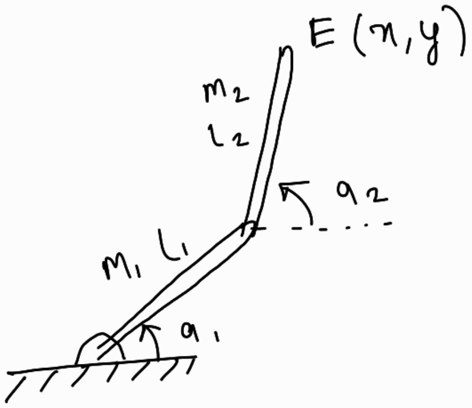


$$x = l_1 \cos q_1 + l_2 \cos q_2$$
$$y = l_1 \sin q_1 + l_2 \sin q_2$$

$$\Downarrow$$

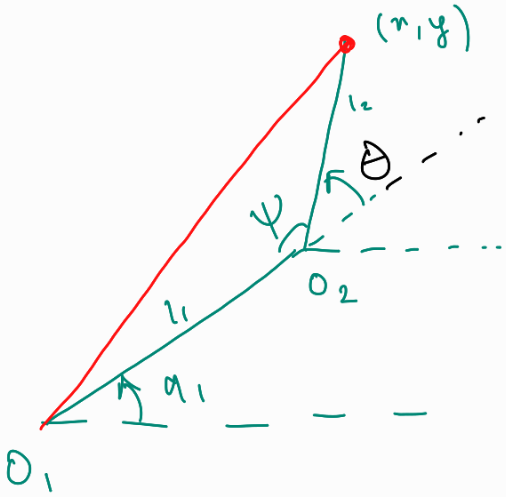
$$\left.\begin{aligned} x &= l_1 cq_1 + l_2 cq_2 \\ y &= l_1 sq_1 + l_2 sq_2 \end{aligned}\right\} (1)$$

$\Downarrow$ differentiating,

$$\left.\begin{aligned} \dot{x} &= -l_1 sq_1 \cdot \dot{q}_1 - l_2 sq_2 \cdot \dot{q}_2 \\ \dot{y} &= l_1 cq_1 \cdot \dot{q}_1 + l_2 cq_2 \cdot \dot{q}_2 \end{aligned}\right\} (2)$$

$\therefore$ End effector velocity $= \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \begin{bmatrix} -l_1 sq_1 & -l_2 sq_2 \\ l_1 cq_1 & l_2 cq_2 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$

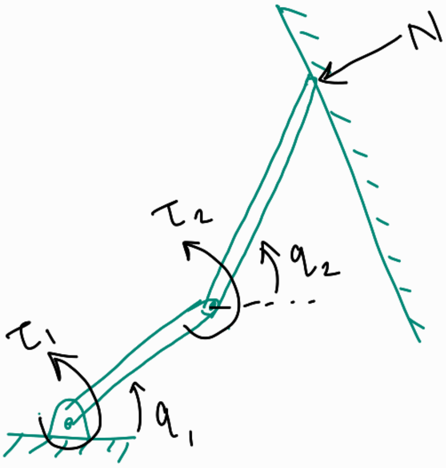
Taking inverse, we get the Joint trajectory.



$$\Rightarrow \quad x^2+y^2 = l_1^2+l_2^2+2l_1l_2\cos\theta$$

$$\left.\begin{aligned}
\theta &= \cos^{-1}\left(\frac{x^2+y^2-l_1^2-l_2^2}{2l_1l_2}\right) \\
q_1 &= \beta-\gamma \\
&= \tan^{-1}\left(\frac{y}{x}\right) - \tan^{-1}\left(\frac{l_2\sin\theta}{l_1+l_2\cos\theta}\right)
\end{aligned}\right\} \quad (3)$$

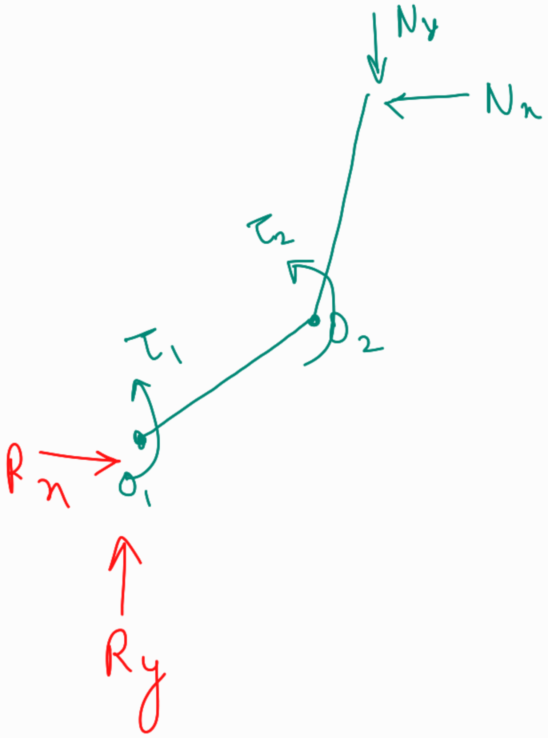
$$\underline{\underline{q_2 = q_1+\theta}}$$



