F1c

NAME: _____ TEACHER: _____

SYDNEY TECHNICAL HIGH SCHOOL

Year 10 Yearly Exam

MATHEMATICS

2014

Time Allowed: 2 Hours

Total Marks: 100

Part A: Questions 1-25 multiple choice (25 marks)

Allow approximately 30mins for Part A

Part B: Questions 1-5 (75 marks)

Allow approximately 90mins for Part B

General Instructions:

- There is a multiple choice answer sheet in your answer booklet for Part A, Question 1-25. Shade the correct response.
- All answers to Part B, Questions 1-5 must be written in your answer book, with complete and organised setting out and working.
- Marks will not be given if required working is not shown.
- Use black or blue pen only.
- Calculators may be used.
- Marks are indicated for each question but may be changed.
- **START EACH QUESTION ON A NEW PAGE.**

Part B: All answers and neat setting out to be done in your answer book.

PART A

Allow about 30 minutes for this part.

Answers on answer sheet provided.

1. Which of the following events would be LEAST likely to occur?

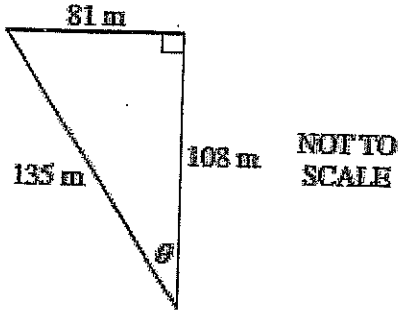
(A) Tossing a fair coin and obtaining a head

(B) Rolling a standard six-sided die and obtaining a 3

(C) Randomly selecting the letter 'G' from the 26 letters of the alphabet

(D) Winning first prize in a raffle of 100 tickets in which you have 4 tickets

2. What is the value of θ, to the nearest degree?



(A) 31° (B) 37° (C) 49° (D) 53°

3. Jackson works the number of hours shown in the table below.

Hours worked			
Employee	Normal hours	Hours at time and a half	Gross wage
Jackson, W	28	6	$592

According to the information in the table, what was the hourly rate at which Jackson was paid?

(A) $16.00

(B) $17.41

(C) $18.50

(D) $20.82

4. $(x^2 + y^2)(x^2 - y^2) =$

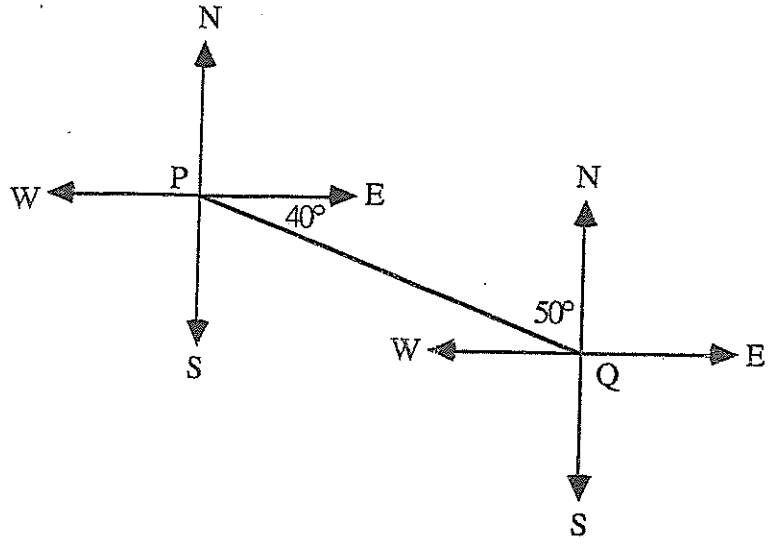
(A) $(x^2 + y^2)(x - y)^2$

(B) $(x + y)^3 (x - y)$

(C) $(x^2 + y^2)(x - y)(x + y)$

(D) $(x + y)^2 (x - y)^2$

5.



The bearing of P from Q is

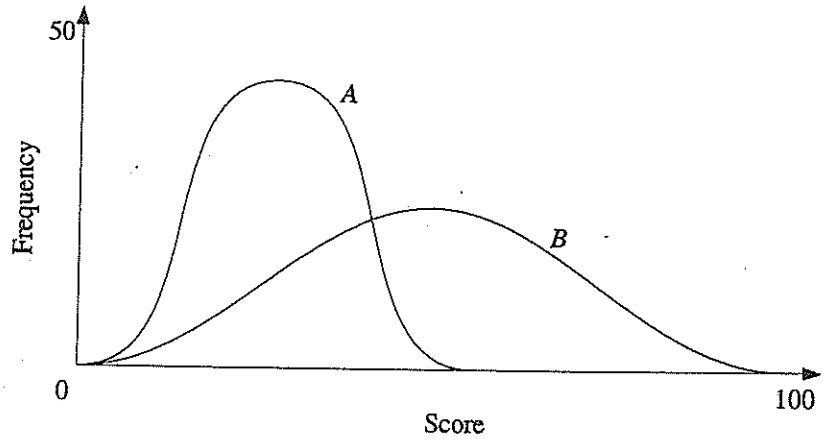
(A) 040°

(B) 050°

(C) 130°

(D) 310°

6.



The graph shows the frequency curves for two sets of test results, A and B.

$\bar{x}_A$ = mean of A $\quad$ σ_A = standard deviation of A

$\bar{x}_B$ = mean of B $\quad$ σ_B = standard deviation of B

Which of the following is true?

(A) $\bar{x}_A > \bar{x}_B$ and $\sigma_A < \sigma_B$ $\quad$ (B) $\bar{x}_A < \bar{x}_B$ and $\sigma_A > \sigma_B$

(C) $\bar{x}_A < \bar{x}_B$ and $\sigma_A < \sigma_B$ $\quad$ (D) $\bar{x}_A > \bar{x}_B$ and $\sigma_A > \sigma_B$

7.

Polly borrowed \$11 000. She repaid the loan in full at the end of two years with a lump sum of \$12 000.

What annual simple interest rate was she charged?

(A) 4.17% $\quad$ (B) 4.55% $\quad$ (C) 8.33% $\quad$ (D) 9.09%.

