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1 $\lim_{x\to 0}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right.$$

$$\lim_{x\to a}\frac{(a+2x-3x)(\sqrt{3a+x}+2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$\lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(3a-3x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$1/3\ \lim_{x\to a}\frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$1/3\cdot\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$$

$$1/3\times\frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$1/3\times\frac{2}{\sqrt{3}}$$

$$\therefore\ \frac{2}{3\sqrt{3}}$$

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2. $$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

$$\lim_{y\to 0}\frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$\lim_{y\to 0}\frac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})} = \frac{1}{2a}$$

3. $$\lim_{x\to \pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$$

By substituting $x-\frac{\pi}{6}=h$

$x=h+\pi/6$ where $h\to 0$

$$\lim_{h\to 0}\frac{\cos(h+\pi/6)-\sqrt{3}\sin(h+\pi/6)}{\pi-6\left(h+\frac{\pi}{6}\right)}$$

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$$\therefore \lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\sin h \cos\frac{\pi}{6} + \cos h \cdot \sin\frac{\pi}{6}}{\pi - 6\left(\frac{6h+\pi}{6}\right)}$$

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$$\therefore \lim_{h \to 0} \frac{\cos\frac{\sqrt{3}}{2}h - \sin\frac{h}{2} - \sin\frac{3h}{2} - \cos\frac{\sqrt{3}}{2}h}{-6h}$$

$$\therefore \lim_{h \to 0} \frac{-\sin\frac{4h}{2}}{-6h}$$

$$\therefore \lim_{h \to 0} \frac{\sin 4h}{\frac{3}{2}h}$$

$$\frac{1}{3} \lim_{h \to 0} \frac{\sin h}{h} = \frac{1}{3}$$

4) $$\lim_{x \to \infty} \left[ \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right]$$

By rationalizing numerator & Denominator

$$\lim_{x \to \infty} \left[ \frac{\sqrt{x^2+5} - \sqrt{x^2-3}}{\sqrt{x^2+3} - \sqrt{x^2+1}} \right] \times \frac{\sqrt{x^2+5} + \sqrt{x^2-3}}{\sqrt{x^2+5} + \sqrt{x^2-3}} \times \frac{\sqrt{x^2+3} + \sqrt{x^2+1}}{\sqrt{x^2+3} + \sqrt{x^2+1}}$$

$$\lim_{x \to \infty} \left[ \frac{(x^2+5-x^2+3)(\sqrt{x^2+3} + \sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5} + \sqrt{x^2-3})} \right]$$

$$\lim_{x \to \infty} \frac{\cancel{8}^{\,4} (\sqrt{x^2+3} + \sqrt{x^2+1})}{\cancel{2}(\sqrt{x^2+5} + \sqrt{x^2-3})}$$

$$4 \lim_{x \to \infty} \frac{\sqrt{x^2(1+\frac{3}{x^2})} + \sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})} + \sqrt{x^2(1-\frac{3}{x^2})}}$$

$$= 4$$

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5.) i.) $$f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}, & \text{for } 0 < x \le \dfrac{\pi}{2} \\[2ex] \dfrac{\cos x}{\pi - 2x}, & \text{for } \dfrac{\pi}{2} < x < \pi \end{cases} \quad \text{at } x = \frac{\pi}{2}$$

$$f\left(\frac{\pi}{2}\right) = \frac{\sin 2\left(\frac{\pi}{2}\right)}{\sqrt{1-\cos 2\left(\frac{\pi}{2}\right)}} \qquad \therefore f\left(\frac{\pi}{2}\right) = 0$$

$f$ at $x = \frac{\pi}{2}$ define.

ii) $$\lim_{x \to \frac{\pi}{2}^+} f(x) = \lim_{x \to \frac{\pi}{2}^+} \frac{\cos x}{\pi - 2x}$$

By substituting method $x - \frac{\pi}{2} = h$

$x = h + \pi/2$

$h \to 0$

$$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2\left(h + \frac{\pi}{2}\right)}$$

$$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$$

$$\lim_{h \to 0} \frac{\cos(h + \pi/2)}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h . \cos\frac{\pi}{2} - \sin h . \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h \to 0} \frac{\cos h . -\sin h}{-2h}$$

$$\frac{1}{2} \lim_{h \to 0} \frac{-\sin h}{-2h} = \frac{1}{2}$$

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b. $\lim_{x \to \pi/2} f(x) = \lim_{x \to \pi/2} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x \to \pi/2} \frac{2 \sin x \cdot \cos x}{\sqrt{2 \sin^2 x}}$

$\lim_{x \to \pi/2} \frac{2 \cancel{\sin x} \cdot \cos x}{\sqrt{2}\, \cancel{\sin x}}$

$\lim_{x \to \pi/2} \frac{2 \cos x}{\sqrt{2}}$

$\frac{2}{\sqrt{2}} \lim_{x \to \pi/2} \cos x$

$\because$ LHL $\neq$ RHL . f is not continuous at $x = \pi/2$ .

5.
ii

$$f(x) = \begin{cases} \frac{x^2-9}{x-3} & 0 < x < 3 \\ = x+3 & 3 \leq x \leq 6 \\ = \frac{x^2-9}{x+3} & 6 \leq x < 9 \end{cases}$$ at $x=3$ & $x=6$

$f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x+3) = 6$

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$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{x^2-9}{x-3} = \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$$

$$LHL = RHL$$

$f$ is continuous at $x=3$

for $x=6$

$$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = 3$$

2. $$\lim_{x\to 6^+} \frac{x^2-9}{x+3}$$

$$\lim_{x\to 6^+} \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$\lim_{x\to 6^+} (x-3) = 6-3 = 3$$

$$\lim_{x\to 6^-} x+3 = 3+6 = 9$$

$$LHL \neq RHL$$

$\therefore$ Its not continuous

6.

1) $$f(x) = \left.\begin{array}{ll} \frac{1-\cos 4x}{x^2} & x<0 \\ = k & x=0 \end{array}\right\} \text{at } x=0$$

$f$ is continuous at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = k$$

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$$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = k$$

$$2\lim_{x\to 0} \left(\frac{\sin 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$k = 8$$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$ $\quad x \neq 0$
$\quad = k$ $\quad x = 0$ $\Big\}$ at $x = 0$

Soln:

$$f(x) = (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$$

$$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$$

we know that

$$\lim_{x\to 0} (1+px)^{\frac{1}{px}} = e$$

$$\therefore k = e$$

iii) $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$ $\quad x \neq \pi/3$
$\quad = \cancel{k}$ $\quad x = \pi/3$ $\Big\}$ at $x = \pi/3$

$$x - \frac{\pi}{3} = h \quad \therefore x = h + \frac{\pi}{3}$$

where $h \to 0$

$$f\left(\frac{\pi}{3}+h\right) = \frac{\sqrt{3} - \tan(\pi/3+h)}{\pi - 3(\pi/3+h)}$$

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$$\lim_{h\to 0} \frac{\sqrt{3}-\tan(\pi/3+h)}{\pi-3(\pi/3+h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3}-\dfrac{\tan\frac{\pi}{3}+\tan h}{1-\tan\frac{\pi}{3}.\tan h}}{\pi-\pi-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3}-\sqrt{3}\times\sqrt{3}.\tan h)-(\sqrt{3}+\tan h)}{1-\tan\pi/3.\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3}-3.\tan h)-(\sqrt{3}+\tan h)}{1-\sqrt{3}\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1-\sqrt{3}\tan h)}$$

$$\lim_{h\to 0} \frac{4\tan h}{3h(1-\sqrt{3}\tan h)}$$

$$4/3 \lim_{h\to 0} \frac{\tan h}{h} \quad \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tan h)}$$

$$= 4/3 \times \frac{1}{(1-\sqrt{3}(0))}$$

$$= 4/3$$

7. $f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = 9 & x = 0 \end{cases}$ at $x = 0$

$f(x) = \dfrac{1-\cos 3x}{x\tan x}$

$\lim\limits_{x\to 0} \dfrac{2\sin^2 3/2\, x}{x \tan x} \times x^2$

$\lim\limits_{x\to 0} \dfrac{\dfrac{2\sin^2 \frac{3x}{2}}{x^2}}{x - \dfrac{\tan x}{x^2}} \times x^2$

$= 2 \lim\limits_{x\to 0} \left(\frac{3}{2}\right)^2 = \cancel{2} \times \frac{9}{\cancel{4}2} = \frac{9}{2}$

$\lim\limits_{x\to 0} f(x) = \frac{9}{2}$ $\qquad 9 = f(0)$

$f$ is not continuous at $x = 0$

Redefine $f^n$.

