

### SAM: The Sensitivity of Attribution Methods to Hyperparameters

### Naman Bansal\* Auburn University

### Chirag Agarwal\* University of Illinois at Chicago

Anh Nguyen\* Auburn University

bnaman50@gmail.com

chiragagarwall12@gmail.com

anh.ng8@gmail.com

#### **Abstract**

Attribution methods can provide powerful insights into the reasons for a classifier's decision. We argue that a key desideratum of an explanation method is its robustness to input hyperparameters which are often randomly set or empirically tuned. High sensitivity to arbitrary hyperparameter choices does not only impede reproducibility but also questions the correctness of an explanation and impairs the trust of end-users. In this paper, we provide a thorough empirical study on the sensitivity of existing attribution methods. We found an alarming trend that many methods are highly sensitive to changes in their common hyperparameters e.g. even changing a random seed can yield a different explanation! Interestingly, such sensitivity is not reflected in the average explanation accuracy scores over the dataset as commonly reported in the literature. In addition, explanations generated for robust classifiers (i.e. which are trained to be invariant to pixel-wise perturbations) are surprisingly more robust than those generated for regular classifiers.

#### 1. Introduction

Why did a self-driving car decide to run into a truck [29]? Why is a patient being predicted to have breast cancer [59] or to be a future criminal [2]? The explanations for such predictions made by machine learning (ML) models can impact our lives in many ways, under scientific [53, 37], social [18] or legal [24, 19] aspects.

A popular medium for visually explaining an image classifier's decisions is an *attribution map i.e.* a heatmap that highlights the input pixels that are the evidence for and against the classification outputs [35]. Dozens of attribution methods (Fig. 1) have been proposed [44] and applied to a variety of domains including natural images [35], medical brain scans [25], text [12], videos [50], and speech [14]. Notably, attribution maps have been useful *e.g.* in localizing malignant tumors in a breast x-ray scan [41] or in revealing biases in object recognition models [30, 31]. Yet are these

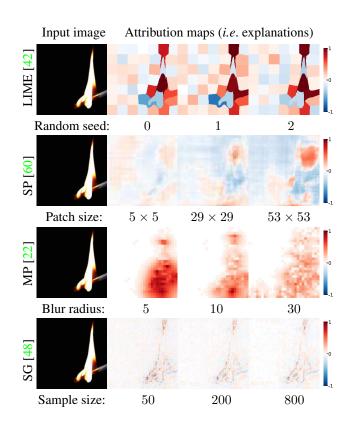


Figure 1: Attribution maps by four methods to explain the same prediction (match stick: 0.535) made by a ResNet-50 classifier to an ImageNet image. In each row, the explanations are generated by running the default settings of a method while varying only one common hyperparameter. All 12 explanations are unique and can be interpreted differently. **LIME:** an explanation changes when one re-runs the algorithm with a different random seed. **SP:** the positive evidence for the fire (top-right red blob) grows together with the patch size. **MP:** attribution maps become more scattered as the Gaussian blur radius increases. **SG:** heatmaps becomes smoother as the number of samples increases.

explanations reliable enough to convince medical doctors or judges to accept a life-critical prediction by a machine [32]? First, ML techniques often have a set of hyperparame-

<sup>\*</sup>Equal contribution. CA performed this work during his internship at Auburn University.

ters to be tuned empirically and most attribution methods are not an exception. Second, a major cause of the current replication crisis in ML [27] is that many methods, *e.g.* in reinforcement learning, are notoriously sensitive to hyperparameters—a factor which is also often overlooked in the interpretability field. Aside from being faithful, an explanation needs to be reproducible and invariant to arbitrary hyperparameter choices. In this paper, we studied an important question: *How sensitive are attribution maps to their hyperparameters?* on 7 well-known attribution methods and found that:<sup>1</sup>

- 1. Gradient heatmaps, for *robust* image classifiers *i.e.* models trained to ignore adversarial pixel-wise noise [21], exhibit visible structures (Fig. 3) in stark contrast to the noisy, uninterpretable gradient images for regular classifiers reported in prior work [48] (Sec. 4.1).
- 2. The gradient images from a robust and a regular classifier are different but would appear  $\sim 1.5 \times$  more similar, potentially causing misinterpretation, under several prior methods that attempted to de-noise the original explanations [49, 48] (Sec. 4.2).
- 3. For many attribution methods [42, 22, 60], their output heatmaps can change dramatically (Fig. 1) when a common hyperparameter changes (Sec. 4.3). This sensitivity of an individual explanation also translates into the sensitivity of its accuracy scores (Sec. 4.5).
- 4. Some hyperparameters cause up to  $10 \times$  more variation in the accuracy of explanations than others (Sec. 4.6).
- 5. Explanations for robust classifiers are not only more invariant to pixel-wise image changes (Sec. 4.1) but also to hyperparameter changes (Sec. 4.3)

