

Mass defect and binding energy

A nuclear chain reaction can produce a vast amount of energy in a short space of time. When uranium-235 undergoes fission, large quantities of energy are released. To obtain some idea of the actual energy produced in such a reaction, it is necessary to understand what is going on in the nucleus of an atom when it is split.

To be stable, the mass of a nucleus must be less than the total mass of the nucleons that make it up. This difference in mass, which is called the mass defect, contributes to the binding energy that holds the nucleons together in the nucleus. The binding energy represents the amount of energy that must be put into a nucleus in order to break it apart into its constituent protons and neutrons.

Calculating the mass defect

To calculate the mass defect we need exact measurements of the mass of the uranium-235 atom and its component nucleons. These are listed below in *unified atomic mass units* (u). Using this scale a neutral carbon-12 atom is assigned the exact value of 12.000 000 u.

	mass (u)
neutral ^{235}U atom (this includes the electrons)	235.043 924
proton	1.007 276
neutron	1.008 665
electron	0.000 549

A neutral uranium-235 atom contains 92 protons, 92 electrons and 143 neutrons, so the mass of the component particles can be totalled as follows:

	mass (u)	subtotal (u)
protons	$92 \times 1.007\,276$	92.669 392
neutrons	$143 \times 1.008\,665$	144.239 095
electrons	$92 \times 0.000\,549$	0.050 508
total mass (A)		236.958 995
mass of neutral ^{235}U atom (B)		235.043 924
mass defect (A - B)		1.915 071

The mass defect is the difference between the mass of the neutral ^{235}U atom and the sum of its component particles.

To convert the mass defect from unified atomic mass units (u) to kilograms, use the conversion factor of $1\text{ u} = 1.6605 \times 10^{-27}\text{ kg}$.

$$\begin{aligned}\text{mass defect} &= 1.915\,071 \times (1.6605 \times 10^{-27})\text{ kg} \\ &= 3.179\,975 \times 10^{-27}\text{ kg}\end{aligned}$$

Calculating the binding energy

Once the mass defect has been calculated, it can be converted into binding energy using Einstein's famous equation $E = mc^2$.

$$\begin{aligned}\text{binding energy} &= 3.179\,975 \times 10^{-27} \times (3.00 \times 10^8)^2 \\ &= 2.86 \times 10^{-10}\text{ J}\end{aligned}$$

The binding energy is usually quoted per nucleon, as this gives a better idea about the stability of nuclei and a guide to which elements are most likely to undergo fission. As there are 235 nucleons in ^{235}U (92 protons and 143 neutrons), the binding energy per nucleon is given by:

$$\begin{aligned}\text{binding energy} &= 2.86 \times 10^{-10} / 235\text{ J per nucleon} \\ &= 1.22 \times 10^{-12}\text{ J per nucleon}\end{aligned}$$

An alternative way of expressing energy is in electron volts (eV). The electron volt is defined as the energy acquired by an electron as it moves through a potential of one volt. The conversion factor for Joules to electron volts is given by $1\text{ eV} = 1.60 \times 10^{-19}\text{ J}$.

MeV are more commonly used, where $1\text{ MeV} = 10^6\text{ eV} = 1.6 \times 10^{-13}\text{ J}$.

In the case of ^{235}U , the binding energy in MeV is:

$$\begin{aligned}\text{binding energy} &= 2.86 \times 10^{-10} / 1.6 \times 10^{-13}\text{ MeV} \\ &= 1.79 \times 10^3\text{ MeV} \\ &= 1.79 \times 10^3 / 235\text{ MeV per nucleon} \\ &= 7.62\text{ MeV per nucleon}\end{aligned}$$

In a typical ^{235}U fission reaction such as the following, about 200 MeV of energy is liberated per uranium atom:

