

一、公式推导

线特征残差的雅可比

$$\text{一维残差: } d_e = \frac{|(\tilde{p}_i - p_b) \times (\tilde{p}_i - p_a)|}{|p_a - p_b|}$$

$$\hat{x} = \frac{(\tilde{p}_i - p_b) \times (\tilde{p}_i - p_a)}{|p_a - p_b|}$$

$$J_e = \frac{\partial d_e}{\partial T} = \frac{\partial |x|}{\partial T} = \frac{\partial |x|}{\partial x} \frac{\partial x}{\partial T} = \frac{x}{|x|} \frac{\partial x}{\partial T} = \frac{x}{|x|} \frac{\partial x}{\partial \tilde{p}_i} \cdot \frac{\partial \tilde{p}_i}{\partial T}$$

$$\frac{\partial x}{\partial \tilde{p}_i} = \frac{1}{|p_a - p_b|} \left(\frac{\partial (\tilde{p}_i - p_b)^\wedge (\tilde{p}_i - p_a)}{\partial \tilde{p}_i} + \frac{(\tilde{p}_i - p_b)^\wedge \partial (\tilde{p}_i - p_a)}{\partial \tilde{p}_i} \right)$$

$$= \frac{1}{|p_a - p_b|} \left(-(\tilde{p}_i - p_a)^\wedge + (\tilde{p}_i - p_b)^\wedge \right)$$

$$= \frac{(p_a - p_b)^\wedge}{|p_a - p_b|}$$

$$\frac{\partial \tilde{p}_i}{\partial R} = \frac{\partial (R p)}{\partial R} = \lim_{\phi \rightarrow 0} \frac{\exp(\phi^\wedge) \exp(\phi^\wedge) p - \exp(\phi^\wedge) p}{\phi}$$

$$= \lim_{\phi \rightarrow 0} \frac{(1 + \phi^\wedge) \exp(\phi^\wedge) p - \exp(\phi^\wedge) p}{\phi}$$

$$= \lim_{\phi \rightarrow 0} \frac{\phi^\wedge R p}{\phi} = \lim_{\phi \rightarrow 0} \frac{-(R p)^\wedge \phi}{\phi} = -(R p)^\wedge$$

$$\frac{\partial \tilde{p}_i}{\partial t} = \frac{a(t)}{t} = I$$

$$J_e = \frac{x}{|x|} \cdot \frac{(p_a - p_b)^\wedge}{|p_a - p_b|} [-(R p)^\wedge I]$$

面特征残差的雅可比

= 面残差: $J_H = \frac{\partial d_H}{\partial T} = \frac{\partial d_H}{\partial \hat{p}_i} \cdot \frac{\partial \hat{p}_i}{\partial T}$, 右边第二项与线特征一致.

$$\hat{x} = (\hat{p}_i - p_j) \cdot \frac{(p_l - p_j) \times (p_m - p_j)}{|(p_l - p_j) \times (p_m - p_j)|}$$

$$\text{则 } \frac{\partial d_H}{\partial \hat{p}_i} = \frac{\partial |x|}{\partial \hat{p}_i} = \frac{\partial |x|}{\partial x} \frac{\partial x}{\partial \hat{p}_i} = \frac{x}{|x|} \frac{\partial x}{\partial \hat{p}_i}$$

$$\frac{\partial x}{\partial \hat{p}_i} = \frac{(p_l - p_j) \times (p_m - p_j)}{|(p_l - p_j) \times (p_m - p_j)|}$$

$$J_H = \frac{x}{|x|} \cdot \frac{(p_l - p_j) \times (p_m - p_j)}{|(p_l - p_j) \times (p_m - p_j)|} \begin{bmatrix} -(R_p)^T & I \end{bmatrix}$$

