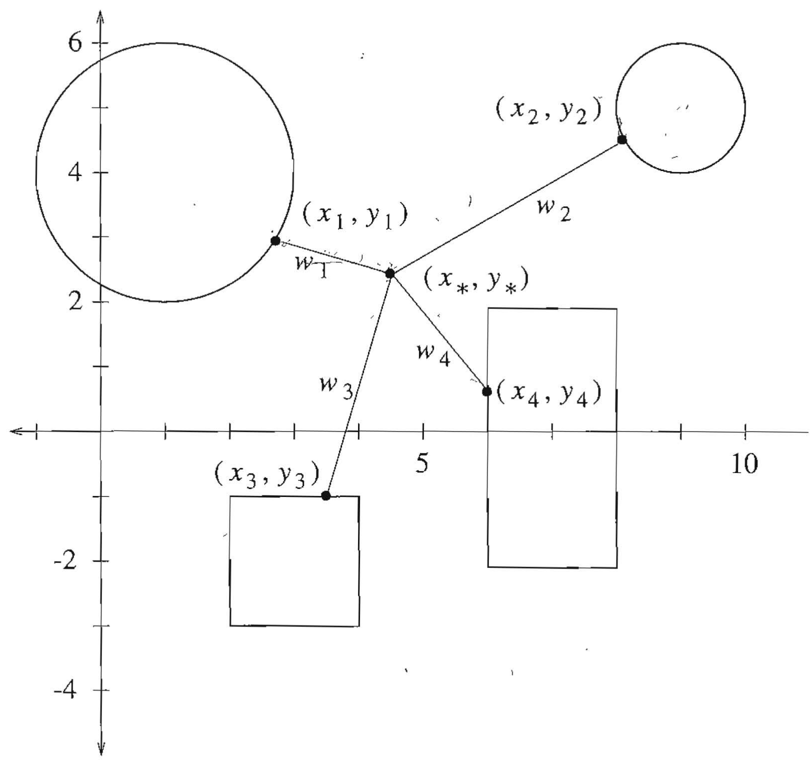
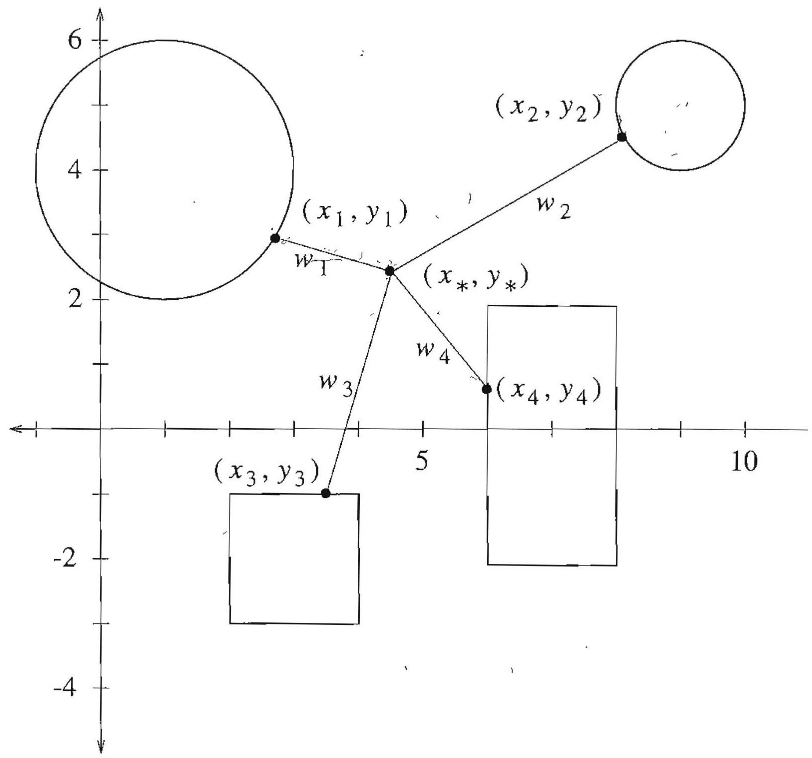
A Optimal Location Problem

Suppose that four buildings are to connected by heating and air-conditioning supply ducts. The positions of the buildings are illustrated in the figure below. The first two buildings are circular: one at $(1,4)$ with radius 2, the second at $(9,5)$ with radius 1. The third building is square with the sides of length 2 centered at $(3,-2)$. The fourth building is rectangular with height 4 and width 2, centered at $(7,0)$. The supply ducts will be originating from a central location (x_0, y_0), and will connect to building i at position(x_i, y_i). The objective is to design this system so that the total length of the ducts (assumed to be proportional to total cost) is minimized.



Math Model for the Optimal Location Problem



One formulation of this problem is:

$$\begin{array}{ll} \underset{x_0, y_0, x_i, y_i, i=1,\ldots,4}{\text{minimize}} & f = \sum_{i=1}^{4} w_i \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} \\ \text{subject to} & \\ & (x_1 - 1)^2 + (y_1 - 4)^2 \leq 4 \\ & (x_2 - 9)^2 + (y_2 - 5)^2 \leq 1 \\ & 2 \leq x_3 \leq 4 \\ & -3 \leq y_3 \leq -1 \\ & 6 \leq x\_4 \leq 8 \\ & -2 \leq y_4 \leq 2 \end{array}$$