Q(a)s) $\frac{dy}{dx} + \frac{3y}{x} = \frac{e^x}{x^3}$

$P(x) = \frac{3}{x}$ & $I = e^{\int P(x)dx}$

$I = e^{\int 3/x \, dx}$

$I = \int \frac{3}{x} dx$

$I = 3 \int \frac{1}{x} dx$

$I = 3 \ln x$

$e^{3\ln x} = e^{\ln x^3} = x^3$

$x^3 \frac{dy}{dx} + \frac{3y}{x} = \cancel{x^3} \frac{e^x}{\cancel{x^3}}$

$x^3 y = e^x$

$\int x^3 y = \int e^x dx$

$x^3 y = e^x + c$

$y = \frac{e^x + c}{x^3}$

Q No 2) $\frac{dy}{dx} - \frac{3y}{x+1} = (x+1)^4$

Sol.

$P(x) = -\frac{3}{x+1}$; $I = e^{\int P(x)\,dx}$

$I = e^{\int (-3/x+1)\,dx}$

$= \int -\frac{3}{x+1}\,dx$

$= -3 \int \frac{1}{x+1}\,dx$

$= -3 \ln|x+1|$

$= e^{-3 \cdot \ln(x+1)} = (x+1)^{-3} = \frac{1}{(x+1)^3}$

$\frac{1}{(x+1)^3} \frac{dy}{dx} - \frac{1}{(x+1)^3} \frac{3y}{(x+1)} = \frac{(x+1)^4}{(x+1)^3}$

$\frac{1}{(x+1)^3} y = \int x+1\,dx$

$\frac{1}{(x+1)^3} y = \frac{x^2}{2} + x + c$

$y = (x+1)^3 \left(\frac{x^2}{2} + x + c\right)$

Q No 4) $x^2 y'' - 6xy' - 18y = 0$

let $y = x^r$

$x^2 [x^r]'' - 6x [x^r] - 18y = 0$ ①

$y = x^r$

$\frac{dy}{dx} = r' x^{r-1}$

$y'' = r(r-1) x^{r-2}$

now put in eq ②

$\cancel{x^2 [r x^{r-1}] - 6x [}$

$x^2 [r(r-1) x^{r-2}] - 6x [r x^{r-1}] - 18 x^r = 0$

$(r^2 - r) x^r - 6 r x^r - 18 x^r = 0$

$(r^2 - r - 6r - 18) x^r = 0$

$r^2 - 7r - 18$

$\cancel{r(r-1)}$

$r^2 + 2r - 9r - 18$

$r(r+2) - 9(r+2)$

$(r-9)(r+2) = 0$

$r = 9, \quad r = -2$

so $y = c_1 x^{-2} + c_2 x^{9}$

Q(no) $x^2 y'' - 7xy' + 16y = 0$

Sol.

Let $y = x^r$

$x^2 [x^r]'' - 7x [x^r]' + 16 x^r = 0$ — ①

$y = x^r$

$\frac{dy}{dx} = r x^{r-1}$

$y'' = r(r-1) x^{r-2}$

now put in eq ①

$x^2 [r(r-1) x^{r-2}] - 7x [r x^{r-1}] + 16 x^r = 0$

$r(r-1) x^r - 7 r x^{r} + 16 x^r$

$[r^2 - r] x^r - 7r x^r + 16 x^r = 0$

$(r^2 - r - 7r + 16) x^r$

$(r^2 - 8r + 16) x^r$

$r^2 - 4r - 4r + 16$

$r(r-4) - 4(r-4)$

$(r-4)(r-4)$

$r = 4$

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$y_1(x) = x^4$

Now let $y = x^4 u$

$x^2 [x^4 u]'' - 7x [x^4 u]' + 16 [x^4 u] = 0$

$y = x^4 u$

$\frac{dy}{dx} = 4x^3 u + u x^4$

$y'' = \cancel{12x^2 u + u x^4}$ $12x^2 u + u 4x^3 + u x^4$

$+ 4x^3 u$

$y'' = 12x^2 u + 8x^3 u + u x^4$

$x^2 [12x^2 u + 8x^3 u + u x^4] - 7x [4x^3 u + u x^4]$

$+ 16 [x^4 u] = 0$

$12x^4 u + 8x^5 u + u x^6 - 28x^4 u - u 7x^5$

$+ 16 x^4 u$

$u x^6 + 8x^5 u - 7x^5 u$

$u x^6 + x^5 u = 0$

let $u = v$

$v x^6 + x^5 v = 0$

$1 dv = - x^5$

$$\frac{1}{v}\frac{dv}{dx} = \cancel{x^5} - \frac{1}{x}$$

$$\int \frac{1}{v} = \int -\frac{1}{x}\,dx$$

$$\ln|v| = -\ln|x| + c_0$$

$$v = e^{-\ln|x| + c_0} = c_2/x$$

$$\int \frac{c_2}{x} = c_2 \int \frac{1}{x} = c_2 \ln|x| + c_1$$

$$y = x^4 u = x^4 [c_2 \ln|x| + c_1]$$

$$= x^4 c_1 + x^4 c_2 \ln|x|$$

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