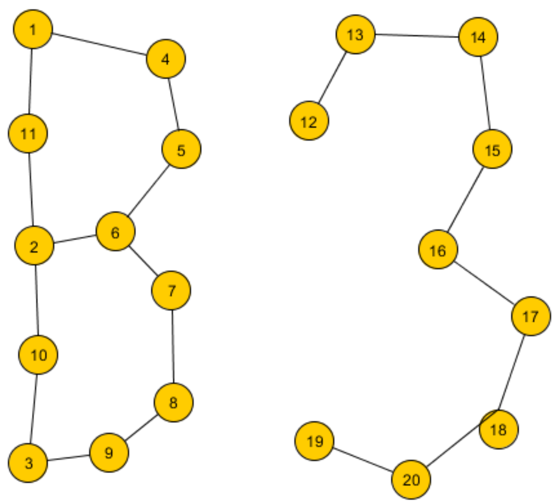


GROUP B-3

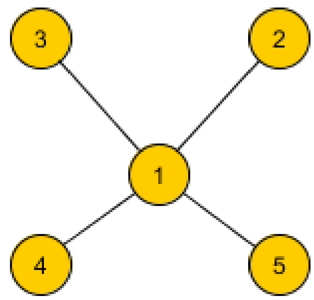
MA 2053: Graph Theory

Research Project I

Related Definitions

Degree of a Vertex (Local Degree)

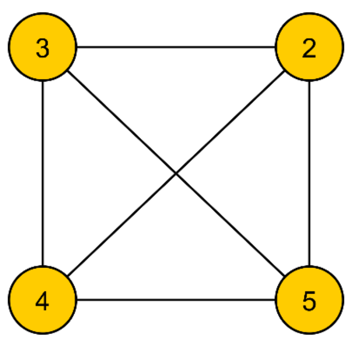
Number of graph edges which touch a particular vertex. [1]



(Degree of the Vertex 1 is 4)

Regular Graph

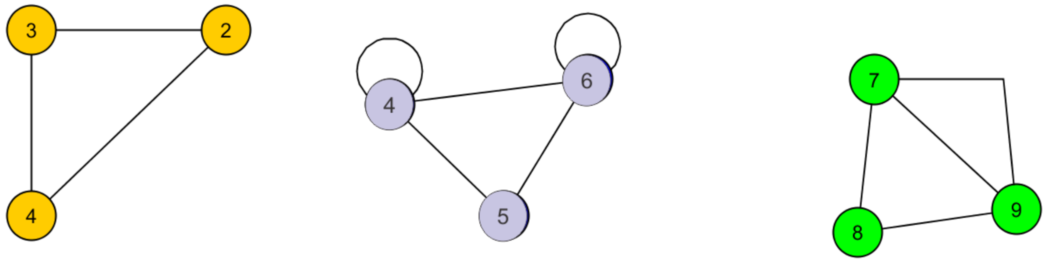
A graph is said to be a regular graph if each vertex has the same number of neighboring vertices in other words the same degree. [2]



(2, 3, 4, 5 all have degree of 3)

Simple Graph

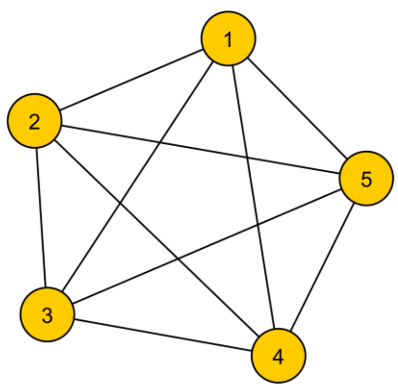
An un-weighted, undirected graph containing no graph loops or multiple edges. Either be connected or disconnected.[3]



(Only the yellow graph is a simple graph)

Complete Graph (K_n)

