Integration

Trapezoidal & Simpson's Rules Indefinite Integrals Definite Integrals – Even & Odd functions Areas enclosed by the x-axis, y-axis Volumes Substitution Method Trapezoidal & Simpson's Rules

Trapezoidal Rule	Simpson's Rule		
$A \approx \frac{h}{2} \left(y_{1st} + y_{last} + 2(others) \right)$	$A \approx \frac{h}{3} \left(y_{1st} + y_{last} + 2(odd) + 4(even) \right)$	$y_1 = 1^{st}$	
	$A \approx \frac{h}{3} \left(y_{1st} + y_{last} + 4(odd) + 2(even) \right)$	$y_0 = 1^{st}$	

For n number of strips, you must need n+1 number y values Height is the difference of 2 x values.

Example 1

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x	0	1	2	3	4		
У	1	2	4	8	16		

4 strips

By Trapezoidal Rule

A
$$\approx \frac{1}{2} (1 + 16 + 2(2 + 4 + 8))$$

 $\approx \frac{1}{2} (1 + 16 + 28)$
 $\approx 22\frac{1}{2}$

By Simpson's Rule

A
$$\approx \frac{1}{3} (1 + 16 + 2(4) + 4(2 + 8))$$

 $\approx \frac{1}{3} (1 + 16 + 8 + 40)$
 $\approx 21\frac{2}{3}$

Indefinite Integrals

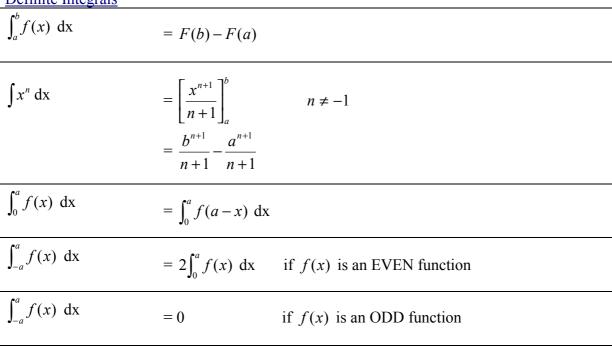
$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \qquad n \neq -1$$

$$\int \frac{1}{x} dx = \ln|x| + c$$

$$\int (ax+b)^n dx = \frac{(ax+b)^{n+1}}{a(n+1)} + c$$

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

Definite Integrals



Areas enclosed by the x-axis, y-axis

Areas above the x-axis will give a positive result

Area =
$$\int_a^b f(x) dx$$

Areas below the x-axis will give a negative result. Take the Absolute value.

Area =
$$\left| \int_{a}^{b} f(x) \right| dx$$

When finding areas for both above and below, separate them into different Areas.

To find the areas between a curve and the y-axis, we change the subject of the equation of the curve to x. x = f(y)

Area =
$$\int_a^b f(y) dy$$
 OR = $\int_a^b x dy$

Similarly to the x-axis,

Areas to the right are Positive Areas to the left are Negative

Volumes

Rotation about the x-axis

Volume =
$$\pi \int_a^b y^2 dx$$

Rotation about the y-axis

Volume =
$$\pi \int_a^b x^2 dy$$

Substitution Method

$$\triangleright$$
 Let $u =$

Find
$$\frac{du}{dx}$$

 \triangleright Find dx or du

Example 1

Find
$$\int_2^3 x(x^2 - 3) \, \mathrm{d}x$$

Let
$$u = x^2 - 3$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$x = 3$$
; $u = 3^2 - 3$

$$x = 2$$
; $u = 2^2 - 3$
 $u = 1$

$$\int_{2}^{3} x(x^{2} - 3) dx = \frac{1}{2} \int_{2}^{3} (x^{2} - 3) 2x dx$$

$$= \frac{1}{2} \int_{1}^{6} u du$$

$$= \frac{1}{2} \left[\frac{u^{2}}{2} \right]^{6}$$

$$= \frac{1}{2} \left[\frac{6^{2}}{2} - \frac{1^{2}}{2} \right]$$

$$= 8\frac{3}{4}$$