8.

Students studying vocational education courses were surveyed about their living arrangements.

	Females	*Males*	*Totals*
Living with parent(s)	46	155	201
Not living with parent(s)	182	122	304
Totals	228	277	505

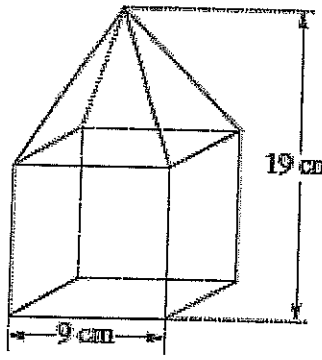
One of these students is selected at random.

What is the probability that this student is male and living with his parent(s)?

(A) 31% (B) 40% (C) 56% (D) 77%

9.

A square pyramid fits exactly on top of a cube to form a solid.

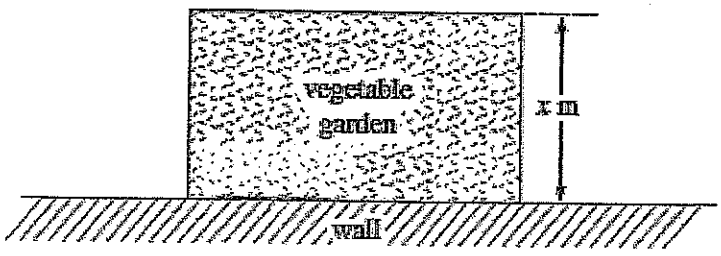


What is the volume of the solid?

(A) 513 cm^3 (B) 999 cm^3 (C) 1242 cm^3 (D) 1539 cm^3

10.

Leanne wants to build a rectangular vegetable garden in her backyard. She has 20 metres of fencing and will use a wall as one side of the garden. The plan for her garden is shown, where x metres is the width of her garden.

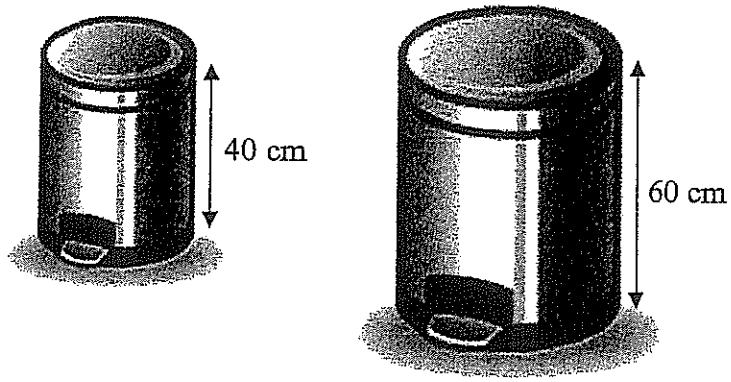


Which equation gives the area, A, of the vegetable garden?

(A) $A = 10x - x^2$ (B) $A = 10x - 2x^2$ (C) $A = 20x - x^2$ (D) $A = 20x - 2x^2$

11.

These two rubbish bins are similar, with heights as shown.



What is the ratio of the area of the lids of the bins?

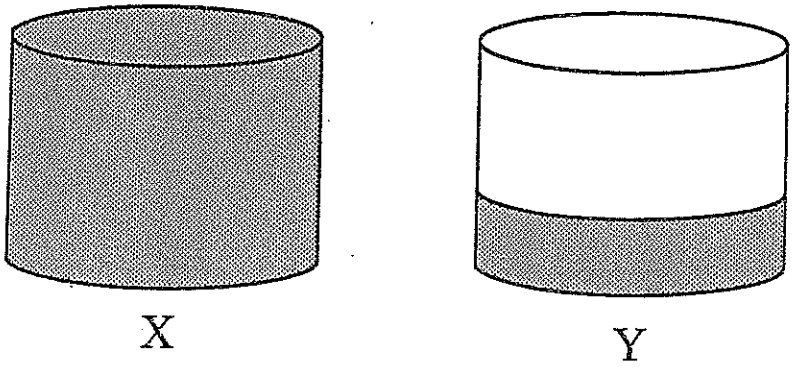
(A) 1:2

(B) 2:3

(C) 4:9

(D) 8:27

12.



Two identical drums X and Y contain water. Drum X is full and drum Y is $\frac{1}{3}$ full.

Half of the water in drum X is poured into drum Y. Half of the water *now* in drum Y is poured back into drum X.

Drum X is now

(A) $\frac{5}{12}$ full (B) $\frac{2}{3}$ full (C) $\frac{11}{12}$ full (D) full

13.

There are two identical blocks of clay.

All of the clay in the first block is used to make 40 identical models of the Opera House.

The second block is used to make smaller models with dimensions half those of the original models.

What is the maximum number of smaller models that can be made?

(A) 20 (B) 80 (C) 160 (D) 320

14.

The July sale prices for properties in a suburb were:

$552 000, $595 000, $607 000, $607 000, $682 000 and $685 000.

On 1 August, another property in the same suburb was sold for over one million dollars.

If this property had been sold in July, what effect would it have had on the mean and median sale prices for July?

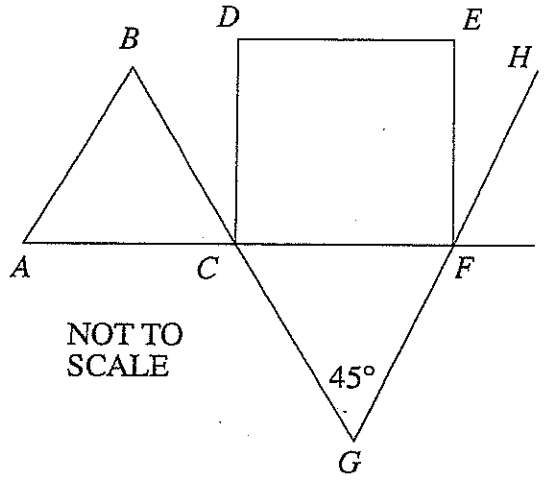
(A) Both the mean and the median would have changed.

