S O L U T I O N S

Introduction to University Mathematics 2017

MATLAB Worksheet I

Complete the following tasks and hand in before the end of the lab session.

1. Basic calculator. MATLAB can be used as a basic calculator. For example, to evaluate

$$5 \times \left(2.4 - 1.3 + \frac{3}{8}\right)$$

type `5*(2.4 - 1.3 + 3/8)` in the command window (try it).

You should get 7.3750. Note that spaces are ignored by MATLAB, but they can help make complicated codes more readable. Also note that division takes precedence over $+$ and $-$ in the brackets, so there is no need to type `+(3/8)`. However, use brackets when in doubt on what MATLAB might do first.

Evaluate the following on MATLAB. Use brackets wisely.

$\left(9.7 - \frac{2}{15}\right) \times \frac{19}{8}$ Ans $=$ **22.7208**

$\frac{2 \times (3.142 - 1.75)}{\frac{7}{12} + 2.15} + \frac{5}{12}$ Ans $=$ **1.4352**

$1 + \frac{2}{3 + \frac{4}{5+\frac{6}{7}}}$ Ans $=$ **1.5430**

2. Format. By default, MATLAB does all calculations in the so-called *double-precision* system. In this system, every real number is stored as a sequence of 64 binary numbers (zeroes and ones). This is roughly equivalent to 15 or 16 significant digits.

But we don't often need to see so many digits - sometimes a few decimal places will do. MATLAB users have the option to choose which `format` they'd like to see.

Try experimenting with more calculations. How many decimal places is displayed in the default format? Ans: 4

Suppose we want to see more decimal places, or work with the so-called *scientific* format (*e.g.* 2.1×10^{-12}). How do we change the display format? Well, just type

`doc format`

which brings up a webpage-style documentation. Alternatively, type 'format' into the search box in the top right-hand corner.

Read the help document and experiment with different formats.

(a) Evaluate 1/701, giving your answer in the specified formats:

LONG Ans = **0.001426533523538**

SHORT Ans = **0.0014**

SHORTE Ans = $\mathbf{1.4265 \times 10^{-3}}$

(b) What format option will produce the following output? Ans: **rat**

```
>> 5/16+2/7
ans =
      67/112
```

Now change the format back to `short` (though you might need to change it later).

At this point, your screen might be a bit cluttered. You might like to start with a clean screen by typing `clc`. No need to fear that you might forget some previous command - they can be accessed simply by pressing the *up arrow*. Try it.

3. Constants. MATLAB has a stock of constants, which include

`pi`	π	
`i`	$\sqrt{-1}$	
`j`	$\sqrt{-1}$	(Engineers use j instead of i. Try `i*j`.)
`inf`	∞	(You should never have to type this)
`eps`	'machine epsilon'	(Typical size of error when the computer does rounding off)
`NaN`	Not a Number	(Something undefined - something has gone wrong in your code)

Note that these are *reserved* names: as far as possible we must try not to create variables that have these names, or there will be chaos! More on this in the coming weeks.

(a) Find the numerical value of the machine epsilon. Ans: $\mathbf{2.2204 \times 10^{-16}}$**. Actually it is exactly** 2^{-52} **(see Q3a)**

(b) Write down an operation which produces `Inf`. Ans: 1/0

(c) Write down an operation which produces `NaN` Ans: 0/0

(d) Evaluate $\dfrac{1-3i}{4+i}$. Give your answer to 4 decimal places. Ans: **0.0588 – 0.7647i**

(e) Use MATLAB to find a rational approximation of π. Explain how you did this. To how many decimal places is this approximation accurate?

With `format rat`, we get $\pi \approx 355/113$**. To see the accuracy, use `format long` to compare this fraction with actual value of `pi` to see that it is accurate to 6 decimal places.**

(f) MATLAB does not store a value for $e = 2.718\ldots$. Instead, MATLAB has the function `exp(x)` which calculates e^x.

Check that `exp(0)` and `exp(1)` are what you expect. Many students make the mistake of typing `e^x` instead of `exp(x)`. But MATLAB doesn't understand what this means (try it). You've been warned!

Write down the value of $1/e^4$ (to 4 decimal places). Ans: `exp(-4)` ≈ 0.0183.

