Q no 1) $T(n) = \begin{cases} c & n=1 \\ 2T\left[\frac{n}{2}\right]+c & n>1 \end{cases}$

$T(n) = 2T\left(\frac{n}{2}\right)+c$ —— ①

$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right)+c$ —— ②

$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right)+c$ —— ③

Resub ② in ①

$T(n) = 2\left[2T\left(\frac{n}{4}\right)+c\right]+c$

$T(n) = 4T\left(\frac{n}{4}\right)+2c+c$

$T(n) = 4T\left(\frac{n}{4}\right)+3c$

$T(n) = 2^2T\left(\frac{n}{2^2}\right)+3c$ —— ④

Put 3 in ④

Date ___/___/20___

$$T(n) = 2^2\left[2T\left(\frac{n}{8}\right)+c\right]+3c$$

$$T(n) = 2^3T\left(\frac{n}{2^3}\right) + 4c+3c$$

$$T(n) = 2^3\,T\left(\frac{n}{2^3}\right) + 7c$$

$$T(n) = 2^3\,T\left(\frac{n}{2^3}\right) + (2^3-1)c$$

$K=3$

$$T(n) = 2^k\,T\left(\frac{n}{2^k}\right) + (2^k-1)c$$

$$T(n) = 2^{\log_2 n}\,T\left(\frac{n}{n}\right) + (2^{\log_2 n}-1)c$$

$n = 2^k$

$\log_2 n = \log_2 2^k$

$\log_2 n = k\,\log_2 2$

1

$\boxed{k = \log_2 n}$

$$T(n) = n\,T(1) + (n-1)c$$

Dropping constant

$O(n) \Rightarrow$ time complexity

Linear

Que 2 $T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{2n}{3}\right) + n$

$T\left(\frac{n}{3}\right) \approx T\left(\frac{2n}{3}\right) + n$

$T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{n}{3}\right) + n$

$T(n) = 2T\left(\frac{n}{3}\right) + n$ — 1

$T\left(\frac{n}{3}\right) = 2T\left(\frac{n}{3^2}\right) + \frac{n}{3}$ — 2

Resub in 1

$T(n) = 2\left[2T\left(\frac{n}{3^2}\right) + \frac{n}{3}\right] + n$

$T(n) = 4T\left(\frac{n}{3^2}\right) + \frac{5n}{3}$ — 3

Put $T\left(\frac{n}{9}\right)$ in 1

$T\left(\frac{n}{9}\right) = 2T\left(\frac{n}{3^3}\right) + \frac{n}{9}$ — 4

Put 4 in 3

$$T(n) = 4\left[2T\left(\frac{n}{3^3}\right) + \frac{n}{9}\right] + \frac{5n}{3}$$

$$T(n) = 2^3 T\left[\frac{n}{3^3}\right] + \frac{7n}{9} + \frac{5n}{3}$$

$$T(n) = 2^3\, T\left(\frac{n}{3^3}\right) + \frac{19n}{9}$$

$$T(n) = 2^k\, T\left(\frac{n}{3^k}\right) + \frac{3^k - 2^k}{3^k/3}\, n$$

$$T(n) = 2^{\log_3 n}\, T\left(\frac{n}{n}\right) + \frac{3^{\log_3 n} - 2^{\log_3 n}}{3^{\log_3 n}/3}\, n$$

$n = 3^k$

$\log n = \log 3^k$

$\log n = k \log 3$

$k = \log_3 n$

$$T(n) = 2^{\log_3 n}\, T(1) + \sum_{i=0}^{\log_3(n)-1} \frac{2^i \cdot 2^i}{3} \cdot n$$

Since $2^{\log_3(n)} = n^{\log_3(2)}$

$$T(n)\ n^{\log_3(2)} + \sum_{i=0}^{\log_3(n)-1} \left(\frac{4}{3}\right)^i n$$

Time complexity $O(n^{\log_3(4)})$

Its a polynomial time complexity