QUESTION SET FOR TUTORIAL -6 : QUEUING THEORY

Model 1

1. At what average rate must a clerk at a super market work in order to ensure a probability of 0.90 so that the customer will not have to spend more than 12 min.? It is assumed that there is only one counter at which customers arrive in a Poisson fashion at an average rate of 15 per hour The length of service by the clerk has an exponential distribution

2. Consider a self-service store with one cashier. Assume Poisson arrivals and exponential service times .Suppose that an average nine customers arrive every 5 min. and that the cashier can serve 10 in 5 min. Find:

(a) average number of customers in the system

(b) probability of having more than 10 customers in the system, and

(c) probability that a customer has to queue for more than 2 min.

If the service can be speed up to 12 in 5 min. by using a different cash register, what will be the effect of this on the quantities (a), (b) and (c)

3. Customer arrive at a box office window, being manned by a single individual, according to a Poisson input process with a mean rate of 30 per hour The time required to serve a customer has an exponential distribution with a mean of 90 seconds Find the average time spent by a customer. Also determine the average number of customers in the system and the average queue length

4. The mean rate of arrival of planes at an airport during the peak period is 20 per hour, and the actual number of arrivals in any hour follows a Poisson distribution The airport can land 60 planes per hour on an average, in good weather, and 30 planes per hour in bad weather The actual number landing in any hour follows Poisson distribution with these respective averages. When there is congestion, the planes are forced to fly over the field in the stack awaiting the landing of other planes that arrived earlier

(a) How many planes would be flying over the field in the stack, on an average, in good weather conditions and in bad weather conditions?

(b) How long would a plane be in the stack and in the process of landing in good and in bad weather?

5. A repair shop, attended by a single mechanic, has an average of four customers an hour who bring small appliances for repair The mechanics inspects them for defects and for this he takes six min. on an average. Arrivals are Poisson and service rate has an exponential distribution. You are required to:

(a) Find the proportion of time during which there is no customer in the shop

(b) Find the probability of finding at least one customer in the shop

(c) Calculate the average number of customers in the system

(d) Find the average time spent by a customer in the shop including service

6. In a bank, cheques are cashed at a single 'teller' counter. Customers arrive at the counter in a Poisson manner at an average rate of 30 customers per hour. The teller takes, on an average, a minute and a half to cash a cheque The service time has been shown to be exponentially distributed

(a) Calculate the % of time the teller is busy

(b) Calculate the average time a customer is expected to spend in the system.

7. In a tool crib manned by a single assistant, operators arrive at the tool crib at the rate of 10 per hour. Each operator needs 3 min, on an average, to be served. Find out the loss of production due to the time lost in waiting for an operator in a shift of 8 hours, if the rate of production is 100 units per shift

8. Trucks arrival at a factory for collecting finished goods that are supposed to be transported to distant markets. As and when they come they are required to join a waiting line and are served on first come, first served basis Trucks arrive at the rate of 10 per hour whereas the loading rate is 15 per hour It is also given that arrivals are Poisson and loading is exponentially distributed. Transporters have complained that their trucks have to wait for nearly 12 mins at the plant. Examine whether the complaint is justified Also determine the probability that the loaders are idle in the above problem.

9. On an average 96 patient per 24 hour day require the service of an emergency clinic. Also on an average, a patient requires 10 min. of active attention. Assume that the facility can handle only one emergency at a time. Suppose that it costs the clinic Rs 100 per patient treated to obtain an average servicing time of 10 min. and that each min. of decreases in this average time would cost Rs 10 per patient treated, how much would have to be budgeted by the clinic to decreases the average size of the queue from 4/3 patients to 1/2 patient?

10. In a service department manned by one server, on an average one customer arrives every 10 min. It has been found out that each customer requires 6 min. to be served Find out

(a) Average queue length

(b) Average time spent in the system

(c) Probability that there would be two customers in the queue

11. A fertilizer company distributes its product by trucks that are loaded at its only loading station. Both, company trucks and contractor's trucks are used for this purpose. It was found that on an average, every 5 min. one truck arrived and the average loading time was 3 min. Out of these trucks 40% belong to the contractors. Making suitable assumptions, determine

(a) The probability that a truck has to wait

(b) The waiting time of a truck that waits

(c) The expected waiting time of contractor's trucks per day

12. Customers arrive at a one window drive in bank according to a Poisson distribution with mean of 10 per hour. Service time per customer is exponential with a mean of 5 min. The space in front of the window, including that for the serviced car, can accommodate a maximum of 3 cars The other cars can wait outside this space

(a) What is the probability that an arriving customer can drive directly to the space in front of the window?

(b) What is the probability that an arriving customer will have to wait outside the indicated space?

(c) How long is an arriving customer expected to wait before starting service?

