GUC
German University in Cairo
Faculty of Engineering and Material Science
Department of Mechatronics Engineering

Control Engineering (ENME503)

Assignment #1

Dynamic Systems: Modelling and Analysis

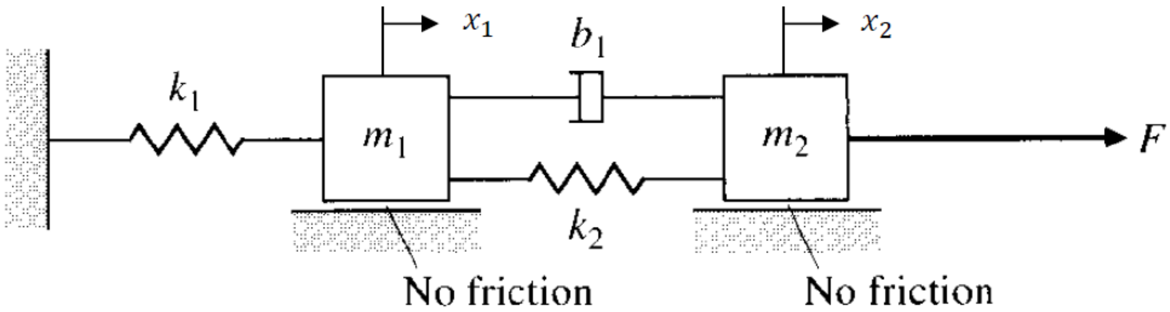
Due Date: Thursday, 4-November-2021

Name:	
GUC ID:	
Tutorial Number:	

Problem 1:

Write the Equations of Motion (EOM) for the two Degrees of Freedom (DOF) system shown in the Figure.

(Note: Show your steps)



Solution:

Problem 2:

A mass - spring - damper system has a mass of 200 Kg, a damping coefficient of 100 N.m/s and an undamped natural frequency of 2 rad/s.

Find the following:

a) The stiffness coefficient
b) The damping ratio
c) The damped natural frequency
d) The system's damping type
e) Plot the system's response using MATLAB for the following initial conditions

$$x(0) = 2\ [m] \qquad , \qquad v(0) = 0\ \ [m/s]$$

(Note: Append your code and the results of the simulation)

Hint: Please check the MATLAB Guide posted on the CMS to get started with MATLAB.

Solution:

Problem 3:

For a second-order system,

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = F(t)$$

a) In case of an **undamped** system (i.e. $\zeta = 0$), find the **particular solution** if the natural frequency is $\omega_n = 4\, rad/s$ and the external force is $F(t) = 2\cos(4t)$.
Hint: Review exponential input theorem (special case $P(\alpha)$=0) in Lecture 3.

b) In case of an **undamped** system (i.e. $\zeta = 0$) with a natural frequency of $\omega_n = 4\, rad/$, find the **total solution** if the external force is $F(t) = 2\cos(3t) + \sin(t)$ and the initial conditions are $x(0) = 1$ and $\dot{x}(0) = 0$.
Hint: Use the principle of superposition.

Solution:

Problem 4:

In Lecture 3, you have obtained the general form of the total solution for an **underdamped** system under **harmonic excitation** as follows.

$$x(t) = \underbrace{Ae^{-\zeta\omega_n t}\sin(\omega_d t + \phi)}_{\text{homogeneous or transient solution}} + \underbrace{X\cos(\omega t - \theta)}_{\text{particular or steady state solution}}$$

$$\phi = \tan^{-1}\frac{\omega_d(x_0 - X\cos\theta)}{v_0 + (x_0 - X\cos\theta)\zeta\omega_n - \omega X\sin\theta}, \quad A = \frac{x_0 - X\cos\theta}{\sin\phi},$$

$$\theta = \tan^{-1}\frac{2\zeta\omega_n\omega}{\omega_n^2 - \omega^2} \quad \text{and} \quad X = \frac{f_0}{\sqrt{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}}$$

Ignoring the transient solution,

a) Plot using MATLAB the steady-state solution (i.e. x(t) against time) for different damping ratios (i.e. $\zeta = 0.1, 0.25, 0.5, 0.75, 0.9$) and a fixed natural and forcing frequency $\omega_n = \omega = 2\ rad/s$. Plot the first 10 seconds.
b) Plot using MATLAB the steady-state solution (i.e. x(t) against time) for different forcing frequencies (i.e. $\omega = 0.1, 0.5, 1, 2, 4, 8\ rad/s$) and a fixed damping ratio of $\zeta = 0.2$ and a fixed natural frequency of $\omega_n = 2\ rad/s$. Plot the first 10 seconds.

Append your code and results of the simulation.

Hint: Please check the MATLAB Guide posted on the CMS to get started with MATLAB.

Solution: