Name.................................... Teacher......................................

Sydney Technical High School

Mathematics
Preliminary Assessment Task 1
March 2011

General Instructions

- Working Time – 70 minutes.
- Write using a blue or black pen.
- Approved calculators may be used.
- All necessary working should be shown for every question. Marks may be deducted for careless or poorly arranged work.
- Begin each question on a new page.

Total marks (63)

- Attempt Questions 1-8.
- All questions are of equal value.
- Mark values are shown with the questions

Qu 1	Qu 2	Qu 3	Qu 4	Qu 5	Qu 6	Qu 7	Qu 8	Total
								63

Question 1 (8 marks)

a) Simplify $\sqrt{75} + \sqrt{32} - \sqrt{27}$ 2

b) Express $0.1\overset{..}{2}\overset{}{5}$ as a fraction in simplest form. 1

c) Evaluate $\dfrac{\sqrt{16.98 + 9.074}}{4.99}$ correct to 3 significant figures 2

d) Simplify $\dfrac{x^2 + 12x + 36}{x^2 + 6x}$ 2

e) Fully factorise $8 - 27d^3$ 1

Question 2 (start a new page) (8 marks)

a) Solve for x :
    (i) $\dfrac{2x}{3} - 4 = x + 2$ 2
    (ii) $(x + 3)^2 = 7$ 2

b) Express $\dfrac{x+1}{x^2-1} - \dfrac{x-1}{x+1}$ as a fraction in its lowest terms 3

c) Express $a^{-3} \times (8a^6)^{\frac{1}{3}}$ in simplest form, without the use of negative indices 1

Question 3 (start a new page) (8 marks)

a) Solve simultaneously $x - 4y + 12 = 0$ and $y = 3 - 2x$. 3

b) Solve $2x^2 - 5x - 12 = 0$ 2

c) Solve and sketch the solution set of $|4 - 3x| < 7$ 3

Question 4 (start a new page)   (8 marks)

a) Sketch the function and state the domain and range of the function   3

$$y = \frac{1}{2x - 3}$$

b) When a number is subtracted from its square the result is 56. Write an equation to
    represent this information and hence find the answer.   3

c) Solve for x: $3^x = 9^{x-2}$ 2

Question 5 (start a new page)   (8 marks)

a) Explain whether $f(x) = x^3 - x$ is an odd function, even function or neither.   2

b) State the domain and range of the function $y = 2^{-x}$ 2

c) Draw a neat sketch of $y = x^2 - 7x + 12$ showing x and y intercepts and vertex   4
    Hence state the domain and range.

Question 6 (start a new page)   (8 marks)

a) Simplify $\dfrac{m^3 + m^2}{x^2 - x} \div \dfrac{m+1}{x - x^3}$ as a single fraction in simplest form.   3

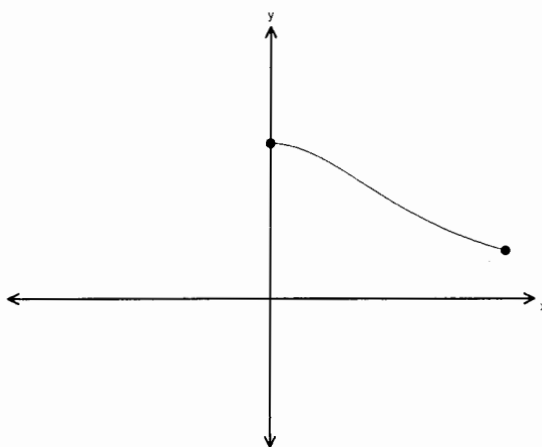b) Fully factorise $w^4 - 16$ 2

c) Show that $\dfrac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}}$ can be expressed in the form $a + b\sqrt{6}$ and find a and b.   3

Question 7 (start a new page) (7 marks)

a) Solve $x^2 - 3x - 1 = 0$ correct to 2 decimal places. **2**

b) Solve $|2x - 2| = 6x + 10$ Check your solution **3**

c) i) What is the condition for an even function **1**
   ii) Complete the graph of the function on your answer sheet, so it represents
   an even function **1**

Question 8 (start a new page) (8 marks)

a) If $f(x) = 3x^2 - 5x + 4$ and $g(x) = 2x + 10$ find:
   i) $f(2) + f(-2)$ **1**
   ii) the values of x for which $f(x) = 6$ **2**
   iii) the values of x for which $f(x) = g(x)$ **2**

b) Show the region of the number plane where the following hold simultaneously: **3**
   $$(x - 2)^2 + y^2 \leq 4$$
   $$y \leq 0$$