Forces applied by manipulator

$$F_x = -N_x$$

$$F_y = -N_y$$

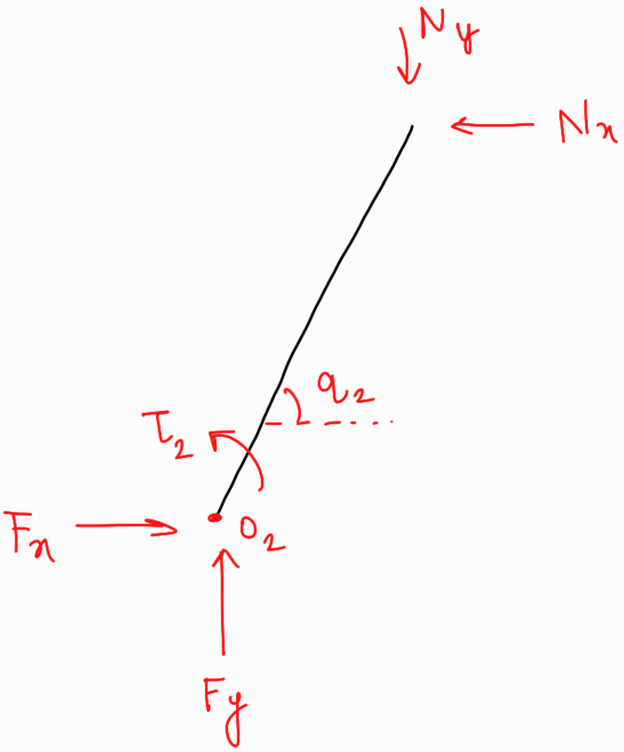


Neglect gravity

Drawing the FBD :-

(each link separately

Link-2

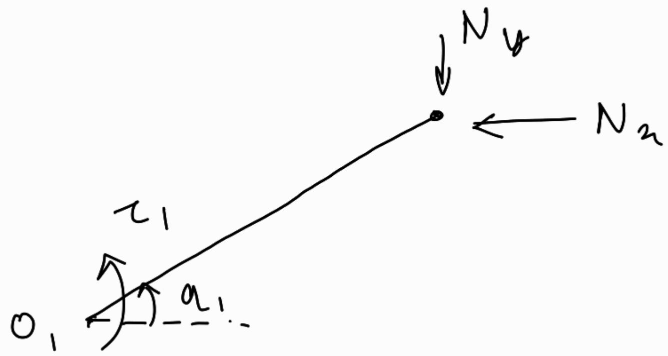


$$\Sigma M_{O_2} = 0$$

CCW +ve

$$+ N_y l_2 c q_2 - N_x l_2 s q_2 = \tau_2$$

Link-2



$$\Sigma M_{O_1} = 0$$

$$\Rightarrow \left.\begin{aligned} N_y l_1 c q_1 - N_x l_1 s q_1 &= \tau_1 \\ N_y l_2 c q_2 - N_x l_2 s q_2 &= \tau_2 \end{aligned}\right\} \quad (4)$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} = \begin{bmatrix} -l_1 s q_1 & l_1 c q_1 \\ -l_2 s q_2 & l_2 c q_2 \end{bmatrix} \begin{bmatrix} N_x \\ N_y \end{bmatrix}$$

Lagrange's Equations

$$\mathcal{L} = K - V$$

$\mathcal{L}$ → Lagrange operator

K → kin. energy

V → Potential energy

$$\boxed{\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = Q_i'} \quad (5)$$

From Lagrange's equations:— (we get individual Torque equations without FBD.

$$
\left.
\begin{aligned}
&\frac{1}{3} m_1 l_1^2 \ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2}{2} \ddot{q}_2 \cos(q_2 - q_1) \\
&\quad - \frac{m_2 l_1 l_2}{2} \dot{q}_2 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) + \frac{m_1 g l_1}{2} \cos q_1 \\
&\quad + m_2 g l_1 \cos q_1 = \tau_1 \\[2ex]
&\frac{1}{3} m_2 l_2 \ddot{q}_2 + \frac{m_2 l_2^2 \ddot{q}_2}{4} + \frac{m_2 l_1 l_2}{2} \ddot{q}_1 \cos(q_2 - q_1) \\
&\quad - \frac{m_2 l_1 l_2}{2} \dot{q}_1 (\dot{q}_2 - \dot{q}_1) \sin(q_2 - q_1) \\
&\quad + m_2 g \frac{l_2}{2} \sin q_2 = \tau_2
\end{aligned}
\right\} \quad (6)
$$