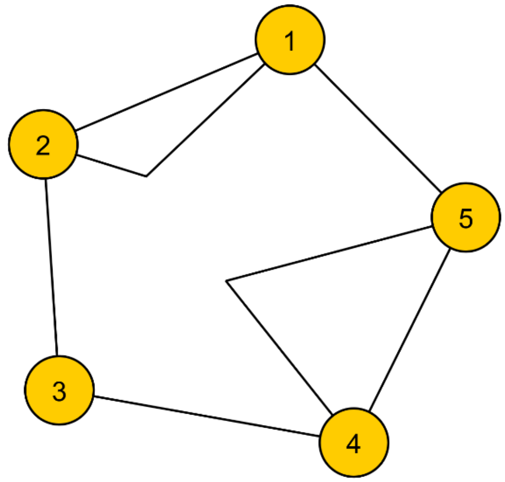
In the mathematical field of graph theory, a complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.[4]



(Each vertex is connected to the other vertices)

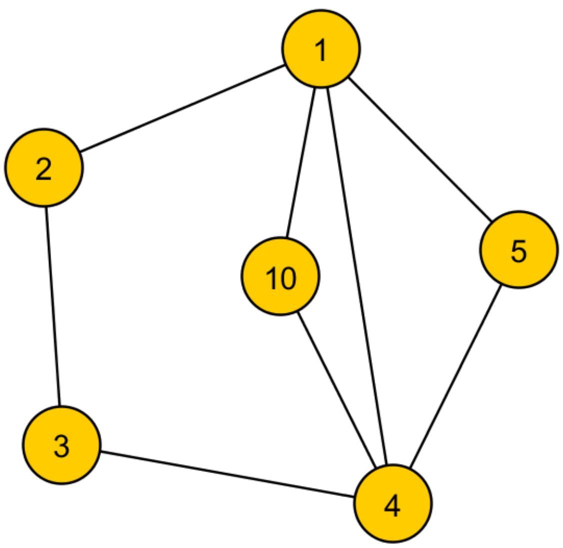
Multigraph

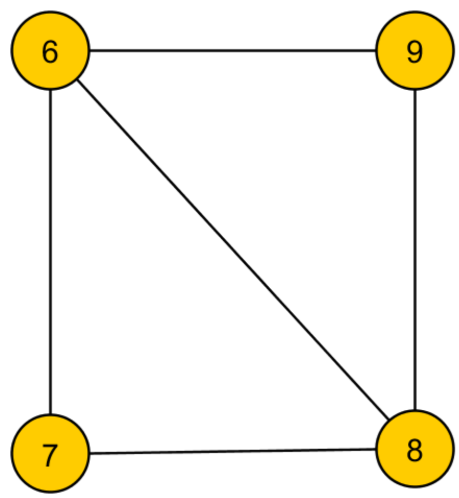
If a graph has multiple edges (i.e. edges which has the same two nodes) they are called multi graphs. [6]



Euler Graph

Euler graph is a graph whose vertices have even numbered degrees. Note that the degrees of each vertex need not be equal, just even. [7]

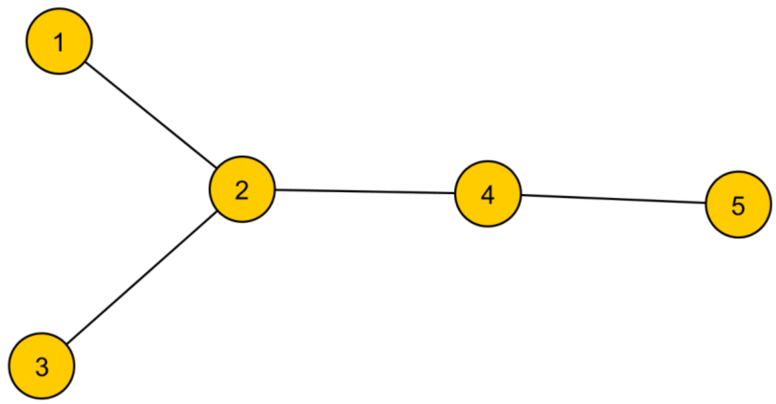




(Left graph is an Euler graph which had vertices with degrees 2 and 4, the right graph is not a Euler graph because it’s local degrees are 2 and 3)

Degree sequence

In an undirected Graph the degree sequence is the non-increasing sequence of vertex degrees in that graph. [5]



(The degree sequence of this graph is {3, 2, 1, 1, 1})

Graphical Degree Sequence

If a degree Sequence is realizable via a graph. That degree sequence is called a Graphical Degree Sequence.

