

Assignment No. 3)

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Ques)

Shear Matrix:

$$\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Unit square vertices

$$A = (0, 0)$$

$$B = (1, 0)$$

$$C = (1, 1)$$

$$D = (0, 1)$$

Now, shear Matrix with each vertices

$$A' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$B' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$C' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$D' = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Now Translate vertex by vector $(2, 3)$

$$\begin{array}{l} A'' = (2, 3) \\ B'' = (3, 3) \\ C'' = (5, 4) \\ D'' = (4, 4) \end{array} \quad \left. \vphantom{\begin{array}{l} A'' \\ B'' \\ C'' \\ D'' \end{array}} \right\} \text{Final vertices}$$

Q No 2)

$$\frac{(x-1)^2}{4} + \frac{(3y'+2)^2}{16} = 1$$

substitute $y = 3y'$

$$\frac{(x-1)^2}{4} + \frac{9\left(y' + \frac{2}{3}\right)^2}{16} = 1$$

$$\frac{(x-1)^2}{4} + \frac{\left(y' + \frac{2}{3}\right)^2}{9} = \frac{16}{9}$$

$$\frac{(x-1)^2}{4} + \frac{\left(y + \frac{2}{3}\right)^2}{9} = \frac{16}{9}$$

$$y' = y \rightarrow$$

Now Multiply 9 on b.s

$$4(x-1)^2 + 9\left(y+\frac{2}{3}\right)^2 = 16$$

Final equation of the stretched ellipse
along the y-axis by a factor of 3

Q No 3)

Shear Matrix:

$$\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

Transform a point (x, y) to a new point (x', y')

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ -2x+y \end{bmatrix}$$

$$x' = x$$

$$y' = -2x + y$$

$$\text{eq.} = (x+1)^2 + (y-2)^2 = 9$$

$$= (x+1)^2 + ((-2x+y)-2)^2 = 9$$

$$= (x+1)^2 + (y' + 2x - 2)^2 = 9$$

$$\text{Expand} = (x+1)^2 = x^2 + 2x + 1$$

$$\text{Expand} = (y' + 2x - 2)^2$$

$$(y' + 2x - 2)^2 = (y')^2 + (2x - 2)^2 + 2y'(2x - 2)$$

$$= y'^2 + (2x - 2)^2 + 2y'(2x - 2)$$

$$= y'^2 + 4x^2 - 8x + 4 + 4xy' - 4y'$$

$$x^2 + 2x + 1 + y'^2 + 4x^2 - 8x + 4 + 4xy' - 4y' = 9$$

$$5x^2 + y'^2 + 4xy' - 4y' - 6x + 5 = 9$$

$$5x^2 + y'^2 + 4xy' - 4y' - 6x - 4 = 0$$

Equation of the sheared circle

① No 4)

$$x = -2$$

$$\text{point } (x, 2x - 3)$$

$$x = -2 \text{ is } x - (-2) = x + 2$$

$$-2 - (x + 2) = -4 - x$$

$$x' = -4 - x$$

$$x = -4 - x'$$

Now substitute $x = -4 - x'$ in the equation $y = 2x - 3$

$$y = 2(-4 - x') - 3$$

$$y = -8 - 2x' - 3$$

$$y = -2x' - 11$$

~~x =~~ The reflection of the line $y = 2x - 3$ and along the line $x = -2$ is

$$y = -2x - 11$$