二、公式与代码的对应关系

线特征的残差和雅可比

```
virtual bool Evaluate(double const* const* parameters, double* residuals, double** jacobians) const
{
    Eigen::Map<const Eigen::Quaterniond> q_last_curr(parameters[0]);
    Eigen::Map<const Eigen::Vector3d> t_last_curr(parameters[0] + 4);

    Eigen::Vector3d lp;
    Eigen::Vector3d lp_r;
    lp = q_last_curr * curr_point + t_last_curr;
    lp_r = q_last_curr * curr_point;

    Eigen::Vector3d nu = (lp - last_point_b).cross(lp - last_point_a);
    Eigen::Vector3d de = last_point_a - last_point_b;
    Eigen::Vector3d re = last_point_b - last_point_a;

    // 对应线特征残差
    Eigen::Vector3d residualsVector;
    residualsVector << nu.x() / de.norm(), nu.y() / de.norm(), nu.z() / de.norm();
    residuals[0] = residualsVector.norm();

    if(jacobians != NULL)
    {
        if(jacobians[0] != NULL)
        {
            if(residuals[0] != 0)
            {
                nu = residualsVector / residuals[0];

                Eigen::Matrix3d skew_re = skew(re);
                Eigen::Matrix3d skew_de = skew(de);
                Eigen::Matrix3d skew_lp_r = skew(lp_r);

                Eigen::Matrix<double, 3, 6> dp_by_so3;
                dp_by_so3.block<3,3>(0,0) = -skew_lp_r;
                (dp_by_so3.block<3,3>(0,3)).setIdentity();
                Eigen::Map<Eigen::Matrix<double, 1, 7, Eigen::RowMajor>> J_se3(jacobians[0]);

                J_se3.setZero();
                J_se3.block<1,6>(0,0) = nu.transpose() * skew_de * dp_by_so3 / de.norm();
            }
        }
    }
}
```

$$d_e = \frac{|(\tilde{p}_i - p_b) \times (\tilde{p}_i - p_a)|}{|p_a - p_b|}$$

$$X = \frac{(\tilde{p}_i - p_b) \times (\tilde{p}_i - p_a)}{|p_a - p_b|}$$

$$J_e = \frac{X}{|X|} \cdot \frac{(p_a - p_b)^n}{|p_a - p_b|} [-(RP)^T I]$$

面特征的残差和雅可比

```
virtual bool Evaluate(double const* const* parameters, double* residuals, double** jacobians) const
{
    Eigen::Map<const Eigen::Quaterniond> q_last_curr(parameters[0]);
    Eigen::Map<const Eigen::Vector3d> t_last_curr(parameters[0] + 4);
    Eigen::Vector3d lp = q_last_curr * curr_point + t_last_curr;
    Eigen::Vector3d lp_r = q_last_curr * curr_point;

    Eigen::Vector3d de = (last_point_j - last_point_l).cross(last_point_j - last_point_m);
    de.normalize();

    double nu = (lp - last_point_j).dot(de);

    residuals[0] = nu;

    if(jacobians != NULL)
    {
        if(jacobians[0] != NULL)
        {
            if(residuals[0] != 0)
            {
                nu = nu / residuals[0];

                Eigen::Matrix3d skew_lp_r = skew(lp_r);

                Eigen::Matrix<double, 3, 6> dp_by_so3;
                dp_by_so3.block<3,3>(0,0) = -skew_lp_r;
                (dp_by_so3.block<3,3>(0,3)).setIdentity();

                Eigen::Map<Eigen::Matrix<double, 1, 7, Eigen::RowMajor>> J_se3(jacobians[0]);

                J_se3.setZero();
                J_se3.block<1,6>(0,0) = de.transpose() * dp_by_so3;
            }
        }
    }
}
```

$$X = (\tilde{p}_i - p_j) \cdot \frac{(p_i - p_j) \times (p_m - p_j)}{|(p_i - p_j) \times (p_m - p_j)|}$$

$$J_H = \frac{X}{|X|} \cdot \frac{(p_i - p_j) \times (p_m - p_j)}{|(p_i - p_j) \times (p_m - p_j)|} [-(RP)^T I]$$

三、参数块和残差块的调用

参数块的调用

```
ceres::Problem::Options problem_options;  
  
ceres::Problem problem(problem_options);  
// problem.AddParameterBlock(para_q, 4, new PoseSE3Parameterization());  
// problem.AddParameterBlock(para_t, 3);  
problem.AddParameterBlock(parameters, 7, new PoseSE3Parameterization());
```