### 2. Methods and Related Work

Let  $f: \mathbb{R}^{d \times d \times 3} \to [0,1]$  be a classifier that maps a color image  $\boldsymbol{x}$  of spatial size  $d \times d$  onto a probability of a target class. An attribution method is a function A that takes three inputs—an image  $\boldsymbol{x}$ , the model f, and a set of hyperparameters  $\mathcal{H}$ —and outputs a matrix  $\boldsymbol{a} = A(f, \boldsymbol{x}, \mathcal{H}) \in [-1,1]^{d \times d}$ . Here, the explanation  $\boldsymbol{a}$  associates each input pixel  $x_i$  to a scalar  $a_i \in [-1,1]$ , which indicates how much  $x_i$  contributes for or against the classification score  $f(\boldsymbol{x})$ .

**Methods** Attribution methods can be categorized into two main types: (1) exact and (2) approximate approaches. **Exact approaches** may derive an attribution map by upsampling a feature map of a convolutional network [62], or from the analytical gradients of the classification w.r.t. the input *i.e.*  $\nabla_x f$  [47, 8], or by combining both the gradients and

the feature maps [45]. These approaches enjoy fast derivation of explanations and have no hyperparameters in principles. However, they require access to the internal network parameters—which may not be available in practice. Also, taking gradients as attributions faces several issues: (1) gradient images are often noisy [48] limiting their utility; (2) gradient saturation [51] *i.e.* when the function f flattens within the vicinity of a pixel  $x_i$ , its gradient becomes nearzero and may misrepresent the actual importance of  $x_i$ ; (3) sudden changes in the gradient  $\partial f / \partial x_i$  (e.g. from ReLUs [36]) may yield misleading interpretation of the attribution of pixel  $x_i$  [46].

Therefore, many approximate methods have been proposed to modify the vanilla gradients to address the aforementioned issues [48, 51, 13]. Among gradient-based methods, we chose to study the following four representatives. **Gradient** [47, 8] The gradient image  $\nabla_x f$  quantifies how a small change of each input pixel modifies the classification and therefore commonly serves as an attribution map. **SmoothGrad (SG)** [48] proposed to smooth out a gradient image by averaging out the gradients over a batch of  $N_{SG}$ noisy versions  $\boldsymbol{x}_n$  of the input image  $\boldsymbol{x}_0$ . That is, an SG heatmap is  $\frac{1}{N_{SG}} \sum_{1}^{N_{SG}} \nabla_{\boldsymbol{x}} f(\boldsymbol{x}_0 + \epsilon)$  where  $\epsilon \sim \mathcal{N}(0, \sigma)$ . **Gradient**  $\odot$  **Input** (GI) [46] As gradients are often noisy and thus not interpretable [48], element-wise multiplying the gradient image with the input i.e.  $\nabla_x f \odot x$  can yield less-noisy heatmaps in practice. Here, the input image acts as a model-independent smoothing filter. GI is an approximation of a family of related LRP methods [13] as shown in [11] and is also a representative for other explicit gradientbased extensions [25, 61, 34, 46].

**Integrated Gradients (IG)** [51] In order to ameliorate the gradient saturation problem [51], IG intuitively replaces the gradient in GI [46] with an average of the gradients evaluated for  $N_{IG}$  images linearly sampled along a straight line between the original image  $\boldsymbol{x}$  and a zero image. IG is intuitively a smooth version of GI and depends on the sample size  $N_{IG}$  while GI has no hyperparameters.

Furthermore, there exist other approximate methods that attempt to compute the attribution of an input region by replacing it with zeros [60, 42], random noise [17], or blurred versions of the original content [22]. These methods inherently depend on many empirically-chosen hyperparameters. Among the family of perturbation-based methods, we chose to study the following three famous representatives.

**Sliding Patch (SP)** [60] slides a square, occlusion patch of size  $p \times p$  across the input image and records the prediction changes into an attribution map. This approach is applicable to any black-box classifier f and widely used [25, 4, 38, 11]. **LIME** [42] Instead of a square patch, LIME generates  $N_{LIME}$  masked images  $\{\bar{x}^i\}$  by masking out a random set of S non-overlapping superpixels in the input image. Intuitively, the attribution for a superpixel k is proportional to

<sup>&</sup>lt;sup>1</sup>Code is available at https://github.com/anguyen8/sam

the average score  $f(\bar{x}^i)$  over a batch of  $N_{LIME}$  perturbed images where the superpixel k is not masked out.

