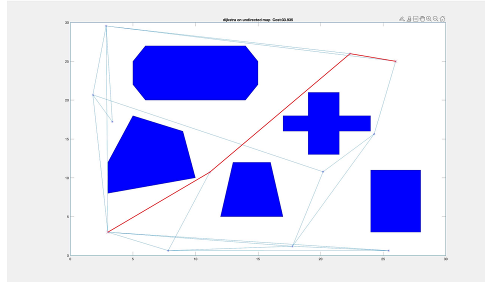
Assignment6

Saturday, 4 March 2023 2:18 PM

1.What are your observations by changing the number of nodes? How many edges were constructed for respective *'nnodes' for* Part-A and Part-B?

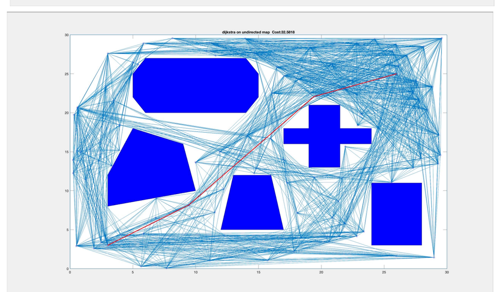
By changing the no. of nodes we found out that

① If we sample very few nodes then it might happen that the no. of nodes is too less and (obstructed by obstacles) in some cases that an edge cannot be formed between them. Thus we may not get any path connecting start node & end node.



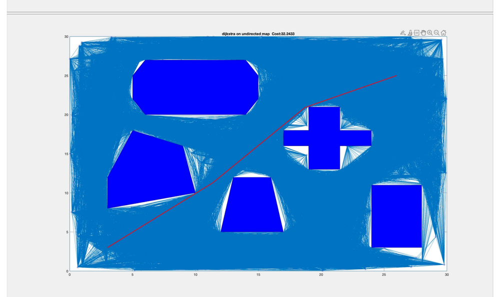
for 10 nodes

② As the no. of nodes sampled, increases we get a path which closely resembles the true shortest path with least cost. It has been observed that with the increase in the no. of nodes sampled the path passes through the points which are in close proximity with the vertices of the obstacles. From this observation it can be extrapolated that if sampled all possible points the path will pass "through" the obstacle vertices and that path will be the least cost path.



for 100 nodes

edges constructed for 'n' nodes in part A & B – ${}^{n\,nodes}C_2$ – No. of edges passing through the obstacles.



for 500 nodes

In case of part B the "no. of edges passing through the obstacles" will be more. ∴ The total edges constructed will be lesser as compared to part A.

2.How do you approach growing *CO (configuration space of obstacles)* space and configuration-space boundary for Part-B?

Ⓐ Growing configuration space of obstacles –

① The approach used for growing the CO is that of Taking the "Minkowski sum".
It is defined as.
for $S \,\&\, T \subseteq \mathbb{R}^2$
$$S \oplus T = \{p+q \mid p \in S, q \in T\}$$

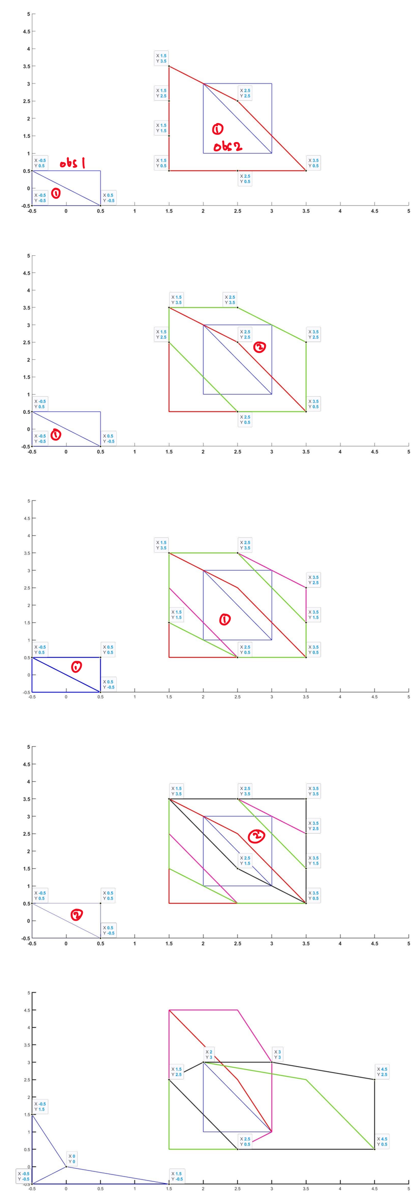
This is basically like vector addition of each point in sets S & T.