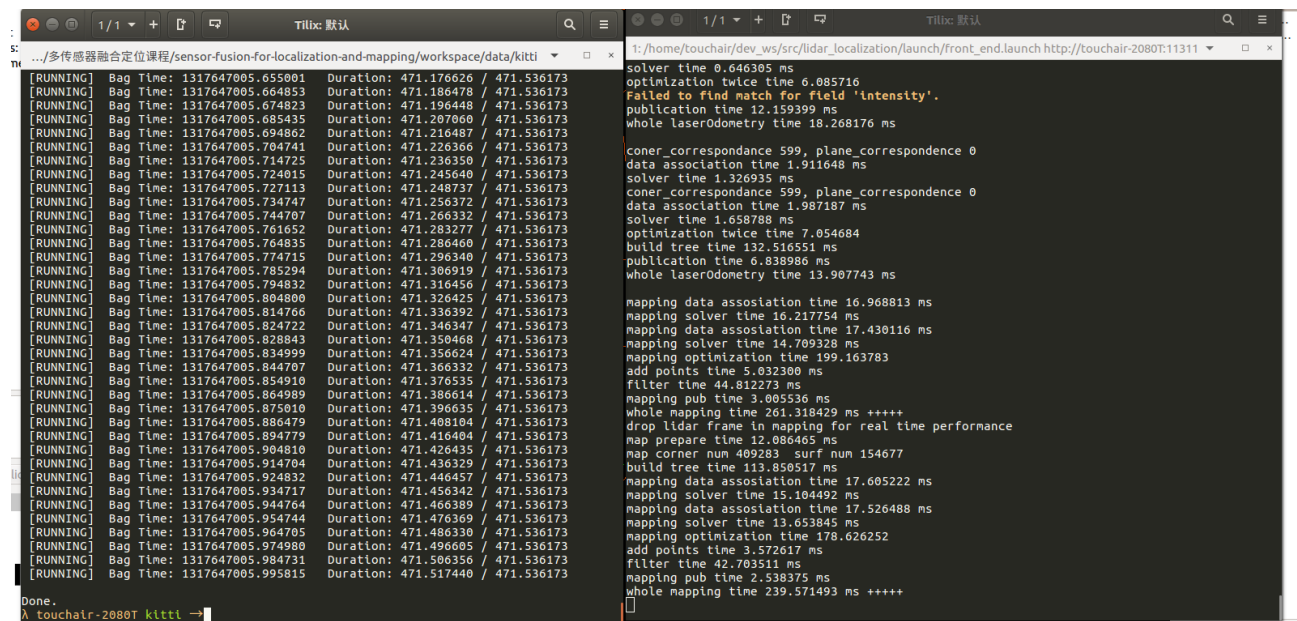
线残差块的调用

```
ceres::CostFunction *cost_function = new EdgeCostFunction(curr_point, last_point_a,  
last_point_b, s);  
  
// problem.AddResidualBlock(cost_function, loss_function, para_q, para_t);  
problem.AddResidualBlock(cost_function, loss_function, parameters);  
corner_correspondence++;
```

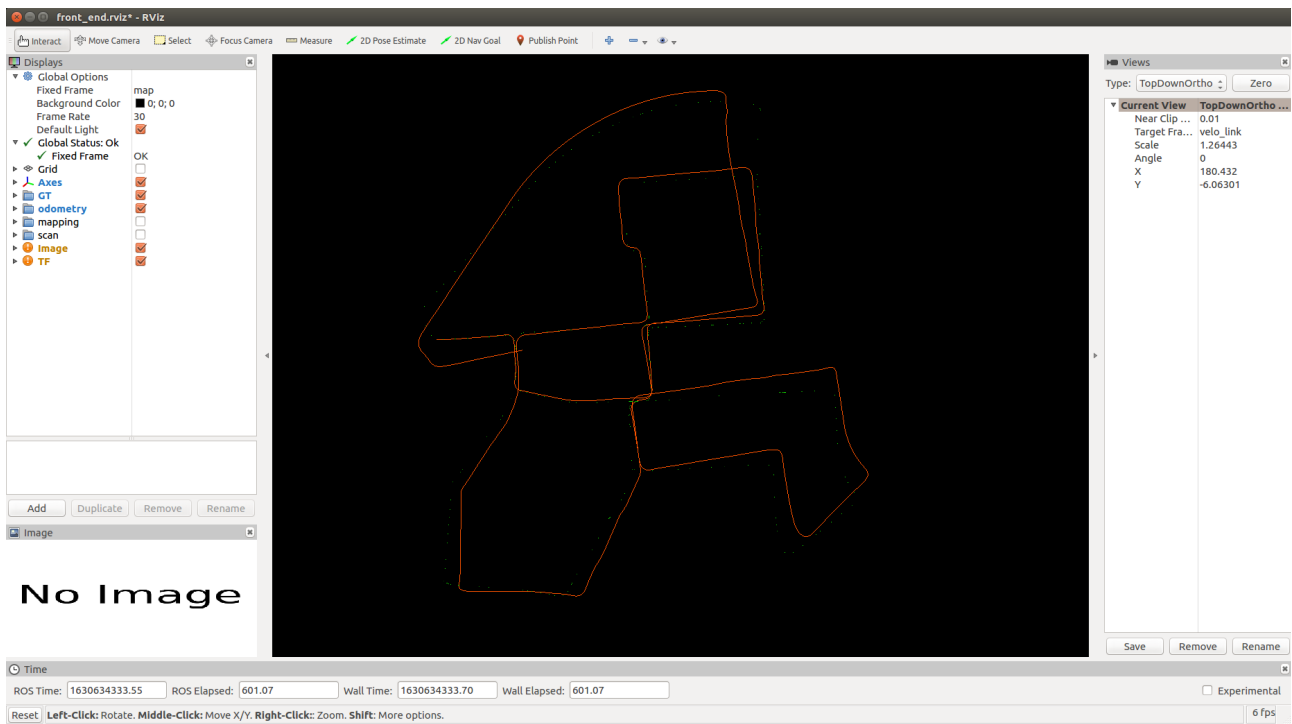
面残差块的调用

```
ceres::CostFunction *cost_function = new PlaneCostFunction(curr_point, last_point_a,  
last_point_b, last_point_c, s);  
  
// problem.AddResidualBlock(cost_function, loss_function, para_q, para_t);  
problem.AddResidualBlock(cost_function, loss_function, parameters);  
plane_correspondence++;
```

