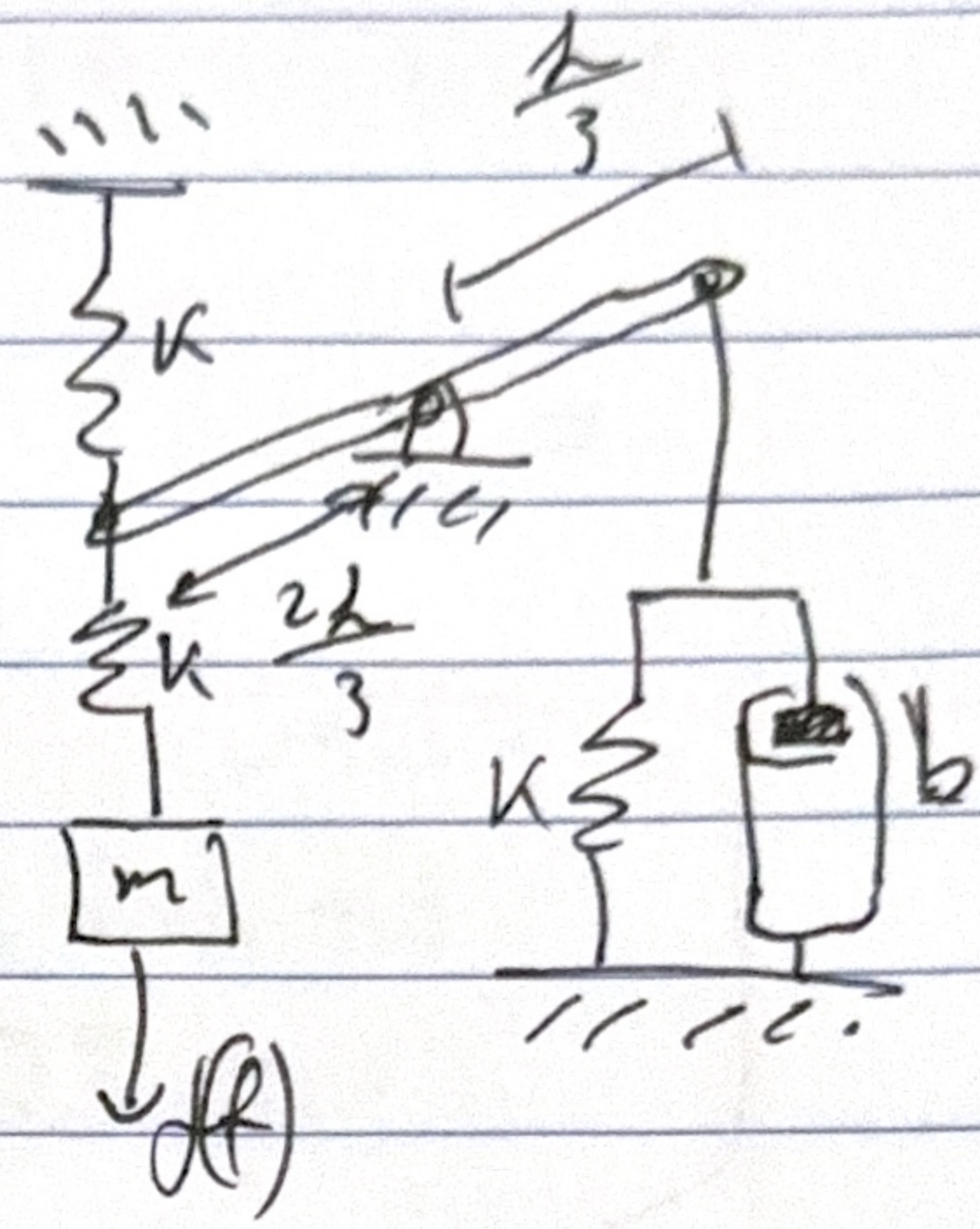
2/21/26 Homework-4 SDR



1) Degree of freedoms

$\theta(t)$ - rotation of bar

$x(t)$ = Vertical displacement of left mass (m)