$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x\tan x} & x \neq 0 \\ = \dfrac{9}{2} & x = 0 \end{cases}$

Now $\lim\limits_{x\to 0} f(x) = f(0)$

$f$ has removable discontinuity $x=0$

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7. ii $f(x) = \dfrac{(e^{3x}-1)\sin x}{x^2}$ $\quad x \neq 0$
$\quad\;\; = \pi/6 \qquad x = 0$ $\qquad x = 0$

$$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\cancel{\lim_{x\to 0}} \cancel{(e^{3x}-1)} \cancel{\sin(\pi}$$

$$\lim_{x\to 0} \frac{e^{3x}-1}{x} \quad \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x\to 0} 3 \cdot \frac{e^{3x}-1}{3x} \quad \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3\lim_{x\to 0} \frac{e^{3x}-1}{3x} \quad \lim_{x\to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$3 \log \frac{\pi}{180} = \frac{\pi}{60} = f(0)$$

$f$ is continuous at $x = 0$

8.) $f(x) = \dfrac{e^{x^2} - \cos x}{x^2}$ $\quad x = 0$

is continuous at $x=0$

$\therefore$ <u>Given</u>: $f$ is continuous at $x=0$

$$\lim_{x\to 0} f(x) = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$$

$$\lim_{x\to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$$

$$\lim_{x\to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$$

$$\lim_{x\to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x\to 0} \frac{1 - \cos 2x}{x^2}$$

$$\log e + \lim_{x\to 0} \frac{2\sin^2 x/2}{x^2}$$

$$\log e + 2\lim_{x\to 0} \left(\frac{\sin x/2}{x}\right)^2$$

multipdy with 2

$$1 + \cancel{2} \times \frac{1}{\cancel{4}\,2} = 3/2 = f(0)$$

9.) $f(x) = \dfrac{\sqrt{2} - \sqrt{1 + \sin x}}{\cos^2 x}$ $\quad x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$$\lim_{x\to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$$

$$\lim_{x\to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x\left(\sqrt{2} + \sqrt{1+\sin x}\right)}$$

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$$\lim_{x \to \pi/2} \frac{1 + \sin x}{1 - \sin^2 x\left(\sqrt{2} + \sqrt{1 + \sin x}\right)}$$

$$\lim_{x \to \pi/2} \frac{\cancel{1 + \sin x}}{(1 - \sin x)\cancel{(1 + \sin x)}\left(\sqrt{2} + \sqrt{1 + \sin x}\right)}$$

$$\lim_{x \to \pi/2} \frac{1}{(1 - \sin x)\left(\sqrt{2} + \sqrt{1 + \sin x}\right)}$$

$$= \frac{1}{2\left(\sqrt{2} + \sqrt{2}\right)}$$

$$= \frac{1}{4\sqrt{2}}$$

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PRACTICAL NO. 2 

Topic : Derivative.

Q1.) Show that the foll function defined from IR to IR are differentiable.

i) $\cot x$

$f(x) = \cot x$

$$f(a) = \lim_{x \to 0} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to 0} \frac{\cot x - \cot a}{x-a}$$

$$= \lim_{x \to 0} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x-a}$$

$$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a) \tan x \cdot \tan a}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a+h-a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$$

formula : $\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$$

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$$= \lim_{h \to 0} \frac{(a-\alpha-h) - (1+\tan a . \tan(a+h))}{h \times \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{-\tan h}{h} \times \frac{1-\tan a \tan(a+h)}{\tan(a+h) \tan a}$$

$$= -1 \times \frac{1+\tan^2 a}{\tan^2 a}$$

$$= \frac{-\sec^2 a}{\tan^2 a}$$

$$= -\frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$$

$$= -\operatorname{cosec}^2 a$$

$$Df(a) = -\cos^2 a$$

$\therefore$ f is differentiable $\forall a \in R$

$$f(x) = \operatorname{cosec} x$$

$$Df(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$$

$$= \lim_{x \to 0} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x-a}$$

$$= \lim_{x \to 0} \frac{\sin a - \sin x}{(x-a)\sin a . \sin x}$$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$f(h) = \lim_{x \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a . \sin(a+h)}$$

formula: $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$



$$\lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right)\cdot \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \cdot \sin(a+h)}$$

$$= \lim_{h \to 0} \frac{-\sin\frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2\cos\left(\frac{2a+h}{2}\right)}{\sin a \cdot \sin(a+h)}$$

$$= \frac{-\frac{1}{2} \times 2\cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$$

$$= -\cot a \cdot \operatorname{cosec} a$$

iii) $\sec x$

$f(x) = \sec x$

$$f(a) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x-a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cdot \cos x}$$

Put $x - a = h$

$x = a+h$

as $x \to a$, $h \to 0$

$$f(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cdot \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin -\frac{h}{2}}{\cos a \cdot \cos(a+h) \times \frac{-h}{2}} \times \frac{-1}{2}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cdot \cos(a+0)}$$

$$= \tan a \cdot \sec a$$

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Q2. If $f(x) = 4x+1 \quad x \leq 2$

$\quad\quad\quad = x^2+5 \quad x > 0$ , at $x=2$

Soln:

$$f(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4\times2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$$

$$f(2^-) = 4$$

$$f(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$$

$$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2}$$

$$= 4$$

RHD = LHD

f is differential at $x = 2$

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Q3. If $f(x) = 4x+7 \qquad x<3$

$= x^2+3x+1 \qquad x \geq 3 \qquad$ at $x=3$

Soln:

$$f(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\times3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)\cancel{(x-3)}}{\cancel{x-3}}$$

$$= 3+6 \qquad = 9$$

$$f(3^-) = \lim_{x\to 3^-} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x+7-19}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4x-12}{x-3}$$

$$= \lim_{x\to 3^-} \frac{4\cancel{(x-3)}}{\cancel{x-3}}$$

$$= 4$$

$\therefore$ It is not differentiable at $x=3$

Q4. If $f(x) = 8x - 5 \quad x \leq 2$

$= 3x^2 - 4x + 7 \quad x > 2 \quad$ at $x = 2$

find if $f$ is differential

Soln:

$$f(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x + 7 - 11}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3 \times 2 + 2 = 8$$

$$f(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8\cancel{(x-2)}}{\cancel{x-2}}$$

$$= 8$$

$\therefore$ It is differentiable at $x = 8$

# PRACTICAL NO. 3

Q. Find the intervals in which function is **42** increasing or decreasing.

1. $f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

$f$ is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$\left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{5/3}\right) > 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \qquad \sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

$f$ is decreasing iff $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$\therefore (x - \sqrt{5/3})(x + \sqrt{5/3}) < 0$

$+ \quad - \quad +$

$-\sqrt{5/3} \qquad \sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2. $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$f(x)$ is increasing iff $f'(x) > 0$

$\therefore 2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and $f$ is decreasing iff $f'(x) \nless 0$

$\therefore 2x - 4 < 0$

$2(x - 2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

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3.) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$\because f$ is increasing iff $f'(x) > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$2(3x^2 + x - 10) > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$(x+2)(3x-5) > 0$

$+ \quad - \quad +$

$-2 \quad 5/3$ $\qquad x \in (-\infty, -2) \cup (5/3, \infty)$

$f$ is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(x+2)(3x-5) < 0$

$+ \quad - \quad +$

$-2 \quad 5/3$ $\qquad x \in (-2, 5/3)$

4.) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$\because f$ is increasing iff $f'(x) > 0$

$3(x^2 - 9) > 0$

$(x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \quad 3$ $\qquad x \in (-\infty, -3) \cup (3, \infty)$

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$$3x^2 - 27 < 0$$
$$3(x^2 - 9) < 0$$
$$(x-3)(x+3) < 0$$

$+ \quad - \quad +$

$-3 \quad 3$

$x \in (-3, 3)$

5.) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because f$ is increasing iff $f'(x) > 0$

$$6x^2 - 18x - 24 > 0$$
$$6(x^2 - 3x - 4) > 0$$
$$x^2 - 4x + x - 4 > 0$$
$$x(x-4) + 1(x-4) > 0$$
$$(x+1)(x-4) > 0$$

$+ \quad - \quad +$

$-1 \quad 4$

$x \in (-\infty, -1) \cup (4, \infty)$

$f$ is decreasing iff $f'(x) < 0$

$$6x^2 - 18x - 24 < 0$$
$$6(x^2 - 3x - 4) < 0$$
$$x(x-4) + 1(x-4) < 0$$
$$(x-4)(x+1) < 0$$

$+ \quad - \quad +$

$-1 \quad 4$

$x \in (-1, 4)$

Q2.

