{5,8,0}\pi_5+\mu_{13,8,2}\pi_{13}$
2,1,2	$(\mu_{9,1,2}+\mu_{9,6,2})\pi_9$	=	$\lambda_{5,9,0}\pi_5+\lambda_{8,9,1}\pi_8$
1,2,1	$(\lambda_{10,11,1}+\mu_{10,1,1})\pi_{10}$	=	$\lambda_{4,10,0}\pi_4+\mu_{13,10,2}\pi_{13}$
1,2,2	$(\mu_{11,1,2}+\mu_{11,6,2})\pi_{11}$	=	$\lambda_{4,11,0}\pi_4+\lambda_{10,11,1}\pi_{10}$
2,2,1	$(\lambda_{12,13,1}+\mu_{12,1,1})\pi_{12}$	=	$\lambda_{3,12,0}\pi_3$
2,2,2	$(\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2})\pi_{13}$	=	$\lambda_{3,13,0}\pi_3+\lambda_{12,13,1}\pi_{12}$
And normalization condition			

$$+\frac{\mu_{3,2,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\Bigg]$$

$$+\frac{\mu_{9,6,2}}{\mu_{9,1,2}+\mu_{9,6,2}}\Bigg[\frac{\lambda_{5,9,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}$$

$$\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\lambda_{8,9,1}}{\lambda_{8,9,1}+\mu_{8,1,1}}\Bigg(\frac{\lambda_{5,8,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}$$

$$\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\mu_{13,8,2}}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$

$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)\Bigg)\Bigg]\Bigg]$$

$$+\frac{\mu_{13,6,2}}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$

$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\mu_{11,6,2}}{\mu_{11,1,2}+\mu_{11,6,2}}\left(\lambda_{4,11,0}+\frac{\lambda_{10,11,1}\lambda_{4,10,0}}{\lambda_{10,11,1}+\mu_{10,1,1}}\right)$$

$$\left(\frac{1}{\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0}}\right)$$

$$\left(\lambda_{1,4,0}+\frac{\mu_{3,4,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\lambda_{10,11,1}}{\mu_{11,1,2}+\mu_{11,6,2}}\left(\frac{\mu_{13,10,2}}{\lambda_{10,11,1}+\mu_{10,1,1}}\right)$$

$$\left(\frac{1}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\right)\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$

$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)\Bigg)\Bigg\}$$

$$\mathrm{F}=\left(\frac{1}{\mu_{7,1,2}}\right)\Bigg\{\frac{\lambda_{2,7,0}}{\lambda_{2,6,0}+\lambda_{2,7,0}+\mu_{2,1,0}}$$

$$\Bigg[\lambda_{1,2,0}+\frac{\mu_{5,2,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}$$

$$\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\mu_{4,2,0}}{\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0}}$$

$$\left(\lambda_{1,4,0}+\frac{\mu_{3,4,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\mu_{3,2,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\Bigg]$$

$$+\frac{\lambda_{6,7,1}}{\lambda_{6,7,1}+\mu_{6,1,1}}\Bigg[\frac{\lambda_{2,6,0}}{\lambda_{2,6,0}+\lambda_{2,7,0}+\mu_{2,1,0}}$$

$$\Bigg[\lambda_{1,2,0}+\frac{\mu_{5,2,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}$$

$$\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\mu_{4,2,0}}{\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0}}$$

$$\left(\lambda_{1,4,0}+\frac{\mu_{3,4,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\mu_{3,2,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\Bigg]$$

$$+\frac{\mu_{9,6,2}}{\mu_{9,1,2}+\mu_{9,6,2}}\Bigg[\frac{\lambda_{5,9,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}$$

$$\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\lambda_{8,9,1}}{\lambda_{8,9,1}+\mu_{8,1,1}}\Bigg(\frac{\lambda_{5,8,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}$$

$$\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\mu_{13,8,2}}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$