So there's 2 DOF

$y_R = \frac{L}{3}\theta$

$y_L = \frac{-2L}{3}\theta$

b) $K_{eq} = \frac{K}{2}$ $F_s = \frac{-k}{s}(x - y_L) = F_s = -\frac{K}{2}\left(x + \frac{2L}{3}\theta\right)$

$m\ddot{x} = -\frac{K}{2}\left(x + \frac{2L}{3}\theta\right) + f(t)$ $I = \frac{1}{12}mL^2$

$= m\ddot{x} + \frac{K}{2}x + \frac{KL}{3}\theta = f(t)$

left spring torque

$F_L = \frac{K}{2}\left(x + \frac{2L}{3}\theta\right)$ $T_L = \frac{2L}{3} \cdot \frac{K}{2}\left(x + \frac{2L}{3}\theta\right)$

$= \frac{KL}{3}x + \frac{2KL^2}{9}\theta$

Right spring torque

$F_R = -Ky_R = -K\frac{L}{3}\theta$ $\tau_R = \frac{-KL^2}{9}\theta$

Damping torque

$F_d = -b\frac{L}{3}\dot{\theta}$ $\tau_d = \frac{-bL^2}{9}\dot{\theta}$

Σ Torque

$I\ddot{\theta} = \tau_L + \tau_R + \tau_d$ next page

$$\frac{1}{12}ml^2\ddot{\theta} = \frac{kl}{3}x + \frac{2kl^2}{9}\theta - \frac{kl^2}{9}\theta - \frac{bl^2}{9}\dot{\theta}$$

$$\Rightarrow \frac{1}{12}ml^2\ddot{\theta} + \frac{bl^2}{9}\dot{\theta} - \frac{kl}{3}x = 0$$

$\Rightarrow$ EOM

$$= m\ddot{x} + \frac{k}{2}x + \frac{kl}{3}\theta = f(t)$$

$$= \frac{1}{12}ml^2\ddot{\theta} + \frac{bl^2}{9}\dot{\theta} - \frac{kl^2}{9}\theta - \frac{kl}{3}x = 0$$

c)

$$ms^2X + \frac{k}{2}X + \frac{kl}{3}\Theta = F(s)$$

$$\frac{1}{12}ml^2s^2\Theta + \frac{bl^2}{9}s\Theta - \frac{kl^2}{9}\Theta - \frac{kl}{3}X = 0$$

$$X = \frac{F(s) - \frac{kl}{3}\Theta}{ms^2 + \frac{k}{2}} \qquad = \left(\frac{1}{12}ml^2s^2 + \frac{bl^2}{9}s - \frac{kl^2}{9}\right)\Theta$$