② We need to take inversion of S i.e $-S = \{-p \mid p \in S\}$. Here every point in set S has it's sign reversed.

③ Let R be our polygonal robot & P be the obstacle then,
$$M(P) = P \oplus -R(0,0) = P \ominus R(0,0)$$
This is also known as "Minkowski difference"

The Minkowski difference between P & R(0,0) gives us the "forbidden space"

forbidden space – $C(P) = \{(x,y) \mid R(x,y) \cap P \neq \phi\}$



Step by step addition of each of triangles of obs1 & obs2

And formation of convex Hulls

Red convex hull due to obs1 Δ1 obs2 Δ1

Green → obs1 Δ1 obs2 Δ2

pink → obs1 Δ2 obs2 Δ1

black obs1 Δ2 obs2 Δ2

union of all this will give final Minkowski region i.e grown obstacle

Some procedure as above but with one of the obj being non convex

Our Implementation –

The minkowski sum method is implemented using the "Star algorithm". for convex polygonal obstacles and robot.

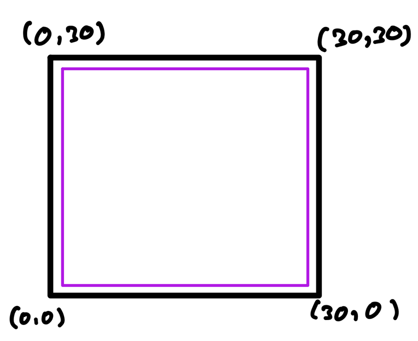
In our case as we have few non convex polygons as well, such as a "plus" sign object. "✚" we are approaching it by considering this non-convex object as a "union of convex sets". This is done by using "Triangulation" function

i) This function "triangulizes" any given polygon thus converting into the union of convex sets (In this case triangles.)

ii) After which, we take combinations of the vertices of a triangle from object 1 with each of the vertices from triangles of object 2 we do this procedure for all the triangles present in object 1 (Here object 1 is robot and object 2 is obstacle)

iii) The combinations of the vertices thus taken are added. mimicing the minkowski sum. and for each of the triangle we form a convex hull. The union of all such convex hulls over all the triangles provide us with the → "Required grown obstacle"

Ⓑ Growing configuration space of the boundary –




As a result of applying minkowski sum to the polygonal robot we know that the workspace "shrinks".
To implement this we came up with following approach.

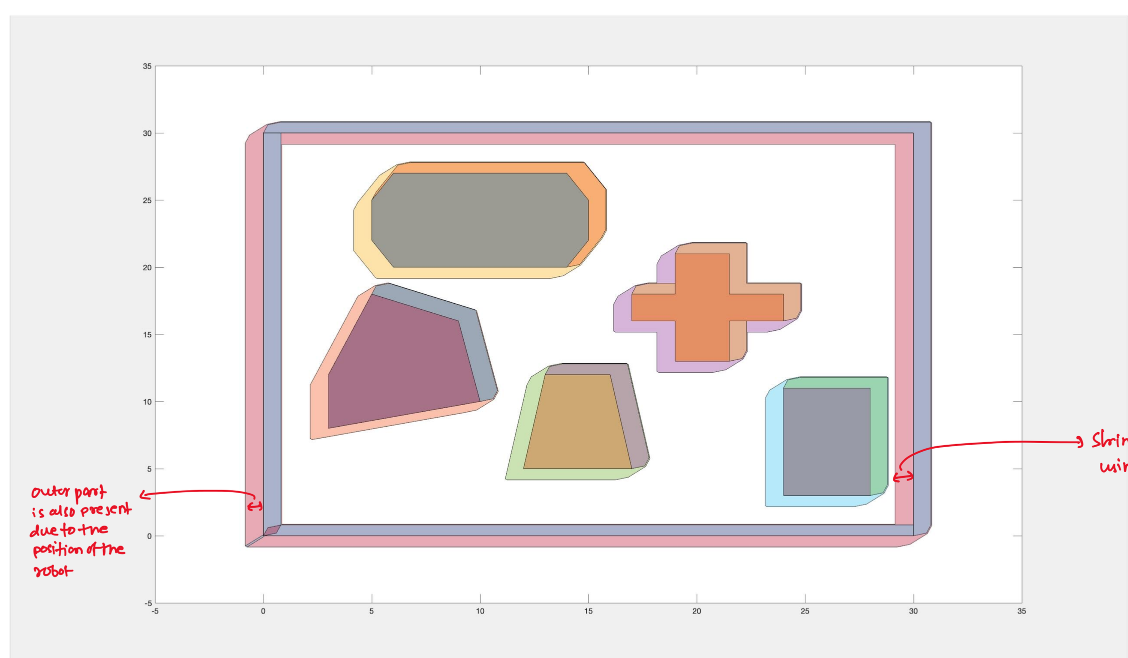
① consider the whole boundary of the workspace as an obstacle with a hole inside it. And the walls of that obstacle are "infinitesimly thin"