END OF EXAMINATION

☺   ☺   ☺

QUESTION 1

a) $\sqrt{75} + \sqrt{32} - \sqrt{27} = 5\sqrt{3} + 4\sqrt{2} - 3\sqrt{3}$

$$= \underline{\underline{2\sqrt{3} + 4\sqrt{2}}}$$

b) Let $x = 0.1\dot{2}\dot{5}$

$10x = 1.\dot{2}\dot{5}$

$1000x = 125.\dot{2}\dot{5}$

$990x = 124$

$x = \dfrac{124}{990}$

$\therefore 0.1\dot{2}\dot{5} = \underline{\underline{\dfrac{62}{495}}}$

$\therefore \underline{\underline{1.02}}$ (3 sig fig)

d) $\dfrac{(x+6)(x+6)}{x(x+6)} = \underline{\underline{\dfrac{x+6}{x}}}$

e) $2^3 - (3d)^3 = \underline{\underline{(2-3d)(4 + 6d + 9d^2)}}$

Question 2

a) i) $\dfrac{2x}{3} - 4 = x + 2$

$2x - 12 = 3x + 6$

$-18 = x$

$\therefore x = -18$

ii) $\underline{\underline{(x-11)(x+8)}}$

b) $\dfrac{(x+1)}{(x-1)(x+1)} - \dfrac{(x-1)}{x(x+1)}$

$\dfrac{x(x+1) - (x-1)^2}{x(x-1)(x+1)}$

$\dfrac{x^2 + x - [x^2 - 2x + 1]}{x(x-1)(x+1)}$

$\dfrac{x^2 + x - x^2 + 2x - 1}{x(x-1)(x+1)}$

$\dfrac{3x - 1}{x(x-1)(x+1)}$

c) $\dfrac{x+1}{(x+1)(x-1)} - \dfrac{x-1}{x+1} = \dfrac{(x+1)^2 - (x-1)^2}{(x+1)(x-1)}$

$$= \dfrac{4x}{(x+1)(x-1)}$$

d) $a^{-3} \times 2a^2 = 2a^{-1}$
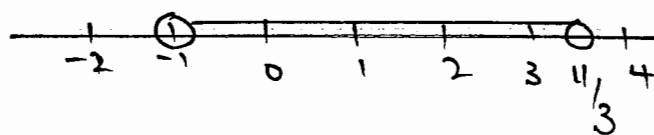
$$= \underline{\underline{\dfrac{2}{a}}}$$

Question 3

a) $x - 4(3 - 2x) + 12 = 0$

$x - 12 + 8x + 12 = 0$

$9x = 0$

$\therefore x = 0$

$\underline{\underline{y = 3}}$

b) $x = \dfrac{5 \pm \sqrt{25 - 4 \cdot 2 \cdot {}^-12}}{4}$

$x = \dfrac{5 \pm \sqrt{121}}{4}$

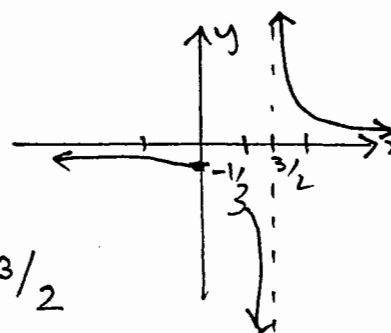$\underline{\underline{x = 4, \ -3/2}}$

c) $\quad 4 - 3x < 7 \qquad 4 - 3x > -7$

$\quad\quad -3x < 3 \qquad\quad -3x > -11$

$\quad\quad x > -1 \qquad\quad\quad x < {}^{11}/3$

$\quad\quad\quad -1 < x < {}^{11}/3$

Question 4

a) $2x - 3 = 0$

$2x = 3$

$\therefore x = {}^3/2$

$\therefore D: \ x \neq {}^3/2$

$R: \ \text{all } y, \ y \neq 0$

a) Let x be number
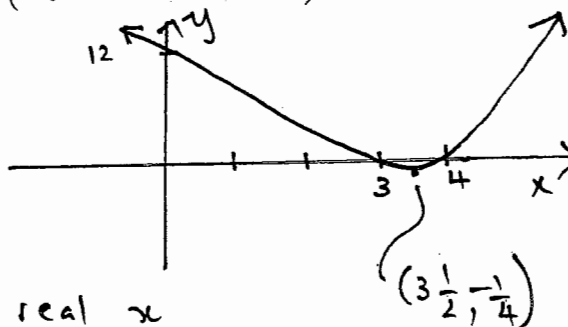$$x^2 - x = 56$$
$$x^2 - x - 56 = 0$$
$$(x + 7)(x - 8) = 0$$
$$\underline{x = -7, \quad x = 8}$$

