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Assignment#03

Q#01

$C = T_0 - T_A$

$$T = T_A + (T_0 - T_A)\, e^{kt}$$

$= T$

$t=0 \quad T(0) = 300^\circ$

$t=3 \quad T(3) = 200$

$t=? \quad T_A = 70^\circ$

at $t=0$

$T(0) = T_A + (T_0 - T_A)\, e^{kt}$

$300 = 70 + (300 - 70)\, e^{k(0)}$

$T(0) = T_A + C\, e^{kt}$

$300 = 70 + C\, e^{k(0)}$

$300 - 70 = C\, e^{0}$

$230 = C$

$T = T_A + 230\, e^{kt}$

at $t = 3$

$$T(3) = T_A + 230e^{k(3)}$$

$$200 = 70 + 230e^{3k}$$

$$200 - 70 = 230e^{3k}$$

$$130 = 230e^{3k}$$

$$\frac{130}{230} = e^{3k}$$

$$\ln\frac{130}{230} = \ln(e^{3k})$$

$$\ln 130 = 3k \ln e$$

$$0 = 3k(1)$$

$$0 = 3k$$

$$k = 0$$

$$130 = 230e^{3k}$$

$$\frac{130}{230} = e^{3k}$$

$$\frac{13}{23} = e^{3k}$$

$$\ln\left(\frac{13}{23}\right) = \ln(e^{3k})$$

$$\ln\left(\frac{13}{23}\right) = 3k$$

$$\frac{1}{3}\ln\left(\frac{13}{23}\right) = k$$

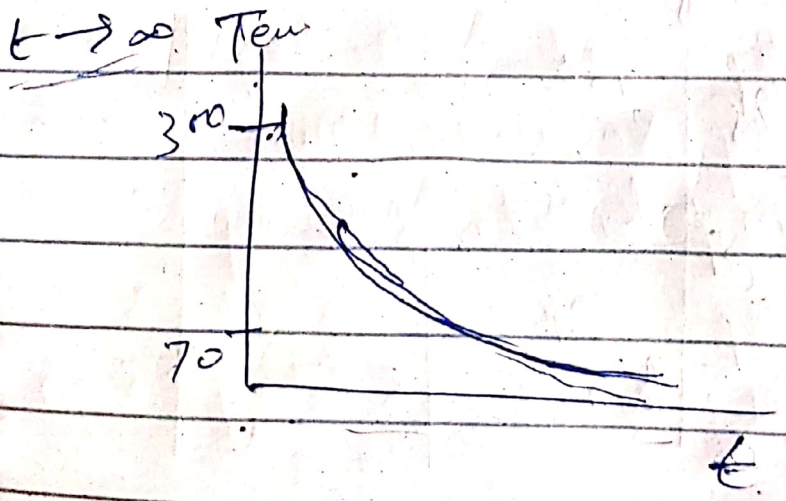
$$-0.19018 = k$$

$$T = T_A + 230\, e^{-0.19018\, t}$$

$$T = 70 + 230\, e^{-0.19018\, t}$$

at what time t?
to cool it at 70°.

$$\lim_{t \to \infty} T(t) = 70$$



Q#02

$t=0 \quad T=100^\circ, \quad T_A=5^\circ$

$t=10, \quad T=60$

$$T = T_A + Ce^{kt}$$

$$100 = 5 + Ce^{k(0)}$$

$$100-5 = C$$

$$\boxed{95 = C}$$

$$T = T_A + 95e^{kt}$$

$t=10$

$$60 = 5 + 95e^{k(10)}$$

$$60-5 = 95e^{10k}$$

$$55 = 95e^{10k}$$

$$\frac{55}{95} = e^{10k}$$

$$\frac{11}{19} = e^{10k}$$

$$\frac{1}{10}\ln\left(\frac{11}{19}\right) = k$$

$$k = -0.05465$$

$$T = 5 + 95e^{-0.05465t}$$

$$T = 20^\circ$$

$$20 = 5 + 95e^{-0.05465(t)}$$

$$20 - 5 = 95e^{-0.05465t}$$

$$15 = 95e^{-0.05465t}$$

$$\frac{15}{95} = e^{-0.05465t}$$

$$\ln\left(\frac{15}{95}\right) = -0.05465t$$

$$-\frac{1}{0.05465}\ln\left(\frac{15}{95}\right) = t$$

$$t = 33.77$$