$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)\Bigg)\Bigg]\Bigg]$$

$$+\frac{\mu_{13,6,2}}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$

$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\mu_{11,6,2}}{\mu_{11,1,2}+\mu_{11,6,2}}\left(\lambda_{4,11,0}+\frac{\lambda_{10,11,1}\lambda_{4,10,0}}{\lambda_{10,11,1}+\mu_{10,1,1}}\right)$$

$$\left(\frac{1}{\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0}}\right)$$

$$\left(\lambda_{1,4,0}+\frac{\mu_{3,4,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$+\frac{\lambda_{10,11,1}}{\mu_{11,1,2}+\mu_{11,6,2}}\left(\frac{\mu_{13,10,2}}{\lambda_{10,11,1}+\mu_{10,1,1}}\right)$$

$$\left(\frac{1}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\right)\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$

$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)\Bigg]\Bigg]\Bigg\}$$

$$G=\left(\frac{1}{\lambda_{8,9,1}+\mu_{8,1,1}}\right)\Bigg[\frac{\lambda_{5,8,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}$$
$$\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$
$$+\frac{\mu_{13,8,2}}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$
$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)\Bigg]\Bigg]$$

$$H=\left(\frac{1}{\mu_{9,1,2}+\mu_{9,6,2}}\right)\Bigg\{\frac{\lambda_{5,9,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}$$
$$\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$
$$+\frac{\lambda_{8,9,1}}{\lambda_{8,9,1}+\mu_{8,1,1}}\Bigg[\frac{\lambda_{5,8,0}}{\lambda_{5,8,0}+\lambda_{5,9,0}+\mu_{5,1,0}+\mu_{5,2,0}}$$
$$\left(\lambda_{1,5,0}+\frac{\mu_{3,5,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$
$$+\frac{\mu_{13,8,2}}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$
$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)\Bigg]\Bigg]\Bigg\}$$

$$I=\left(\frac{1}{\lambda_{1011,1}+\mu_{10,1,1}}\right)\Bigg[\frac{\lambda_{4,10,0}}{\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0}}$$
$$\left(\lambda_{1,4,0}+\frac{\mu_{3,4,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$
$$+\frac{\mu_{13,10,2}}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$
$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$J=\Bigg\{\frac{1}{\mu_{11,1,2}+\mu_{11,6,2}}\left(\lambda_{4,11,0}+\frac{\lambda_{10,11,1}\lambda_{4,10,0}}{\lambda_{1011,1}+\mu_{10,1,1}}\right)$$
$$\left(\frac{1}{\lambda_{4,10,0}+\lambda_{4,11,0}+\mu_{4,1,0}+\mu_{4,2,0}}\right)$$
$$\left(\lambda_{1,4,0}+\frac{\mu_{3,4,0}\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$
$$+\frac{\lambda_{10,11,1}}{\mu_{11,1,2}+\mu_{11,6,2}}\left(\frac{\mu_{13,10,2}}{\lambda_{10,11,1}+\mu_{10,1,1}}\right)$$
$$\Bigg[\frac{1}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$
$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)\Bigg]\Bigg]$$

$$K=\left(\frac{\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

$$L=\left(\frac{1}{\mu_{13,1,2}+\mu_{13,10,2}+\mu_{13,6,2}+\mu_{13,8,2}}\right)\left(\lambda_{3,13,0}+\frac{\lambda_{12,13,1}\lambda_{3,12,0}}{\lambda_{12,13,1}+\mu_{12,1,1}}\right)$$
$$\left(\frac{\lambda_{1,3,0}}{\lambda_{3,12,0}+\lambda_{3,13,0}+\mu_{3,1,0}+\mu_{3,2,0}+\mu_{3,4,0}+\mu_{3,5,0}}\right)$$

In which

$$\pi_1=\frac{1}{\{1+A+B+C+D+E+F+G+H+I+J+K+L\}} \quad (1)$$

$$\pi_2=A\pi_1 \quad (2)$$

$$\pi_3=B\pi_1 \quad (3)$$

$$\pi_4=C\pi_1 \quad (4)$$

$$\pi_5=D\pi_1 \quad (5)$$

$$\pi_6=E\pi_1 \quad (6)$$

$$\pi_7=F\pi_1 \quad (7)$$

$$\pi_8=G\pi_1 \quad (8)$$

$$\pi_9=H\pi_1 \quad (9)$$

$$\pi_{10}=I\pi_1 \quad (10)$$

$$\pi_{11}=J\pi_1 \quad (11)$$

$$\pi_{12}=K\pi_1 \quad (12)$$

$$\pi_{13}=L\pi_1 \quad (13)$$

After identifying the steady state probability, the following section will construct numerical experiments to create a base case, followed by the creation of sensitivity to investigate the impact of parameters on the total expected disruption costs. The purpose of determining the steady-state probabilities is to assess the likelihood that the system will be in a different condition over a long period.