(B) Neither the mean nor the median would have changed.

(C) The mean would have changed and the median would have stayed the same.

(D) The mean would have stayed the same and the median would have changed.

15.



ABC is an equilateral triangle.

$CDEF$ is a square.

BCG, ACF, and GFH are straight lines.

$\angle CGF = 45°$.

The size of $\angle EFH$ is

(A) 15°

(B) $22\frac{1}{2}°$

(C) 30°

(D) 45°

16.

Iliana buys several items at the supermarket. The docket for her purchases is shown.

What is the amount of GST included in the total?

(A) $1·15

(B) $1·27

(C) $1·55

(D) $1·71

XYZ Supermarket

27/04/03 12.53

MILK 1 L	$1.19
* PEPSI 1.25 L	$1.29
* DISINFECTANT	$7.23
* TEA TREE OIL	$4.13
SPINACH	$1.64
SOUP	$1.57
TOTAL	$17.05

10% GST INCLUDED IN COST OF TAXABLE ITEMS

* = TAXABLE ITEMS

17.

An enterprise agreement has the following annual salary arrangements:

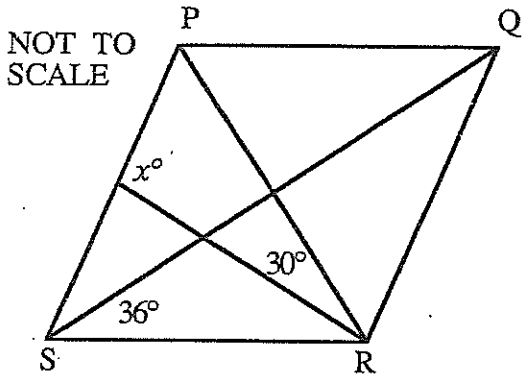
Base Salary	
Step 1	\$35 000
Step 2	\$40 000
Step 3	\$45 000

Leadership Allowance	
Leader 1	\$5000
Leader 2	\$7500
Leader 3	\$10 000

George's employer pays 6% more than the enterprise agreement. He is on Step 3 and receives an allowance for Leader 2. What is George's gross monthly pay?

(A) \$4375.00 (B) \$4412.50 (C) \$4600.00 (D) \$4637.50

18.



PQRS is a rhombus. Find the value of x.

(A) 90
(B) 96
(C) 102
(D) 108

19.

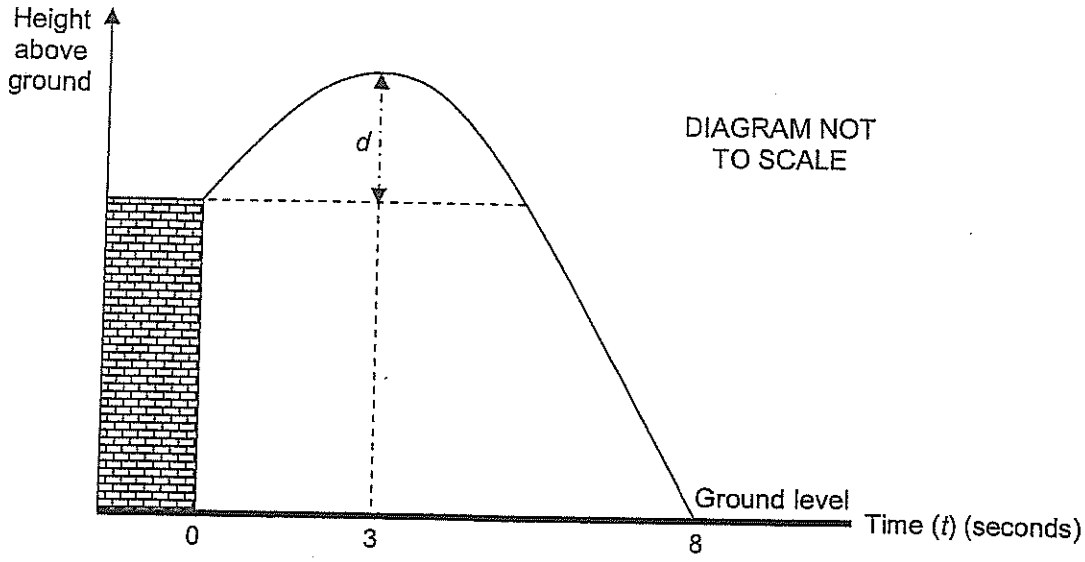
Two unbiased dice, each with faces numbered 1, 2, 3, 4, 5 and 6, are rolled.

What is the probability of obtaining a sum of 6?

(A) $\frac{1}{6}$ (B) $\frac{1}{12}$ (C) $\frac{5}{12}$ (D) $\frac{5}{36}$

20.

The path of an object thrown from the top of a building is shown.



The equation of the object's path is given by $h = 16 + 6t - t^2$, where h is the height of the object above the ground and t is the time in seconds that the object is in the air.

If the object reaches its maximum height above the ground after 3 seconds, what is the vertical distance (d) in metres that the object reaches above the building?

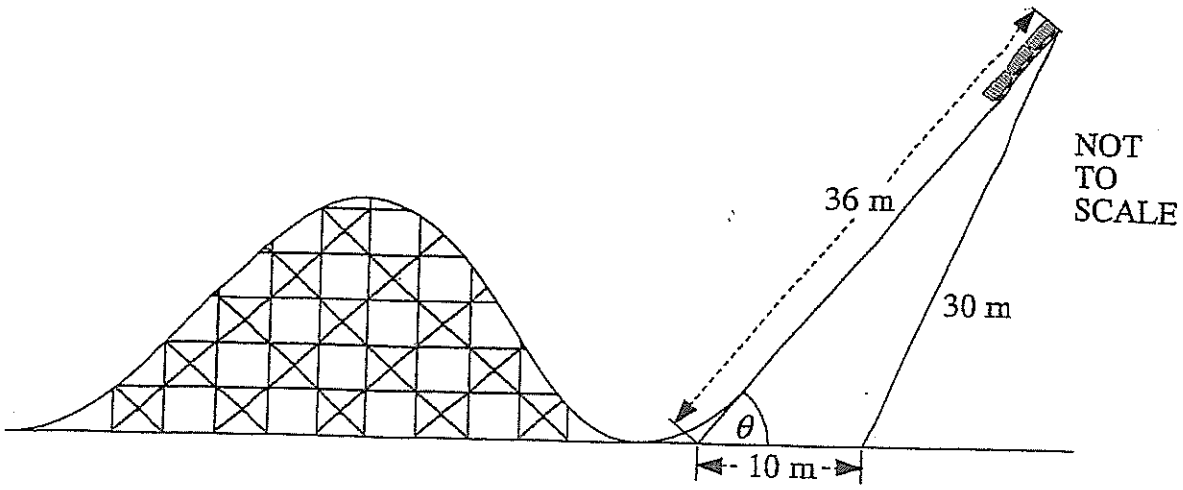
(A) 5

(B) 6

(C) 8

(D) 9

21.