**Meaningful-Perturbation** (MP) [22] finds a minimal Gaussian blur mask of radius  $b_R$  such that when applied over the input image would produce a blurred version that has a near-zero classification score. MP is the basis for many extensions [56, 40, 15, 57, 54, 9]. In this paper, we evaluate MP sensitivity to three common hyperparameters: the blur radius  $b_R$ , the number of steps  $N_{iter}$ , and the random seed (which determines the random initialization).

See Sec. S1 for a detailed description of all methods.

Explanation sensitivity First, recent work has argued that some attribution methods have a problem of being highly sensitive to small pixel-wise changes in the input image [28, 10, 23]. Our results suggest that such sensitivity to image changes also depends on the classifier f. That is, gradient-based explanations of a robust classifier stay more consistent when the input image is perturbed with pixel-wise noise (Sec. 4.1). Second, some attribution methods were found to behave akin to an edge detector i.e. producing similar explanations despite that f's parameters are randomized to various degrees [8]. In sum, previous work has studied the sensitivity of explanations to input image changes [28, 10, 23] and classifier changes [8]. In this paper, we present the first systematic study on the sensitivity of explanations to changes in the hyperparameters  $\mathcal{H}$ , which are often randomly or heuristically tuned [48, 58].

#### 3. Experiment framework

**Explanation evaluation metrics** Currently, there is not yet a common ground-truth dataset for evaluating the accuracy of attribution methods [18]. However, researchers often approximate explanation correctness via two main techniques: (1) object localization [62]; and (2) Insertion & Deletion [38]. The localization error measures how accurately an attribution map localizes the main object in the input image [62]—a reasonable approximation for the ImageNet images [43], which are object-centric and paired with human-labeled segmentation masks. We did not use evaluation metrics like Pointing Game accuracy [61] and Saliency Metric [17] as they are derivatives of the localization task. The **Deletion** metric [38] measures the classification score changes as we gradually zero out the input pixels in the descending order of their attributions. The idea is if the attribution values correctly reflect the discriminative power of the input pixels, knocking out the highest-attribution pixels should quickly cause the probability to approach zero. In contrast, **Insertion** [38] tests whether inserting the highestattribution pixels into a zero image would quickly increase the probability. We used all three above mentioned metrics<sup>2</sup>

to quantify how much the variation of explanations translates into the sensitivity of their accuracy (Sec. 4.5).

Classifiers All of our experiments were conducted on two groups of classifiers: (a) GoogLeNet [52] & ResNet-50 [26] (hereafter, ResNet) pre-trained on the 1000-class 2012 ImageNet dataset [43]; and (b) the robust versions of them *i.e.* GoogLeNet-R & ResNet-R that were trained to also be invariant to small adversarial changes in the input image [20]. We obtained the two regular models from the PyTorch model zoo [39], the ResNet-R from [20], and we trained GoogLeNet-R by ourselves using the code released by [20]. While the two robust classifiers are more invariant to pixelwise noise they have lower ImageNet validation-set accuracy scores (50.94% and 56.25%) than those of the original GoogLeNet & ResNet (68.86% and 75.59%).

**Datasets** From the 50,000 ImageNet validation-set images, we randomly sampled a set of 1735 images that all four models correctly classify. We used this set of images in all experiments throughout the paper.

**Similarity metrics** To quantify the sensitivity of attribution maps, we followed Adebayo et al. [8] and used three measures<sup>3</sup> that cover a wide range of similarity notions: Spearman rank correlation, Pearson correlation of the histogram of gradients (HOGs), and the structural similarity index (SSIM). To quantify the sensitivity of the accuracy scores of explanations, we used the standard deviation (std).

### 4. Experiments and Results

# 4.1. Gradient maps of robust classifiers are smooth and insensitive to pixel-wise image noise

Gradient saliency maps of image classifiers are (1) notoriously noisy [47, 48, 13] limiting their utility and (2) sensitive to input changes [10]. Therefore, a number of techniques have been proposed to de-noise the gradient images [46, 48, 49, 45]. However, are these smoothing techniques necessary for gradients of robust classifiers?

First, we observed, for the first time, that the vanilla gradients of robust classifiers consistently exhibit visible structures (see the outline of the goblet in Fig. 3c & e), which is surprising! They are in stark contrast to the noisy gradients of regular classifiers (Fig. 3b & d).

Second, we found that the gradient explanations of robust classifiers are significantly more invariant to a large amount of random noise added to the input image. Specifically, for each image x in the dataset, we added noise  $\sim \mathcal{N}(0, 0.1)$  to generate a noisy version  $x_n$  (Fig. 3; bottom) and measured the similarity between the saliency maps for the pair  $(x, x_n)$  using all three similarity metrics described in Sec. 3. Across all images and all three quantitative metrics, the gradients of robust classifiers are substantially more invariant to noise than their regular counterparts

<sup>&</sup>lt;sup>2</sup>We used the Insertion and Deletion code by the authors [38].