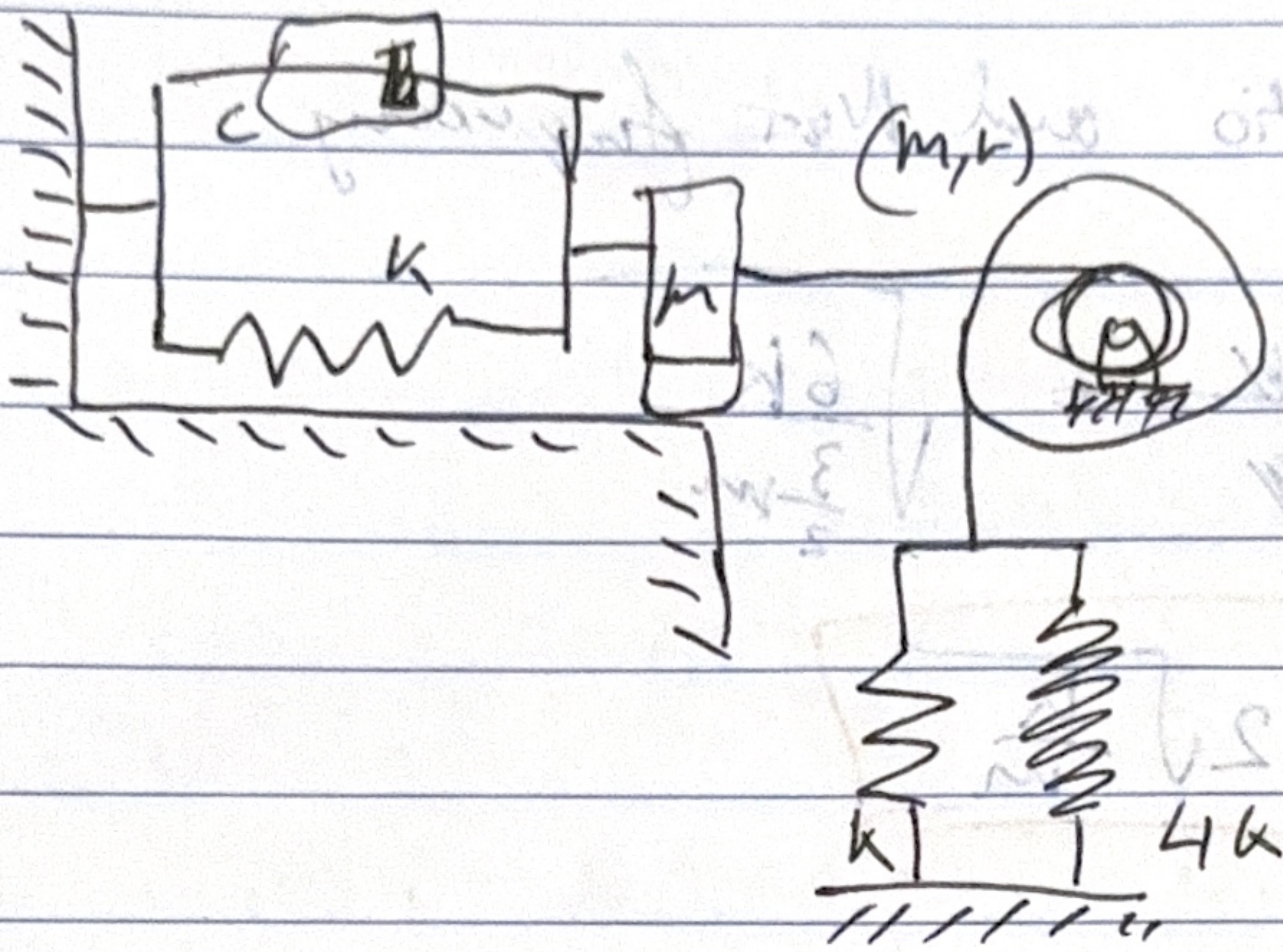
$$\frac{1}{12}ml^2s^2 + \frac{bl^2}{9}s - \frac{kl^2}{9}\Theta = \qquad = \frac{kl}{3}X$$

$$\frac{kl}{3}\,\frac{F(s) - \frac{kl}{3}\Theta}{ms^2 + \frac{k}{2}}$$

too much more work

$$\boxed{\frac{\Theta(s)}{F(s)} = \frac{\frac{kl}{3}}{\left(ms^2 + \frac{k}{2}\right)\left(\frac{1}{12}ml^2s^2 + \frac{bl^2}{9}s - \frac{kl^2}{9}\right) + \frac{k^2l^2}{9}}}$$

2)



$x = r\theta$

$\dot{x} = r\dot{\theta}$

$\ddot{x} = r\ddot{\theta}$

$k_v = k + 4k = 5k$

$y = r\theta = x$

KE ~~Vertical~~ Vertical restoring force

$F_v = 5kx$

$T_b = \frac{1}{2} m\dot{x}^2$

$I = \frac{1}{2} mr^2$ Disk

$T_d = \frac{1}{2} I\dot{\theta}^2 = \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\dot{x}}{r}\right)^2 = \frac{1}{4} m\dot{x}^2$

total KE

$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{4} m\dot{x}^2 = \frac{3}{4} m\dot{x}^2$ $\quad m_{eq} = \frac{3}{2} m$