The diagram shows a fun-park ride. The angle θ is closest to

(A) 46°

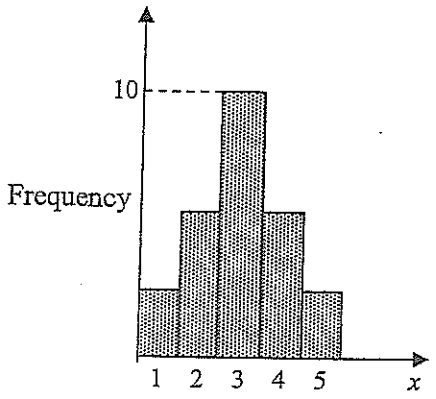
(B) 56°

(C) 72°

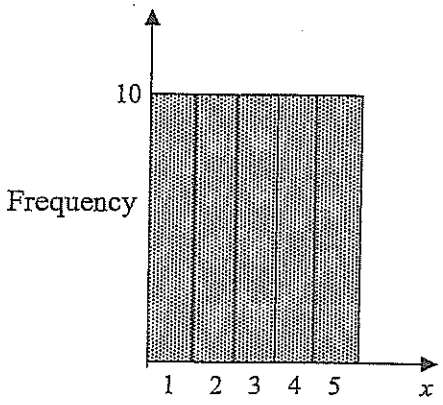
(D) 74°

22. Which one of the following data distributions has the greatest standard deviation?

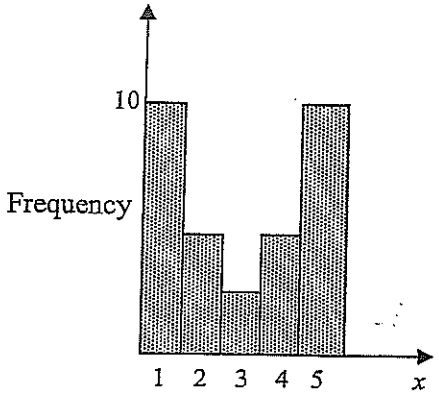
(A)



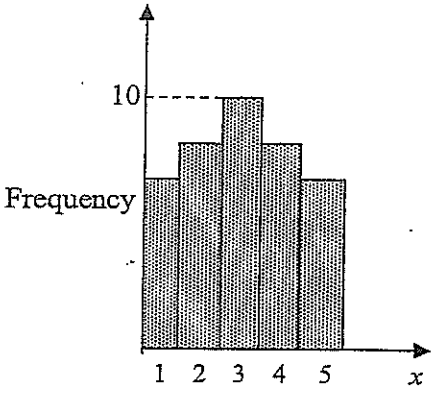
(B)



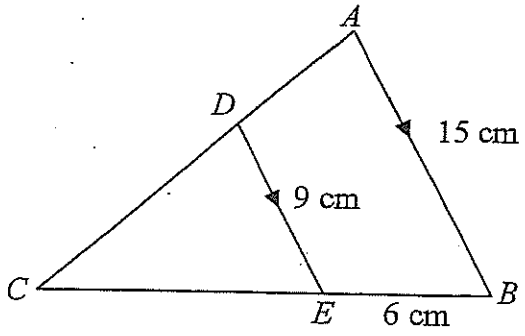
(C)



(D)



23. In the diagram below ABC is a triangle and $AB \parallel DE$



Given that $AB = 15$ cm, $DE = 9$ cm and $BE = 6$ cm, what is the value of BC?

A. 3.6 cm

B. 6 cm

C. 9 cm

D. 15 cm

24. The midpoint of (a,b) and $(5,-3)$ is $(-1,4)$. What are the values of a and b?

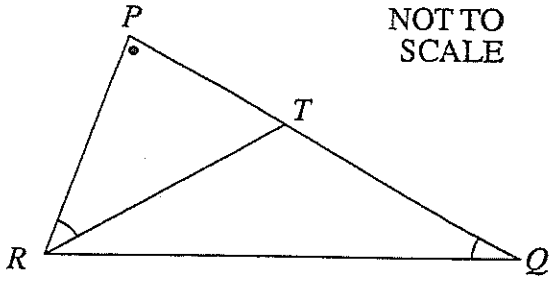
A. $a=2,\ b=\frac{1}{2}$

B. $a=-7,\ b=11$

C. $a=11,\ b=-10$

D. $a=3,\ b=3\frac{1}{2}$

25.



NOT TO SCALE

ΔPQR is similar to ΔPRT where $\angle PQR = \angle PRT$.

Then $\frac{QR}{RT} =$

(A) $\frac{PQ}{PT}$ (B) $\frac{PR}{PT}$ (C) $\frac{PT}{PR}$ (D) $\frac{RT}{PT}$

PART B **All Answers in your answer book.**

Question 1 **(15 marks)**

a) Expand and simplify fully

I. $5(b-2)-4(b+3)$ 1

II. $(7x-1)(x+5)$ 1

b) Express $\frac{4}{x}-\frac{8}{5x}$ as a single fraction. 1

c) Solve $\frac{7x}{2}+2x=12-x$ 2

d) Solve and sketch your solution on a number line.

