

Homework 6

1) Iterative merge sort

```
def merge_sort_iterative(numbers):
    for x in range(0, len(numbers)):
        x *= 2
        for y in range(0, len(numbers), x):
            temp1 = [y:y+(x/2)]
            temp2 = [y+(x/2):x-y]
            low = 0
            mid = 0
            while low < len(temp1) AND mid < len(temp2):
                if temp1[low] < temp2[mid]:
                    mid += 1
                elif temp1[low] > temp2[mid]:
                    temp1[low], temp2[mid] = temp2[mid], temp1[low]
                    low += 1
            numbers[y:y+(x/2)], numbers[y+(x/2):x-y] = temp1, temp2
    return numbers
```

This merge sort will have a runtime complexity of $O(N \log N)$ because the outer loop window sizes grow at a power of 2 so X has $\log N$ iterations. Even though there are multiple windows that are covered by the inner loop all of the windows for a given X cover the input array meaning that the inner loop is $O(N)$. By combining these two loops we get $O(N \log N)$.

2) Simultaneous min / max

```
def recursive_min_max(numbers):  
    sorted = merge_sort(numbers)  
    min = sorted[0]  
    max = sorted[-1]  
    return min, max
```

```
def merge_sort(numbers):  
    if len(numbers) < 2:  
        return numbers
```

```
    middle = len(numbers)/2  
    left = numbers[:middle]  
    right = numbers[middle:]  
    return merge(left, right)
```

```
def merge(left, right)  
    if not(left) or not(right):  
        return left or right  
    merged = []  
    x, y = 0, 0  
    while len(merged) < len(left) + len(right):  
        if left[x] < right[y]:  
            merged.append(left[x])  
            x += 1  
        elif left[x] > right[y]:  
            merged.append(right[y])  
            y += 1  
        if x == len(left) or y == len(right):  
            merged.extend(left[x:] or right[y:])  
            break  
    return merged
```

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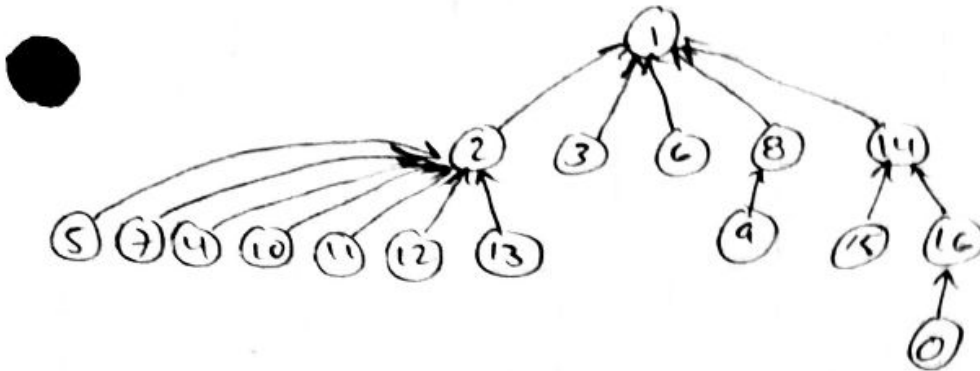
3a) Two numbers that sum up the inputted k $O(N^2)$

```
def sum_equals_k(numbers, k):  
    for (i = 0; i < len(numbers); i++)  
        for(j = 0; j < len(numbers); j++)  
            if numbers[i] + numbers[j] == k:  
                return yes  
    return no
```

