

Online navigation with neuromodulation-based Hebbian plasticity

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1 Introduction

The ability to make decisions for long-term reward maximization is a fundamental aspect of cognition. The brain has evolved a complex web of interconnected regions that work together to express this behaviour under the constraints of biology. The Pre-Frontal Cortex (PFC) is considered a fundamental high-level region for the attention and cognitive control, in particular the medial PFC [1, 2]. Further, the orbitofrontal cortex (OFC) is thought to be involved in motivation and representation of the expected value of the actions, either positive or negative [3, 4, 5], and action selection in uncertain environments [6]. A relevant element for online executive functions is working memory, which is usually defined as the capacity to hold and manipulate information over short periods of time [7]; thus functionality has been associated with the dorsolateral PFC [8, 9, 10, 11]. However, it has also been suggested that the PFC is instead exercising a more top-down control over more sensory regions [12]. This cortical projectons have been proposed to target also the basal ganglia, which are thought to rely on first-order reward statistics, while the OFC is able to capture more complex contextual dynamics [13].

An important ingredient for processing and selecting adaptive behaviours is the neuromodulation. In particular, the catecholamine dopamine has been shown to play a crucial role in learning and memory in numerous brain regions, and across animal species [14, 15]. Important sources are the ventral tegmental area (VTA), locus coeruleus (LC), and the substantia nigra pars compacta (SNpc), which project consistently to the frontal lobe, the basal ganglia (BG), and the hippocampus [16, 17, 18]. Dopamine has been long associated with reward prediction errors, for which dopaminergic neurons have been recorded tuning their phasic firing rate to signal either unexpectedly positive, neutral, or negative feedback [19, 20]. Further, several studies have highlighted the importance of dopamine in modulating processes in the PFC, such as multiple types of synaptic plasticity, balance of neuronal excitation and inhibition, updating and supporting representation in working memory, learning in operant conditioning protocols, and helping value encoding [2, 21, 22, 8, 23].

Regarding decision-making tasks, simple and well-studied ecological settings are foraging problems, *e.g.* food search, where an agent is set to choose between different options to maximize the expected reward. Depending on context, animals have been shown to exhibit different strategies. In this regard, *matching behaviour* is a well-known phenomenon in which the animal's choice behaviour is proportional to the reward probability of the options. This behaviour is thought to be the result of a trade-off between exploration and exploitation [24, 25], where the animal must balance the need to explore new alternatives with the need to exploit the best option found so far. Other options are *input matching*, where social cues are considered, and *probability matching*, where the

animal’s choice behaviour is proportional to the reward probability of the options [26, 27]. A popular formalization of such tasks in optimal decision theory is the “multi-armed bandit problem” (MABP) [28], where an agent is faced with a set of K possible actions, each one associated with an unknown reward probability distribution. The agent has to learn to choose the action that maximizes the expected reward by repeatedly selecting actions and observing the rewards obtained. This problem has been extensively studied in the context of reinforcement learning, and it is considered a fundamental building block for more complex tasks [24].

Extensive research has been conducted on the topic, and several algorithms have been proposed, such as Thompson sampling, ϵ -greedy, UCB1, VDBE, alongside convergence proofs for specific settings [29, 30, 31, 32, 33].

In this work, we focus on a stochastic bandit problem, a more challenging variant of the original task endowed with *concept drift*, where the reward distribution changes over time [34, 35, 36]. We propose a biologically plausible model using spiking neural networks (SNN), obtaining good performance over an arbitrary number of bandit trials. Its architecture is composed of two parts. The first is a working memory component, whose scope is to maintain an active representation of the current stimulus, and it is inspired by the functionality of the dorsolateral PFC. Several previous studies have proposed bio-realistic models of WM [37], including focus on synaptic facilitation [38], random connectivity [39], excitation-inhibition balance (E-I) [40, 41], and echo state networks [42, 43]. Here in particular, we used a three-population SNN exploiting the E-I balance to maintain the stimulus trace, similarly to [44]. The second component is a one-layer hybrid network, and it is where a pre-defined architectural policy assign a subjective value to the available options *i.e.* the bandits. This component is modeled after the interaction between the OFC and the basal ganglia, in particular the striatum [26, 13], given the role of the frontal lobe in reversal learning [45], various network models of the PFC and BG for decision-making have been proposed, centered on the role of dopamine in the basal ganglia [46], spike-time dependent plasticity (STPD) [47], cortico-striatal interaction [48], and task switching processes [49, 50].