1. $y = 3x^2 - 2x^3$

$\therefore f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

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$f$ is concave upward if $f''(x) > 0$

$$(6-12x) > 0$$
$$12(6/12 - x) > 0$$
$$x - 1/2 > 0$$
$$x > 1/2$$
$$\because f''(x) > 0$$
$$x \in (1/2, \infty)$$

2. $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$$f'(x) = 4x^3 - 18x^2 + 24x + 5$$
$$f''(x) = 12x^2 - 36x + 24$$

$f$ is concave upward if $f''(x) > 0$

$$12x^2 - 36x + 24 > 0$$
$$12(x^2 - 3x + 2) > 0$$
$$x^2 - 2x - x + 2 > 0$$
$$x(x-2) - 1(x-2) > 0$$
$$(x-2)(x-1) > 0$$

+ − +

1 2

$$x \in (-\infty, 1) \cup (2, \infty)$$

3. $y = x^3 - 27x + 5$

$$f'(x) = 3x^2 - 27$$
$$f''(x) = 6x$$

$f$ is concave upward iff $f''(x) > 0$

$$6x > 0$$
$$x > 0$$
$$x \in (0, \infty)$$

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4. $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff $f''(x) > 0$

$12x - 18 > 0$

$12(x - 18/12) > 0$

$x - 3/2 > 0 \quad x > 3/2$

$\therefore x \in (3/2, \infty)$

5. $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward iff $f''(x) > 0$

$f''(x) > 0$

$12x + 2 > 0$

$12(x + 2/12) > 0$

$x + 1/6 > 0$

$x < -1/6$

$f''(x) \not> 0$

There exist no interval.

23/4/2020

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# PRACTICAL NO. 4

Q1.)

i) $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - 32/x^3$

consider, $f'(x) = 0$

$\therefore 2x - 32/x^3 = 0$

$2x = 32/x^3$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + 96/x^4$

$f''(2) = 2 + 96/(2)^4$

$= 2 + 96/16$

$= 8 > 0$

$\therefore$ f has minimum value at $x = 2$

$f''(-2) = 2 + 96/-2^4$

$= 2 + 96/16$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ function reaches minimum values at $x = 2$ and $x = -2$.

2.) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

consider, $f'(x) = 0$

$\therefore -15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x = \pm 1$

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$f''(x) = -30x + 60x^3$

$f''(1) = -30(1) + 60(1)^3$

$= 30 > 0$

$\therefore$ f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$\therefore f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore$ f has the maximum value of 5 at $x = -1$ and has minimum value 1, at $x = 1$.

3.) $f(x) = x^3 - 3x^2 + 1$

$\therefore f'(x) = 3x^2 - 6x$

consider, $f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$3x = 0$ or $x - 2 = 0$

$x = 0$ or $x = 2$

$$f''(x) = 6x - 6$$

$$f''(0) = 6(0) - 6$$

$$= -6 < 0$$

$\therefore$ f has maximum value at $x = 0$

$$f''(2) = 6(2) - 6$$

$$= 12 - 6$$

$$= 6 > 0$$

$\therefore$ f has minimum value at $x = 2$

$$f(2) = (2)^3 - 3(2)^2 + 1$$

$$= 8 - 3(4) + 1$$

$$= 9 - 12$$

$$= -3$$

$\therefore$ f has maximum value 1 at $x = 0$

& f has minimum value $-3$ at $x = 2$

4) $f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

consider, $f'(x) = 0$

$$6x^2 - 6x - 12 = 0$$

$$6(x^2 - x - 2) = 0$$

$$x^2 + x - 2x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$x = 2$ or $x = -1$

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$$f''(x) = 12x - 6$$

$$f''(2) = 12(2) - 6$$

$$= 24 - 6$$

$$= 18 > 0$$

$f$ has minimum value at $x = 2$

$$f(2) = 2(2)^3 - 3(2)^2 - 12(2) + 1$$

$$= 2 \times 8 - 3(4) - 24 + 1$$

$$= 16 - 12 - 24 + 1$$

$$= -19$$

$$f''(-1) = 12(-1) - 6$$

$$= -12 - 6$$

$$= -18 > 0$$

$\therefore$ $f$ has maximum value at $x = -1$

$$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$$

$$= -2 - 3 + 12 + 1$$

$$= \underline{\underline{8}}$$

$\therefore$ $f$ has maximum value $8$ at $x = -1$

& $f$ has minimum value $-18$ at $x = 2$

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Q2.

(i) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0 + \frac{9.5}{55}$$

$$x_1 = 0.1727$$

$$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$$

$$= 0.0051 - 0.0895 - 9.4985 + 9.5$$

$$= -0.0329$$

~~$f'(x_1) = 3(0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$~~

$$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$$

$$= 0.0895 - 1.0362 - 55$$

$$= 55.9467$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 0.1727 - \frac{0.0329}{55.9467}$$

$$= 0.1712$$

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$$f(x_2) = (0.1712)^3 - 3(0.1712)^2 - 55(0.1712) + 9.5$$
$$= 0.0050 - 0.0879 - 9.416 + 9.5$$
$$= 0.0011$$

$$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$$
$$= 0.0879 - 1.0272 - 55$$
$$= -55.9393$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 0.1712 + \frac{0.0011}{55.9393}$$
$$= 0.1712$$

The root of the equation is 0.1712

(ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$$f(2) = \cancel{3}\,(2)^3 - 4(2) - 9$$
$$= 8 - 8 - 9$$
$$= -9$$

$$f(3) = 3^3 - 4(3) - 9$$
$$= 27 - 12 - 9$$
$$= 6$$

Let $x_0 = 3$ be the initial approximation.

$\therefore$ By Newton's method

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$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$= 3 - \frac{6}{23}$$

$$= 2.7392$$

$$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$$

$$= 20.5528 - 10.9568 - 9$$

$$= 0.596$$

$$f'(x_1) = 23(2.7392)^2 - 4$$

$$= 22.5096 - 4$$

$$= 18.5096$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 2.7392 - \frac{0.596}{18.5076}$$

$$= 2.7071$$

$$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$$

$$= 19.8386 - 10.8284 - 9$$

$$= 0.0102$$

$$f'(x_2) = 3(2.7071)^2 - 4$$

$$= 21.9861 - 4$$

$$= 17.9851$$

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$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2.7071 - \frac{0.0102}{17.9851}$$

$$= 2.7015$$

$$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$$

$$= 19.7158 - 10.806 - 9$$

$$= -0.0901$$

$$f'(x_3) = 3(2.7015)^2 - 4$$

$$= 21.8943 - 4$$

$$= 17.8943$$

$$x_4 = 2.7015 + \frac{0.0901}{17.8943}$$

$$= 2.7015 + 0.0050$$

$$= 2.7065$$

3) $f(x_1) = x^3 - 1.8x^2 - 10x + 17$

$$f(x) = 3x^2 - 3.6x - 10$$

$$f(1) = (1)^3 - 1.8(7)^2 - 10(1) + 17$$

$$= 1 - 1.8 - 10 + 17$$

$$= 6.2$$

$$f(2) = 2^3 - 1.8(2)^2 - 10(2) + 17$$

$$= 8 \cdot 7.2 - 20 + 17$$

$$= -2.2$$

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Let $x_0 = 2$ be initial appuoximation
By Neuton's method,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_0 - \frac{f(x_2)}{f'(x_2)}$$

$$= 2 - \frac{2.2}{5.2}$$

$$= 2 - 0.4230$$

$$= 1.577$$

$$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577) + 17$$

$$= 3.9219 - 4.4764 - 15.77 + 17$$

$$= 0.6755$$

$$f'(x_1) = 3(1.077)^2 - 3.6(1.577) - 10$$

$$= 7.4608 - 5.6772 - 10$$

$$= -8.2164$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1.577 + \frac{0.6755}{8.2164}$$

$$= 1.577 + 0.872$$

$$= 1.6592$$

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$$f(x_2) = (1.1593)^3 - 1.3(1.6592)^2 - 10(1.6592) + 1.7$$
$$= 4.5677 - 0.9153 - 16.592 + 17$$
$$= 0.0204$$

$$f'(x_2) = 3(1.6592)^2 - 3.6(1.5592) - 10$$
$$= 8.2588 - 5 - 9.7312 - 10$$
$$= -7.7143$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$
$$= 1.6592 + \frac{0.0204}{7.7143}$$
$$= 1.6592 + 0.0026$$
$$= 1.6618$$

$$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10(1.6618) + 17$$
$$= 4.5892 - 4.9708 - 16.618 + 17$$
$$= 0.0004$$
$$f'(x_3) = 3(1.6618)^2 - 3.6(1.8618) - 10$$
$$= 8.2847 - 5.9824 - 10$$
$$= -7.6977$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$$
$$= 1.6618 + \frac{0.0004}{7.6977}$$
$$= \underline{\underline{1.6618}}$$