3.4 The Performance Measures

In this study, the steady-state probabilities are combined with the disruption cost to assist decision-makers in anticipating the effects of unforeseeable events. Simultaneously, they can prepare the required budget once the threat emerge. Hence, the expected disruption costs can be calculated as follows.

$$E[G_1] = (\pi_2 + \pi_6 + \pi_7)C_1 \tag{14}$$

$$E[G_2] = (\pi_5 + \pi_8 + \pi_9)C_2 \tag{15}$$

$$E[G_3] = (\pi_4 + \pi_{10} + \pi_{11})C_3 \tag{16}$$

$$E[G_4] = (\pi_3 + \pi_{12} + \pi_{13})C_4 \tag{17}$$

$$E[C] = E[G_1] + E[G_2] + E[G_3] + E[G_4] \tag{18}$$

The expected disruption cost can be separated to four groups (i.e., G_1, G_2, G_3, and G_4), as shown in Figure 2 and Table 4. It should be noted that the criteria should be $C_4 > C_2, C > C_3$ because the disruption costs when both suppliers are fully disrupted should be the highest one (C_4). On the other hand, the disruption costs when both suppliers are partially disrupted should be the lowest one (C_1). Therefore, $E[C]$ is the performance measure that must be minimized.

4. NUMERICAL ANALYSIS

This section describes the numerical experiments performed to demonstrate the effects of group parameters on the expected cost of disruption separated by group. Then, the total expected disruption cost of the system is measured. Finally, the results are discussed in this section.

4.1 Define the Base Case Values

4.1.1 Define the base case values for transition rates

In Table 5, the initial values for all parameters have

Table 5. Base case values for transition rates

Group	Group Name	No.	Parameters	Value
1	Both suppliers are partly disrupted (G_1)	1	$\lambda_{1,2,0}$	0.03
		2	$\lambda_{2,6,0}$	0.008
		3	$\lambda_{2,7,0}$	0.0005
		4	$\lambda_{6,7,1}$	0.0005
		5	$\mu_{2,1,0}$	0.5
		6	$\mu_{6,1,1}$	0.2
		7	$\mu_{7,1,2}$	0.2
2	1^{st} supplier is completely and 2^{nd} supplier is partly disrupted (G_2)	8	$\lambda_{1,5,0}$	0.008
		9	$\lambda_{5,8,0}$	0.008
		10	$\lambda_{5,9,0}$	0.0005
		11	$\lambda_{8,9,1}$	0.0005
		12	$\mu_{5,1,0}$	0.3
		13	$\mu_{8,1,1}$	0.2
		14	$\mu_{9,1,2}$	0.2
3	1^{st} supplier is partly and 2^{nd} supplier is completely disrupted (G_3)	15	$\lambda_{1,4,0}$	0.008
		16	$\lambda_{4,10,0}$	0.008
		17	$\lambda_{4,11,0}$	0.0005
		18	$\lambda_{10,11,1}$	0.0005
		19	$\mu_{4,1,0}$	0.3
		20	$\mu_{10,1,1}$	0.2
		21	$\mu_{11,1,2}$	0.2
4	Both suppliers are completely disrupted (G_4)	22	$\lambda_{1,3,0}$	0.0005
		23	$\lambda_{3,12,0}$	0.008
		24	$\lambda_{3,13,0}$	0.0005
		25	$\lambda_{12,13,1}$	0.0005
		26	$\mu_{3,1,0}$	0.5
		27	$\mu_{12,1,1}$	0.2
		28	$\mu_{13,1,2}$	0.2