② This idea is represented by the figure along side with the inner region of the purple square being the "free space" for the robot to traverse.

③ But practicly as it is not possible to create such infinitesimally small walled polygon we took the walls as 0.01 unit thick.

④ After which the above mentioned approach for CO of obstacles is directly applicable for this problem.

⑤ Using which we got the "shrinked" boundry region.



outer part is also present due to the position of the robot

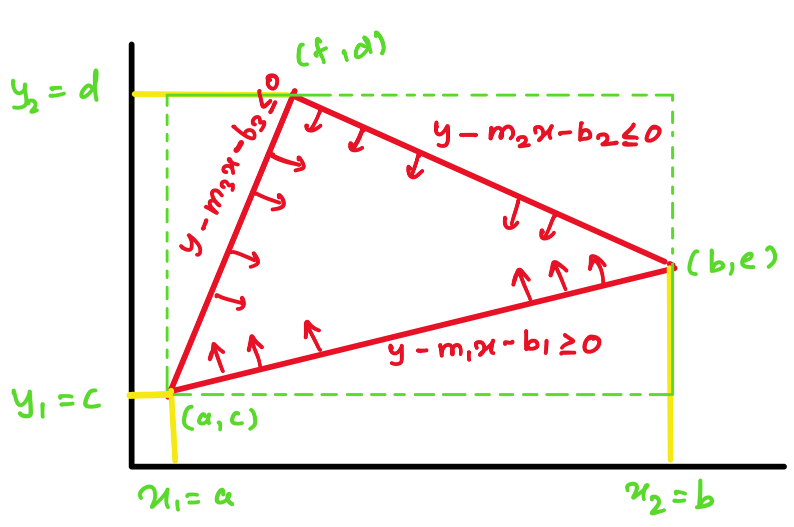
Shrinked boundary using Minkowski Sum

3.What are your observations on grown convex obstacles and non-convex obstacles Part-B?

For grown convex obstacles – ① The in between step of triangulation is not essential for convex polygons.
② We can directly apply Minkowski

For grown non-convex obstacles – ① Application of direct "star algorithm" (Minkowski sum) is not possible in this case, so we first perform triangulation
② Then we apply "star algorithm" on each of the triangles
③ And take the union over all such small triangles to grow the concave obstacle
④ In both the cases no. of edges reduce as we discard those edges which are cutting through the obstacle in the case of Non convex obstacles. These no. of edges will be lesser as there is more chance of an edge passing through an object.

4. For a configuration space with triangular boundary, what is your approach to generate nodes?




Here we are considering a triangular boundary with a triangle connecting 3 vertices (a,c), (b,e), (f,d)

Approach for sampling the nodes from triangular region

① Consider the outer rectangle joining the vertices of the triangle

② We will sample points from this rectangle using uniform sampling within $a \leq x \leq b$ & $c \leq y \leq d$, $U[a,b]$ over $x$ & $U[c,d]$ over $y$

③ But the point will be accepted only if it satisfies the non-linearity constraint of the triangle as follows.

Accept $(x,y)$ where, $x \in U[a,b]$ & $y \in [c,d]$
if and only if
$(y - m_1x - b_1 \geq 0)$ AND $(y - m_2x - b_2 \leq 0)$ AND $(y - m_3x - b_3 \leq 0)$