四、运行结果

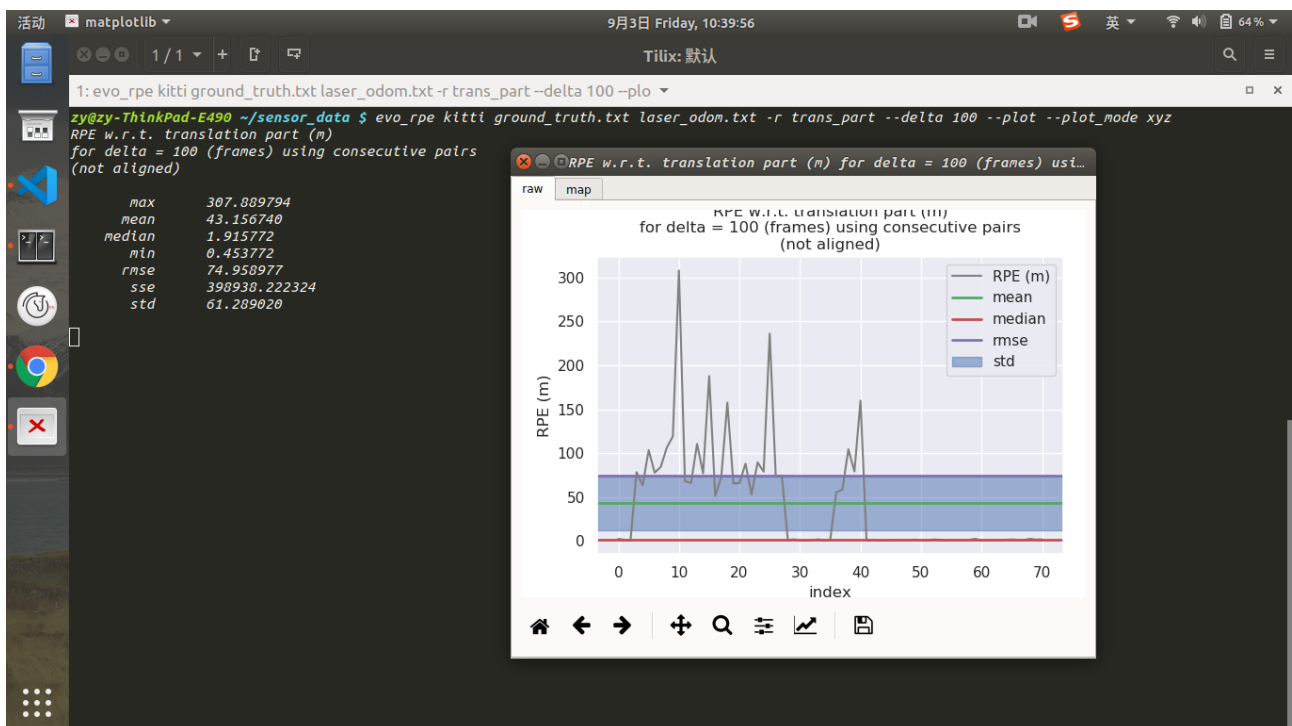


```
.../多传感器融合定位课程/sensor-fusion-for-localization-and-mapping/workspace/data/kitti  
[RUNNING] Bag Time: 1317647005.655001 Duration: 471.176626 / 471.536173  
[RUNNING] Bag Time: 1317647005.664853 Duration: 471.186478 / 471.536173  
[RUNNING] Bag Time: 1317647005.674823 Duration: 471.196448 / 471.536173  
[RUNNING] Bag Time: 1317647005.685435 Duration: 471.207060 / 471.536173  
[RUNNING] Bag Time: 1317647005.694862 Duration: 471.216487 / 471.536173  
[RUNNING] Bag Time: 1317647005.704741 Duration: 471.226366 / 471.536173  
[RUNNING] Bag Time: 1317647005.714725 Duration: 471.236359 / 471.536173  
[RUNNING] Bag Time: 1317647005.724815 Duration: 471.245640 / 471.536173  
[RUNNING] Bag Time: 1317647005.727113 Duration: 471.248737 / 471.536173  
[RUNNING] Bag Time: 1317647005.734747 Duration: 471.256372 / 471.536173  
[RUNNING] Bag Time: 1317647005.744707 Duration: 471.266332 / 471.536173  
[RUNNING] Bag Time: 1317647005.761652 Duration: 471.283277 / 471.536173  
[RUNNING] Bag Time: 1317647005.764935 Duration: 471.286460 / 471.536173  
[RUNNING] Bag Time: 1317647005.774715 Duration: 471.296340 / 471.536173  
[RUNNING] Bag Time: 1317647005.785294 Duration: 471.306919 / 471.536173  
[RUNNING] Bag Time: 1317647005.794832 Duration: 471.316456 / 471.536173  
[RUNNING] Bag Time: 1317647005.804800 Duration: 471.326425 / 471.536173  
[RUNNING] Bag Time: 1317647005.814766 Duration: 471.336392 / 471.536173  