PE left spring $= \frac{1}{2} kx^2$

Bottom spring $= \frac{1}{2}(5k)x^2$ $\quad V = \frac{1}{2}(6k)x^2$

Damping $= \frac{1}{2} c\dot{x}^2$

$\mathcal{L}$ $(m_{eq}\ddot{x} + c\dot{x} + k_{eq}x = 0)$

a) $\boxed{\text{EOM} = \frac{3}{2} m\ddot{x} + c\dot{x} + 6kx = 0}$

b) Damping Ratio and Nat frequency

$$w_n = \sqrt{\frac{K_{eq}}{m_{eq}}} = \sqrt{\frac{6K}{\frac{3}{2}m}}$$

$$\boxed{W_n = 2\sqrt{\frac{k}{m}}}$$

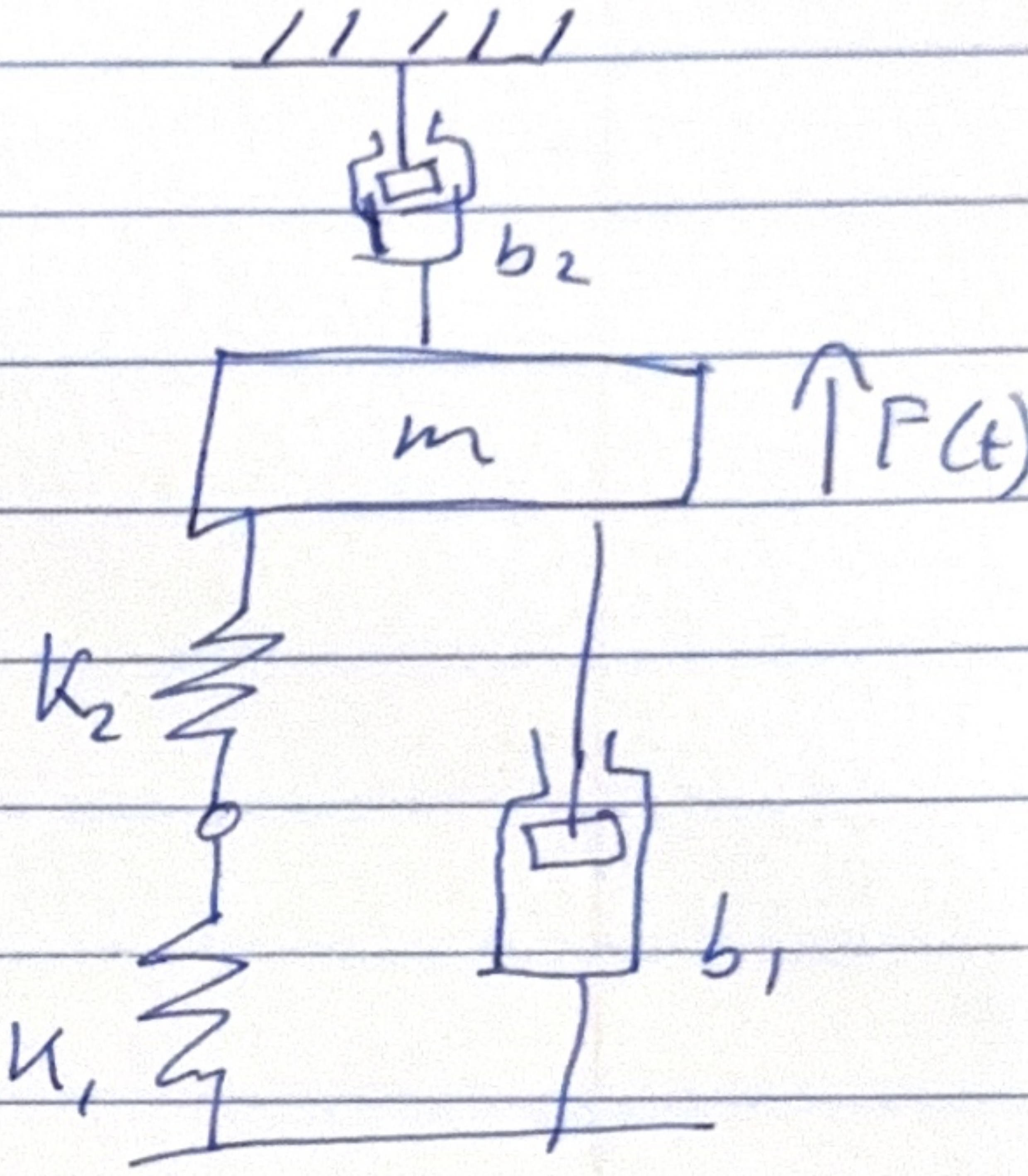
$$\zeta = \frac{c}{2\sqrt{m_{eq} \cdot K_{eq}}} = \frac{c}{2\sqrt{\frac{3}{2}m \cdot 6K}} \quad \frac{3}{2} \cdot 6 = 9$$