$4-6x \leq 3+2x$ 2

e) Make u the subject

$E=\frac{1}{2}m(v^2-u^2)$ 2

f) Factorise fully

I. $9-x^2$ 1

II. $2ax-a+6x-3$ 1

III. $60-4x-x^2$ 1

IV. $3x^2-9x-30$ 1

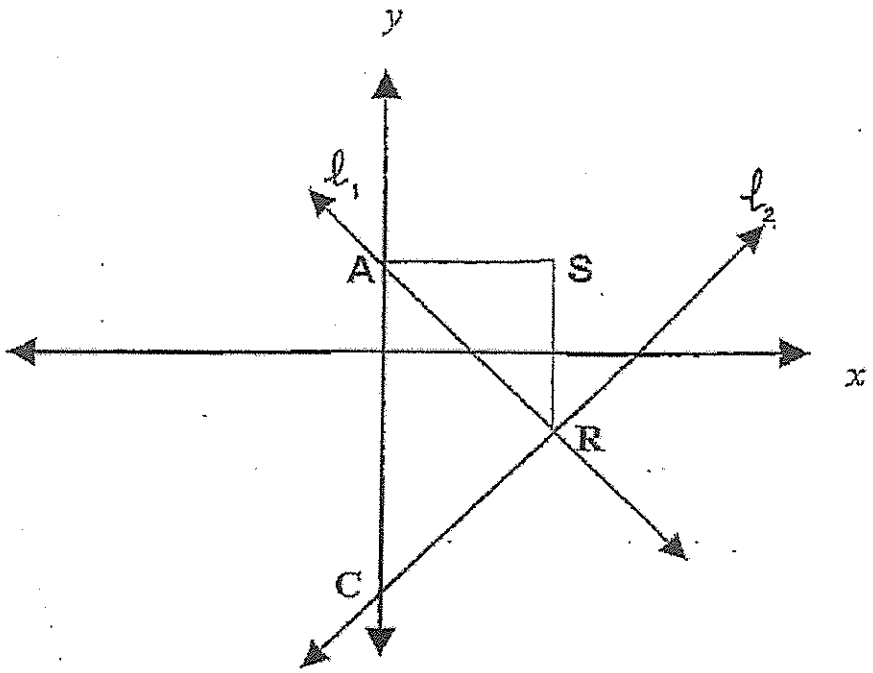
g) Solve

I. $3x^2+8x-3=0$ 1

II. $1-\frac{x^2}{4}=0$ 1

Question 2 **(15 marks) (Start a new page)**

a)



NOT TO SCALE

ℓ_1 Has equation $x + y = 2$ and intersects the y axis at point $A(0,2)$

ℓ_2 Has equation $x - y = 4$ and intersects the y axis at point C

Lines ℓ_1 and ℓ_2 intersect at R

The horizontal line through A intersects the vertical line through R at S

Copy the above diagram into your answer book neatly.

I. Find the co-ordinates of point C. 1

II. Show by using simultaneous equations that R has co-ordinates $(3, -1)$ 1

III. Find the equation of line SR 1

IV. Find the gradient of ℓ_1 1

V. Find the distance AR 1

VI. Find the co-ordinates of S 1

VII. On your diagram shade the region $x - y \geq 4$ 1

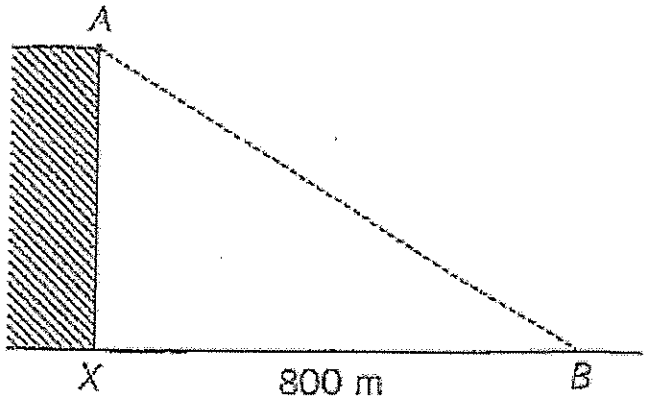
b)

I. Solve $x(x-2) = 8$ 2

II. Solve $3x^2 - x - 1 = 0$ leaving your answer in exact form. 2

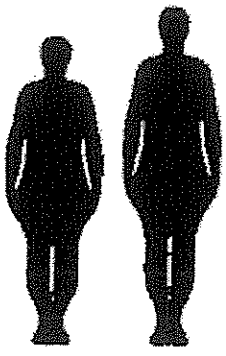
c) The angle of depression from a point A to a ship at point B is 10^0 2

If the distance BX from B to the foot of the cliff at X is 800m, find the height of the cliff, correct to the nearest metre.



d) The corresponding dimensions of figure B are approximately 10% larger than those of figure A 2

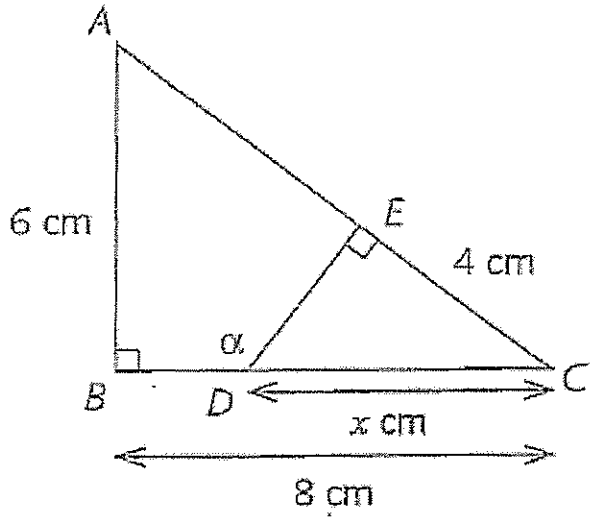
I. What is the percentage increase in body area?
II. What is the percentage increase in body volume?