13. A maintenance service facility has Poisson distribution arrival rates, negative exponential service times, and operates on a first come, first served queue discipline. Breakdowns occur on an average of three per day. The maintenance crew can service, on an average, six machines per day, Find the:

(a) Utilization factor of the service facility

(b) Mean waiting time in the system

(c) Mean number machines in the system

(d) Mean waiting time of machines in the queue

(e) Probability of finding 2 machines in the system

14. Telephone users arrive at a booth following a Poisson distribution with an average time of 5 min. between one arrival and the next the time taken for a telephone call is on an average 3 min. and it follows an exponential distribution. What is the probability that the booth is busy? How many more booths should be established to reduce the total time spent to less than or equal to half of the present total spent time?

15. In a factory, the machine breakdown on an average rate of 10 machines per hours. The idle time cost of a machine is estimated to be 20 per hour. The factory works 8 hours a day. The factory manager is considering 2 mechanics for repairing the machines. The first mechanic A takes about 5 min. on an average, to repair a machine and demands wages of Rs 10 per hour The second mechanic B takes about 4 min. in repairing a machine and demands wages at the rate of Rs 15 per hour. Assuming that the rate of machine breakdown is Poisson Distributed and the repair rate is exponentially distributed, which of the two mechanics should be engaged?

16. A scooter mechanic finds that the time required to repair a scooter is exponentially distributed with mean 30 minutes. The arrival rate of scooters for repair is approximately Poisson distribution with the average of 12 scooters per 8 hours day. Find the expected time for which the mechanic is idle in a day. How many scooters on an average are therein the system? Also find the probability that a customer has to wait.

Model 2

16. If in a period of 2 hours, in a day (08:00 to 10:00 am), trains arrive at the yard every 20 min. but the service time continues to remain 36 min. then calculate, for this period

(a) The probability that the yard is empty, and

(b) The average number of trains in the system, on the assumption that the line capacity of the yard is only limited to 4 trains.

17. At a railway station, only one train is handled at a time the railway yard is sufficient only for two trains to wait while the other is given a signal to leave the station. Trains arrive at the station at an average rate of 6 per hour and the railway station can handle them on an average 12 per hour. Assuming Poisson arrivals and exponential service distribution, find the steady state probabilities for the various number of trains in the system. Also find the average waiting time of a new train arriving at the yard.

18. Assume that good trains are coming in a yard at the rate of 30 trains per day and suppose that the Inter arrival times follow an exponential distribution. The service time for each train is assumed to be exponential with an average of 36 min. If the yard can admit 9 trains at a time (there being 10 lies, one of which is reserved for shunting purpose), calculate the probability that the yard is empty and find the average queue length.

HINTS AND ANSWERS

1. $\lambda = 15/60 = 1/4\,;\ \mu = ?$

$P(W_s \geq 12) = 0.10$ (given)

2. **Case 1:** $\lambda = 9/5\,, \mu = 10/5\,;$

(a) $L_s = 9$ customers **(b)** $P(n > 10) = (0.9)^{11}$

(c) $P\left(W_q > 2\right)$

Case 2: $\lambda = 9/5\,, \mu = 12/5\,;$ calculate (a), (b), (c) again.

3. $\lambda = 30/60 = 0.5\ per\ min;\ \mu = 60/90 = 0.67\ per\ min$

(a) $W_s = 5.88$ min. **(b)** $L_s = 3$

(c) $L_q = 2$

4. $\lambda = 20, \mu = 60$ (in good weather) and 30 (in bad weather)

(a) $L_q = \begin{cases} 1/6\,, \text{(in good weather)} \\ 4/3\,, \text{(in bad weather)} \end{cases}$ **(b)** $W_s = \begin{cases} 1/40\ \text{hour}, \text{(in good weather)} \\ 1/10, \qquad \text{(in bad weather)} \end{cases}$

5. $\lambda = 4\ per\ \text{hour};\ \mu = 60/6 = 10\ per\ \text{hour};\ \rho = \frac{\lambda}{\mu} = 0.4$

(a) $P_0 = 1 - \rho = 0.6$ **(b)** $P(n \geq 1) = (0.4)$

(c) $L_s = 2/3$ **(d)** $W_s = 1/6\ \text{hour or } 10\ \text{min.}$

6. $\lambda = 30\ per\ \text{hour};\ \mu = \frac{60}{(3/2)} = 40\ per\ \text{hour}$

(a) Busy period $= 1 - P_0 = \lambda/\mu = 3/4$, i.e., teller is busy for 75% of its time

(b) $W_s = 1/10\ \text{hour or 6 min}$

7. $\lambda = 10/\ \text{hour};\ \mu = 20\ /\text{hour}$

(a) $W_q = 1/20$ hour; average waiting time per shift is $8/20 = 2/3$ hour;

