MATH 2216: Intro to Abstract Math: Assignment #6

Original Homework due on Monday, October 22th, 2017 at 3:00pm

Problem Set Six: #1-7

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Problem 1

Solution
a.
Let $y \in [0, \infty]$. Then, choose $x = \sqrt{y}$, so that $f(x) = (\sqrt{y})^2 = y = y$, and f is surjective.
b.
Let $z \in [-\infty, \infty]$. Any z can be written as $r(\cos\phi + i\sin\phi)$ with $r \geq 0$ and that is the square of (among others) $\sqrt{r}(\cos\frac{\phi}{2} + i\sin\frac{\phi}{2})$.
Therefore $f(z)$ is surjective.

Problem 2

Solution
a.
Let x be an arbitrary element of $(A\times B)\cup(C\times D)$. That means that x is an element of $(A\times B)\cup(C\times D)$ that isn't special. The defining properties of being in $(A\times B)\cup(C\times D)$, then prove x has to be in $(A\cup C)\times(B\cup D)$ then any element in $(A \times B) \cup (C \times D)$ must be in $(A \cup C) \times (B \cup D)$.
The first thing we can say for sure is that x must be in either $A \times B$ or in $C \times D$. Both of these are sets of ordered pairs, so x must be an ordered pair (w, z) with either w in A and z in B or w in C and z in D.
If the first, then w is in $A \cup C$ and z is in $B \cup D$, so $x = (w, z)$ is in $(A \cup C) \times (B \cup D)$. This proves anything in $(A \times B) \cup (C \times D)$ must be in $(A \cup C) \times (B \cup D)$.
b.
Let $(x, y) \in (A \times B) \cap (C \times D)$. Then we may assume that $(x, y) \in (A \times B) \wedge (x, y) \in (C \times D)$. Since (x,y) is in both A and B, we know $(x \in A \wedge y \in B) \wedge (x \in C \wedge y \in D)$. If we put the x's together and the y's together, we obtain $(x \in A \wedge x \in C) \wedge (y \in B \wedge y \in D)$. We may now assume the intersections, $(x \in A \cap C) \wedge (y \in B \cap D)$. Then we know finally that, $(x, y) \in (A \cap C) \times (B \cap D)$.
c.
We must first prove the implication

$$A \subset C, B \subset D \qquad A \times B \subseteq C \times D$$

Assume the left hand side is true.
So we assume $A \subset C$ and $B \subset D$. Now we need to prove $A \times B \subseteq C \times D$. We now need to prove another implication

$$x \in A \times B \qquad x \in CB$$

Again, we assume the left-hand side to be true. And we try to deduce the right-hand side. So assume $x \in AB$. We can write such an element x as a pair (a, b) with $a \in A$ and $b \in B$. Notice that we assumed $A \subset C$ and $B \subset D$. So we have (by definition!) $a \in C$ and $b \in D$. Therefore $x = (a, b) \in CB$. So we succesfully proved the second implication. This means that we proved that $A \times B \subseteq C \times D$. We proved this from the assumption $A \subset C, B \subset D$.

Problem 3

Solution
For $g \circ f$ to be invertible, g only needs to be invertible when restricted the range of f. It may not actually be invertible on its whole domain. For example, let $f(x) = e^x$ and $g(x) = x^2$.

Problem 4

Solution
a.
Let $x \in A$ be arbitrary.. Let there be a trivial $B = f[A] \subseteq Y$. Now what elements of X belong to the set $f^{-1}\big[f[A]\big] = f^{-1}[B]$? By definition $f^{-1}[B] = \{a \in A : f(a) \in B\}$. Is it true that $f(x) \in B$? If so, $x \in f^{-1}[B] = f^{-1}\big[f[A]\big]$, and youll have shown that $A \subseteq f^{-1}\big[f[A]\big]$.
b.
Let $f(A) = x^2$. Then, $f^{-1}(1) \neq= 1$. Values can either be -1, or 1. However, this is not always the case as seen with x^2.

Problem 5

Solution
a.
Take an $x \in f(A)$. Then there is an $y \in A$ such that $f(y) = x$. Now, we also have $y \in B$, since $A \subset B$. Therefore $f(y) \in f(B)$, which means $x \in f(B)$.
b.
You have

$$x \in f^{-1}(A)$$

you can say $f(x) \in A$ and conversely. Thus

$$x \in f^{-1}(A) \implies f(x) \in A \implies f(x) \in B \implies x \in f^{-1}(B)$$

Problem 6

Solution
a.
Apply distributing rule: $(1^2 + 2 \cdot 1 + 2i + (2i)^2$

$$\begin{aligned} &= 1 + 1 \cdot 2 \cdot 2i + (2i)^2 \\ &= 1 + 4i - 4 \\ &= -3 + 4i \end{aligned}$$

Problem 6 continued on next page. . .

b.

$$
\begin{aligned}
&= \frac{1}{1+2i} \\
&= \frac{1 \cdot (1-2i)}{(1+2i)(1-2i)} \\
&= \frac{1-2i}{5}
\end{aligned}
$$

Problem 7

Let $z = a + bi \in \mathbb{C}$ for $a, b \in \mathbb{R}$. If $z \in \mathbb{R}$, then $b = 0$ and $z = a$. Then we know that $z = a - 0i = z$. On the contrary, $\overline{_{z=z}}$, we have $a + bi = a - bi$, then we know that $b = 0$. Finally, $z = a \in \mathbb{R}$.