c) $$3^x = 3^{2(x-2)}$$
$$x = 2x - 4$$
$$\therefore \quad 4 = x$$
$$ii \quad \underline{x = 4}$$

Question 5

a) $$f(x) = x^3 - x$$
$$f(-x) = (-x)^3 + x$$
$$\quad = -x^3 + x$$
$$-f(-x) = x^3 - x$$
$$\therefore \quad \underline{f(x) = -f(-x) \; odd}$$

b)

D: all real x
R: $y > 0$
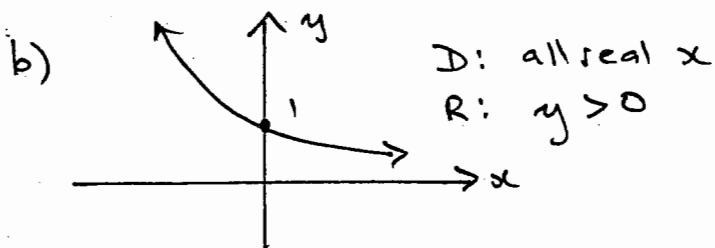
c) $$y = x^2 - 7x + 12$$
$$y = (x - 3)(x - 4)$$

$\left(3\frac{1}{2}, -\frac{1}{4}\right)$

D: all real x
R: $y \geqslant -4$

Question 6

a) $$\frac{m^2(m+1)}{x(x-1)} \times \frac{x(1-x^2)}{(m+1)}$$
$$\frac{m^2(1-x)(1+x)}{-(1-x)}$$
$$\underline{-m^2(1+x)}$$

b)

$$w^4 - 16$$
$$(w^2 - 4)(w^2 + 4)$$
$$\underline{(w-2)(w+2)(w^2+4)}$$

c) $$\frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} + 2\sqrt{3}} \times \frac{3\sqrt{2} - 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}}$$

$$\frac{18 - 6\sqrt{6} - 6\sqrt{6} + 12}{6}$$
$$\frac{30 - 12\sqrt{6}}{6}$$
$$5 - 2\sqrt{6}$$
$$\therefore \underline{a = 5 \qquad b = -2}$$

Question 7

a) $$x = \frac{3 \pm \sqrt{9 - 4 \cdot 1 \cdot -1}}{2}$$
$$x = \frac{3 \pm \sqrt{13}}{2}$$
$$x = 3 \cdot 30, \quad - \cdot 30$$

b) $2x - 2 = 6x + 10$ $2x - 2 = -6x - 10$

  $-12 = 4x$

  $x = -3$

check:

  $|-6-2| = -18 + 10$ $8x = -8$

  $8 \neq -8$ $x = -1$

  $\therefore x = -3$ not solution   check:

  $|-2-2| = -6 + 10$

  $4 = 4$

  true

$\therefore$ $\underline{\underline{x = -1 \text{ only solution}}}$

c) i) $f(x) = f(-x)$

  ii)

b)
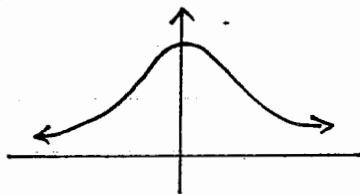
Question 8

a) i) $f(2) + f(-2)$

  $= (12 - 10 + 4) + (12 + 10 + 4)$

  $= \underline{\underline{28}}$

  ii) $3x^2 - 5x + 4 = 6$

  $3x^2 - 5x - 2 = 0$

  $(3x + 1)(x - 2) = 0$

  $\underline{x = -\tfrac{1}{3}, \quad x = 2}$

  iii) $3x^2 - 5x + 4 = 2x + 10$

  $3x^2 - 7x - 6 = 0$

  $(3x + 2)(x - 3) = 0$

  $\underline{\underline{x = -\tfrac{2}{3}, \quad x = 3}}$