<sup>&</sup>lt;sup>3</sup>We used the implementation by scikit-image [55].

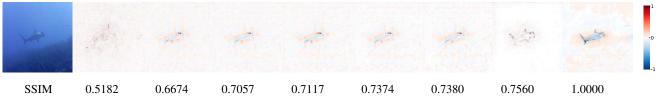


Figure 2: The SmoothGrad [48] explanations (b–g) for a prediction by ResNet are becoming increasingly similar to the explanation for a different prediction by a ResNet-R as we increase  $N_{SG}$ —a hyperparameter that governs the smoothness of SG explanations. Similarly, under GuidedBackprop (GB) [49], the explanation appears substantially closer to that of a different model (h vs. i) compared the original heatmaps (b vs. i). Below each heatmap is the SSIM similarity score between that heatmap and the ResNet-R heatmap (i). See more examples in Fig. S14.

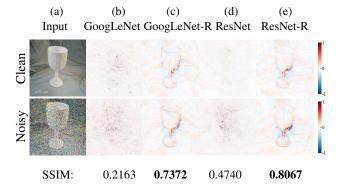


Figure 3: **Top:** The gradients of robust classifiers (c & e) reflect the structure of the goblet in an example input image (a), which is in stark contrast to the commonly reported noisy gradients of regular classifiers (b & d). **Bottom:** The gradients of robust classifiers remain similar before and after the addition of noise to the input image (c & e—higher SSIM scores). An SSIM similarity score is for the two images in each column.

(Figs. 5a & S13). For example, the average similarity of the gradient pairs from robust models is  $\sim 36 \times$  higher than that of the counterparts under the Spearman rank correlation (Fig. S13; leftmost bars). This result interestingly show that the gradients of *robust* models are fairly insensitive to minor pixel-wise image changes—a concern in [28, 10, 23].

# **4.2.** De-noising explanations may cause misinter-pretation

We have shown that the vanilla gradients of robust classifiers can be fairly smooth (Sec. 4.1). That result naturally raises a follow-up question: Do the smoothing techniques [48, 45, 49] improve or mislead our interpretation of explanations? To shed light on that question, we quantify the similarity between (a) the de-noised explanations by SG [48] for a regular classifier and (b) the vanilla gradient saliency maps for a robust classifier.

**Experiment** For each image, we generated SG explanations for regular models by sweeping across a range of the

sample size  $N_{SG} \in \{0, 50, 100, 200, 500, 800\}$ . Here,  $N_{SG} = 0$  yields the vanilla gradient. We measured the similarity between each SG heatmap of a regular model and the vanilla gradient of a robust counterpart model (*e.g.* ResNet vs. ResNet-R).

**Results** We observed that as the sample size  $N_{SG}$  increases, the resultant explanations of ResNet become increasingly more similar to the explanation of ResNet-R—a completely different classifier! That is, the SSIM similarity between two heatmaps increases up to  $\sim 1.4 \times$  (Fig. 2; b–g) on average. This monotonic trend is also observed across three similarity metrics and two pairs of regular vs. robust models (Fig. S3).

Additionally, we generated an explanation using another popular explanation method, GuidedBackprop (GB) [49], which *modifies* the gradient by only letting the *positive* forward activations and backward gradients to flow through during backpropagation. Across the dataset, the average similarity between a pair of (ResNet GB heatmap, ResNet-R gradient heatmap) is 0.377 while the original similarity between the vanilla gradients of two models is only 0.239.

In sum, our result shows that two explanations from two completely different classifiers (ResNet vs. ResNet-R) may become substantially more similar under explanations techniques (here, SG and GB) that attempt to heuristically denoise heatmaps, potentially misleading user interpretation. We reached the same conclusion by comparing GI and its approximate version *i.e.* IG [51] (see Sec. S3).