(A degree sequence like {7, 1} cannot be realized into a graph)

Theorems & Proofs

Vizing's Theorem: The edges of a graph G can be properly colored using At least Δ (G) colors and at most (Δ (G) + 1) [8]

$$\Delta (G) \leq X' (G) \leq \Delta (G) + 1. [8]$$

Proof:

The lower bound is trivial. For the upper bound we do induction on the number of edges.

Suppose we have a coloring of all but one edge xy ε E (G) using {1, 2, ..., ∆(G)+1} colors . Then we wish to recolor so all the edges are colored. Note that one color is unused (“missing") at every vertex. Let xy_0 be the uncolored edge. We construct a sequence of edges xy_0, xy_1... and a sequence of Colors c_0, c_1... as follows.

Pick c_i to be a color missing at y_i. Let xy_{i+1} be an edge with color c_i. We stop with k = i when either c_i is a color unused at x, or c_k is already used on xy_j for j<k . If c_k was a color unused at x then we recolor xy_i with c_i for $0 \le i \le k$. This finishes the easy case where we can recolor the edges touching x to give a coloring for G. Otherwise we recolor xy_i with ci for $0 \le i < j$ and unicolor xy_j. Notice that c_k (red) is missing at both y_i and y_k. Let blue be a color unused at x.

1. If red is missing at x, we color xy_j red.
2. If blue is missing at y_j we color xy_j blue.
3. If blue is missing at y_k we color xy_i with ci for $j \le i < k$ and color xy_k blue. (None of the xy_i, $j \le i < k$ are red or blue.)

If none of the above hold, then we consider the sub graph of red and blue edges. The components

Of this sub graph are paths or cycles. The vertices x, y_i, y_k are the end vertices of paths. Therefore they cannot all belong to the same component [8]

Theorem: A r-regular graph G with n vertices has nr/2 edges.

Proof:

Let us count the number of edges by adding up the degrees of each vertices to get *nr.* Note for any edge E joining vertices x and y.

E is counted twice: once from the point of view of x and then from y.

Hence the total number of edges in the graph is obtainable by $[\frac{nr}{2}]$.

Questions

There exists a 4-regular simple graph with 12 edges.

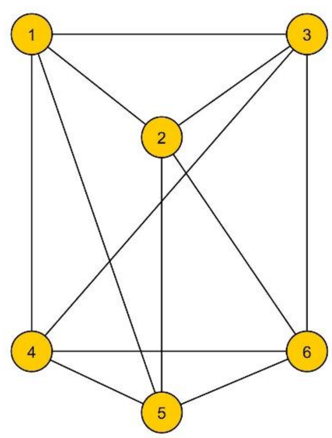
If there is a 4-regular simple graph with 12 edges the number of vertices in it will be given by the equation.

$$E = n * r/2$$

$$12 = n * 4 / 2$$

$$n = 6$$

Let us now construct a 4-regular simple graph with 12 edges and 6 vertices.



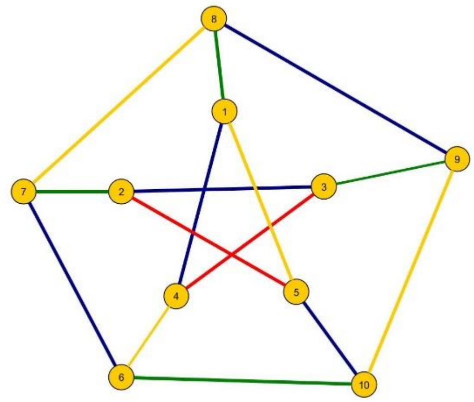
The edge chromatic number of a 3-regular graph is 3.