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# PRACTICAL NO. 5



1.) solve the following:

i) $\int \frac{dx}{\sqrt{x^2+2x-3}} \, dx$

$= \int \frac{1}{\sqrt{x^2+2x-3}}$

$= \int \frac{1}{\sqrt{x^2+2x+1-4}} \, dx$

$= \int \frac{1}{\sqrt{(x-1)^2-4}} \, dx = \int \frac{1}{(x+1)^2-(2)^2} \, dx$

$a^2+2ab+b^2 = (a+b)^2$

substitute,

$x+1 = t$

$dx = \frac{1}{t} \, dt \qquad t = 1, \ t = x+1$

$\int \frac{1}{\sqrt{t^2-4}} \, dt$

$\log(t+\sqrt{t^2-4})$

$\because \left[\int \frac{1}{\sqrt{x^2 a^2}} \, dx = \log|x+\sqrt{x^2-a^2}|\right]$

$t = x+1$

$\log(|x+1+\sqrt{(x+1)^2-4}|)$

$\log(|x+1+\sqrt{x^2+2x-3}|)+6$

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2.) $\int (4e^{3x}+1)\,dx$

$I = \int (4e^{3x}+1)\,dx$

$= \int 4e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= 4\int e^{3x}\,dx + \int 1\,dx$

$= \frac{4e^{3x}}{3} + x \qquad \left[\because \int e^{ax}\,dx = \frac{1}{a}e^{ax}\right]$

$= \frac{4e^{3x}}{3} + x + C$

3.) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin x + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin x + 5x^{1/2}\,dx$

$= \int 2x^2\,dx - 3\int \sin x\,dx + 5\int x^{1/2}\,dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10\sqrt[3]{x}}{3} + c$

$\left[\because \int x^n\,dx = \frac{x^{n+1}}{n+1} + c \quad \int \sin x\,dx = -\cos x\,dx\right]$

$= \frac{2x^3+10\sqrt[3]{x}}{3} + 3\cos x + C$

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4.) $\int \frac{x^3+3x+4}{\sqrt{x}}\,dx$

$$I = \int \frac{x^3+3x+4}{\sqrt{x}}\,dx$$

$$= \int \left(\frac{x^3}{\sqrt{x}} + \frac{3x}{\sqrt{x}} + \frac{4}{\sqrt{x}}\right)dx$$

$$= \int \frac{x^3}{x^{1/2}}\,dx + \int \frac{3x}{x^{1/2}}\,dx + \int \frac{4x}{x^{1/2}}\,dx$$

$$= \int x^{5/2}\,dx + 3\int x^{1/2}\,dx + 4\int x^{-1/2}\,dx$$

$$= \frac{x^{7/2}}{7/2} + \frac{3x^{3/2}}{3/2} + 4\,\frac{x^{1/2}}{1/2}$$

$$= \frac{2x^{7/2}}{7} + 2x^{3/2} + 8\sqrt{x} + C$$

5.) $\int t^7 \times \sin(2t^4)\,dt$

put $v = 2t^4$

$dv = 2 \times 4t^3$

$$= \int t^7 \times \sin(2t^4) \times \frac{1}{2\times 4t^3}\,du$$

$$= \int t^4 \sin(2\cancel{t}) \times \frac{1}{2\times 4}\,du$$

$$= \int t^4 \sin(x^4)\,\frac{1}{8}\,du$$

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$$= \frac{t^4 \times \sin(2t^4)}{8} \, du$$

Substitute $t^4$ with $\frac{u}{2}$

$$= \int \frac{u/2 \times \sin(2 \, u/2)}{8} \, du$$

$$= \int \frac{u/2 + \sin(u)/2}{8} \, du$$

$$= \frac{1}{16} \int u * \sin(u) \, du$$

$$= \frac{1}{16} \left( u \times (-\cos(u)) - \int -\cos(u) \, du \right.$$

$[\because \int u \, dv = u.v - \int v \, du$ where $u = u$

$dv = \sin(u) \times dv$

$du = 1 dv \quad v = -\cos(u)$

$$= \frac{1}{16} \times \left( u \times (-\cos v) \right) + \int \cos(u) \, du$$

$$= \frac{1}{16} \times \left( 4 \, (-\cos(u)) + \sin(u) \right)$$

substituting $u = 2t^4$

$$= \frac{1}{16} \left( 2t^4 (-\cos(2t^4)) + \sin(2t^4) \right)$$

$$= \frac{-t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + c$$

6. $\int \sqrt{x}\,(x^2-1)\,dx$

$I = \int \sqrt{x}\,(x^2-1)\,dx$

$= \int x^{1/2}\,(x^2-1)\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}dx - \int x^{1/2}dx$

$= \dfrac{x^{5/2+1}}{5/2+1} - \dfrac{x^{1/2+1}}{1/2+1}$

$= \dfrac{x^{7/2}}{7/2} - \dfrac{x^{3/2}}{3/2}$

$= \dfrac{2x^{7/2}}{7} - \dfrac{2x^3}{3} + c$

7. $\int \dfrac{\cos x}{3\sqrt{\sin(x)^2}}\,dx$

$I = \int \dfrac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \dfrac{\cos x}{\sin x^{3/2}}\,dx$

put $t = \sin x$

$dt = \cos x\,dx$

$= \dfrac{1}{(t)^{2/3}}\,dt$

$= \dfrac{t^{-2/3+1}}{-2/3+1} = \dfrac{t^{1/3}}{1/3} = 3\sqrt[3]{t} + C$

$= 3\sqrt[3]{\sin x} + C$

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9.) $\int e^{\cos^2 x} \sin 2x \, dx$

$I = \int e^{\cos^2 x} \sin 2x \, dx$

put $\cos^2 x = t$

$2\cos x(-\sin x)\,dx = dt$

$-\sin 2x \, dx = dt$

$\sin 2x \, dx = dt$

$\sin 2x \, dx = -dt$

$\therefore \int e^t (-dt)$

$= -\int e^t \, dt$

$= -e^t + c \qquad [\because \int e^x dx = e^x + c]$

$= -e^{\cos^2 x} + c$

10.) $\int \frac{x^2 - 2x}{x^3 - 3x^2 + 1} \, dx$

put $x^3 - 3x^2 + 1 = t$

$(3x^2 - 6x)\,dx = dt$

$3(x^2 - 2x)\,dx = dt$

$(x^2 - 2x)\,dx = \frac{dt}{3}$

$\int \left(\frac{1}{t}\right) \frac{dt}{3}$

$\frac{1}{3} \int \left(\frac{1}{t}\right) dt$

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$= \frac{1}{3} \log |t| + c \quad \left[ \because \int \left(\frac{1}{x}\right) dx = \log |x| + c \right]$

