

CSE3500-001 Homework #3

Exercises 3:

1, 2, 5, 6, 7.

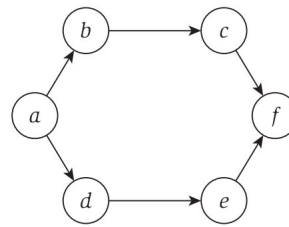


Figure 3.10 How many topological orderings does this graph have?

Exercises

1. Consider the directed acyclic graph G in Figure 3.10. How many topological orderings does it have?
2. Give an algorithm to detect whether a given undirected graph contains a cycle. If the graph contains a cycle, then your algorithm should output one. (It should not output all cycles in the graph, just one of them.) The running time of your algorithm should be $O(m + n)$ for a graph with n nodes and m edges.
5. A binary tree is a rooted tree in which each node has at most two children. Show by induction that in any binary tree the number of nodes with two children is exactly one less than the number of leaves.
6. We have a connected graph $G = (V, E)$, and a specific vertex $u \in V$. Suppose we compute a depth-first search tree rooted at u , and obtain a tree T that includes all nodes of G . Suppose we then compute a breadth-first search tree rooted at u , and obtain the same tree T . Prove that $G = T$. (In other words, if T is both a depth-first search tree and a breadth-first search tree rooted at u , then G cannot contain any edges that do not belong to T .)
7. Some friends of yours work on wireless networks, and they're currently studying the properties of a network of n mobile devices. As the devices move around (actually, as their human owners move around), they define a graph at any point in time as follows: there is a node representing each of the n devices, and there is an edge between device i and device j if the physical locations of i and j are no more than 500 meters apart. (If so, we say that i and j are "in range" of each other.)

They'd like it to be the case that the network of devices is connected at all times, and so they've constrained the motion of the devices to satisfy

the following property: at all times, each device i is within 500 meters of at least $n/2$ of the other devices. (We'll assume n is an even number.) What they'd like to know is: Does this property by itself guarantee that the network will remain connected?

Here's a concrete way to formulate the question as a claim about graphs.

Claim: Let G be a graph on n nodes, where n is an even number. If every node of G has degree at least $n/2$, then G is connected.

Decide whether you think the claim is true or false, and give a proof of either the claim or its negation.