Let G be a 3-regular graph. According to Vizing Theorem,

$$3 \leq X'(G) \leq 4$$

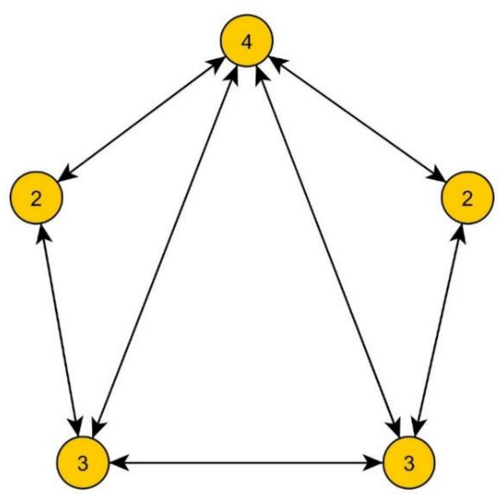
Hence there is a possibility for 3-regular graphs with an edge chromatic number which is not 3. We shall find such an example,

One such example is the famous Peterson Graph,

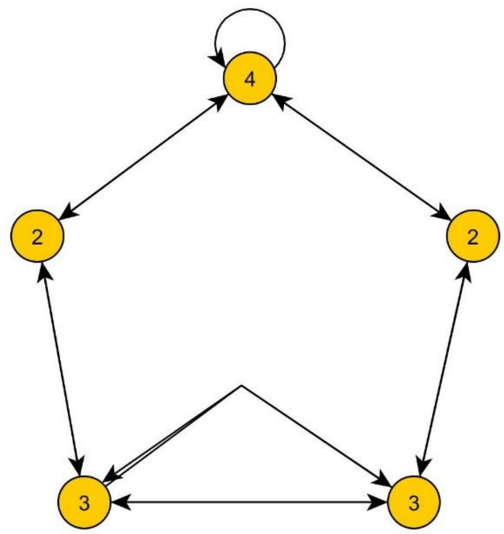


If a degree sequence is graphic, then every graph with that degree sequence is simple.

Let us begin by creating a graphic degree sequence. Using this graph,



Here the degree Sequence of the graph is {4, 3, 3, 2, and 2}. Next we shall replace the two edges inside the pentagonal circular graph in the diagram with a loop and a multi edge as follows.



Still we see that the graph realizes the same degree sequence {4, 3, 3, 2, and 2} but is not a simple graph. Hence the statement is false.

K_n Is an Euler Graph

K_n is defined as a complete graph, which is a graph of n vertices where each and every vertex is adjacent to other vertices. Let us take one of the simplest complete graphs K2,

Here the degree of each vertex is 1 which is not even and hence the graph is not an Euler Graph.

Application Question

Suppose you are a member of the organizing committee of an Inter University tennis league. In this league each of the players in the league must play at least one match with all the other players.

The organizers want the tournament to finish as soon as possible, and you are tasked with scheduling the matches.

The players need to rest between matches and each player can play only one match per week. How would you prepare the match draws?

True False Questions

1. The number of additional edges needed to combine two complete graphs $\boldsymbol{K_n}$ and $\boldsymbol{K_m}$ is equal to $\boldsymbol{n} \times \boldsymbol{m}$.
2. The number of edges in a simple graph is bounded.
3. When you remove a vertex from a complete graph. The resulting graph is also a complete graph.

References

[1] . Weisstein, Eric W. "Vertex Degree." From *MathWorld*--A Wolfram Web Resource. [Online]. Available: http://mathworld.wolfram.com/VertexDegree.html

[2]. Meringer, Markus and Weisstein, Eric W. "Regular Graph." From *MathWorld*--A Wolfram Web Resource. [Online]. Available :http://mathworld.wolfram.com/RegularGraph.html

[3]. Weisstein, Eric W. "Simple Graph." From *MathWorld*--A Wolfram Web Resource. [Online]. Available: http://mathworld.wolfram.com/SimpleGraph.html

[4]. Complete graph. (2014, July 25). In *Wikipedia, The Free Encyclopedia*. [Online]. Available: http://en.wikipedia.org/wiki/Complete_graph

[5]. Multigraph. (2014, June 6). In *Wikipedia, The Free Encyclopedia*. [Online]. Available: http://en.wikipedia.org/wiki/Multigraph

[6]. Weisstein, Eric W. "Euler Graph." From *MathWorld*--A Wolfram Web Resource. [Online]. Available: http://mathworld.wolfram.com/EulerGraph.html

[7] Degree (graph theory). (2014, June 7). In *Wikipedia, The Free Encyclopedia*. [Online]. Available: http://en.wikipedia.org/wiki/Degree_%28graph_theory%29

[8] John Fremlin. Edge colouring and Vizing's theorem. [Online]. Available: http://john.fremlin.de/schoolwork/graph/graph-theory/node12.html

Additional Bibliography

David Joyner, Minh Van Nguyen, David Phillips (2013 May 10) *“Algorithmic Graph Theory and Sage”* Self Published under GPL.