Resubstituting $x^3 - 3x^2 + 1 = t$

$\therefore \frac{1}{3} \log |x^3 - 3x^2 + 1| + c$

AK
06/02/2020

# PRACTICAL NO. 6

## Application of integration & Numeric Integration.

Q1. Find length of the foll :-

i. $x = t - \sin t$ ; $y = 1 - \cos t$  $x \in [0, 2\pi]$

Soln :

$$\text{length} = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^3}$$

$$dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2 dt}$$

$$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^2 t + \sin^2 t \quad + dt}$$

$$= \sqrt{1 - 2\cos t + dt}$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t \, dt}$$

$$= \int_0^{2\pi} 2 \sin \frac{1}{2} \; dt$$

$$= \left[ -9 \cos \tfrac{1}{2} \right]_0^{2\pi}$$

$$= (-4 \cos \pi) + 4\cos 0$$

$$= 8 \text{ units.}$$

ii) $y = \sqrt{4 - x^2}$ $\qquad x \in [-2, 2]$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4 - x^2}}$$

$$= \frac{-x}{\sqrt{4 - x^2}}$$

$$L = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4 - x^2 + x^2}{4 - x^2}}\, dx$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right.$$

$$= 2\left[\pi/2 + \pi/2\right] = 2\pi$$

iii) $y = x^{3/2}$ $\qquad \in [0, 4]$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$L = \int_{0}^{4} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{0}^{4} \sqrt{1 + 9/4}\, dx$$

$$= \frac{1}{2}\left[\frac{(4 + 9x)^{3/2}}{3/2} \times 1/9\right]_0^4$$

$$= 1/27\left[(4 + 9x)^{3/4}\right]_0^4$$

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$$= -\frac{1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

8/3

$$L = \frac{1}{27}\left(40^{3/2} - 8\right) \text{ units.}$$

iv) $x = 3\sin t \quad , \; y = 3\cos t \qquad t \in (0, 2\pi)$

$$\frac{dx}{dt} = 3\cos t \quad ; \; \frac{dy}{dt} = -3\sin t$$

$$l = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin^2 t)}\; dt$$

$$= \int_0^{2\pi} 3\sqrt{x}\; dt$$

$$= 3\int_0^{2\pi} 3.\, dt$$

$$= 6\pi \text{ units}$$

v) $x = \frac{1}{6}y^3 + \frac{1}{2}y \qquad \text{on} \qquad y \in (1, 2)$

$$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}.$$

$$\frac{dx}{dy} = \frac{y^2 - 1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\; dy$$

$$= \int_1^2 \sqrt{1 + \frac{(y^2)^2}{4y^4}}\; dy$$

$$= \int_1^2 \sqrt{\frac{(y^4-1)+4y^4}{4y^4}}\ dy$$

$$= \int_1^2 \frac{y^4+1}{y^2}\ dy$$

$$= \frac{1}{2}\int_1^2 y^2\ dy + \frac{1}{2}\int_1^2 y^{-2}\ dy$$

$$= \frac{1}{2}\left[\frac{y^3}{3} - \frac{y^{-1/2}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left[\frac{7}{3} + \frac{1}{2}\right]$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$= \frac{17}{12} \text{ units}$$

Q.II. 1.) $\int_0^4 x^2\, dx$ $\qquad n = 4$

$$l = \frac{4-0}{4} = 1$$

| | | | | | |
|---|---|---|---|---|---|
| $x$ | 0 | 1 | 2 | 3 | 4 |
| $y$ | 0 | 1 | 4 | 9 | 16 |

$$\int_0^4 x^2\ dx = \frac{1}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2(y_2)\right]$$

$$= \frac{1}{3}\left[16 + 4(16) + 8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2\,dx = 21.533$$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\,dx \qquad n = 6$

$$L = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$ | 0 | 0.1745 | 0.3491 | 0.5236 | 0.6981 | 0.8727 | 1.39 |
| $y$ | 0 | 0.4167 | 0.5848 | 0.7071 | 0.8087 | 0.8752 | 0.99 |
| | $y_0$ | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ |

$$\int_0^{\pi/3} \sqrt{\sin x} = \frac{L}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= \frac{\pi}{54} \times 11.7614$$

$$= 0.6843$$

# PRACTICAL NO. 7

## TOPIC : Differential Equation.



Q1. Solve the following differential equation.

(i) $x\frac{dy}{dx} + y = e^x$

Soln. Dividing by $x$

$$\frac{dy}{dx} + \frac{y}{x} = \frac{e^x}{x}$$

comparing with $\frac{dy}{dx} + p(x)y = Q(x)$

$$If = e^{\int p\,dx}$$

$$= e^{\int 1/x\,dx}$$

$$= \int e^{\log x}\,dx = x$$

$$Y(If) = \int Q(I.f)x\,dx + c$$

$$Y(x) = \int \frac{e^x}{x} \cdot x\,dx + c$$

$$Y(x) = \int e^x + c$$

$$Y(x) = e^x + c$$

(ii) $e^x \frac{dy}{dx} + 2y = 1$

Dividing by $e^x$.

$$\frac{dy}{dx} + 2 \cdot y = \frac{1}{e^x}$$

comparing with

$$\frac{dy}{dx} + p(x)y = Q(x)$$

$$If = e^{\int p\,dx}$$

$$= e^{\int 2\,dx}$$

$$= e^{2x}$$

$$y(If) = \int Q(If)x\,dx + c$$

$$y(e^{2x}) = \int \frac{1}{e^{x}} \cdot e^{2x} \cdot x\,dx + c$$

$$= \int e^{2x} - e^{x}\,dx + c$$

$$= \int e^{x}\,dx + c$$

$$ye^{2x} = e^{x} + c$$

(iii) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

Soln:

$$\frac{x\,dy}{dx} = \frac{\cos x}{x} - 2y$$

$$\frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$$

comparing with

$$\frac{dy}{dx} + P(x)y = Q(x)$$

$$If = e^{\int p\,dx}$$

$$= e^{\int \frac{2}{x}\,dx}$$

$$= e^{2\ln x} = x^2$$

$$Y(If) = \int Q(x)\,(If)\,dx + c$$
$$= \int \frac{\cos x}{x^2} - x^2\, dx + c$$
$$= \int \cos x + c$$
$$x^2 y = \sin x + c$$

(iv) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

Soln:

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3} \quad (\div \text{ by } x \text{ on both sides})$$

$$P(x) = 3/x \quad Q(x) = \frac{\sin x}{x^3}$$

$$I.f = e^{\int P\,dx}$$
$$= e^{\int 3/x\,dx}$$
$$= e^{3/x\,dx}$$
$$= e^{\ln x^3}$$
$$If = x^3$$

$$Y(I.f) = \int Q(x)\,(I.f)\,dx + c$$
$$= \int \frac{\sin x}{x^3}\,dx + c$$
$$= \int \sin x\,dx + c$$
$$x^3 y = -\cos x + c$$

(v) $e^{2x} dy + 2e^{2x} y = 2x$

Soln:

$$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$$
$$If = e^{\int P(x)\,dx}$$
$$= e^{\int 2\,dx} = e^{2x}$$

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$$Y(I.f) = \int Q(x)(I.f)\,dx + c$$
$$= \int 2x e^{-2x} e^{2x}\,dx + c$$
$$= \int 2x\,dx + c$$
$$y e^{2x} = x^2 + c$$

(vi) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

Soln:

$$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$$
$$\frac{\sec^2 x\,dx}{\tan x} = \frac{-\sec^2 y\,dy}{\tan y}$$
$$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$$
$$\log|\tan x| = -\log|\tan y| + c$$
$$\log|\tan x - \tan y| = c$$
$$\tan x \cdot \tan y = e^c$$

(vii) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1 = v$

Differentiating on both sides

$$x-y+1 = v$$
$$1 - \frac{dy}{dx} = \frac{dv}{dx}$$
$$1 - \frac{dv}{dx} = \frac{dy}{dx}$$

$$1-\frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1-\sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v \, dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + c$$

(viii) $$\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$$

Put $2x+3y = v$

$$2+\frac{3dy}{dx} = \frac{dv}{dx}$$

$$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$$

$$\frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$$

$$\frac{dv}{dx} = \frac{v-1}{v+2}+2$$

$$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$$

$$= \frac{3v+3}{v+2} = \frac{3(v+1)}{v+2}$$

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$$= \int \left(\frac{v+2}{v+1}\right) dv = 3dx$$

$$= \int \frac{v+1}{v}\, dx + \int \frac{1}{v+1}\, dv = 3x$$

$$v + \log|x| = 3x + c$$

$$2x + 3y + \log|2x + 3y + 1| = 3x + c$$

$$3y = x - \log|2x + 3y + 1| + c$$

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# PRACTICAL NO. 8

Q. Using Euler's Methods, find :-

(i) $\frac{dy}{dx} = y + e^{x} - 2$ , $y(0) = 2$ , $h = 0.5$

Find $y(2)$.

