With this model the fishes attempt to stay a constant distance from each other and move in the same direction by the use of 3 mechanisms, orientation, repulsion and attraction.



Here we see the different zones around an invidiual fish

Repulsion If a fish is within the radius of repulsion then an angle will be calculated away from that fish and averaged with the current fishes angle.

Attraction if a fish is within the radius of attraction an angle will be calculated towards that fish and averaged with the current fishes angle.

Orientation If a fish is whitin the radius of orientation then the direction of that fish will be averaged with the current fishes direction.

This is done for each fish. Given that fishes have a position x and constant speed v. We calculate the distance from fish i to fish j with the Eucledian metric

$$d_{ij} = \sum_{k=1}^{k=dim} (x_j[k] - x_i[k])^2$$

The vector from fish i to fish j is defined by $\overrightarrow{r}_{ij} = x_j - x_i(2)$. Each fish is moving in a direction represented by a unit vector \overrightarrow{dir}_i , to calculate the repulsive component we first find the set of fishes in the repulsive zone S_{rep} . We then calculate the cumulative direction away from S_{rep} followed by a normalization to get the average direction away from fishes in the repulsive zone, our repulsive unit velocity becomes

$$\overrightarrow{v}_{rep} = \frac{\sum_{j \in S_{rep}} -r_{ij}}{|\sum_{j \in S_{rep}} -r_{ij}|}$$

This is done similarly with attraction

$$\overrightarrow{v}_{att} = \frac{\sum_{j \in S_{att}} \overrightarrow{r}_{ij}}{|\sum_{j \in S_{att}} \overrightarrow{r}_{ij}|}$$

Finally we have orientation, taking the average direction of all fish in the orientation zone

$$\overrightarrow{v}_{ori} = \frac{\sum_{j \in S_{ori}} \overrightarrow{dir}_j}{|\sum_{j \in S_{ori}} \overrightarrow{dir}_j|}$$

Finally we change the direction by a weighted average

$$\overrightarrow{dir}_{i}(t + \Delta t) = \frac{\omega_{s}\overrightarrow{dir}_{i}(t) + \omega_{r}\overrightarrow{v}_{rep} + \omega_{a}\overrightarrow{v}_{att} + \omega_{o}\overrightarrow{v}_{ori}}{|\omega_{s}\overrightarrow{dir}_{i}(t) + \omega_{r}\overrightarrow{v}_{rep} + \omega_{a}\overrightarrow{v}_{att} + \omega_{o}\overrightarrow{v}_{ori}|}$$

Where we introduce ω_s our self weight, ω_r repulsion weight, ω_a attraction weight, ω_o orientation weight.

Further we introduce some stochastic behaviour into our model. Every timestep each fish's velocity is altered by adding a random velocity with normally distributed components

$$\overrightarrow{v}_i(t + \Delta t) = |\overrightarrow{v}_i(t)| \overrightarrow{dir}_i(t + \Delta t) + \omega_{vn} * N(\mu_v, \sigma_v^2)$$

where ω_{vn} scales our noise. We do the same with our positions

$$\overrightarrow{x}_i(t) = \overrightarrow{x}_i(t) + \omega_{pn} N(\mu_p, \sigma_p^2)$$

Then finally we update the positions

$$x_i(t + \Delta t) = x_i(t) + \Delta t * \overrightarrow{v}_i(t + \Delta t)$$

, where Δt is our time step and ω_{pn} scales our positional noise. In this model we have introduced a max velocity v_{max} if any fish exceeds this its velocity will be reduced to v_{mean} our mean velocity. The fishes positions and velocities are normally distribitued initially.