A B

Question 3 **(15 marks) (Start a new page)**

a)



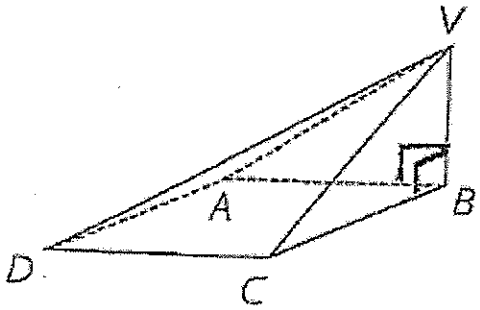
I. Prove that $\triangle DEC$ is similar to $\triangle ABC$ (giving reasons) 2

II. Hence find x 1

III. Use trigonometry to find α correct to nearest minute. 2

b) Waleed owns a portfolio of shares that he purchased for $38,000. For the first four years, the portfolio appreciated in value at an average of 8% each year, but for the next four years, it depreciated in value at an average of 8% each year. Calculate the value of the portfolio, correct to the nearest dollar, at the end of these eight years. 2

c)



In the pyramid above, VB is perpendicular to the square ABCD, AB = 12m and VB = 5m

I. Calculate the surface area of the pyramid. 2

II. Calculate the volume of the pyramid. 2

d) The curved surface area of a cone is $80\pi\ cm^2$ and the area of the circular base is $16\pi\ cm^2$.

I. Calculate the radius of the base, of the cone. 1

II. Calculate the exact perpendicular height of the cone. 2

III. Calculate the volume of the cone, correct to the nearest cm^3 1

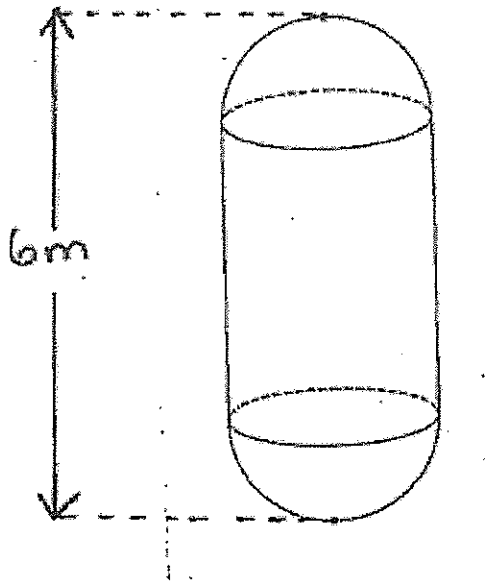
Question 4 **(15 marks)** **(Start a new page)**

a) A storage tank, shown below, is constructed as a cylinder with a hemisphere at each end of the cylinder. The radius of the cylinder is 1.5m and the overall length of the tank is 6m.

Calculate:

I. The surface area of the tank (nearest m^2) 2

II. The volume of the tank (nearest m^3) 2



b) For the parabola $y = x^2 - x - 12$ find.

I. The x intercepts 1

II. The co-ordinates of the vertex. 2

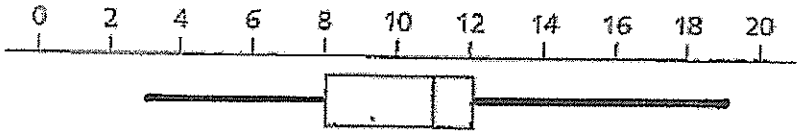
III. Sketch the parabola. Indicate and label the above information and also find where the parabola cuts the y axis. 2

c) In a group of 100 students, 60 study mathematics, 50 study physics and 20 study both mathematics and physics.

I. Draw a Venn diagram to represent this information. 1

II. One student is selected at random. What is the probability that he studies neither physics or mathematics. 1

d) In a market survey, 200 people were asked how many hours of television they watched in the previous week. The results are presented in the box and whisker plot below.



I. What is the maximum number of hours anyone watched television? 1

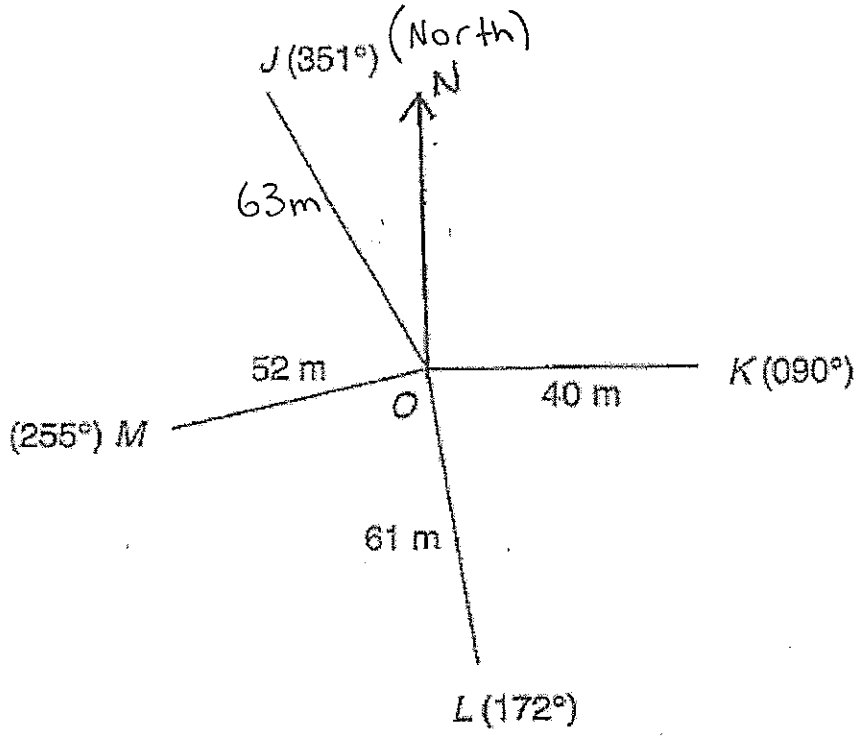
II. How many people watched more than 8hrs of television? 1

III. What is the interquartile range? 1

IV. How many people watched between 8hrs and 11hrs of television? 1

Question 5 **(15 marks) (Start a new page)**

a) A radial survey of an area of land is shown below.