4. Scientific calculator. Here are examples of some maths operations and their commands. Like all MATLAB functions and variables, these *are* case-sensitive. Study each entry below very carefully.

2^{12}	`2^12`	$3^{-1/3}$	`3^(-1/3)`
3.5×10^{-12}	`3.5e-12`	$\sqrt{2}$	`sqrt(2)`
$\sin x$	`sin(x)`	$\operatorname{cosec} x$	`csc(x)`
$\cos x$	`cos(x)`	$\sec x$	`sec(x)`
$\tan x$	`tan(x)`	$\cot x$	`cot(x)`
e^x	`exp(x)`	$\ln x$	`log(x)`
$\lvert x\rvert$	`abs(x)`	$n!$	`factorial(n)`

Note that all angles in the trigonometric functions are assumed to be in radian.

(a) Use MATLAB to evaluate the following. Leave answers to 4 dec. pl. where needed.

$$\begin{aligned}
\tan(5.3 \times 10^{50}) &= \mathbf{3.9970} \\
\sin i &= \mathbf{1.1752i} \\
2^{4^3} &= \mathbf{1.8447 \times 10^{19}} \qquad \text{– type } \texttt{2^(4^3)} \\
\left(2^4\right)^3 &= \mathbf{4096} \\
4^{5/2} &= \mathbf{32} \\
4^5/2 &= \mathbf{512} \\
i^i &= \mathbf{0.2079} \quad \textbf{(In fact } i^i = e^{-\pi/2}\textbf{)} \\
\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}} &= \mathbf{1.9904} \textbf{ (this sequence converges to 2)} \\
\sqrt[3]{328509} &= \mathbf{69} \\
\ln\cos 10^\circ &= \mathbf{-0.0153} \quad \textbf{(convert } 10^\circ = \pi/18\textbf{)} \\
\log_{10}(-\pi) &= \mathbf{0.4971{+}1.3644i} \\
\log_2(\text{machine epsilon}) &= \mathbf{-52}
\end{aligned}$$

Considering the last part, deduce the exact value of machine epsilon. 2^{-52}

(b) Calculate $\sin 0$ and $\sin \pi$ on MATLAB. Why do you think we get different answers?

pi isn't exactly π due to a round-off error

(c) The value of the factorial, $n!$, grows notoriously quickly. Read the help document on the factorial.

Beyond a certain integer, n, MATLAB is no longer able to calculate the exact value of $n!$ (and can only give an approximation). This is because the result requires more than 64 binary numbers to store exactly.
What is the largest value of n such that $n!$ is exact in MATLAB?

ANS: $n =$ **21**

Beyond another integer, N, MATLAB cannot even give an approximation for $N!$ as it's too large for MATLAB to handle in double precision (and MATLAB reports an `Inf` instead). What is the largest value of N such that $N!$ can be approximated by MATLAB?

ANS: $N =$ **170**

(d) Use MATLAB to evaluate the binomial coefficient $\binom{60}{50}$. How did you do it?

Either do `factorial(60)/(factorial(50)*factorial(10))` **or** `nchoosek(60,50)`**. The answer is 7.5394×10^{10}.**

5. Help! Explore MATLAB's help documents and answer how the following questions can be solved in MATLAB. In each case, **write down the command used**, and not just the answer. The first one has been done for you.

(a) Calculate $\sinh(1.5 \times 10^{-5})$.

Command: `sinh(1.5e-5)`. Answer $= 1.5000 \times 10^{-5}$

(b) Calculate $\sin(10°)$ without converting $10°$ to radian first.

`sind(10)=0.1736`

(c) Calculate $\tan^{-1}(0.5)$.

`atan(0.5)=0.4636`

(d) Find the remainder when 5×10^{30} is divided by 299.

rem(5×10^{30}, 299)=8

(e) Is 1234567 a prime number?

Use `factor(1234567)` to see that $1234567 = 127 \times 9721$, so clearly not a prime. Alternatively, `isprime(1234567)=0` (the `0` here means false, see logical class in worksheet 2.)

(f) How many prime numbers there are between 1000 and 5000?

length(primes(5000))-length(primes(1000)) = 501