Homework 1 System dynamics & Responec. MECE 3400

1) a $x^2 - (750mm)x + (2500mm^2) = 0$

$x^2 - 750x + 2500 = 0$

$a = 1$

$b = -750$

$c = 2500$

$$x = \frac{-(-750mm) \pm \sqrt{(750mm)^2 - 4(1)2500}}{2(1)}$$

$$x = \frac{750mm \pm \sqrt{562500mm^2 - 10000mm^2}}{2}$$

$$x = \frac{750mm \pm \sqrt{552500mm^2}}{2}$$

$$x = \frac{750mm \pm 743.3mm}{2}$$

$x = 746.65, 3.35$ mm

b) $750 = 2500\cos\theta$

$\cos\theta = \frac{750}{2500} = 0.3$ $\quad \theta = \cos^{-1}(0.3)$

$\theta = 2\pi - \cos^{-1}(0.3)$ $\quad [0, 2\pi]$

$\theta = 1.2661$ rad, 5.0171 rad

c) $(1000m) = (2000m)\sin(\theta - \frac{\pi}{3}rad)$

$\frac{1000}{2000} = \frac{2000}{2000}\sin(\theta - \frac{\pi}{3})$ $\quad \sin(\theta - \frac{\pi}{3}) = 0.5$

$\phi = \theta - \frac{\pi}{3}$ $\quad \sin\phi = 0.5$ $\quad \phi = \frac{\pi}{6} + 2k\pi$

$\phi = \frac{\pi}{6}, \frac{5\pi}{6}$ $\quad \phi = \pi - \frac{\pi}{6} + 2k\pi$

$\theta \rightarrow \frac{\pi}{3} = \frac{\pi}{6} \Rightarrow \theta = \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

$\theta - \frac{\pi}{3} = \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{3} + \frac{5\pi}{6} = \frac{7\pi}{6}$