been established, assuming the system will operate regularly at a performance level of approximately 95% in the long period of time. All the base case values in Table 5 must make up a proportion of state (0,0,0), i.e., π_1 of no less than 0.95 (95%). However, if disruptions occur, the system may transition to other states in varying proportions, as demonstrated in Table 6. By utilizing the base case values from Table 5, the steady-state probability derived from equations (1)-(13) can be computed. The results are in Table 6, which summarizes the probability and explains the outcomes. These results explain the probability in the long-term period. The probability that the system will operate normally (π_1) is approximately 0.95390910. At about 0.01281551 and 0.00078382 are the chances that both suppliers will be partially (π_2) and completely (π_3) disrupted, whereas a manufacturer will generally be operating. The likelihood that the first supplier will be partly disrupted, but the second supplier will be wholly disrupted (π_4), and the probability that the first supplier will be disrupted entirely but the second supplier will be partly disrupted (π_5) while a manufacturer is still normally operating are at about 0.01531571. An estimated 0.05518 percent of all members will be partly disrupted (π_6). The probability that both suppliers will be partially disrupted, and a manufacturer wholly disrupted (π_7) is nearly 0.00003342. The likelihood that the first supplier will experience a complete disruption, the second supplier, and a manufacturer will experience a partial disruption (π_8) is approximately 0.00061161. The probability that the first supplier and a manufacturer will be wholly disrupted while the second supplier will be partly disrupted (π_9) is about 0.00001991. The likelihood that the first supplier and a manufacturer will experience partial disruption (π_{10}) is nearly 0.00061161. The possibility that a first supplier will be partly disrupted, a second supplier, and a manufacturer will be disrupted entirely (π_{11}) is estimated at 0.00000001. The probability that both suppliers will be completely disrupted, and one manufacturer will be partly disrupted is around 0.00003127 (π_{12}). Lastly, the likelihood of a complete system disruption (π_{13}) is about 0.00000051.

Table 6. Steady-state probabilities (π_j) for a base case

π_i	Steady state probability for a base case
π_1	0.95390910
π_2	0.01281551
π_3	0.00078382
π_4	0.01531571
π_5	0.01531571
π_6	0.00055182
π_7	0.00003342
π_8	0.00061161
π_9	0.00001991
π_{10}	0.00061161
π_{11}	0.00000001
π_{12}	0.00003127
π_{13}	0.00000051
Total	1.00000000

4.1.2 Define the base case values for disruption costs

To determine the total expected disruption cost ($E[C]$), we first established base case values for each group's disruption cost, as detailed in Table 7. We assumed that the proposed supply chain network would be subject to random disruptions that would leave suppliers unable to provide raw materials to manufacturers. We then established the highest disruption cost (C_4) at $100,000 per day if both suppliers were completely disrupted (G_4), followed by the disruption cost (C_2) of $75,000 per day if the first supplier was completely disrupted while the second supplier was only partly disrupted (G_2) and the disruption cost (C_3) of $75,000 per day if the first supplier was only partly disrupted while the second supplier was completely disrupted (G_3). The disruption costs for Group 1 (G_1), when both suppliers were only partly disrupted, were assumed at $50,000 per day ($C_1$).

Using equation (14), we calculated the expected cost of disruption for Group 1 ($E[G_1]$) at $670.04. Using equation (15), the expected cost of disruption for Group 2 ($E[G_2]$) is $1,196.04. Using equation (16), the expected disruption cost for Group 3 ($E[G_3]$) is $1,194.55. Finally, using equation (17), the expected cost of disruption for Group 4 ($E[G_4]$) is $81.56. By adding these expected costs and using equation (18), the total expected cost of disruption ($E[C]$) is $3,142.19. Table 8 provides a summary of these calculations.

Table 7. Base case values for disruption costs

Parameters	Disruption costs (per day) for a base case (×$1,000)
C_1	50
C_2	75
C_3	75
C_4	100

Table 8. The Total expected disruption cost for a base case

$E[G_i]$	Expected disruption costs (per day) for a base case (× $1,000)
$E[G_1]$	0.670
$E[G_2]$	1.196
$E[G_3]$	1.195
$E[G_4]$	0.082

Total, $E[C]$	3.142

4.2 Results and Discussions

The values of each parameter in G_1 to G_4 used in this study are shown in Tables 9 to 12, and the results are depicted in Figures 3 to 6. The objective is to determine the effect of group transition rates on expected disruption costs.

4.2.1 The Effect of Transition Rate in G_1 on the Expected disruption costs

Observe the effect of transition rates in G_1 on the expected disruption costs, which consist of the seven parameters shown in Table 9. The results of the expected disruption costs when increasing the transition rates in G_1 are depicted in Figure 3.