# 4.3. Gradient-based attribution maps are sensitive to hyperparameters

In practice, attribution methods often have various hyperparameters that are either randomly set (*e.g.* a random seed [42]) or empirically tuned (*e.g.* the number of optimization steps [22]). It is important to understand how such choices made by the end-user vary the explanations (Fig. 1), which impedes reproducibility and can impair users' trust *e.g.* a medical doctor's trust in a model's explanation of its prediction [32, 18]. Here, we quantify the sensitivity of attribution maps generated by two representative meth-

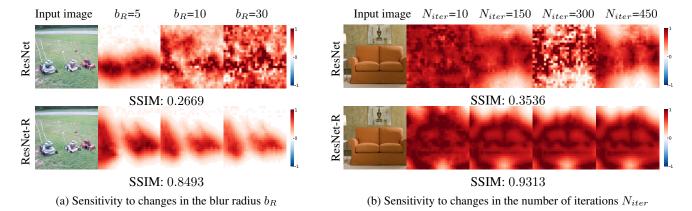


Figure 4: MP attribution maps generated for a regular model (ResNet) are highly sensitive to changes (i.e. low SSIM scores) in the Gaussian blur radius  $b_R$  (a) and in the number of iterations  $N_{iter}$  (b). In contrast, the same MP explanations for a robust model (ResNet-R) are substantially more stable (see Fig. S8 for quantitative results). Two reference images in this figure are the top-2 that cause the largest differences between the SSIM scores of ResNet vs. ResNet-R heatmaps. MP being more unstable with ResNet compared to that with ResNet-R can be seen quantitatively in the loss plot (Fig. S9) and qualitatively in the evolution of the MP heatmaps (Fig. S10). See Fig. S16 for more examples of the blur sensitivity experiments.

ods (SG and MP) as a common hyperparameter changes. In all experiments, we compare the average pair-wise similarity between a *reference* heatmap—the explanation generated using the default settings provided by the authors—and those generated by changing one hyperparameter.

#### 4.3.1 SmoothGrad is sensitive to sample sizes

SG was created to combat the issue that gradient images for image classifiers are often too noisy to be human-interpretable—an issue reported in many previous papers [48, 49, 13, 47] and also shown in Sec. 4.1. While SG does qualitatively sharpen the explanations [48] (see Fig. 2b vs. c), the method also introduces two hyperparameters (1) the sample size  $N_{SG}$  and (2) the Gaussian std  $\sigma$  that were empirically tuned [48]. Here, we test the sensitivity of SG explanations when varying these two hyperparameters.

**Experiment** To test the sensitivity to sample sizes, we measure the average pair-wise similarity between a reference heatmap at  $N_{SG} = 50$  (Fig. S12b; ii)—*i.e.* the default value in [48]—and each of the four heatmaps generated by sweeping across  $N_{SG} \in \{100, 200, 500, 800\}$  (Fig. S12b; iii—vi) on the same input image.  $\sigma$  is constant at 0.15.

**Results** We found that the SG explanations for robust models exhibited near-maximum consistency (Fig. S12a; all scores are near 1.0). In contrast, the robustness of SG when running on regular models is consistently lower under all three metrics (Fig. S12a; light vs. dark red or light vs. dark green). SG heatmaps for robust classifiers appear sharper and less noisy compared to those of regular models (Fig. S12b; top vs. bottom). Furthermore, while SG heatmaps may appear qualitatively stable (Fig. S12b; ii–vi), the actual pixel-wise variations are not. For example, the  $L_1$  pixel-wise difference between the ResNet heatmaps at the

two extreme settings (i.e.  $N_{SG} = 50$  vs. 800) is over  $5 \times 10^{-2}$  larger than the difference between the respective ResNet-R explanations (Fig. S12b; vii).

In sum, we showed that it is non-trivial how to tune a hyperparameter, here  $N_{SG}$ , to yield an accurate explanation because the heatmaps vary differently for different classifiers. Similarly, we further found SG heatmaps to be highly sensitive to changes in the amount of noise *i.e.* Gaussian std  $\sigma$  (Sec. S4.1) added to the input image.

# 4.3.2 Meaningful-Perturbation is sensitive to the number of iterations, the Gaussian blur radius, and the random seed

MP [22] is a representative of a family of methods that attempt to learn an explanation via iterative optimization [56, 40, 15, 57, 54, 9]. However, in practice, optimization problems are often non-convex and thus the stopping criteria for iterative solvers are heuristically set. For instance, it can be controlled by a pre-defined number of iterations  $N_{iter}$ . Also, MP learns to blur the input image to minimize the classification scores and thus depends on the Gaussian blur radius  $b_R$ . Here, we test MP sensitivity to three common hyperparameters:  $N_{iter}$ ,  $b_R$ , and the random seed which governs random initializations.

**Experiment** In order to test the sensitivity to the number of iterations, we measure the average similarity between a reference heatmap at  $N_{iter}=300$  which is the default seting in [22] and each of the three heatmaps generated by sweeping across  $N_{iter} \in \{10, 150, 450\}$  (Fig. 4b) on the same input image. To measure the sensitivity to the blur radius settings, we repeated a similar comparison to the above for a reference heatmap at  $b_R=10$  and other heatmaps by sweeping across  $b_R \in \{5, 30\}$  (Fig. 4a). For other hyperpa-

rameters, we used all default settings as in [22].

