Assignment 1

ITR

Shashank Ghosh
21110196

Q.6

Soln:- Taking R_0^1 as $R_{z,\theta}$

Two vectors are orthogonal when they are $\perp$ to each other.

$$R_0^1 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Here, $\vec{c_1} = \begin{bmatrix} c \\ s \\ 0 \end{bmatrix}$, $\vec{c_2} = \begin{bmatrix} -s \\ c \\ 0 \end{bmatrix}$, $\vec{c_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

So, $\vec{c_1} \cdot \vec{c_2} = -\cos\theta\sin\theta + \cos\theta\sin\theta = 0$

$\vec{c_2} \cdot \vec{c_3} = 0$

$\vec{c_3} \cdot \vec{c_1} = 0$

$\therefore$ the columns are orthogonal.

Note:- The columns of Rotation Matrix

R describe the cordinate axes of the rotated cordinate system in terms of the original cordinate system. Each column of the rotation matrix corresponds to one of the new cordinate axes after rotation is applied. Hence, the columns are always orthogonal, since the axes of frame are $\perp$ to each other

Q.7

Soln:- Let $R_0^1 = R_{z,\theta}$

$$R_0^1 = R_{z,\theta}$$

$$= \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det[R_0^1] = \cos\theta\,[\cos\theta] - \sin\theta\,(-\sin\theta)$$

$$= \cos^2\theta + \sin^2\theta = \underline{1}$$

Similarly it is true for any axis & angle, not just z & θ

<u>Note:-</u> determinant of Rotation Matrix is one because it represents rotation & No scaling of the vector's Magnitude.