$$\boxed{\text{Damping ratio} = \frac{c}{6\sqrt{mk}}}$$

3) $K_1 = 1200\ N/m$ $\qquad b_1 = 100N/(m/s)$

$K_2 = 3600\ N/m$ $\qquad b_2 = 120N/(m/s)$

$F(t) = F_0 \sin(\omega t) \uparrow$



$F(t) = F_0 \sin(\omega t)$ $\qquad F_0 = 100\ N$

$w = 5\ rad/s$

a) $$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2}$$

$$K_{eq} = \frac{K_1 K_2}{K_1 + K_2} = \frac{1200 \cdot 3600}{1200 + 3600} = \frac{4320000}{4800} = 900\ N/m$$

$$b_{eq} = b_1 + b_2 = 100 + 120 = 220\ N\cdot s/m$$

$$m\ddot{x} + b_{eq}\dot{x} + K_{eq}x = F_0 \sin(\omega t)$$

$$\frac{100\ddot{x} + 220\dot{x} + 900x = 100\sin(5t)}{100}$$

$$\boxed{= \ddot{x} + 2.2\dot{x} + 9x = \sin(5t)}$$

b) $x_1 = x$ $\qquad \dot{x}_1 = x_2$ $\qquad \dot{x}_2 = -9x_1 - 2.2x_2 + \sin(5t)$

$x_2 = \dot{x}$

$$\boxed{\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -9x_1 - 2.2x_2 + \sin(5t) \end{aligned}}$$

4) La place transformation:

a) $$F(s) = \frac{2s+1}{s(s+4)(s+1)}$$

$$\frac{2s+1}{s(s+4)(s+1)} = \frac{A}{s} + \frac{B}{s+4} + \frac{C}{s+1} \cdot (s+4) \cdot (s+1)$$

$$= 2s+1 = A(s+4)(s+1) + Bs(s+1) + Cs(s+4)$$

$$= A(s^2+5s+4) + B(s^2+s) + C(s^2+4s)$$

$$(A+B+C)s^2 + (5A+B+4C)s + 4A$$

$$\begin{cases} A+B+C=0 \\ 5A+B+4C=2 \\ 4A=1 \end{cases} \quad A=\frac{1}{4} \quad B+C=-\frac{1}{4} \quad B+4C=\frac{3}{4} \quad B=-\frac{1}{4}-\frac{1}{3}=-\frac{7}{12}$$

$$F(s) = \frac{1}{4s} - \frac{7}{12}\cdot\frac{1}{s+4} + \frac{1}{3}\cdot\frac{1}{s+1}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s}\right) = 1 \qquad \mathcal{L}^{-1}\left(\frac{1}{s+a}\right) = e^{-at}$$

$$\boxed{f(t) = \frac{1}{4} - \frac{7}{12}e^{-4t} + \frac{1}{3}e^{-t}}$$

b) $$F(s) = \frac{2s}{s^2+6s+13}$$

$$s^2+6s+13 = (s+3)^2+4$$

$$2s = 2(s+3)-6$$

$$F(s) = \frac{2(s+3)}{(s+3)^2+4} - \frac{6}{(s+3)^2+4}$$

$$\mathcal{L}(e^{-at}\cos bt) = \frac{s+a}{(s+a)^2+b^2}$$

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$$\mathcal{L}(e^{-at}\sin bt) = \frac{b}{(s+a)^2+b^2}$$

$a=3$
$b=2$

$$\frac{2(s+3)}{(s+3)^2+4} = 2e^{-3t}\cos(2t)$$

$$\frac{6}{(s+3)^2+4} = 3\cdot\frac{2}{(s+3)^2+4} \Rightarrow 3e^{-3t}\sin(2t)$$

$$\boxed{f(t) = 2e^{-3t}\cos(2t) - 3e^{-3t}\sin(2t)}$$