**Results** We found that MP explanations are sensitive to changes in the blur radius but interestingly in opposite ways for two different types of classifiers. That is, as we increase  $b_R$ , the heatmaps for ResNet tend to be more noisy and sparse; however, those for ResNet-R become gradually more localized and smoother (Fig. 4a; top vs. bottom). See Fig. S16 for more examples.

Across the number of iterations, MP explanations for regular classifiers vary dramatically. In contrast, the heatmaps for robust models are 1.4× more consistent under SSIM similarity metrics (Figs. 4b & S10). The MP optimization runs for robust models converged substantially faster within only  $\sim 10$  steps (compared to the default  $N_{iter} = 300$  [22]) which can be seen in both the loss plot (Fig. S9) and the sequence of heatmaps (Fig. S10). This inconsistent behavior of MP suggests that when comparing MP explanations between these two classifiers, an enduser may draw an entirely different conclusion depending on when optimization stops (which is heuristically chosen). Sensitivity to the random seed Our previous experiments followed exactly the setup in [22] where the authors used a blur circular mask that suppresses the target probability by 99% as the initial heatmap. This initialization, however, strongly biases the optimization towards a certain type of explanation. To avoid that, in practice, MP users randomly initialize the explanation before optimization [16]. By running experiments similar to the previous ones, we found that MP is also sensitive to the random seed, which controls the random initializations. That is, on average across 3 similarity metrics, heatmaps for robust classifiers are  $1.22 \times$  more consistent than those for regular classifiers (see Sec. S4.3 for more details and Fig. S7 for results).

In sum, consistent with SG results (Sec. 4.3.1), robust classifiers yield more stable explanations than regular models for the three aforementioned hyperparameters of MP (Fig. S8). That is, not only the gradients of robust classifiers are more interpretable but also more invariant to pixel-wise image changes, yielding more robust explanations (Fig. 4b).

### 4.4. Non-gradient attribution maps are sensitive to hyperparameters

#### 4.4.1 Sliding-Patch is sensitive to the patch size

Sec. 4.3 shows that *gradient*-based explanation methods are sensitive to hyperparameters and their sensitivity depends on the robustness of the gradients with respect to the input changes (Sec. 4.3.2). Here, we test whether methods that are *not gradient-based* would have similar shortcomings. We chose SP [60] which slides a square patch of size  $p \times p$  across the input image and records the classification probability changes into the corresponding cells in the attribution map. While SP has been widely used [60, 4, 11], it remains unknown how to choose the patch size.

Input image  $5 \times 5$   $17 \times 17$   $29 \times 29$   $41 \times 41$   $53 \times 53$ 



Figure 6: SP explanations are sensitive to patch sizes. **Zoomed-out:** SP attribution region (red squares) for a tennis ball of size  $19 \times 19$  (rendered on a  $224 \times 224$  zero image)

grows as the patch size increases. **Zoomed-in:** SP outputs blank heatmaps at patch sizes of  $5 \times 5$ ,  $17 \times 17$ , and  $29 \times 29$ , which are much smaller than the size of the tennis ball (here,

 $84 \times 84$ ) in a zoomed-in version of the top image.

To understand the relation between SP patch size and the size of the object in an input image, we generated two images, each containing a tennis ball of size  $19 \times 19$  or  $84 \times 84$  on a zero background of size  $224 \times 224$  (Fig. 6). We ran SP on these two images sweeping across 5 patch sizes of  $p \times p$  where  $p \in \{5, 17, 29, 41, 53\}$ . We observed that the heatmaps tend to be blank when the patch size is much smaller than the object size (Fig. 6; zoomed-in) because the occlusion patch is too small to substantially change the classification score. In contrast, if the patch size is much larger than the object size (Fig. 6; zoomed-out), the attribution areas tend to be exaggerated *i.e.* even larger than the object size (Fig. 6; the size of the red square increases from left to right). Therefore, SP explanations are subject to errors as the size of the object in the image is unknown.

**Sensitivity to large changes** To quantify the sensitivity of SP explanations to the patch size, here, we measure the average similarity between a reference SP attribution map at p=29 and each of the four attribution maps generated by sweeping across  $p \in \{5, 17, 41, 53\}$  on the same input image. This set of patch sizes covers a large range of settings (hence, denoted by SP-L) used in the literature [11, 60, 4]. We kept the stride constant at 3. We observed that across all classifiers, SP is highly sensitive to changes within the SP-L set. In contrast to the case of gradient-based methods, SP explanations for robust classifiers are not significantly more consistent than those for regular models (Fig. S11). Compared to other methods, SP sensitivity to patch sizes is higher than the sensitivity of SG and MP (Fig. 5a; SP-L bars are the shortest on average). See Fig. \$15 for more examples on sensitivity to large changes in patch size.

