Q.No 1

$$\frac{dy}{dx} + \frac{y}{x} = y^9$$

divide y^9 on both sides

$$\frac{1}{y^9}\frac{dy}{dx} + \frac{1}{x}\frac{y}{y^9} = 1$$

let $z = y^{1-n} \Rightarrow y^{1-9} = y^{-8}$

$$\frac{1}{y^9}\frac{dy}{dx} + \frac{1}{x}y^{-8} = 1$$

$$\frac{1}{y^9}\frac{dy}{dx} + \frac{1}{x}z = 1$$

we know

$$z = y^{-8}$$

$$\frac{dz}{dx} = -8y^{-9}\frac{dy}{dx}$$

Multiply -8 on b.s

$$\frac{1}{y^9}\frac{dy}{dx} + \frac{8}{x}z = -8$$

$$\frac{dz}{dx} + \frac{8}{x}z = -8$$

integrating factor

$P(x) = \frac{8}{x}$ $\quad I = e^{\int P(x)\,dx}$

$I = e^{\int (8/x)\,dx}$

$I = \int \frac{8}{x}\,dx$

$8 \int \frac{1}{x}\,dx$

$I = 8 \ln x$

$e^{8 \ln x} = 8$

$\frac{dx}{dz} + \frac{8}{x} z = 8$

$E(z) = \int 16\,dx$

$8z = 16x$

$z = 8x$

$\frac{y^{-8}}{-8} = 8x$

$\frac{1}{y^8} = -8x$

$\frac{z = y^{-8}}{z = \frac{1}{y^8}}$

$\frac{1}{y^8} = -8x + c$

QNo2)

$$\frac{dy}{dx} + x^5 y = x^5 y^7$$

$z = y^{1-n}$ $\quad n = 7 \quad z = y^{-6} = \frac{1}{y^6}$

divide y^7 on b.s

$$\frac{1}{y^7}\frac{dy}{dx} + x^5 \frac{1}{y^6} = x^5$$

$$\frac{1}{y^7}\frac{dy}{dx} + x^5 z = x^5 \quad \text{---(1)}$$

we know that

$$z = y^{-6}$$

$$\frac{dz}{dx} = -6y^{-7}\frac{dy}{dx}$$

Now multiply -6 on b.s of eq (1)

$$-6\frac{1}{y^7}\frac{dy}{dx} - 6x^5 z = -6x^5$$

$$\frac{dz}{dx} - 6x^5 z = -6x^5$$

by ~~integra~~ seperable variable

$$\frac{dz}{dx} = 6x^5 z - 6x^5$$

$$\frac{dz}{dx} = 6x^5(z-1)$$

$$\frac{1}{(z-1)} \frac{dz}{dx} = 6x^5$$

$$\int \frac{1}{z-1} dx = \int 6x^5 dx$$

$$\ln(z-1) = \frac{6x^6}{6} + c$$

$$\ln(z-1) = x^6 + c$$

$$z-1 = e^{x^6} + c$$

$$z = e^{x^6+c} + 1$$

$$z = y^{-6}$$

$$y^{-6} = e^{x^6+c} + 1$$

$$y = (e^{x^6+c} + 1)^{(-1/6)}$$

① $\frac{dy}{dx} = (x^2+6)(y-7)$

$\frac{dy}{(y-7)} = x^2+6\,dx$

$\int \frac{1}{y-7}\,\frac{dy}{dx} = \int x^2+6\,dx$

$\ln|y-7| = \frac{x^3}{3} + 6x + c$

$y-7 = e^{x^3/3+6x+c}$

$y = e^{x^3/3+6x+c} + 7$

② $\frac{dy}{dx} = ye^x$

$\frac{dy}{y} = e^x dx$

$\frac{1}{y}\frac{dy}{dx} = e^x dx$

$\int \frac{1}{y} = \int e^x dx$

$\ln y = e^x + c$

$y = e^{ex+c}$

③ $\frac{dy}{dx} = xy - 3x + 4y - 12$

$\frac{dy}{dx} = x(y-3) + 4(y-3)$

$\frac{dy}{dx} = (x+4)(y-3)$

$\frac{dy}{(y-3)} = (x+4)\,dx$

$\frac{1}{y-3}\,dy = x+4\,dx$

$\int \frac{1}{y-3} = \int x+4\,dx$

$\ln|y-3| = \frac{x^2}{2} + 4x + c$

$y-3 = e^{x^2/2 + 4x + c}$

$y = e^{x^2/2 + 4x + c_1} + 3$