Multivariable Calculus

Assignment No: 1

→ Equation of Line:

① Find V.E = ? P.E = ?

Point (5, 1, 0)

plane : $2x - y + z = 1$

$v = 2i - j + k$

P.E:

$x = x_0 + v_1 t$; $x = 5 + 2t$

$y = y_0 + v_2 t$; $y = 1 - t$

$z = y_{2z_0} + v_3 t$; $z = 0 + t$

$V.E = xi + yj + zk = (5+2t)i + (1-t)j + (0+t)k$

plane : $2x - y + z = 1$

Pa x-intercept put $y = 0, z = 0$

$2x - 0 + 0 = 1$

$2x = 1$, $x = \frac{1}{2}$ $(\frac{1}{2}, 0, 0)$ → point

y-intercept put $x = 0, z = 0$

$2(0) - y + 0 = 1$

$y = -1$, $(0, -1, 0)$

Equation of plane:

① P(1, 7, 2)

$x = 1 + t$ e of p: $A(x - x_0) + B(y - y_0) + C(z - z_0)$

$y = 2t$

$z = 8 + t - 3t$

$\vec{v}$: ~~$i + j - k$~~ $i + 2j - 3k$

putting values

$1(x - 1) + 2(y - 7) + -3(z - 2) = 0$

$x - 1 + y - 7 - 2z + 2 = 0$

$x + 2y + 3z = 1 + \frac{14}{3} - 6$

$x + 2y + 3z = 6 - 19$

② P(6, 0, -2)

Line $x = 4 - 2t$,

$y = 3 + 5t$

$z = 7 + 4t$

$v = -2i + 5j + 4k$

putting values

$-2(x - 6) + 5(y - 0) + -2(z + 4) = 0$

$-2x + 12 + 5y + 2z - 68 = 0$

$-2x + 5y - 2z = -12 + 8$

$-2x + 5y - 2z = -4$

$3x + 6y - 3z = 4$

x-intercept put $y = 0, z = 0$

$3x + 6(0) - 3(0) = 4$

$3x = 4 \quad \therefore x = \left(\frac{4}{3}, 0, 0\right)$ Q

$PQ\left(\frac{4}{3}i + 0j - K\right)$

$\overrightarrow{PQ} \cdot \vec{n} = \frac{4 + 0 + 3}{3 - 6 - 3} = 7$

$\frac{7}{-6} \cdot [3, -6, -3]$

$\frac{-7}{2}, \frac{7}{2}, \frac{7}{2}$

$\text{Proj} \left| \frac{\overrightarrow{PQ} \cdot \vec{n}}{|\vec{n}|^2} \cdot \vec{n} \right|$

$\text{distance} = \sqrt{\left(\frac{-7}{2}\right)^2 + (7)^2 + \left(\frac{7}{2}\right)^2}$

$= \sqrt{\frac{49}{4} + 49 + \frac{49}{4}}$

$= \sqrt{\frac{147}{2}}$

2) $P(-2,4,0)$ $Q(1,1,1)$

$P(2,3,4)$ and $Q(3,-1,8)$

perpendicular or not

First we find vector

$\vec{v_1} = \vec{PQ}$

$\vec{v_2} = 3i - 3j + k$

$\vec{w} = i - 4j + 4k$

Take a cross product.

$\vec{v} \times \vec{v}$.

$$\begin{vmatrix} i & j & k \\ 3 & -3 & 1 \\ 1 & -4 & 4 \end{vmatrix}$$

$i(-12+4) - j(12-1) + k(-12+3)$

$\vec{c} = -8i - 11j - 9k$

$\vec{v} \cdot \vec{c} =$

$-24 + 33 - 9$

$= 0$

~~so they are not perpendicular~~

$\vec{w} \cdot \vec{c} = -8 + 44 - 36$

$= 0$

(4) Find equation of plane P(1, 0, -2)

Q(3, 4, 0)

first we find vector

$\overrightarrow{PQ} = v$

$v = 2i + 4j - 2k$

putting values

$2(x-1) + 4(y-0) - 2(z-2) = 0$

$2x - 2 + 4y - 2z + 4 = 0$

$2x + 4y - 2z = 2 - 4$, -2

Distance from point to Line & point to p

S(3, -2, 7)

P(4, 3, 7)

$\vec{n}(3i - 2j + 7k)$

$x = 4 + 3t$

$y = 3 - 2t$

$z = 7 + 7t$

$d = \frac{|\overrightarrow{PS} \cdot \vec{n}|}{|\vec{n}|}$

$|\vec{n}| = \sqrt{(3)^2 + (-2$

$\sqrt{9 + 4 + 4}$

$\sqrt{32}$

$\overrightarrow{PS} = -1i + -5j + 0k$

$\overrightarrow{PS} \cdot \vec{n} = -3 + 10 + 0$

$|PS \cdot \vec{n}| = \sqrt{(3)^2 + (10)^2 + (0)^2}$

(3) $x = 3 + 2t$ $\quad x = 2x + 4$

$y = t$ $\quad y =$

$z = 8 - t$ $\quad z =$

plane $2x + 4y + 8z = 17$

-> x-intercept Put $y = 0, z = 0$

$2x + 4(0) + 8(0) = 17$

$2x = 17 \quad x = \frac{17}{2}, \left(\frac{17}{2}, 0, 0\right)$

-> y-intercept put $x = 0, z = 0$

$2(0) + 4y + 8(0) = 17$

$4y = 17, \; y = \frac{17}{4}, \left(0, \frac{17}{4}, 0\right)$

-> z-intercept put $x = 0, y = 0$

$2(0) + 4(0) + 8z = 17$

$8z = 17, \; z = \frac{17}{8} \left(0, 0, \frac{17}{8}\right)$

$\vec{v} = 2i + j - k$

$P = (3, 0, 8)$

$2(x - 3) + 1(y - 0) - 1(z - 8) = 0$

$2x - 6 + y - z + 8 = 0$

$2x + y - z = 6 - 8 \quad -2$