**Sensitivity to small changes** We further repeated the previous experiment but comparing the similarity of SP explanations at p = 53 with those generated at  $p \in \{52, 54\}$  i.e. a

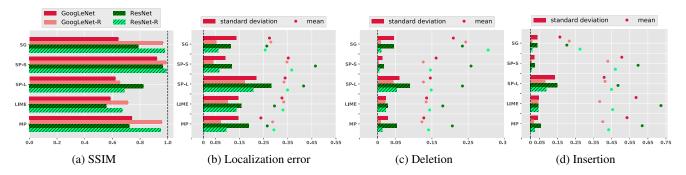


Figure 5: Average sensitivity of an individual attribution map measured in the pixel space (a) and three accuracy metric spaces: the Localization error (b), Deletion (c) and Insertion (d) scores (Sec. 4.5). The results were produced by varying the random seed of LIME and MP (bottom two rows), the patch size in SP (SP-S and SP-L), and the sample size of SG (top row). SP-S and SP-L are two variants of the SP experiments (Sec. 4.4.1). For Localization performance of SP-S (b), even a change of  $\pm 1$ px in patch size results in a std of  $\sim 10\%$  for GoogLeNet (dark red) and ResNet (dark green). Compared to regular models, robust models (here, GoogLeNet- and ResNet-R) cause the attribution maps to be more consistent pixel-wise under hyperparameter changes—*i.e.* higher SSIM scores (a)—and also more consistent in the three accuracy metrics—*i.e.* lower standard deviations (b–d). See Table S1 for the exact numbers.

small range (hence, denoted by SP-S). We observed that SP explanations are not 100% consistent even when the patch dimension changes within only  $\pm 1$ px (Fig. 5a; SSIM scores for SP-S are < 1.0).

### 4.4.2 LIME is sensitive to random seeds and sample sizes

LIME [42] is a black-box explanation method. Instead of masking out a single square patch (as in SP), which can yield the "square artifact" (Fig. 6; zoomed-out), LIME masks out a finite set of random *superpixels*.

Our experiments show that LIME is highly sensitive to its two common hyperparameters. First, LIME attribution maps interestingly often change as the random seed (which controls the random sampling of superpixel masks) changes! Second, LIME is also sensitive to the changes in the number of perturbation samples. See Sec. S4.2 for more details. Aligned with the results with SP (Sec. 4.4.1), here, we did not find robust classifiers to yield more stable LIME heatmaps than regular classifiers consistently under all three similarity metrics. An explanation is that GoogLeNet-R and ResNet-R are robust to pixel-wise changes but not patchwise or superpixel-wise changes (as done by SP and LIME) in the input image. See Fig. S17 for a list of the most sensitive cases across all the LIME sensitivity experiments.

# 4.5. How do the accuracy scores of an explanation vary when a hyperparameter changes?

In Sec. 4.3 and Sec. 4.4, we have shown that many attribution methods are highly sensitive to changes in their common hyperparameters. For example, under SSIM, the average explanation consistency is often far from the maxi-

mum (Fig. 5a; GoogLeNet and ResNet scores are far below 1.0). However, there is still a need to quantify how the variation in pixel-wise heatmaps translates into the variation in accuracy scores. That is, two heatmaps that are different pixel-wise may have the same accuracy score. Therefore, it is important for users to understand: How much does the correctness of an explanation varies, on average, when a given hyperparameter changes? To answer that, here, we quantify the variance of three explanation accuracy scores (i.e. the Localization error, Insertion, and Deletion scores described in Sec. 3) upon varying the most common hyperparameters of the considered attribution methods: (1) the sample size in SG (Sec. 4.3.1); (2) the patch size in SP (Sec. 4.4.1; both sweeping across a small range *i.e.* SP-S and a large range i.e. SP-L); (3) the random seed in LIME (Sec. 4.4.2); and (4) the random seed in MP (Sec. 4.3.2).

**Experiment** For each hyperparameter, we swept across N values to generate the corresponding N explanations for each input image. Using an accuracy metric, we evaluated each set of N attribution maps per image to produce N accuracy scores. From the N scores, we then obtained a mean and a std, for each image. From the per-image means and standard deviations, we then calculated the global mean and average std across the dataset (Fig. 5). We repeated the same procedure for each accuracy metric and each classifier.