Soln : $f(x) = y + e^{x} - 2$ , $x_0 = 0$

$y(0) = 2$ , $h = 0.5$ $\therefore y(0.2) = ?$

| $n$ | $x_n$ | $y(0)$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 2 | 1 | 2.5 |
| 1 | 0.5 | 2.5 | 2.14787 | 3.5743 |
| 2 | 1 | 3.5743 | 4.2925 | 5.7205 |
| 3 | 1.5 | 5.7205 | 8.2021 | 9.8215 |

$y(2) = 9.8215$

(ii) $\frac{dy}{dx} = 1 + y^2$ , $y(0) = 0$ $h = 0.2$. find $y(1)$

Soln: $y_2 = 0$ , $y_0 = 0$ , $h = 0.2$

| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 1 | 0.2 |
| 1 | 0.2 | 0.2 | 1.04 | 0.408 |
| 2 | 0.4 | 0.408 | 1.1664 | 0.6412 |
| 3 | 0.6 | 0.6412 | 1.4111 | 0.9234 |
| 4 | 0.8 | 0.9234 | 1.8596 | 1.2939 |
| 5 | 1.0 | 1.2939 | | |

$y(1) = 1.2939$

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(iii) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ , $y(0) = 1$ , $h = 0.2$ Find $y(1)$

$y(0) = 1$ , $x_0 = 0$ , $h = 0.2$

| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 |
| 1 | 0.2 | 1 | 0.4472 | 1.0894 |
| 2 | 0.4 | 1.0894 | 0.6059 | 1.2105 |
| 3 | 0.6 | 1.2105 | 0.7040 | 1.3513 |
| 4 | 0.8 | 1.3313 | 0.7694 | 1.5051 |
| 5 | 1 | 1.5051 | | |

$\therefore y(1) = 1.5051$

(iv) $\frac{dy}{dx} = 3x^2 + 1$ , $y(1) = 2$ find $y(2)$

For $h = 0.5$ , $h = 0.25$ , ~~[illegible]~~

$\therefore y_0 = 2$ , $x_0 = 1$

| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 0 | 1 | 2 | 4 |
| 1 | 1.5 | 1.5 | 4 | 7.875 |
| 2 | 2 | 2 | 7.875 | |

$y(2) = 7.875$

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for $h = 0.25$

| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 2 | 4 | 3 |
| 1 | 1.25 | 3 | 5.8875 | 4.4218 |
| 2 | 1.5 | 4.4218 | 59.6589 | 19.3360 |
| 3 | 1.75 | 19.3360 | 1122.648 | 299.9966 |
| 4 | 2 | 299.9966 | | |

$\therefore y(1) = 299.9966$

(v) $\frac{dy}{dx} = \sqrt{x}y + 2$ , $y(1) = 1$ find $y(1.2)$, $h = 0.$

<u>Soln:</u> $y(0) = 1$ $x(0) = 1$ $h = 0.2$

| $n$ | $x_n$ | $y_n$ | $f(x_n, y_n)$ | $y_{n+1}$ |
|---|---|---|---|---|
| 0 | 1 | 1 | 3 | 3.6 |
| 1 | 1.2 | 3.6 | | |

$\therefore y(1) = 3.6$

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23/01/2020

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# PRACTICAL NO. 9

TOPIC: Limits & Partial Order Derivative

Q1.) Evaluate the foll limits :-

(i) $\lim_{(x,y)\to(-4,-1)} \dfrac{x^3-3y+y^2-1}{xy+5}$

$$\frac{(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$$

$$\frac{64+3+1-1}{4+5}$$

$$\frac{67}{9}$$

(ii) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2+y^2-4x)}{x+3y}$

$$\frac{(0+1)(2+0^2-4(2))}{2+3(0)}$$

$$\frac{1(4+0-8)}{8}$$

$$=\frac{-4}{2}$$

$$=-2$$

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(iii) $\lim_{(x,y,z)\to(1,1,1)} \dfrac{x^2 - y^2 z^2}{x^3 - x^2 yz}$

$\lim_{(x,y,z)\to(1,1,1)} \dfrac{(x)^2 - (yz)^2}{x^2(x - yz)}$

$\lim_{(x,y,z)\to(1,1,1)} \dfrac{(x+yz)(x-yz)}{x^2(x-yz)}$ $\left[\because a^2 - b^2 = (a+b)(a-b)\right]$

$\lim_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x}$

$\therefore \dfrac{1+(1)(1)}{(1)^2} = 1$

Q2. Find $f_x, f_y$ for the following:

(i) $f(x,y) = xy\,e^{x^2+y^2}$

$f(x) = \dfrac{\partial f}{\partial x}$

$= \dfrac{\partial(xy\,e^{x^2+y^2})}{\partial x}$

$= y\,\dfrac{\partial(x\,e^{x^2+y^2})}{\partial x}$

$= y\left[x \cdot \dfrac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2}\dfrac{d}{dx}(x)\right]$

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$$= y[x \cdot e^{x^2+y^2} \cdot 2x + e^{x^2+y^2}(1)]$$

$$= y \cdot e^{x^2+y^2}[2x+1]$$

Now,

$$f(y) = \frac{\partial f}{\partial y}$$

$$= \frac{\partial (xy\, e^{x^2+y^2})}{\partial y}$$

$$= x \cdot \frac{\partial}{\partial y}(y \cdot e^{x^2+y^2})$$

$$= x\left[y \frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2} \cdot \frac{d}{dy}(y)\right]$$

$$\because \frac{d}{dx}(uv) = uv' + vu'$$

$$= x \cdot [2y^2 \cdot e^{x^2+y^2} + e^{x^2+y^2}]$$

$$= x \cdot e^{x^2+y^2}[2y^2+1]$$

(ii) $f(x,y) = e^x \cos y$

$$\therefore f(x) = e^x \cos y$$

$$f(y) = e^x \frac{d}{dy}(\cos y)$$

$$= e^x(-\sin y)$$

$$= -e^x \sin y$$

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3y^2 - 3x^2y + y^3+1)}{\partial x}$$

$$= \underline{3x^2y^2 - 3(2x)y}$$

$$= 3x^2y^2 - 6xy$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3y^2 - 3x^2y + y^3+1)}{\partial y}$$

$$= x^3(2y) - 3(1)x^2 + 3y^2$$

$$= 2x^3y - 3x^2 + 3y^2$$

Q3. Using defination, find values of $f_x$, $f_y$ at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

Soln: $f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$$(a,b) = (0,0)$$

$$\therefore f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$\lim_{h\to 0} \frac{2h-0}{2} = 2$$

similarly, $f_y(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$$= \lim_{h\to 0} \frac{0-0}{h} = 0$$

$\therefore f_x = 2$, $f_y = 0$

Q4. Find all second order partial derivaties of f. Also find whether $f_{xy} = f_{yx}$

(i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial (y^2 - xy)}{x^2}$$

$$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy)\frac{d}{dx}(x^2)}{(x^2)^2}$$

$$\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= \frac{x(xy - 2y^2)}{x4}$$

$$f_x = \frac{xy - 2y^2}{x3}$$

$$f_y = \frac{\partial f}{\partial y} = \partial\left(\frac{y^2 - xy}{x^2}\right) = \frac{\partial\left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right)}{\partial y}$$

$$= \partial\left(\frac{y^2}{x^2} - \frac{y}{x}\right) / \partial y$$

$$= \frac{1}{x^2} 2y - \frac{1}{x}$$

$$\therefore f_y = \frac{2y - x}{x^2}$$

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$$f(xx) = \frac{\partial\left(\frac{xy - 2y^2}{x^3}\right)}{\partial x}$$

$$= \frac{x^3 \frac{d}{dx}(xy - 2y^2) - (xy - 2y^2)\frac{d}{dx}(x^3)}{(x^3)^2}$$

$$= \frac{x^3(y) - (xy)y^2(3x^2)}{6}$$

$$= \frac{x^3y - 3x^3y + 6x^2y^2}{6}$$

$$= \frac{6x^2y^2 - 2x^3y}{x^6}$$

$$= \frac{x^2(6y^2 - 2xy)}{x^6}$$

$$= \frac{6y^2 - 2xy}{x^4}$$

$$f(yy) = \frac{\partial\left(\frac{2y - x}{x^2}\right)}{\partial y}$$

$$= \cancel{\frac{\frac{\partial}{x^2}(2y - x)}{\partial y}} = \frac{1}{x}(2) = \frac{2}{x^2}$$

$$f(xy) = \frac{\partial\left(\frac{xy - 2y^2}{x^3}\right)}{\partial y} = \frac{\partial\left(\frac{xy}{x^3} - \frac{2y^2}{x^3}\right)}{\partial y}$$

$$= \frac{\partial\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right)}{\partial y}$$

