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L.A

# Assignment No 4

Name: Affan Ahmad

Enroll: 03-134221-003

BSCS 5-A

## Question No 1.

1) An eigen values A is a scaler $\lambda$,
$\det(\lambda I - A) = 0$

2) The eigenvector A corresponding to $\lambda$
non zero solution $(\lambda I - A)x = 0$

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 0$$

$$\det \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix} - \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = 0$$

$$\det \begin{bmatrix} \lambda-2 & -1 & 0 \\ 0 & \lambda-2 & 0 \\ 0 & 0 & \lambda-2 \end{bmatrix} = 0$$

$$(\lambda-2)(\lambda-2)(\lambda-2)+0+0=0$$

$$(\lambda-2)^3=0$$

repeated zero of degree 3 $\boxed{\lambda=2}$

Now find the eigen vector

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-y=0$$

$$y=0$$

$x = t$

$z = \cancel{8}t$

$$X \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ 0 \\ t \end{bmatrix}$$

$$t \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

independant, so the dimension of the eigenspace is 2

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Question No 2)

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 5 & -10 \\ 1 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 \end{bmatrix}$$

$$A - \lambda I = \begin{bmatrix} 1-\lambda & 0 & 0 & 0 \\ 0 & 1-\lambda & 5 & -10 \\ 1 & 0 & 2-\lambda & 0 \\ 0 & 0 & 0 & 3-\lambda \end{bmatrix}$$

Taking determinant

$\det(A - \lambda I) = (1-\lambda)((1-\lambda)[(2-\lambda)(3-\lambda)+0)$
$-0) - 0) - 0$

$= (1-\lambda)[(1-\lambda)(6-5\lambda+\lambda^2)-0)+0$

$= (1-\lambda)((1-\lambda)(6-5\lambda+\lambda^2))$

$= (1-\lambda)(1-\lambda)(6-5\lambda+\lambda^2)$

$= (1-\lambda)^2(6-5\lambda+\lambda^2)$

$\lambda_1 = 1$ (Multiplicity 2)

$\lambda_2 = 3$

where $x$ is the eigenvecto corresponding to eigenvalue $\lambda$

for $\lambda = 1$, the matrix $A - \lambda I$ :

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & -10 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$$

row reduce this matrix

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 5 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 + x_3 = 0$

$5x_3 - 10x_4 = 0$

$x_1 = -x_3$

$x_4 = \frac{1}{2} x_3$

$\boxed{\text{Put } x_3 = 2}$

Let

$\boxed{x_3 = 2}$

$x_1 = -2$

$x_4 = \frac{1}{2} x_3$

$\lambda = 1 \Rightarrow$ eigenvector

$$v_1 = \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \end{bmatrix}$$

for $\lambda = 1$

choosing $x_3 = 1$

$$v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1/2 \end{bmatrix}$$

for $\lambda = 3$

$$\begin{bmatrix} -2 & 0 & 0 & 0 \\ 0 & -2 & 5 & -10 \\ 1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

so row reduce

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & -2 & 5 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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$x_1 - x_3 = 0$

$-2x_2 + 5x_3 - 10x_4 = 0$

$x_1 = x_3$

$x_2 = \frac{5}{2} x_4$

$$v_3 = \begin{bmatrix} 2 \\ 5 \\ 2 \\ 2 \end{bmatrix} \quad \text{for } \lambda = 3$$

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Question No 3)

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix}$$

$\det(A - \lambda I)$:

$$\begin{bmatrix} 2-\lambda & -1 \\ 1 & 2-\lambda \end{bmatrix}$$

$$= (\lambda - 2)^2 + 1$$

$$= \lambda^2 - 4\lambda + 5$$

$\lambda_1 = 2 + i$

$\lambda_2 = 2 - i$

$(A - (2+i)I)x = 0 \quad \begin{bmatrix} -i & -1 \\ 1 & -i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$x_1 = i x_2$

$\lambda_2 = 2 - i$

$(A - (2-i)I)x = 0$

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$$\begin{bmatrix} i & -1 \\ 1 & i \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} ix_1 & - x_2 \\ x_1 & + ix_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$x_1 = -ix_2$$

$u_1 = (i, 1)$

$u_2 = (-i, 1)$