Regarding our model, the novelty relies in two main elements. The first is the decision making process itself, implemented as a dynamic population interaction between the neural traces active in working memory and the values represented in the hybrid network, similarly to bump attractor networks for perceptual computations [51, 52]. The second is learning, which is applied to the hybrid network, and it is based on a neuromodulated Hebbian-like synaptic plasticity rule with special weight-dependent kernels for long-term potentiation (LTP) and long-term depression (LTD). This choice is motivated by the asymmetry of the dopamine signal in supporting LTP or LTD [20, 19, 18, 53].

2 Methods

2.1 Related work

There exists various flavours of this problem, with the simplest having a stationary reward distribution, while the more challenging ones have *concept drift*, where the reward distribution changes over time. Over the years, several algorithms have been proposed, alongside with their theoretical guarantees. In this regard, Thompson sampling is a popular algorithm that has been shown to achieve near-optimal regret bounds in the stochastic setting [54], which a Bayesian approach the idea of maintaining a posterior distribution over the reward probabilities of the actions, and selecting actions according to the posterior distribution. Another popular algorithm is the Upper Confidence Bound (UCB) algorithm, which has been shown to achieve near-optimal regret bounds in the adversarial setting [55]. The algorithm is based on the idea of maintaining an upper confidence bound on the reward probabilities of the actions, and selecting actions according to the upper confidence bound.

Despite the success of these algorithms in solving the k-armed bandit problem, they lack biological plausibility. In contrast, the brain has evolved a complex network of interconnected regions that work together to solve this task. In particular, the dopamine-acetylcholine system has been shown to play a crucial role in learning and decision making [56].

2.2 Binomial K-armed bandit problem

The standard formulation of the task is structured as a set of $\{1 \dots K\}$ levers (or arms), with an associated reward distribution $\mathbf{p} = \{p_1, \dots p_K\}$. At each iteration, the agent pulls a lever and collect a possible reward drawn as a Bernoulli variable $R \sim \mathcal{B}(\{0, 1\}, p_k)$. The agent’s objective is maximizing the total reward $\sum_t^T R_t$, after a certain number T of trials. Importantly, the agent is unaware of the true reward probability distribution, and thus has to make its decisions following a certain policy, usually denoted as π . In the reinforcement learning literature, the policy is often defined as a distribution over the actions, here the levers K , given the current state, which in this case can be the history of past actions and rewards up to time $t \leq T$. Given the inherent stochasticity of the feedbacks from the environment, the definition of the policy is affected by the so-called exploration-exploitation trade-off, which here is phrased as the contrast between the option of the lever with the known highest expected reward versus the option to explore other levers, so to gather more information. A common approach is the ϵ -greedy policy, where the choice to explore is selected with a probability ϵ . Moreover, it is often preferable to have a more explorative behaviour early during the training, with the intent to have a good sample size for the empirical reward distribution, which can be later exploited for maximizing reward.

Another important concept in multi-armed bandit problems is the *regret*. Intu-

itively, it is defined as the deviation of the total reward obtained by the agent from the optimal reward that could have been obtained by always choosing the lever with the highest expected reward. Formally, the regret is defined as:

$$\rho = R^* - \sum_t^T R_t \quad (1)$$

where R^* is the reward obtained by always choosing the lever with the highest expected reward $R^* = T \max_k \{p_k\}$, and R_t is the empirical reward obtained up to time t by following policy π as $R_t = \sum_{t=1}^T \pi_\theta(t)$. The regret is a measure of the performance of the agent, and it is often used to compare different algorithms. The goal of the agent is to minimize the regret, and thus maximize the total reward.

2.2.1 Non-stationary environment

The setting we consider in this work is a non-stationary environment with Binomial rewards. In particular, the agent is evaluated over a number T_{trials} of *trials*, each composed by an arbitrary number T of *rounds*; each *trial* is characterized by a different reward distribution $\mathbf{p} \sim \mathcal{U}(0, 1)^K$ (although in practice the bounds have been set to $(0.1, 0.8)$ such that the distributions are less trivial).

Our goal in this work is to investigate the performance of the agent in a non-stationary environment with Binomial reward distributions, meaning that its underlying distribution changes over time. We choose this setting as it resembles an ecological scenario in which an animal has to forage in a patchy environment, where the reward of a given patch can change over time.

In particular, we consider two different types of non-stationarity.

zero-steps distribution shift: the reward distribution changes immediately at the end of a trial i to a new one $i + 1$ as $\pi_i \rightarrow \pi_{i+1}$.

epsilon-steps distribution shift: the reward distribution π_t at time t changes gradually over time, and it tends to a target distribution $\hat{\pi}_i$ as $\tau_\pi \hat{\pi}_t = \hat{\pi}_i - \pi_t$. Once the distance is below a threshold ϵ as $|\hat{\pi}_i - \pi_t| < \epsilon$, the target distribution is changed to a new one $\hat{\pi}_i \rightarrow \hat{\pi}_{i+1}$.