Loss of production due to waiting $= \left(\frac{2}{5}\right) \times \left(\frac{100}{8}\right) = 5$ units

8. $\lambda = 10/\ \text{hour};\ \mu = 15/\ \text{hour}$

(a) $W_q = 10/75\ \text{hour or 8 min}, \quad W_s = 12\ min$

(b) Idle time $P_0 = 1 - \rho = 5/15$ or 33.33%

9. $\lambda = 4/hour$; $\mu = 6/$ hour

(a) $L_q = 4/3$ Patients

But if L_q is changed from $4/3$ to $1/2$, then new value of μ will be

$\frac{1}{2} = \frac{\lambda^2}{\mu'(\mu'-\lambda)} =$ or $\mu' = 8$ patients/hour

Thus average Time of treatment required is $1/\mu' = 7.5$ min.

A decrease in the average time of treatment required is $(10 - 7.5) = 2.5$ min.

Revised budget per patient $=$ Rs $(100 + 2.5 \times 10) =$ Rs 125

10. $\lambda = 60/10 = 6/$ hour; $\mu = 60/6 = 10$ /hour

(a) $L_q = 0.9$ Customers **(b)** $W_s = 1/8$ hour or 15 min

(c) $P_2 = (\lambda/\mu)^2(1 - \lambda/\mu) = 0.144$ or 1.44%

11. $\lambda = 60/5 = 12/$ hour; $\mu = 60/3 = 20/$ hour

(a) $1 - P_0 = \lambda/\mu = 0.6$ **(b)** $W_q = 3/40$ hour or 4.5 min

(c) Total waiting time $=$ (number of trucks per day) $\times$ (per cent contractors trucks) $\times$ Expected waiting time for a truck

$$= (12 \times 24) \times \frac{40}{100} \times \frac{\lambda}{\mu(\mu-\lambda)} = 8.64 \text{ hours per day}$$

12. $\lambda = 10$ /hour; $\mu = 60/5 = 12$ /hour

(a) $P(n < 3) = 1 - P(n \geq 3) = 0.42$

(b) $P(n > 3) = ?$ **(c)** $W_q = 5/12$ hour or 25 min

13. $\lambda = 3$ /day; $\mu = 6$ /day

(a) $\rho = \lambda/\mu = 3/6$ or 50% **(b)** $W_s = 1/3$ day **(c)** $L_s = 1$ machine

(d) $W_q = 1/6$ day **(e)** $P_2 = (\lambda/\mu)^2(1 - \lambda/\mu) = 0.125$

14. $\lambda = 12/$ hours; $\mu = 20$ /hour

(a) Busy period $1 - P_0 = \lambda/\mu = 0.60$ **(b)** $W_s = 1/8$ hour

(c) $W_s' = \frac{1}{\mu'-\lambda}$ or $\frac{1}{\mu'-12} = \frac{1}{16}$, i.e., $\mu' = 28/$ hour

Thus, number of booths required to achieve new service rate is $= 28/20 = 1.41$ booths

15. Mechanic A: $\lambda = 10$ /hours; $\mu = 12/$ hour

Total cost $=$ Total wages $+$ cost of non productive time

$= (\text{hourly rate} \times \text{no of hours}) + L_s(\text{cost of idle machine hour}) \times (\text{number of hours})$

$= 10 \times 8 + \frac{\lambda}{\mu-\lambda} \times 20 \times 8$

$= 640 + \frac{10}{12-10} \times 160 =$ Rs 880

Similarly calculate total cost for mechanic B and decide which should be employed.

16. $\lambda = \frac{12}{8}$ per hour; $\mu = 2/$ hour

(a) L_s (b) idle time per hour = $1 - \rho$ (c) $P_w = \rho$

17. $\lambda = 1/20$; $\mu = 1/36$; $\rho = 36/20 = 1.8(> 1)$ and $N = 4$

(a) $P_0 = \frac{\rho-1}{\rho^{N+1}-1} = 0.04$ **(b)** $L_s = \sum_{n=0}^{4} nP_n = 2.9 = 3$

18. $\lambda = 6$; $\mu = 12$; $\rho = \lambda/\mu = 0.5$ and $N = 3$

(a) $P_0 = \frac{\rho-1}{\rho^{N+1}-1} = 0.53$ **(b)** $L_s = \sum_{n=0}^{3} nP_n = 0.74$

(c) $L_q = L_s - \frac{\lambda}{\mu} = 0.24$ **(d)** $W_q = L_q/\lambda = 0.04$

18. $\lambda = 30$ trains per day; $\mu = 40$ trains per day; $\rho = \frac{\lambda}{\mu} = 0.75, P_0 = 0.2649, L_q = 1.6533$