Assignment #02

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Q.No1

$$\frac{dy}{dx} + \frac{3y}{x} = \frac{e^x}{x^3}$$

$$P = \frac{3}{x} \quad , \quad Q = \frac{e^x}{x^3}$$

$$I = e^{\int p x dx}$$

$$e^{\int \frac{3}{x} dx} = \int \frac{3}{x} dx = 3 \ln x$$

$$I = e^{3 \ln x} = e^{\ln x^3}$$

$$\boxed{I = x^3} \quad \text{multiply with } x^3$$

$$x^3 \frac{dy}{dx} + 3x^2 y = e^x$$

$$x^3 y = \int e^x dx$$

$$x^3 y = e^x + c$$

$$\boxed{y = \frac{e^x + c}{x^3}}$$

(b)

$$\frac{dy}{dx} - \frac{3y}{x+1} = (x+1)^4$$

$$P = -\frac{3}{x+1}, \quad Q = (x+1)^4$$

$$I = e^{\int px\, dx}$$

$$I = e^{\int -\frac{3}{x+1}\, dx}$$

$$-3\int \frac{1}{x+1}\, dx = -3\ln(x+1) = \ln(x+1)^{-3}$$

$$I = e^{\ln(x+3)^{-3}} = \frac{1}{(x+1)}$$

$$\frac{1}{(x+3)} \frac{dy}{dx} - \frac{3y}{(x+1)^4} = (x+1)$$

$$\frac{1}{(x+1)^3}\, y = \int x+1\, dx$$

$$\frac{y}{(x+1)^3} = \frac{x^2}{2} + x + C$$

$$y = (x+1)^3\left(\frac{x^2}{2} + x + C\right)$$

(c) (c) part is same as (b)

Q#02 Cauchy-Euler D.E.

(a) $x^2y'' - 6xy' - 18y = 0$

Let $y = x^r$

$$x^2[x^r]'' - 6x[x^r]' - 18x^r = 0$$

$$x^2[rx^{r-1}]' - 6x(rx^{r-1}) - 18x^r = 0$$

$$x^2[r(r-1)x^{r-2}] - 6x(rx^{r-1}) - 18x^r = 0$$

$$(r^2 - r - 6r - 18)x^r = 0$$

$$r^2 - 7r - 18 = 0$$

$$r^2 + 2r - 9r - 18 = 0$$

$$r(r+2) - 9(r+2) = 0$$

$$(r+2)(r-9) = 0$$

$$r = -2, \quad r = 9$$

=> these are distinct possible values for r

(b) $x^2y'' - 7xy' + 16y = 0 \quad , \quad x > 0$

$y = x^r$

$x^2[x^r]'' - 7x[x^r]' + 16x^r = 0$

$x^2[r x^{r-1}]' - 7x(r x^{r-1}) + 16x^r = 0$

$x^2[r(r-1)(x^{r-2})] - 7x(r x^{r-1}) + 16x^r = 0$

$(r^2 - r)x^r - 7r x^r + 16x^r = 0$

taking x^r common.

$(r^2 - r - 7r + 16)x^r = 0$

$r^2 - 8r + 16 = 0$

$(r-4)^2 = 0$

$(r-4)(r-4) = 0 \quad , \quad \boxed{r = 4}$

therefore the solution is $y(x) = x^4$

let $y(x) = x^4 u(x)$

differentiate it

$y(x) = x^4 u(x)$

$y'(x) = [x^4 u]' = 4x^3 u + x^4 u'$

$y''(x) = [4x^3 u + x^4 u']'$

$$= 12x^2u + 8x^3u' + x^4u''$$

Now take actual equation.

$$x^2y'' - 7xy' + 16y = 0$$

Now Substitute

$$x^2[12x^2u + 8x^3u' + x^4u''] - 7x[4x^3u + x^4u']$$
$$+ 16[x^4u] = 0$$

$$12x^4u + 8x^5u' + x^6u'' - 28x^4u - 7x^5u' + 16x^4u = 0$$

$$x^6u'' + [8x^5 - 7x^5]u' + [12x^4 - 28x^4 + 16x^4]u = 0$$

$$x^6u'' + x^5u' = 0$$

let

$$u' = V$$

$$x^6V' + x^5V = 0$$

$$x^6\left(\frac{dV}{dx}\right) + x^5V = 0$$

$$x^6\left(\frac{dV}{dx}\right) = -x^5V$$

$$\left(\frac{1}{V}\right)\left(\frac{dV}{dx}\right) = -\frac{x^5}{x^6}$$

$$\left(\frac{1}{V}\right)\left(\frac{dV}{dx}\right) = -\frac{1}{x}$$

Integrating on b/s, then

$$\int\left(\frac{1}{V}\right)\left(\frac{dV}{dx}\right)dx = \int\left(-\frac{1}{x}\right)dx$$

$$\ln|V| = -\ln|x| + C_0$$

As

$$\log_e 10 = 2.305$$ * As we know the rule.

$$10 = e^{2.305}$$

$$\log_e|V| = -\ln|x| + C_0$$

$$V = e^{-\ln|x| + C_0}$$

$$V = e^{-\ln x} + e^{C_0} \rightarrow \text{constt.}$$

$$V = C_2 e^{-\ln x}$$

$$V = C_2/x$$

$\because u' = V$

Now;

$$u(x) = \int u'(x)dx = \int V(x)\,dx$$

$$u(x) = \int \frac{C_2}{x}\,dx$$

$$u(x) = c_2 \ln(x) + c_1$$

General solution is.

$$y(x) = x^4 u(x)$$

$$= x^4 (c_2 \ln|x| + c_1)$$

$$y(x) = c_1 x^4 + c_2 x^4 \ln|x|$$