**Introduction to University Mathematics 2018**

MATLAB Worksheet V

# S O L U T I O N S

▶ **TASK 1**: Modify `emac.mlx` so that the command `emac(x)` returns an estimate of $e^x$ using the Maclaurin series with terms up to $x^5$ (I'll call this the *5th-order* series). Test it to make sure that it is more accurate than the cubic series.

```
1  function  y = emac(x)
2  % emac(x) gives the value of e^x using Maclaurin series up to x^5
3  y=1 + x + x^2/2 + x^3/6 + x^4/24 + x^5/120;
4  end
```

▶ **TASK 2**: Modify `emac.mlx` so that it gives an output of the form `[y,err]`, where `y` = 5th-order Maclaurin estimate for $e^x$, and `err` = the fractional error.

```
1  function  [y,err] = emac(x)
2  % emac(x) gives an array of two numbers [y,err]
3  % y=Maclaurin's series evalauted at x, err = fractional error
4  y =1 + x + x^2/2 + x^3/6 + x^4/24 + x^5/120;
5  err = y/exp(x) -1;
6  end
```

Alternatively, in line 5, let `A=exp(x)` then calculate `err= (y-A)/A`.
Avoid calculating `exp(x)` twice. Always avoid redundant calculations.

▶ **TASK 3**: Modify `emac.mlx` so that it takes 2 inputs `(x,N)` (where `x` is a number, and `N` is an integer) and produces two outputs `[y,err]`, as described above.

```
1  function  [y,err] = emac(x,N)
2  % emac(x) gives an array of two numbers [y,err]
3  % y=Maclaurin's series of degree N evaluated at x, err = fractional error
4  A=[0:N];
5  B=x.^A./factorial(A);
6  y=sum(B);
7  err=y/exp(x) -1;
8  end
```

**Marking scheme for `emac.mlx` [10 marks]**

- A very good professional looking code = 10 marks
- 50% maximum if it gives the wrong result
- 40% maximum if it does not run
- No useful annotation = $-1$ mark
- Misleading annotation (e.g. telling the user to use `emac(x)`)= $-1$ mark
- Missing semicolons = $-1$ mark
- No mention of what $N$ is = $-1$ mark.
- Inaccurate info on what $N$ means (*e.g.* telling the user that $N$ is the number of terms)= $-0.5$ mark.
- Calculating `exp(x)` twice = $-0.5$ mark
- Other errors will be penalised at the marker's discretion.

However, If your code gives the correct numerical answers with no serious error, the minimum you will get is 5/10.

▶ **TASK 4**: Plot the graph of the function $y = e^x$, *smoothly* over the interval $-3 \leq x \leq 3$. Use `linspace`.

```
x = linspace(-3,3);
y = exp(x);
plot(x, y)
xlabel('x-axis')
ylabel('y-axis')
title('function y=e^x');
```

▶ **TASK 5**: Plot all these 3 functions on the same set of axes (with $-3 \leq x \leq 3$).

- $y = 1 + x$,
- $y = 1 + x + \frac{1}{2}x^2$
- $y = e^x$

```
x = linspace(-3,3);
y1 = 1+x;
y2 = 1+x+0.5*x.^2;
y3 = exp(x);
plot(x, y1, 'b*-', x , y2, 'mo--', x, y3, 'kx:')
xlabel('x-axis')
ylabel('y-axis')
title('three functions')
legend('linear','quadratic','exponential')
```

▶ **TASK 6**: Create a plot similar to the one below.

```
x = -3:0.1:3;
y1 = exp(x);
% use eloop function to create three y vectors
y2 = eloop(x,2);
y3 = eloop(x,3);
y4 = eloop(x,4);
% plot using different line types and colors
plot(x,y1,'k-',x,y2,'b--',x,y3,'r:',x,y4,'m-.')
xlabel('x')
ylabel('y')
title('e^x and its Maclaurin series of various orders')
legend('e^x','up to x^2','up to x^3','up to x^4','Location','northwest')
```

By the way, the code `eloop.mlx` can actually be much improved by avoiding the factorial altogether. Can you see how? (again, avoid redundant calculations)

Here is a list of some common mistakes.

- All curves drawn in solid lines.
- The legend shows the wrong order or is misleading (e.g. "$x^2$" instead of "terms up to $x^2$").
- Curves plotted with near-invisible lines.
- Using thick, attention-grabbing lines for unimportant curves, and thin, faint lines for important curves (like $y = e^x$).
- Legend box placed at a strange position.
- Typos in title/legend.

### Interpretation

This figure tells us that higher-order Maclaurin series are better approximations to $e^x$ (if $N$ is large, the $N$th order curve will closely match $y = e^x$).

Amazingly, the Maclaurin series for $e^x$ converges for *all* $x$ (not just small $x$ – as you can tell from the graph. You will see this in Calculus).

### Marking scheme for the graph [10 marks]

- Severe penalty for giving wrong mathematical information such as ploting the wrong graphs or a wrong label in the legend. ($-2$ or $-3$ marks)
- Less severe penalty ($-0.5$ or $-1$ mark) for aesthetics offence such as indistinguishable or invisible lines.