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$$\frac{1}{x^2} - \frac{1}{x^3} \quad 2(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3 - 4yx^2}{x6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

$$f(yx) = \frac{\partial\left(\frac{2y-x}{x^2}\right)}{\partial x}$$

$$= \frac{\partial\left(\frac{2y}{x^2} - \frac{x}{x^2}\right)}{\partial x} = \frac{\partial\left(\frac{2y}{x^2} - \frac{1}{x}\right)}{\partial x}$$

$$= 2y\left(-\frac{2}{x^3}\right) - \left(-\frac{1}{x^2}\right)$$

$$= -\frac{4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{-4yx^2 + x^3}{x^6}$$

$$= \frac{x^2(x-4y)}{x^6}$$

$$= \frac{x-4y}{x^4}$$

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$$\therefore f(xy) = f(yx) = \frac{x-4y}{x^4}$$

Hence verified.

(ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial\left((x^3+3x^2y^2) - \log(x^2+1)\right)}{\partial x}$$

$$= 3x^2 + 3(2x)y^2 - \frac{1}{x^2+1}(2x)$$

$$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$= 0 + 3(2y)(x^2) + 0$$

$$f(y) = 6x^2y$$

$$f(xx) = \frac{\partial}{\partial x} fx \qquad = \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 6x + 6y^2(1) - 2\left[\frac{x^2+1(1) - x(2x)}{(x^2+1)^2}\right]$$

$$\left[\frac{d}{dx} u/v = \frac{v.u' - u.v'}{v^2}\right]$$

$$6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)$$

$$= 6x + 6y^2 - 2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$$

$$f(yy) = \frac{\partial fy}{\partial y} \qquad = \frac{\partial(6x^2y)}{\partial y}$$

$$= 6x^2(1) \qquad = 6x^2$$

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$$f(xy) = \frac{\partial\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)}{\partial y}$$

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$$= 0 + 6x(2y)$$

$$= 12xy$$

$$f(xy) = f(yx) = 12xy$$

Hence verified.

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial x}$$

$$= \cos(xy)(y) + e^{x+y}(1)$$

$$= y\cos xy + e^{x+y}$$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$$

$$= \cos(xy)(x) + e^{x+y}(1)$$

$$= x\cos xy + e^{x+y}$$

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$$f(xx) = \frac{\partial}{\partial x} fx$$

$$= \frac{\partial (y\cos xy + e^{x+y})}{\partial x}$$

$$= y\cos xy\,(y) + e^{x+y}(1)$$

$$= y^2 \cos xy + e^{x+y}$$

$$f(yy) = \frac{\partial fy}{\partial y} = \frac{\partial (x\cos xy + e^{x+y})}{\partial y}$$

$$= x\cos xy\,(x) + e^{x+y}(1)$$

$$= x^2 \cos xy + e^{x+y}$$

$$f(xy) = \frac{\partial fx}{\partial y} = \frac{\partial (y\cos xy + e^{x+y})}{\partial y}$$

$$= y[-\sin(xy)(x) + \cos xy(1)] + e^{x+y}(1)$$

$$\left[\because \frac{d}{dx} uv = u.v' + v.u'\right]$$

$$= -xy\sin(xy) + \cos xy + e^{x+y}$$

$$f(yx) = \frac{\partial fy}{\partial x} = \frac{\partial (x\cos xy + e^{x+y})}{\partial x}$$

$$= \cos xy\,(1) + x(-\sin(xy))(y) + e^{x+y}$$

$$= -xy\sin(xy) + \cos(xy) + e^{x+y}$$

$$f(xy) = f(yx) = -xy\sin(xy) + \cos(xy) + e^{x+y}$$

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Q5. Find the linearization of $f(x,y)$ at given pt.

(i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$$f(1,1) = \sqrt{(1)^2+(1)^2}$$
$$= \sqrt{2}$$

$$f(x) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2x = \frac{x}{\sqrt{x^2+y^2}}$$

$$f(y) = \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = \frac{y}{\sqrt{x^2+y^2}}$$

$$f_x(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$f_y(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$$

$$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$$

$$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$$

$$= \frac{2+x-1+(y-1)}{\sqrt{2}}$$

$$= \frac{2+x-1+(y-1)}{\sqrt{2}}$$

$$= \frac{2+x+y-2}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$$

(ii) $f(x,y) = 1 - x + y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

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$$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2} + 0 \cdot \left(\sin\left(\frac{\pi}{2}\right)\right)$$

$$= 1 - \frac{\pi}{2}$$

$$f(x) = -1 + y\cos x$$

$$f(y) = 1$$

$$fx\left(\frac{\pi}{2}, 0\right) = -1 + 0 \cdot \cos\left(\frac{\pi}{2}\right)$$

$$= -1$$

$$fy\left(\frac{\pi}{2}, 0\right) = 1$$

$$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$$

$$= 1 - \frac{\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y-0)$$

$$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$$

$$= y - x + 1$$

(iii) $f(x,y) = \log x + \log y$ at $(1,1)$

$$f(1,1) = \log(1) + \log(1)$$

$$= 0$$

$$\cancel{f(x)}\ fx(1,1) = 1$$

$$f(y) = \frac{1}{y}$$

$$fy(1,1) = 1$$

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ns: $L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a-b)(y-b)$

$= 0 + 1(x-1) + (y-1)$

$= x - 1 + y - 1$

$= \underline{x + y - 2}$

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# PRACTICAL NO. 10



Q1. Find the directional derivative of the following fn. at given points & in the direction of given vector.

**Soln:**

$f(x, y) = x+2y-3$ $\quad a = (1,-1) \quad u = 3i - j$

Here, $u = 3i - j$ not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}(3,-1)$

$= \left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = 1+2(-1)-3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1+\frac{3}{\sqrt{10}}\right), \left(-1, \frac{-h}{\sqrt{10}}\right)$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{-4 + h/\sqrt{10} + 4}{h}$

$= \frac{1}{\sqrt{10}}$

2. $f(x) = y^2 - 4x + 1 \qquad a(3,4) \qquad u = i + 5j$

Here $u = i + 5j$ is not unit vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

$u$ is $\dfrac{u}{|u|} = \dfrac{1}{\sqrt{26}}(1, 5)$

$= \left(\dfrac{1}{\sqrt{26}}, \dfrac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5$

$f(a + hu) = f(3,4) + h\left(\dfrac{1}{\sqrt{26}}, \dfrac{5}{\sqrt{26}}\right)$

$= f\left(3 + \dfrac{h}{\sqrt{26}}, 4 + \dfrac{5h}{\sqrt{26}}\right)$

$f(a + hu) = \left(4 + \dfrac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \dfrac{h}{\sqrt{26}}\right) + 1$

$= 16 + \dfrac{25h^2}{26} + \dfrac{40h}{\sqrt{26}} - 12 - \dfrac{4h}{\sqrt{26}} + 1$

$= \dfrac{25h^2}{26} + \dfrac{40h}{\sqrt{26}} - \dfrac{4h}{\sqrt{26}} + 5$

$= \dfrac{25h^2}{26} + \dfrac{40h - 4h}{\sqrt{26}} + 5$

$= \dfrac{25h^2}{26} + \dfrac{36h}{\sqrt{26}} + 5$

$$D_u f(a) = \lim_{h \to 0} \frac{\dfrac{25h^2}{26} + \dfrac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$= \dfrac{25h}{26} + \dfrac{36}{\sqrt{26}}$

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3.) $2x + 3y$ $\quad a = (1,2)$ , $u = (3i + 4j)$

$u = 3i + 4j$ is not a unit vector

$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

unit vector along $u$ is $\frac{u}{|u|} = \frac{1}{5}(3,4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a + hu) - f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$= f\left(1 + \frac{3h}{5}, 2 + \frac{4}{5}\right)$

$f(a + hu) = 2\left(1 + \frac{3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5} + 8$

$D_u f(a) = \lim\limits_{h \to 0} \frac{18h}{5} + 8 - 8 / h$

$= \frac{18h}{5}$

Q2.) Find gradient vector for the foll.

