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Math 18 Written Homework Five
Note that this homework is four pages long!

1. Suppose $(B - C)D = 0$, where B and C are $m \times n$ matrices and D is invertible. Show that $B = C$.

$(B-C)D = 0$

$(B-C)DD^{-1} = D^{-1} \cdot 0$

$(B-C)I = 0$

$B - C = 0$

$B - C + C = 0 + C$

$B = C$

2. Suppose A is an invertible $n \times n$ matrix. Show that the columns of A must span $\mathbb{R}^n$.

Since A is an nxn invertible matrix, according to IMT, the columns of A span $\mathbb{R}^n$

3. Suppose that H is an $n \times n$ matrix. If the equation $H\vec{x} = \vec{c}$ is inconsistent for some $c \in \mathbb{R}^n$, then what can you say about the equation $H\vec{x} = \vec{0}$? Why?

Since $H\vec{x} = \vec{c}$ is inconsistent for some $c \in \mathbb{R}^n$, then by IMT, the equation $Ax = b$ won't have at least solution for each b. So $H\vec{x} = \vec{0}$ will have only the trivial solution.

4. Suppose A is an $n \times n$ invertible square matrix with the property that the equation $A\vec{x} = \vec{0}$ has only the trivial solution. Without using the Invertible Matrix Theorem, explain directly why the equation $A\vec{x} = \vec{b}$ must have a solution for each $\vec{b}$ in $\mathbb{R}^n$.

$Ax = 0$ has only the trivial solution, so it won't have free variables. Therefore, A has a pivot position in each row Therefore $Ax = b$ is consistent, and the equation has a solution for each b in $\mathbb{R}^n$

5. If the given statement is true, write "True". If the given statement is false, write "False" and explain why it is false using complete sentences with proper grammar and punctuation. Giving an example is a great way to demonstrate that a statement is false!

   (a) Suppose A is an $n \times n$ invertible square matrix. If $A\vec{x} = \vec{0}$ has only the trivial solution, then A is row equivalent to the $n \times n$ identity matrix.

   True

   (b) If the columns of an $n \times n$ matrix A span $\mathbb{R}^n$, then the columns of A are linearly independent.

   True

   (c) If A is an $n \times n$ matrix, then the equation $A\vec{x} = \vec{b}$ has at least one solution for every $\vec{b}$ in $\mathbb{R}^n$.

   False. We need more information in order to use IMT. (E.x. A is invertible)

   (d) If A is an $n \times n$ matrix and the equation $A\vec{x} = \vec{0}$ has a nontrivial solution, then A has fewer than n pivot positions.

   True

   (e) If A is an $n \times n$ matrix and A^T is not invertible, then A is not invertible.

   True

6. Construct a geometric figure that illustrates why a line in $\mathbb{R}^2$ which does **not** pass through the origin is not closed under vector addition. Note that here the "vectors" are the points on your line.

Suppose line $y = mx+b$

Let $(x_1, y_1), (x_2, y_2) \in S$

$y_1 = mx_1 + b \qquad y_2 = mx_2 + b$

$y_1 + y_2 = m(x_1 + x_2) + 2e$

$(x_1 + x_2, y_1 + y_2) \notin S$ So, S is not a subspace

7. Let S be the set of all polynomials of the form $p(t) = at^2$, where a is a real number. Is S a subspace of $\mathbb{P}_2$? Carefully justify your answer. (Recall from class that $\mathbb{P}_2$ is the set of all polynomials of degree less than or equal to two).

$p(t) = at^2 \qquad a \in R$

$p_1(t) = a_1 t^2, \quad p_2(t) = a_2 t^2$

$\alpha P(t) + \beta P_2(t) = \alpha(a_1 t^2) + \beta(a_2 t^2)$

$= (\alpha a_1 + \beta a_2) t^2$

where $\alpha a_1 + \beta a_2 \in R$

so β is a subspace of P_2

8. Let K be the set of all polynomials of the form $p(t) = a + t^2$, where a is a real number. Is K a subspace of $\mathbb{P}_2$? Carefully justify your answer. (Recall from class that $\mathbb{P}_2$ is the set of all polynomials of degree less than or equal to two).

$K = \{ p(t) = a + t^2 \mid a \in R \}$

let $P(t) = a + t^2$ & $q(t) = b + t^2, \ a, b, \in R$

$p(t) + q(t) = a + b + 2t^2 \notin S$

So, S is not ~~closed~~ under multipication

Hence, S is not a subspace

9. Let H be the set of all matrices of the form $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$. Is H a subspace of the set of all 2×2 matrices? Carefully justify your answer.

Show H closed under addition & scalar multiplication

Set $H_1 = \begin{bmatrix} a_1 & b_1 \\ 0 & d_1 \end{bmatrix}$ & $H_2 = \begin{bmatrix} a_2 & b_2 \\ 0 & d_2 \end{bmatrix}$

let $H_1, H_2 \in M_{2\times2}$, $a, b, c \in \mathbb{R}$

then $H_1 + H_2 = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ 0 & d_1 + d_2 \end{bmatrix} \in M_{2\times2}$

$rH = \begin{bmatrix} ra & rb \\ 0 & rd \end{bmatrix}$, Also $\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} = a\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

So subspace H is generated by: $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ & $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

10. If the given statement is true, write "True". If the given statement is false, write "False" and explain why it is false using complete sentences with proper grammar and punctuation. Giving an example is a great way to demonstrate that a statement is false!

(a) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$.

False, e.x. $(1,2) \in \mathbb{R}^2$, but $(1,2) \notin \mathbb{R}^3$

$\therefore$ $\mathbb{R}^2$ is not subspace of $\mathbb{R}^3$

(b) A subset H of a vector space V is a subspace of V if the zero vector is in H and if H is closed under addition.

False, H needs to be also closed under scalar multiplication