It can be seen in Figure 3 that as the transition rates in G_1 increase, the expected disruption costs in G_1 (i.e., $E[G_1]$) decrease while the expected disruption costs in G_2, G_3, and G_4 (i.e., $E[G_2], E[G_3], E[G_4]$) rise. As a result, the system's expected total disruption cost ($E[C]$) decreases. This means that in a scenario where both suppliers are partly disrupted, increasing the value of transition rates in G_1 has a beneficial effect on the expected disruption cost in G_1 but a negative impact on the expected disruption cost G_2, G_3, and G_4. Nonetheless, when taken overall, the expected disruption cost is reduced.

4.2.2 The effect of transition rate in G_2 on the expected disruption costs

Consider the effect of transition rates in G_2 on the expected disruption costs, comprised of the seven para-

Table 9. Data sets of sensitivity analysis for G_1

Group	Group Name	Parameters	Base Case Value	1	2	3	4	5
1	Both suppliers are partly disrupted (G_1)	$\lambda_{1,2,0}$	0.03	0.04	0.05	0.06	0.07	0.08
		$\lambda_{2,6,0}$	0.008	0.009	0.01	0.011	0.012	0.013
		$\lambda_{2,7,0}$	0.0005	0.0006	0.0007	0.0008	0.0009	0.001
		$\lambda_{6,7,1}$	0.0005	0.0006	0.0007	0.0008	0.0009	0.001
		$\mu_{2,1,0}$	0.5	0.6	0.7	0.8	0.9	1
		$\mu_{6,1,1}$	0.2	0.3	0.4	0.5	0.6	0.7
		$\mu_{7,1,2}$	0.2	0.3	0.4	0.5	0.6	0.7

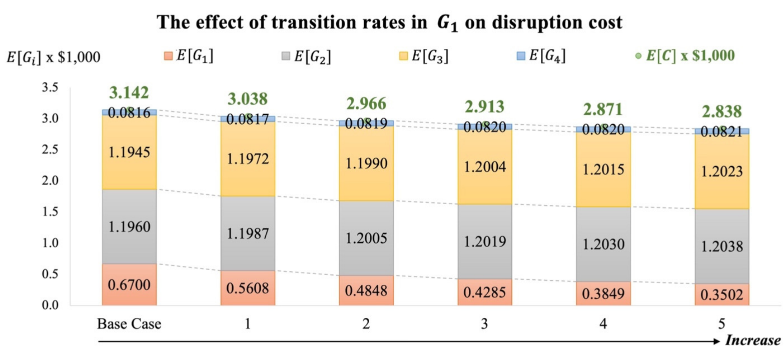

Figure 3. The effect of transition rates in G_1 on the expected disruption cost.

Table 10. Data sets of sensitivity analysis for G_2

Group	Group Name	Parameters	Base Case Value	1	2	3	4	5
2	1^{st} supplier is completely and 2^{nd} supplier is partly disrupted (G_2)	$\lambda_{1,5,0}$	0.008	0.04	0.05	0.06	0.07	0.08
		$\lambda_{5,8,0}$	0.008	0.009	0.01	0.011	0.012	0.013
		$\lambda_{5,9,0}$	0.0005	0.0006	0.0007	0.0008	0.0009	0.001
		$\lambda_{8,9,1}$	0.0005	0.0006	0.0007	0.0008	0.0009	0.001
		$\mu_{5,1,0}$	0.3	0.6	0.7	0.8	0.9	1
		$\mu_{8,1,1}$	0.2	0.3	0.4	0.5	0.6	0.7
		$\mu_{9,1,2}$	0.2	0.3	0.4	0.5	0.6	0.7

