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ASSIGNMENT: 01

Q d5

$$\frac{dy}{dx} - \frac{y}{x} = y^9$$

$n = 9 \quad p(x) = \frac{-1}{x} \quad Q(x) = 1$

Solution:

$$\frac{du}{dx} + (1-n)u\,p(x) = (1-n)\,Q(x).$$

substitute value,

$$\frac{du}{dx} + \frac{8u}{x} = -8$$

~~let u = xv~~

$$\cancel{\frac{du}{dx} = v\frac{dx}{dx} + x\frac{dv}{dx}}$$

$$\cancel{v\frac{dx}{dx} + x\frac{dv}{dx} + \frac{8xv}{x} = -8}$$

$\cancel{\frac{du}{dx}} + c$

$$\frac{du}{dx} + \frac{8u}{x} = -8.$$

$$\boxed{\begin{aligned} u &= y^{1-n} \\ u &= y^{-8} \end{aligned}}$$

$$I = e^{\int P(x)dx} = e^{\int \frac{8}{x} dx}$$

$$I = \int \frac{8}{x} dx = 8\int \frac{1}{x} dx = 8\ln x = \ln x^8$$

$$I = e^{\ln x^8} \Rightarrow \boxed{I = x^8}$$

Multiply on b/s.

$$x^8 \frac{du}{dx} + 8\frac{u}{x} x^8 = -8x^8.$$

$$x^8 \frac{du}{dx} + 8ux^7 = -8x^8.$$

integrate on b/s.

$$\int x^8 \frac{du}{dx} + 8ux^7 dx = \int -8x^8 dx$$

$$x^8 u = -\frac{8}{9} x^9 + c$$

$$u = -\frac{8}{9} x + cx^{-8}.$$

$$u = y^{-8}$$

$$y^{-8} = \frac{-8}{9}x + Cx^{8}$$

$$y = \left(\frac{-8}{9}x + Cx^{8}\right)^{(-1/8)}$$

General Solution:-

$$\boxed{y = \left(\frac{-8}{9}x + Cx^{-8}\right)^{(-1/8)}}$$

Q2 $\frac{dy}{dx} + x^5 y = x^5 y^7$.

$P(x) = x^5$. $Q x = x^1$ $n = 7$.

$u = y^{1-n}$

$u = y^{1-7} = y^{6}$

$y = u^{(\frac{-1}{6})}$

$$\frac{dy}{dx} = \frac{-1}{6} u^{(\frac{-7}{6})} \frac{du}{dx}.$$

$$-\frac{1}{6} u^{(\frac{-7}{6})} \frac{du}{dx} + x^5 u^{(-1/6)} = x^5 u^{(-7/6)}$$

Multiply $= -6(u)^{(7/6)}$

$$\frac{du}{dx} - 6x^5u = -6x^5$$

$$\frac{du}{dx} = (u-1)\,6x^5$$

Seperating variable.

$$\frac{du}{u-1} = 6x^5 dx.$$

$$\int \frac{1}{u-1}\,du = \int 6x^5 dx.$$

$$\ln(u-1) = x^6 + C$$

$$u-1 = e^{x^6} + c$$

$$u = e^{(x^6+c)} + 1$$

substitute back, $y = u^{(-1/6)}$

$$y = e^{(x^6+c)} + 1)^{(-1/6)}.$$

An

Date: __/__/20

Q2 Seperable differential.

1. $dy/dx = (x^2+6)(y-7)$

Sol:-

$$\frac{dy}{dx} = (x^2+6)(y-7)$$

$$\frac{dy}{(y-7)} = (x^2+6)\,dx$$

$$\int \frac{dy}{y-7} = \int (x^2+6)\,dx$$

$$\boxed{\ln(y-7) = \frac{x^3}{3} + 6x + C_1}$$

$$= (y-7) = e^{\frac{x^3}{3} + 6x + C_1}$$ Ans

2. $$\frac{dy}{dx} = y \cdot e^x$$

Sol:-

$$\frac{dy}{dx} = y \cdot e^x$$

$$\frac{1}{y}\,dy = e^x\,dx$$

$$\int \frac{1}{y}\,dy = \int e^x\,dx$$

$$\boxed{\ln|y| = e^x + C_1}$$ Ans $\boxed{y = e^{e^x + C}}$

(iii) $y' = xy - 3x + 4y - 12$

Sol:-

$$y' - 4y = xy - 3x - 12.$$

Integrating factor $= e^{-4x}$. multiply on both side.

$$e^{-4x} y' - 4e^{-4x} y = xe^{-4x} - 3e^{-4x} - 12e^{-4x}$$

$$\frac{d}{dx}(e^{-4x} y) = xe^{-4x} - 3e^{-4x} - 12e^{-4x}$$

$$\int d(e^{-4x} y) = \int (xe^{-4x} - 3e^{-4x} - 12e^{-4x})\,dx.$$

$$e^{-4x} y = -\frac{1}{4} x e^{-4x} - \frac{3}{4} e^{-4x} + 3e^{-4x} + c_1$$

$$y = \frac{-1}{4} x - \frac{3}{4} + 3e^{4x} + c_1 e^{4x}$$

Ans