# Assignment # 4

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## Question NO. 1

$$dp/dt = 0.2P(t)\left(1 - \frac{P(t)}{200}\right), P(0) = 150$$

$$\frac{1000\,dp}{P(P-200)} = dt$$

$$5\int \left(\frac{1}{P-200} - \frac{1}{P}\right) dp = dt$$

$$\int \ln\left|\frac{P-200}{P}\right| = t + C$$

$$\frac{P-200}{P} = Ce^{t/5}$$

$$P(t) = \frac{200}{1 - Ce^{t/5}}$$

$$P(0) = 150 \Rightarrow \frac{200}{1-C} = 150 \Rightarrow C = -1/3$$

$$P(t) = \frac{200}{1 + 1/3\, e^{t/5}}$$

## Question NO. 2

$$P(t) = \frac{K}{1 + Ae^{-kt}}$$

$$A = \frac{K - P_0}{P_0} = \frac{1000 - 1}{1} = 999$$

using formula $P(4) = 50$

$$50 = \frac{1000}{1 + 999e^{-k4}}$$

$$1 + 999^{-4k} = \frac{1000}{50}$$

$$999e^{-4k} = 20$$

$$e^{-4k} = \frac{20}{999}$$

$$-4k = \ln \frac{20}{999}$$

$$k = -3.9110$$

$$P(t) = \frac{K}{1 + Ae^{-kt}}$$

$$P(t) = \frac{1000}{1 + 999e^{-3.9110}}$$

$$\approx 612.7$$

## Question No. 3

Mass m is dropped from helicopter Finds its velocity as f(t) air resistance is proptional to velocity.

$$\frac{dv}{dt} = \frac{kv}{m} = y$$

where y is the gravitational constant
K is the constant of propotionality

$$b = k/m$$

$$dv = (y - bv)$$

$$\int \frac{dv}{g - bv} = \int dt$$

$$-\frac{1}{b} \ln |g - bv| = t + g$$

$$\ln |g - bv| = -bt - bc,$$

$$g - bv = ce^{-bt}$$

$v = 0$ where $t = 0$, then $g = 0$

$$-bv = -g + ge^{-bt}$$

$$v = \frac{g - ge^{-bt}}{10}$$

$$= \frac{mg}{k}\left(1 - e^{-kt/m}\right)$$

Question No. 4

$$\frac{dv}{dt} = g - Bv$$

$$\int \frac{dv}{g - Bv} = \int dt$$

$$-\frac{1}{B} Ln(g - Bv) = t + c'$$

$$Ln(g - Bv) = Bt + c'$$

$$g - Bv = ce^{-bt}$$

$$v(0) = v_1 \Rightarrow g - Bv_1 = C$$

Eq for Skydivers speed at t seconds after parachutes open

$$g - Bv = (g - Bv)\, e^{-Bt}$$

$$Bv = g - (g - Bv)\, e^{-Bt}$$

$$v = g/B \left| 1 - \left(1 - \frac{B}{g} v_1\right) e^{-Bt} \right|$$

$$v = \frac{mg}{k} \left| 1 + \left(\frac{k}{mg} v_1 - 1\right) e^{-kt} \right.$$

As time passes, as t increase, the term $e^{(-k/m)^t}$ goes zero, So, the parachutes speed v slow the $mg/k$ which is terminal speed with parachutes open.