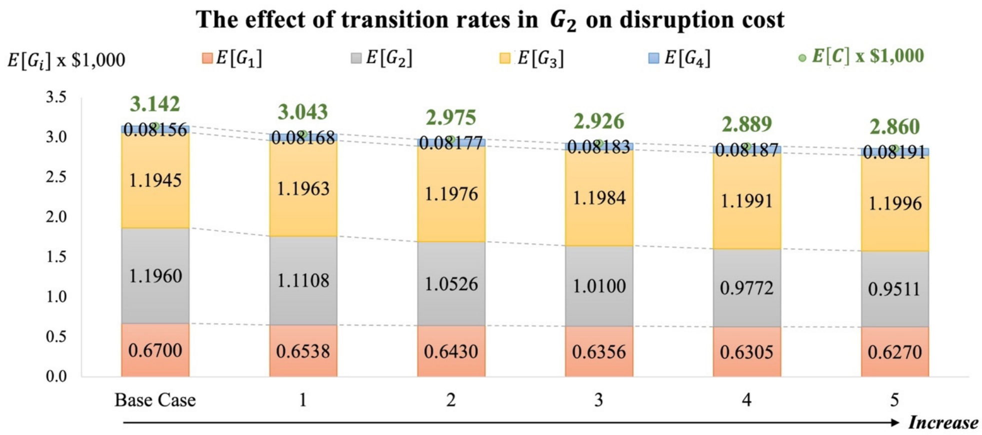

Figure 4. The effect of transition rates in G_2 on the expected disruption cost.

meters in Table 10. Figure 4 depicts the results of expected disruption costs when increasing the transition rates in G_2 .

Figure 4 shows that as G_2 transition rates rise, $E[G_1]$ and $E[G_2]$ decrease, whereas $E[G_3]$ and $E[G_4]$ rise. This leads to a drop in $E[C]$. An increase in the value of transition rates in G_2 has a positive effect on the expected disruption cost in G_1 and G_2 but an adverse impact on the expected disruption cost in G_3 and G_4. This occurs when the first supplier is wholly disrupted, and the second supplier is partly disrupted. However, the total expected disruption cost goes down.

4.2.3 The effect of transition rate in G_3 on the expected disruption costs

Examine the impact of transition rates in G_3 on the expected disruption costs, comprised of seven parameters in Table 11. Figure 5 depicts the expected disruption costs when transition rates in G_3 continue to increase.

Figure 5 demonstrates that as G_3 transition rates increase, $E[G_1]$ and $E[G_3]$ fall while $E[G_2]$ and $E[G_4]$ increase. Because of this, $E[C]$ decreases. The expected disruption cost in G_1 and G_3 improves with a rise in the value of transition rates in G_3, whereas in G_2 and G_4, it worsens. This occurs if there is a partial disruption

Table 11. Data sets of sensitivity analysis for G_3

Group	Group Name	Parameters	Base Case Value	1	2	3	4	5
3	1^{st} supplier is partly and 2^{nd} supplier is completely disrupted (G_3)	$\lambda_{1,4,0}$	0.008	0.009	0.01	0.011	0.012	0.013
		$\lambda_{4,10,0}$	0.008	0.009	0.01	0.011	0.012	0.013
		$\lambda_{4,11,0}$	0.0005	0.0006	0.0007	0.0008	0.0009	0.001
		$\lambda_{10,11,1}$	0.0005	0.0006	0.0007	0.0008	0.0009	0.001
		$\mu_{4,1,0}$	0.3	0.4	0.5	0.6	0.7	0.8
		$\mu_{10,1,1}$	0.2	0.3	0.4	0.5	0.6	0.7
		$\mu_{11,1,2}$	0.2	0.3	0.4	0.5	0.6	0.7

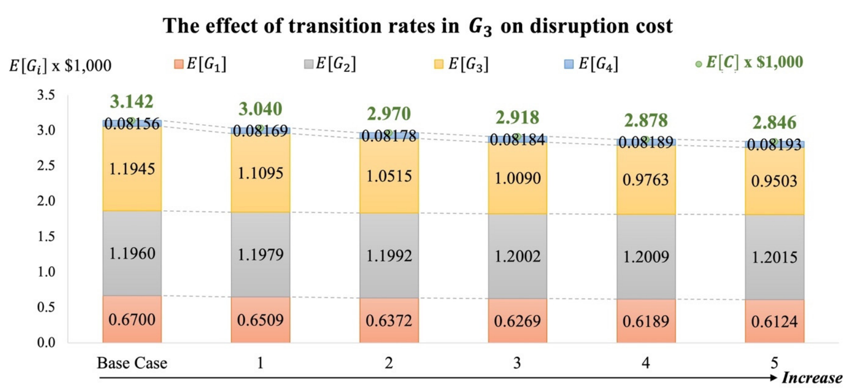

Figure 5. The effect of transition rates in G_3 on the expected disruption cost.