[RUNNING] Bag Time: 1317647005.824722 Duration: 471.346347 / 471.536173  
[RUNNING] Bag Time: 1317647005.828843 Duration: 471.350468 / 471.536173  
[RUNNING] Bag Time: 1317647005.834999 Duration: 471.356624 / 471.536173  
[RUNNING] Bag Time: 1317647005.844707 Duration: 471.366332 / 471.536173  
[RUNNING] Bag Time: 1317647005.854910 Duration: 471.376535 / 471.536173  
[RUNNING] Bag Time: 1317647005.864989 Duration: 471.386614 / 471.536173  
[RUNNING] Bag Time: 1317647005.875010 Duration: 471.396635 / 471.536173  
[RUNNING] Bag Time: 1317647005.886479 Duration: 471.408104 / 471.536173  
[RUNNING] Bag Time: 1317647005.894779 Duration: 471.416404 / 471.536173  
[RUNNING] Bag Time: 1317647005.904810 Duration: 471.426435 / 471.536173  
[RUNNING] Bag Time: 1317647005.914704 Duration: 471.436329 / 471.536173  
[RUNNING] Bag Time: 1317647005.924832 Duration: 471.446457 / 471.536173  
[RUNNING] Bag Time: 1317647005.934717 Duration: 471.456342 / 471.536173  
[RUNNING] Bag Time: 1317647005.944764 Duration: 471.466389 / 471.536173  
[RUNNING] Bag Time: 1317647005.954744 Duration: 471.476369 / 471.536173  
[RUNNING] Bag Time: 1317647005.964705 Duration: 471.486330 / 471.536173  
[RUNNING] Bag Time: 1317647005.974980 Duration: 471.496605 / 471.536173  
[RUNNING] Bag Time: 1317647005.984731 Duration: 471.506356 / 471.536173  
[RUNNING] Bag Time: 1317647005.995815 Duration: 471.517440 / 471.536173  
Done.  
A touchair-2080T ktttl ->
```



五、精度评价

分段误差



总体误差