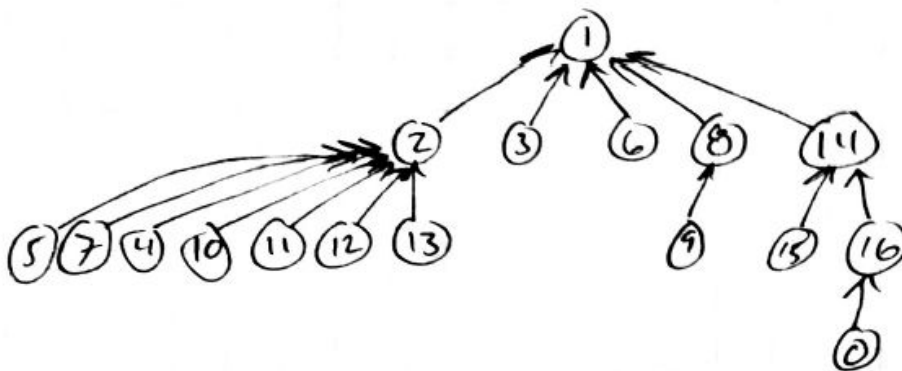
3b) Two numbers that sum up the inputted k $O(N\log N)$

```
def sums_equals_k_sorted(numbers, k):  
    numbers = merge_sort(numbers) # function from Question 2  
    left = 0  
    right = len(numbers)  
  
    while left < right:  
        if numbers[left] + numbers[right]:  
            return yes  
        elif numbers[left] + numbers[right] < k:  
            left += 1  
        else:  
            right -= 1  
    return no
```

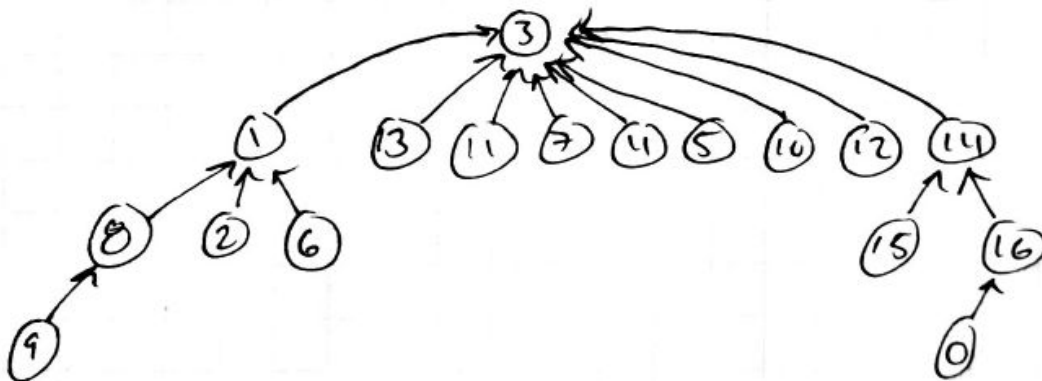
4) a)



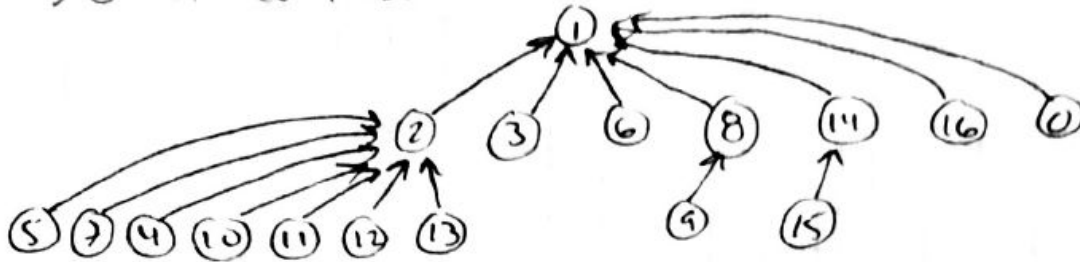
b)



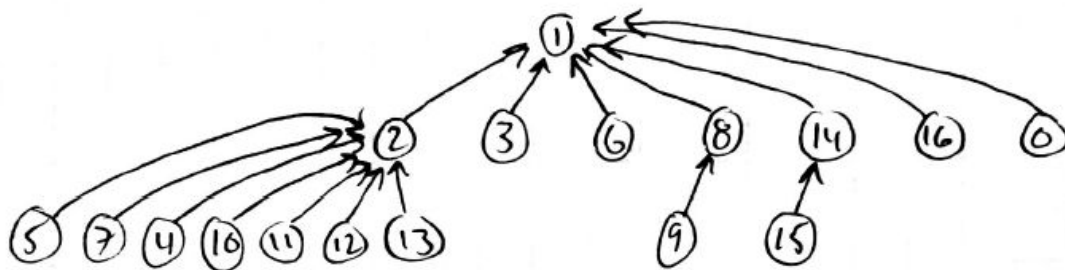
c)



5) a) Path Compression of 4a



b) Path compression of 4b



c) Path compression of 4c