Table 12. Data sets of sensitivity analysis for G_4

Group	Group Name	Parameters	Base Case Value	1	2	3	4	5
4	Both suppliers are completely disrupted (G_4)	$\lambda_{1,3,0}$	0.0005	0.0006	0.0007	0.0008	0.0009	0.001
		$\lambda_{3,12,0}$	0.008	0.009	0.01	0.011	0.012	0.013
		$\lambda_{3,13,0}$	0.0005	0.0006	0.0007	0.0008	0.0009	0.001
		$\lambda_{12,13,1}$	0.0005	0.0006	0.0007	0.0008	0.0009	0.001
		$\mu_{3,1,0}$	0.5	0.6	0.7	0.8	0.9	1
		$\mu_{12,1,1}$	0.2	0.3	0.4	0.5	0.6	0.7
		$\mu_{13,1,2}$	0.2	0.3	0.4	0.5	0.6	0.7

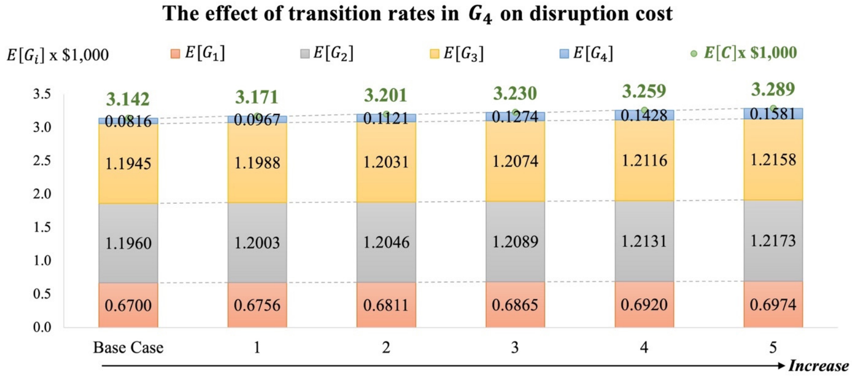

Figure 6. The effect of transition rates in G_4 on the expected disruption cost.

of the first supplier and a total disruption of the second supplier. On the other hand, the overall expected disruption cost drops.

4.2.4 The effect of transition rate in G_4 on the expected disruption costs

Investigate the impact of transition rates in G_4 on the expected disruption costs, which consist of seven parameters in Table 12. Figure 6 displays the expected disruption costs as transition rates in G_4 increase.

As can be seen in Figure 6, the values of $E[G_1]$, $E[G_2]$, $E[G_3]$, and $E[G_4]$ all increase as the transition rates of G_4 increase. As a results, this causes an increase in $E[C]$. If both suppliers apparently lost all of their capacity, it would have a significantly negative impact on the cost of disruption for every group. Furthermore, the total expected disruption cost of the system tends to rise.

5. CONCLUSION

In this study, we construct a Discrete Time Markov Chain model for a two-stage supply chain consisting of two suppliers and a single manufacturer. In the event of disruptions, the managers or practitioners of the supply chain can use the stated steady-state probabilities to prepare for the likelihood that the system will be in each possible state. We include the disruption cost function that can be used to assess the costs of disruptions when taking the transition rates between system states into account. The overall cost of the system can also be estimated by calculating the total expected cost of disruption. This approach can be applied to any industry, as supply chain managers can customize the model to suit their specific supply chain networks by adjusting the relevant transition rates (i.e., parameters) based on desired performance levels. Each transition rate can be adjusted individually or as a group, increasing or decreasing as needed. By doing so, they can estimate the potential disruption costs that may arise in the event of any disruptions in the long-term period.

Through our study, we explore the impact of disruption costs when the transition rate inside and across groups varies. Future research can investigate the effect of disruption time and analyze the transition rates between groups of system states or individual transition rate changes. However, since our study only considers a two-stage supply chain with two suppliers and a single manufacturer, further studies can explore the effect of transition rates on disruption costs or times in multiple-stage supply chain models for a more realistic representation.

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