SS

(i) $f(x,y) = x^y + y^x$ a $(1,1)$

$\cancel{f(x)}$) $f_x = yx^{y-1} + y^x \log y$

$f_y = x^y \log x + xy^{x-1}$

$f(x,y) = (f_x, f_y)$

$= (yx^{y-1} + y^2 \log y,\ x^4 \log x + xy^{x-1})$

$f(1,1) = (1+0,\ 1+0)$

$= (1,1)$

(ii) $f(x,y) = (\tan^{-1} x) \cdot y^2$ a $(1,-1)$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2},\ 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2},\ \tan^{-1}(1)(-2)\right)$

$= \frac{1}{2},\ \frac{\pi}{4}(-2)$

$= \left(\frac{1}{2},\ -\frac{\pi}{2}\right)$

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(iii) $f(x,y,z) = xyz - e^{x+y+z}$ , $a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$f(x,y,z) = f_x, f_y, f_z$

$= f_z - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$f(1,-1,0) = ((-1)(0) - e^{(1+(-1)+0)} (1)(0) - e^{x+(-1)+0}$

$(1)(-1) - e^{1+(-1)+0}$

$= (0 - e^0, 0 - e^0, -1 - e^0)$

$= (-1, -1, -2)$

Q3. Find the equ$^n$ of tangent & noumal to each of the foll:

(i) $x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} + y$

$f_y = x^2(-\sin y) + e^{xy} . x$

$(x_0, y_0) = (1,0)$ $\therefore x_0 = 1, y_0 = 0$

Eq$^n$ of tangent : $f_x (x - x_0) + f_y (y - y_0) = 0$

$f_x(x_0, y_0) = \cos 0 \, 2(0) + e^{\cdot 0}$

$= 1(2) + 0$

$= 2$

$f_y(x_0, y_0) = (1)^2 (-\sin 0) + e^0 . 1$

$= 0 + 1.1$

$= 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$

$\therefore$ It is the required eq$^n$ of tangent.

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Equation of Normal: $ax + by + c = 0$

$= bx + ay + d = 0$

$1(1) + 2(y) + d = 0$

$1 + 2y + d = 0$

$1 + 2(0) + d = 0$ at $(1, 0)$

$d + 1 = 0$

$\therefore d = -1$

(ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$fx = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$fy = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0, y_0) = (2, -2)$ $\therefore x_0 = 2$, $y_0 = -2$

$fx(x_0, y_0) = 2(2) - 2 = 2$

$fy(x_0, y_0) = 2(-2) + 3 = -1$

Equation of tangent:

$fx(x - x_0) + fy(y - y_0) = 0$

$2(x-2) + (1(y+2)) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0$

$\therefore$ The equ. of tangent is found.

Equation of Normal:

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2, -2)$

$-2 - 4 + d = 0$

$$-6+d=0$$
$$\therefore \underline{\underline{d=6}}$$

Q4.) Find the equ. of tangent & noumal to each of the foll :-

i.) $x^2 - 2y^2 + 3y + xz = 7$ at $(2,1,0)$

$$fx = 2x - 0 + 0 + z$$
$$fx = 2x + z$$
$$fy = 0 - 2z + 3 + 0$$
$$= 2z + 3$$
$$fz = 0 - 2y + 0 + x$$
$$= -2y + x$$

$(x_0, y_0, z_0) = (2,1,0) \quad \therefore x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$$fx(x_0, y_0, z_0) = 2(2) + 0 = 4$$
$$fy(x_0, y_0, z_0) = 2(0) + 3 = 3$$
$$fz(x_0, y_0, z_0) = -2(1) + 2 = 0$$

Equation of tangent

$$\cancel{fx}(x_0 - x_0) + fy(y - y_0) + fz(z - z_0) = 0$$
$$= 4x - 8 + 3y - 3 = 0$$
$$= 4x + 3y - 11 = 0$$

$\therefore$ Equ. of tangent is found

Equation of Noumal at $(4, 3, -11)$

$$\frac{x - x_0}{fx} = \frac{y - y_0}{fy} = \frac{z - z_0}{fz}$$
$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+11}{0}$$

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ii) $3xy^2 - xz - y + z = -4$ at $(1,-1,2)$

$3xy^2 - x - y + z + 4 = 0$ at $(1,-1,2)$

$$f_x = 3yz - 1 - 0 + 0 + 0$$
$$= 3yz - 1$$
$$f_y = 3xz - 0 - 1 + 0 + 0$$
$$= 3xz - 1$$
$$f_z = 3xy - 0 - 0 + 1 + 0$$
$$= 3xy + 1$$

$(x_0, y_0, z_0) = (1, -1, 2) \quad \therefore x_0 = 1,\ y_0 = -1,\ z_0 = 2$

$f_x(x_0, y_0, z_0) = 3(-1).(2) - 1 = -7$

$f_y(x_0, y_0, z_0) = 3(1)(2) - 1 = 5$

$f_z(x_0, y_0, z_0) = 3(1)(-1) + 1 = -2$

Equation of tangent $-7(x-1) + 5(y+1) - 2(z-2) =$

$$= \cancel{-7(x+1) + 5(y+x}$$
$$= -7x + 7 + 5y + 5 - 2z + 4 = 0$$
$$= -7x + 5y - 2z + 16 = 0$$

This is required Eq$^n$. of tangent

Equation of Normal : $(-7, 5, -2)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{-2}$$

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5.)
(i) $f(x,y) = 3x^2 + y^2 = 3xy + 6x - 4y$

$fx = 6x + 0 - 3y + 6 = 0$

$= 6x - 3y + 6$

$fy = 0 + 2y - 3x + 0 - 4$

$= 2y - 3x - 4$

$fx = 0$

$6x - 3y + 6 = 0$

$3(2x - y + 2) = 0$

$2x - y + 2 = 0$

$2x - y = -2 \quad \ldots(i)$

$fy = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \quad \ldots(ii)$

multiply eq$^n$. 1 by 2

$4x - 2y = -4$

$y - 3x = 4$

$x = 0$

subs. value of $x$ in eqn 1

$2(0) - y = -2$

$y = 2$

Critical points are $(0,2)$

$r = fxx = 6$

$t = fyy = 2$

$s = fxy = -3$

Here $x > 0$

$= rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

$f$ has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ at $(0,2)$

$= 3(0)^2 + (2)^2 - 3(0)(2) + 6(0) - 4(2)$

$= 0 + 4 - 0 + 0 + 8$

$= -4$

(ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 2y) = 0$

$4x^2 + 3y = 0$ ...(i)

$f_y = 0$

$3x^2 - 2y = 0$ (ii)

solving eqn. ① & ②

$12x^2 - 9y = 0$

$-12x^2 - 8y = 0$

$y = 0$

$$4x^2 + 3(0) = 0$$



$$4x^2 = 0$$

$$x = 0$$

critical pts. in $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} : 0 - 2 = -2$

$s = f_{xy} : 6x - 0 = 6x = 6(0) = 0$

$r$ at $(0,0)$

$= 24(0) + 6(0) = 0$

$\therefore r_e = 0$

$f(x,y)$ at $0,0$

$2(0)^7 + 3(0)^2(0) - (0)$

$0 + 0$

$0$

$$rt - s^2 = 0(-2) - (0)^2$$

$$= 0 - 0$$

$$= 0$$

$\therefore$ Nothing to say $\because r = 0$ & $rt - s^2 = 0$

(iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$

$2x + 2 = 0$

$\boxed{x = -1}$

$f_y = 0$

or $-2y + 8 = 0$

$\boxed{y = +4}$

critical pts are $(-1, -4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$

$rt - s^2 = 2(-2) - 0^2$

$= -4 < 0$

$f(x,y)$ at $(-1, +4)$

$(-1)^2 - (+4)^2 + 2(-1) + 8(+4) - 70$

$\cancel{1 - 16 + (-2) - 32 - 70}$

$\cancel{15 - 2 - 32 - 70}$

$1 + 16 - 2 + 32 - 70$

$17 + 30 - 70$

$47 - 70$

$= -23$

AK
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