We will investigate the performance of the model in both settings, and compare it to the common benchmark algorithms.

2.3 Model description

The model is constructed as a rate network of two populations of neurons M and P , the former representing the memory trace of the K available options (*i.e.* the bandits), and the latter representing the value of the options under the current policy. More formally, the model is described by a set of coupled ordinary differential equations (ODEs) that capture the decision-making process in two distinct neural spaces. The first equation tracks the evolution of the neural activity \mathbf{u} of M , while the second tracks the activity \mathbf{v} of the P . The time constants τ are the same for both equations and are set to $\tau = 10$ ms.

$$\begin{aligned}\tau \dot{\mathbf{u}} &= -\mathbf{u} + \mathbf{v} + \mathbf{I}_{\text{ext}} \\ \tau \dot{\mathbf{v}} &= -\mathbf{v} + \mathbf{z} \odot \mathbf{u}\end{aligned}\tag{2}$$

The external input \mathbf{I}_{ext} is a constant input that is used to set the initial conditions of the neural activity \mathbf{u} . The term \mathbf{z} is a vector that weights the contribution of the active options \mathbf{u} to the value representation \mathbf{v} , and functionally it is the core of the policy adopted by the model. In practice, \mathbf{z} defined as a function of the synaptic weights \mathbf{W}^{MP} from M to P as $\mathbf{z} = \Phi_v(\mathbf{W}^{MP})$. Importantly, the connections are not fully connected, but rather are simply one-to-one mapping between the corresponding neurons in each population, such that the weight matrix \mathbf{W}^{MP} is simply a matrix $K \times 1$, namely a vector. The function Φ_v is chosen to be a sum of a generalized sigmoid and a Gaussian, whose contributions are weighted by a parameter r :

$$\Phi_v(x) = r\gamma_1 \frac{1}{1 + e^{-\beta(x-\alpha)}} + (1-r)\gamma_2 \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

The motivation behind this choice is to express a function that possesses a bounded region (depending on μ, σ) at a high/low peak (depending on the value of γ_2), and a continuous transition to a constant value (depending on the steepness of the sigmoid β , shift α , and intensity γ_1).

2.3.1 Option selection

The decision-making process within a single round is structured in two distinct phases. Initially, the model receives a constant external input targeting all neurons in the memory population M equally. During this phase, I_{ext} works as an equilibrium value while the reciprocal interactions with population P push \mathbf{u} to different values, depending on the current policy encoded in \mathbf{z} . However, in the early rounds the weights \mathbf{W}^{MP} are zero, and thus the contribution from P is null. After a fixed amount of time ~ 5 s, the second phase begins. Here, the external input is removed and the model is left to evolve autonomously, and since there are no recurrent connections in neither population the dynamics is entirely driven by their coupling.

A selection \hat{k} is sampled after another fixed amount of time ~ 5 s, and it is defined according to the following rule:

$$\hat{k} = \begin{cases} \operatorname{argmax}_k \{\mathbf{v}\} & \text{if } \operatorname{argmax}_k \{\mathbf{v}\} = \operatorname{argmax}_k \{\mathbf{u}\} \\ \operatorname{random}(K) & \text{otherwise} \end{cases}$$

The selection rule is simple: if the value representation \mathbf{v} is in agreement with the memory trace \mathbf{u} , then the option with the highest value is selected. Otherwise, a random option is chosen. This rule is a way to express the exploration-exploitation trade-off, and it is dependent on the current policy \mathbf{z} .

2.4 Learning

Given a selected option, the environment (bandit) samples and returns a reward $R \in [0, 1]$. Then, the connections \mathbf{W}^{MP} for the neuron corresponding to the option k are updated according to the following plasticity rule:

$$\Delta \mathbf{W}_k^{MP} = \tilde{\eta}_k \left(R \cdot W^+ - \mathbf{W}_k^{MP} \right) \quad (3)$$

Where W^+ is a constant value that sets the upper bound for the synaptic weights, and it is set to $W^+ = 5$, while $\tilde{\eta}_k$ is the learning rate for the option k determined by a function of the current weights \mathbf{W}_k^{MP} and its shape is the same as Φ_v , but with different parameters.