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Honour Code Pledge

I Y.K.K. Arachchi attest that,

- I have done my fair share of work on this group research project
- I have maintained a record of the relevant sources I read and have contributed to their being cited in the report and listed in the References and Additional Bibliography
- I have NOT accessed anything that the previous batch produced during their research cycles.
- My specific contributions to this entire project (research, poster, report, questions, group organization, etc.) consisted of (give details)
    1. Researching on the solution and related theorems and definition for sub question 1.
    2. Researching and finding good questions.
    3. Type settings and doing graphic design in the report.
    4. Preparing the layout for the poster with Sasini and Samitha.
    5. Developing the presentation script with others.
- Overall my contribution to the group project was roughly 20% of the work.
- I have taken an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honour Code.

Signature and ID #

Details of group members attesting to the truth of the above declaration:

1.
2.
3.
4.

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Honour Code Pledge

S.H. Palihawadana attest that,

- I have done my fair share of work on this group research project
- I have maintained a record of the relevant sources I read and have contributed to their being cited in the report and listed in the References and Additional Bibliography
- I have NOT accessed anything that the previous batch produced during their research cycles.
- My specific contributions to this entire project (research, poster, report, questions, group organization, etc.) consisted of (give details)
    1. Contributed the answer for Question 2
    2. Collaborated with Menaka and Sasini in the Proof Reading process.
    3. Researched and compiled the definitions and properly recorded the sources for citation.
    4. Researched for challenging questions with Yasiru.
    5. Arranged the Layout of the Poster.
    6. Developed the presentation script with my team mates.
- Overall my contribution to the group project was roughly 20% of the work.
- I have taken an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honour Code.

Signature and ID #

Details of group members attesting to the truth of the above declaration:

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I P.L.S Madhumali attest that,

- I have done my fair share of work on this group research project
- I have maintained a record of the relevant sources I read and have contributed to their being cited in the report and listed in the References and Additional Bibliography
- I have NOT accessed anything that the previous batch produced during their research cycles.
- My specific contributions to this entire project (research, poster, report, questions, group organization, etc.) consisted of (give details)
    1. Worked on designing the Poster.
    2. Validated the answers for the Questions.
    3. Proofread the report and the poster content.
    4. Help compile the list of definitions.
    5. Worked on the presentation script and ideas.
    6.
- Overall my contribution to the group project was roughly 20% of the work.
- I have taken an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honour Code.

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Honour Code Pledge

I K. Linganesan attest that,

- I have done my fair share of work on this group research project
- I have maintained a record of the relevant sources I read and have contributed to their being cited in the report and listed in the References and Additional Bibliography
- I have NOT accessed anything that the previous batch produced during their research cycles.
- My specific contributions to this entire project (research, poster, report, questions, group organization, etc.) consisted of (give details)
    1. Contributed answer to question 3
    2. Worked with Yasiru on typing the report.
    3. Cross checked the research done by team mates.
    4. Setup online working frame works (Google docs, Trello worksheet)
    5. Researched definitions and proofs.
    6. Worked on creating the presentation script.
- Overall my contribution to the group project was roughly 20% of the work.
- I have taken an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honour Code.

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Honour Code Pledge

I Menaka Lahiru Sirisena attest that,

- I have done my fair share of work on this group research project
- I have maintained a record of the relevant sources I read and have contributed to their being cited in the report and listed in the References and Additional Bibliography
- I have NOT accessed anything that the previous batch produced during their research cycles.
- My specific contributions to this entire project (research, poster, report, questions, group organization, etc.) consisted of (give details)
    1. Contributed answer to question 4
    2. Proof read the report with Sasini and Samitha.
    3. Researched on the definitions and
    4. Helped Yasiru and Samitha find new questions.
    5. Worked on developing answers for new questions.
    6. Worked on the presentation script and ideas.
- Overall my contribution to the group project was roughly 20% of the work.
- I have taken an active part in seeing to it that others as well as myself uphold the spirit and letter of the Honour Code.

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