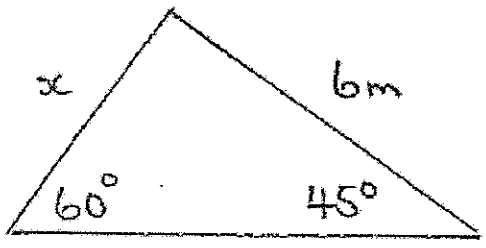
The distance of points K,L,M and J from 0 are shown, as well as their true bearings from 0.

I. Explain why the size of $\angle$JOK is 99° 1

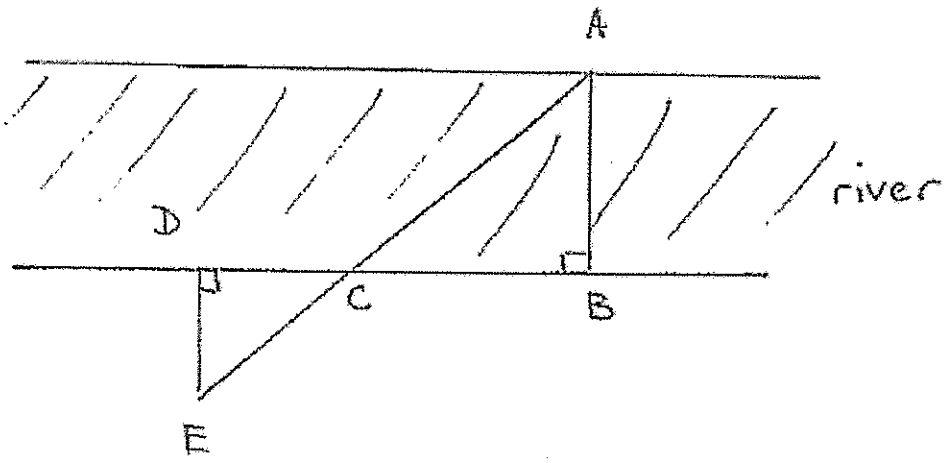
II. Find the area of triangle KOL to the nearest square metre 1

III. Find the distance LM to the nearest metre 2

b) Show the exact value of x in the triangle below is $2\sqrt{6}$ m. 2



c)



This diagram represents a river (shaded) with a tree on the bank at point *A*. A man stands directly opposite *A*, on the opposite bank, at point *B*. He then walks 100m along the bank, to point *C*, where he places a peg. He then walks a further 50m to point *D*, turns 90° and walks 65m to point *E*, where he finds that *E,C* and *A* are in a straight line.

Draw the diagram above in your answer book and show all the above information. Hence find the width of the river. 2

d) i) Expand $\left(\sqrt{x}+\sqrt{y}\right)^2$ 1

ii) Using the expansion above write $\sqrt{16+2\sqrt{55}}$ as the sum of two irrational numbers 2

e) In cricket, batting average $= \frac{total\ number\ of\ runs\ scored}{number\ of\ times\ out}$ in that season. A cricketer scored 1800 runs in a season. If he had been out on one more occasion, his average would have been three runs less.

Let his batting average be x and his number of times out be y. 3

Form 2 simultaneous equations and use them to find his batting average.
(Working out must be shown, marks will not be given for guessing and checking)

SOLUTIONS YEAR 10 : YEARLY 2014

PART A

1	C	6	C	11	C	16	C	21	A
2	B	7	B	12	C	17	D	22	C
3	A	8	A	13	D	18	B	23	D
4	C	9	B	14	C	19	D	24	B
5	D	10	D	15	A	20	D	25	B

PART B

Question 1

a) i) $b - 22$

ii) $7x^2 + 34x - 5$

b) $\frac{12}{5x}$

c) $\frac{7x}{2} + 2x = 12 - x$

$7x + 4x = 24 - 2x$

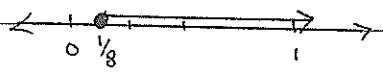
$13x = 24$

$x = \frac{24}{13}$

d) $4 - 6x \leq 3 + 2x$

$-8x \leq -1$

$x \geq \frac{1}{8}$



e) $\frac{2E}{m} = v^2 - u^2$

$u^2 = v^2 - \frac{2E}{m}$

$u = \pm\sqrt{v^2 - \frac{2E}{m}}$

f) i) $(3 - x)(3 + x)$

ii) $a(2x - 1) + 3(2x - 1)$

$(2x - 1)(a + 3)$

iii) $(10 + x)(6 - x)$

iv) $3(x + 2)(x - 5)$

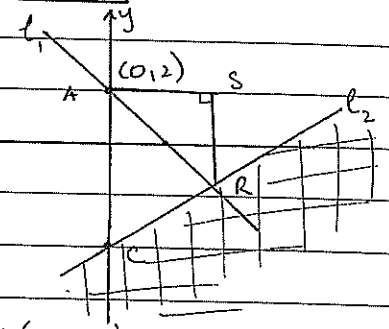
g) i) $(3x - 1)(x + 3) = 0$

$x = 1/3 \quad x = -3$

ii) $\left(1 - \frac{x}{2}\right)\left(1 + \frac{x}{2}\right) = 0$

$x = 2, -2$

Question 2



i) $C(0, -4)$

ii) $x + y = 2$

$x - y = 4$

$2x = 6$

$x = 3 \quad \therefore R(3, -1)$

iii) SR $x = 3$

iv) $m_{l_1} = -1$

v) $AR = \sqrt{(3-0)^2 + (-1-2)^2}$

$= \sqrt{18}$ units (OR $3\sqrt{2}$)

vi) $S(3, 2)$

vii) see diagram

b) i) $x(x - 2)$

$x^2 - 2x - 8 = 0$

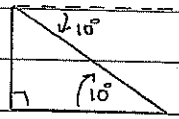
$(x - 4)(x + 2) = 0$

$x = 4, -2$

ii) $x = \frac{1 \pm \sqrt{1 - 4.3.-1}}{6}$

$x = \frac{1 \pm \sqrt{13}}{6}$

c)



$\tan 10° = \frac{h}{800}$

$h = 141$m

d)