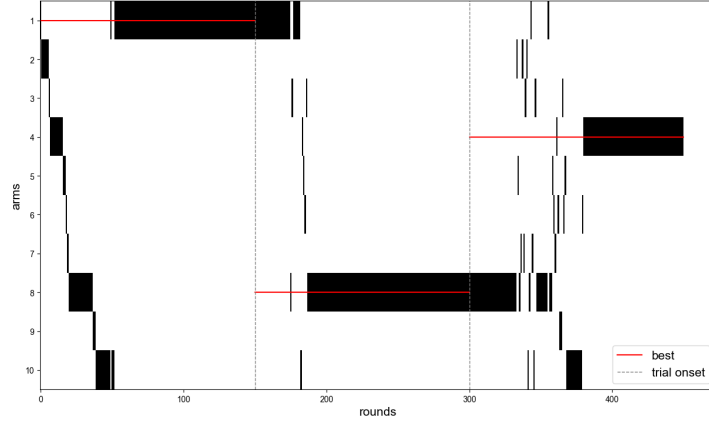


Figure 1: SELECTION EVOLUTION OVER ROUNDS

3 Results

The model has been applied to two non-stationary environments.

3.1 Zero-steps distribution shift

In this first setting, as the end of a trial i the arm distribution changes immediately to a new one $i + 1$ as $\pi_i \rightarrow \pi_{i+1}$.

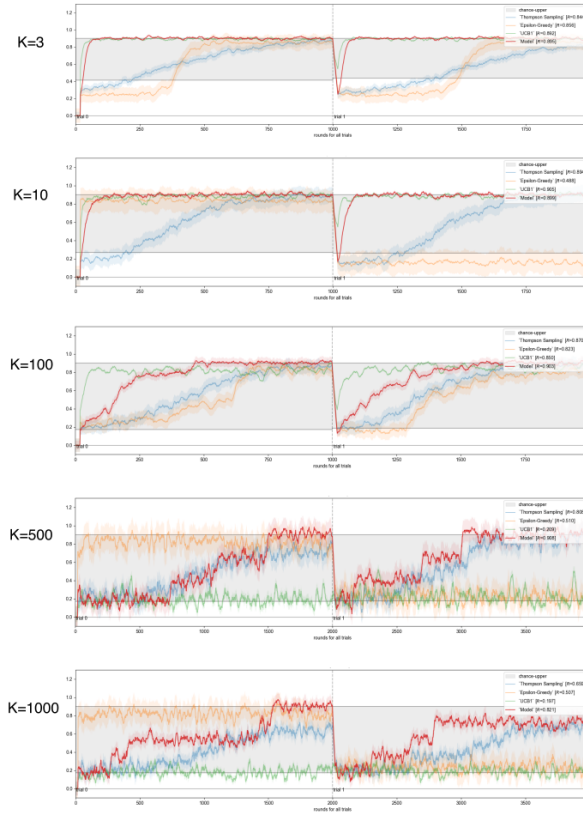


Figure 2: PERFORMANCE WITH VARIABLE NUMBER OF ARMS - each plot is a simulation with K numbers of arms, the x-axis are rounds, the central vertical line signals the start of the second trial, the y-axis is the reward fraction. The shaded area is the reasonable reward range, where the lower bound is the chance level and the upper bound the best reward (following the optimal policy). The model performance is in red, while Upper-Confidence Bound green, Thompson Sampling blue, and Epsilon-Greedy orange.

From figure 2 above, it is clear the ability of the model (in red) to reach almost always the optimal reward policy (*i.e.* the greediest) for all trials, even after the

distribution shift. In comparison, the other algorithms start to struggle when the arms are more 100 and the distribution changes.

3.2 Epsilon-steps distribution shift

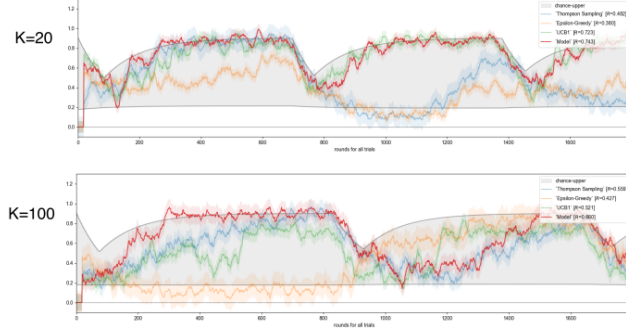


Figure 3: PERFORMANCE WITH VARIABLE NUMBER OF ARMS - *each plot is a simulation with K -numbers of arms, and the rest is also the same as before in 2.* Each trial has 3 rounds, meaning that every three steps the distribution change.

In the setting with a smooth distribution shift the difficulty of the problem is increased, especially since short-sighted greedy behaviours are easily sub-optimal. The model (always in red) is capable of reaching and maintaining a successful profile, even with many arms available.

4 Discussion

In the context of human behaviour, it has been observed that the adopted policies vary considerably [57]. However, the subjects seems able to integrate environmental uncertainty and trial generalization in their strategy, and Bayesian algorithms are generally a good fit for the observed behaviour [58, 59].

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5 Appendix