$\theta = \frac{\pi}{2}, \frac{7\pi}{6}$ rad

d) $(3750\ cm^2) = (2500\ cm^2)\cos\theta + (4300\ cm^2)\sin\theta$

$3750 = 2500\cos\theta + 4300\sin\theta$

$A\cos\theta + B\sin\theta = R\sin(\theta + \phi)$

$R = \sqrt{2500^2 + 4300^2} = 4973.932\ldots = 4974$

$R\sin(\theta + \phi) = R(\sin\theta\cos\phi + \cos\theta\sin\phi)$

$R\cos\phi = 4300$ $\qquad \sin\phi = \frac{2500}{4974} = 30.2°$

$R\sin\phi = 2500$

$\cos\phi = \frac{4300}{4974}$

$3750 = 4974\sin(\theta + 30.2°)$

$\sin(\theta + 30.2) = \frac{3750}{4974} = 0.754$

$\theta + 30.2 = 48.9$ $\qquad \theta + 30.2 = 180 - 48.9 = 131.1$

$\theta = 18.7°, 100.9°$

$\theta = 0.33\ rad, 1.76\ rad$

2) $m\lambda^2 + b\lambda + k = 0$

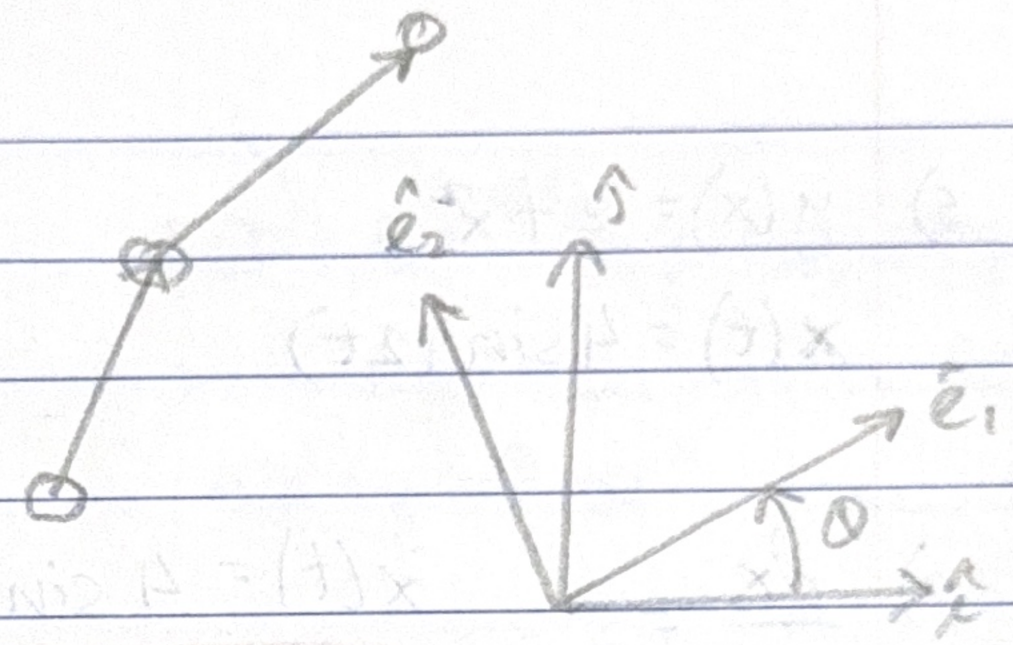
a b c

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4mk}}{2m}$$

3) $\underline{r}_{A/O} = 2i + 4J$

$r_{B/A} = 4\hat{e}_1$

$\theta = 30^\circ$



a) $e_1 = \cos\theta i + \sin\theta J$ $\quad \cos(30)i + \sin(30)J$

$e_2 = -\sin\theta i + \cos\theta J$ $\quad -\sin(30)i + \cos(30)J$

$e_1 = \frac{\sqrt{3}}{2}i + \frac{1}{2}J \quad e_2 = -\frac{1}{2}i + \frac{\sqrt{3}}{2}J$

b) $i = \cos\theta e_1 - \sin\theta e_2$

$J = \sin\theta e_1 + \cos\theta e_2$

$i = \frac{\sqrt{3}}{2}e_1 - \frac{1}{2}e_2$

$J = \frac{1}{2}e_1 + \frac{\sqrt{3}}{2}e_2$

c) $|r|_{A/O} = \sqrt{2^2+4^2} = \sqrt{20}$

$= 2\sqrt{5}$

$|r_{A/O}| = 2\sqrt{5}$

$|r_{B/A}| = 4$

d) $r_{B/A} = 4e_1 = 4\left(\frac{\sqrt{3}}{2}i + \frac{1}{2}J\right) = 2\sqrt{3}\,i + 2J$

$r_{B/O} = r_{A/O} + r_{B/A}$

$= (2+2\sqrt{3})i + (4+2)J$

$r_{B/O} = (2+2\sqrt{3})i + 6J$

e) $|r_{B/O}| = \sqrt{(2+2\sqrt{3})^2+6^2}$

$= \sqrt{52+8\sqrt{3}} = 8.15$ $\quad |r_{B/O}| = 8.15$

from i: $\tan\theta = \frac{6}{2+2\sqrt{3}} =$ $\quad \theta = 47.2^\circ$

5) $y(x) = 2 + x^2$

$x(t) = 4\sin(2t)$

a) $\frac{dx}{dt} = x(t) = 4\sin(2t)$

$= 8\cos(2t)$

b) $\frac{dy}{dx} = y(x) = 2 + x^2$

$= 2x$

c) $\frac{dy}{dt} = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$

$2x \cdot 8\cos(2t)$

$x = 4\sin(2t)$

$= 16 \cdot 4\sin(2t) \cdot \cos(2t)$

$= 64\sin(2t)\cos(2t)$

$\frac{dy}{dt} = 32\sin(4t)$

d) $\frac{d^2y}{dt^2} = 32 \cdot \cos(4t) \cdot 4$

$= 128\cos(4t)$

6) train 300 mph a) $\frac{1584000}{3600}$ ft/h $= 440$ ft/s

b) 482.8 km/h $300 \cdot 1.609$

c) 134.1 m/s $\frac{482803.2}{3600}$ m/s

7) $m\ddot{x} + b\dot{x} + Kx = mg$ $\quad x(t) = Ae^{-\sigma t}\sin(\omega_d t + \phi) + \frac{mg}{K}$

$\sigma = \frac{b}{2m}$ $\quad \omega_n = \sqrt{\frac{K}{m}}$ $\quad \omega_d = \sqrt{\omega_n^2 - \sigma^2}$

$\dot{x} = Ae^{-\sigma t}[\omega_d \cos(\omega_d t + \phi) - \sigma \sin(\omega_d t + \phi)]$

$\ddot{x} = Ae^{-\sigma t}[-(\omega_d^2 + \sigma^2)\sin(\omega_d t + \phi) - 2\sigma\omega_d \cos(\omega_d t + \phi)]$

$m\ddot{x} + b\dot{x} + Kx$

$m\ddot{x} = mAe^{-\sigma t}[-(\omega_d^2 + \sigma^2)\sin - 2\sigma\omega_d \cos]$

$b\dot{x} = bAe^{-\sigma t}[\omega_d \cos - \sigma \sin]$

$Kx = KAe^{-\sigma t}\sin + mg$

$Ae^{-\sigma t}\omega_d \cos(\cdot)\,(-2m\sigma + b)$

$\frac{b}{2m} = \sigma$

$-2m\sigma + b = -b + b = 0$

$Ae^{-\sigma t}\sin(\cdot)\,[-m(\omega_d^2 + \sigma^2) - b\sigma + K]$

$\omega_d^2 = \omega_n^2 - \sigma^2 = \frac{K}{m} - \sigma^2$

$-m(\omega_d^2 + \sigma^2) = -m\omega_n^2 = -K$

$-b\sigma = -\frac{b^2}{2m}$ and $\sigma^2 = \frac{b^2}{4m^2}$

$-K - b\sigma + K = 0$

$m\ddot{x} + b\dot{x} + Kx = mg$

$$x(t) = Ae^{-\sigma t}\sin\left(\sqrt{\omega_n^2 - \sigma^2 t} + \phi\right) + \frac{mg}{K}$$