A : B

1 : 1.1

i) $1^2 : (1.1)^2 \Rightarrow 21\% \uparrow$

ii) $1^3 : (1.1)^3 \Rightarrow 33.1\% \uparrow$

Question 3

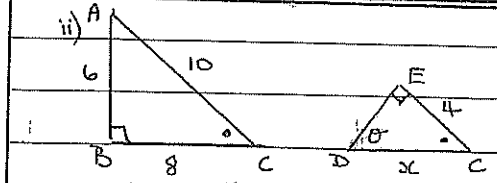
a) i) In Δ's DEC and ABC

$\hat{C}$ is common

$A\hat{B}C = D\hat{E}C \quad (90°)$

$\therefore \Delta DEC \,|||\, \Delta ABC$ (equiangular)

ii)



$\frac{4}{8} = \frac{x}{10}$

$\therefore x = 5$

iii) $\sin\theta = 4/5$

$\theta = 53°8'$

$\therefore \alpha = 126°52'$

b) $A = \left[38000\left(1 + \frac{8}{100}\right)^4\right]\left(1 - \frac{8}{100}\right)^4$

$A = \$37036$

c) i) $SA = 12^2 + 2\left(\frac{12 \times 5}{2}\right) + 2\left(\frac{1}{2} \times 12 \times$

(VA = 13 by pythagoras)

$SA = 144 + 60 + 156$

$= 360\,m^2$

ii) $V = \frac{1}{3} \times 12^2 \times 5$

$= 240\,m^3$

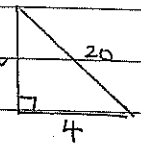
d) i) $16\pi = \pi r^2$

$r = 4$ cm

ii) ~~$\pi r l = 80\pi$~~

$4l = 80$

$l = 20$



$h^2 = 20^2 - 4^2$

$h = \sqrt{384}$ cm

OR

$h = 8\sqrt{6}$

iii) $V = \frac{1}{3} \times \pi \times 4^2 \times \sqrt{384}$

$= 328\,cm^3$

Question 4

a) i) $SA = \left(\frac{4 \times \pi \times (1.5)^2}{2}\right) + \left(2 \times \pi \times 1.5 \times 3\right)$

$= 57\,m^2$

ii) $V = \left(\frac{4}{3} \times \pi \times 1.5^3\right) + \left(\pi \times 1.5^2 \times 3\right)$

$= 35 m^3$

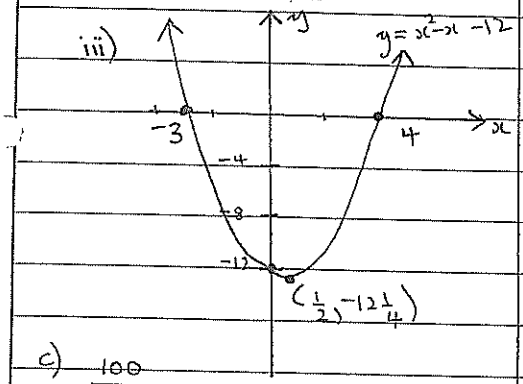
b) $y = x^2 - x - 12$

i) $(x - 4)(x + 3) = 0$

$x = 4, -3$

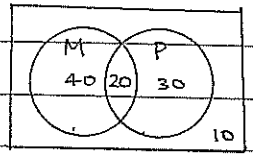
ii) V $\left(\frac{1}{2}, -12\frac{1}{4}\right)$

iii)



c) 100

i)



ii) $P(\text{not P or M}) = \frac{10}{100} = \frac{1}{10}$

d) i) 19 hours

ii) 75% x 200 = 150 people

iii) IQR = 12 − 8 = 4

iv) 25% ⇒ 50 people

Question 5

a) i) $J\hat{O}N = 9^\circ$ ∴ $J\hat{D}K = 99^\circ$

$N\hat{O}K = 90^\circ$

ii) $\triangle KOL = \frac{1}{2} \times 40 \times 61 \times \sin 82^\circ$

$= 1208 m^2$

iii) $LM^2 = 52^2 + 61^2 - 2 \times 52 \times 61 \times \cos 83^\circ$

$LM = 75 m$

b) $\frac{x}{\sin 45^\circ} = \frac{6}{\sin 60^\circ}$

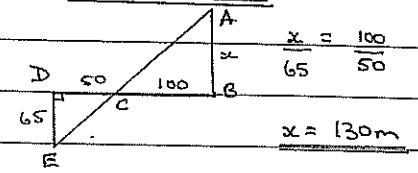
$x = \frac{6 \times \frac{1}{\sqrt{2}}}{\sqrt{3}/2}$

$x = \frac{6}{\sqrt{2}} \times \frac{2}{\sqrt{3}}$

$x = \frac{12}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}}$

$x = 2\sqrt{6} m$

c)



$\frac{x}{65} = \frac{100}{50}$

$x = 130m$

d) i) $(\sqrt{x} + \sqrt{y})^2 = x + 2\sqrt{xy} + y$

ii) $\sqrt{x} + \sqrt{y} = \sqrt{x + y + 2\sqrt{xy}}$

$\therefore \sqrt{16 + 2\sqrt{55}} = \sqrt{x + y + 2\sqrt{(xy)}}$

$x + y = 16 \quad xy = 55$

$x = 11 \quad y = 5$

$\therefore \sqrt{16 + 2\sqrt{55}} = \sqrt{11} + \sqrt{5}$

e) x batting average

y = no. times out

$\therefore x = \frac{1800}{y}$ ——— ①

$x - 3 = \frac{1800}{y+1}$ ——— ②

sub ① into ②

$\frac{1800}{y} - 3 = \frac{1800}{y+1}$

$\frac{1800 - 3y}{y} = \frac{1800}{y+1}$

$(y+1)(1800 - 3y) = 1800y$

$1800y - 3y^2 + 1800 - 3y = 1800y$

$-3y^2 - 3y + 1800 = 0$

$y^2 + y - 600 = 0$

$y = \frac{-1 \pm \sqrt{1 + 4.600}}{2}$

$y = \frac{-1 \pm 49}{2}$

$y = 24$ only $y > 0$

$\therefore x = 75$ (batting average)