**Results** First, we found that changing the tested hyperparameters (*i.e.* which are the most common) does not only change the explanations (Fig. 5a; average SSIM scores are under 1.0) but also their three downstream accuracy scores (Fig. 5b–d; the average std bars are above 0). However, explanation accuracy varies differently between the metrics. That is, compared to the mean scores (Fig. 5; circles), the score variation (in std) are higher for object local-

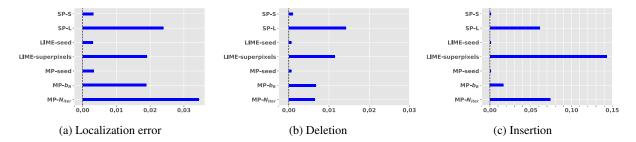


Figure 7: Comparisons of the variation in three accuracy scores of attribution methods when changing different hyperparameters. Here, the horizontal bars show standard deviations (std) for the Localization error (a), Deletion (b) and Insertion (c) scores obtained by marginalizing over all images and classifiers (see Sec. 4.6). Changing the number of superpixels (in LIME) and the number of iterations (in MP) causes the largest sensitivity to the accuracy of the two methods, respectively.

ization (Fig. 5b) and lower for deletion and insertion metrics (Fig. 5c–d). Notably, the localization scores are highly sensitive—the average stds of regular and robust models are  $0.51 \times$  and  $0.31 \times$  of their respective mean accuracy scores.

Second, varying the patch size of SP by only 1px caused a small variation in the explanation (Fig. 5a; mean SSIM scores are  $\approx 1$  for SP-S) but a large variation in object localization performance (Fig. 5b; for SP-S, the stds are  $\sim 10\%$  of the mean statistics).

Third, across all four tested hyperparameters and three accuracy metrics, the correctness of explanations for robust models is on average  $2.4\times$  less variable than that for regular models. In sum, we found that explanations for robust classifiers are not only more consistent but also more similarly accurate upon varying the common hyperparameters (compared to the darker bars *i.e.* regular classifiers, lighter bars are longer in Fig. 5a and shorter in Fig. 5b–d).

# **4.6.** Which hyperparameter when changed causes a higher variation in explanation accuracy?

In Sec. 4.5, we show that the accuracy of an *individual* explanation, on average, can vary substantially as we change a hyperparameter. Here, we ask a different important question: Which hyperparameter when varied causes a higher variation in explanation accuracy? That is, we attempt to compare hyperparameters by computing the marginal effects of changing each hyperparameter to the variation in accuracy scores (when marginalizing over all images and four classifiers).

**Experiment** As a common practice in the literature, for each classifier, we computed an accuracy score for each generated explanation and took a mean accuracy score over the entire dataset. Repeating the computation for N values of each hyperparameter (e.g. N random seeds of LIME), we obtained N mean accuracy scores from which we computed an std s. For each hyperparameter, we averaged over  $\{s\}_4$  i.e. four such stds, each computed for a classifier, yielding one global std, which is used for comparing hyperparameters. Here, we compare the global stds for different hyper-

parameters within and between methods (see Fig. 7): (1) the patch size in SP (Sec. 4.4.1; SP-S and SP-L); (2) the random seed and the number of superpixels in LIME (Sec. 4.4.2); (3) the random seed, the blur radius, and the number of iterations of MP (Sec. 4.3.2).

**SP results** Within SP, we found that varying the patch size across a larger range yields a higher variation in accuracy scores (Fig. 7a; SP-L vs. SP-S).

**LIME results** Our results enable quantitatively comparing the effects of changing different hyperparameters. In LIME, varying the number of superpixels causes far more sensitivity in the correctness of explanations compared to varying the LIME random seed (Fig. 7; row 3 vs. 4). Specifically, the std of Insertion scores when changing the number of superpixels was  $130.5 \times$  higher as compared to the std when changing the random seed (Fig. 7c).

**MP results** In MP, changing the number of optimization iterations causes the largest sensitivity in explanation accuracy (among the three MP hyperparameters). Precisely, the std of Insertion scores, when changing the blur radius  $b_R$  and the number of iterations  $N_{iter}$ , was  $16.6 \times$  and  $74 \times$  higher than that when changing the random seed (Fig. 7c; bottom three rows).

**Across methods** Changing the random seed in LIME vs. in MP (two different methods) interestingly causes a similar variation in all three accuracy metrics (Fig. 7; row 3 vs. 5).

#### 5. Discussion and Conclusion

We present the first thorough study on the sensitivity of attribution methods to changes in their input hyperparameters. Our findings show that the attribution maps for many gradient-based and perturbation-based interpretability methods can change radically upon changing a hyperparameter, causing their accuracy scores to vary as well. We propose to evaluate the sensitivity to hyperparameters as an evaluation metric for attribution methods. It is important to carefully evaluate the pros and cons of interpretability methods with no hyperparameters and those that have.

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