SYDNEY TECHNICAL HIGH SCHOOL

MANNERS MAKYTH MAN

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 2
MARCH 2016

Mathematics Extension 2

Name ....

Teacher ....

General Instructions

- Reading Time - 5 minutes.
- Working Time - 90 min.
- Write using a blue or black pen.
- Board approved calculators may be used.
- The BOSTES reference sheet is provided.
- In Questions 7-10, show relevant mathematical reasoning and /or calculations.
- Begin each question on a fresh sheet of paper.
- Full marks may be not be awarded for careless and illegible writing.

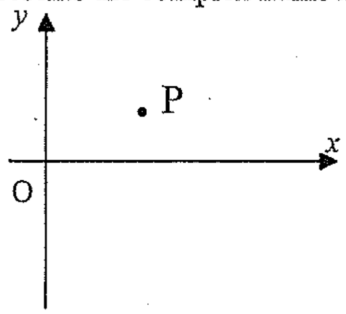
Total marks (68)

- Attempt Questions 1-10.

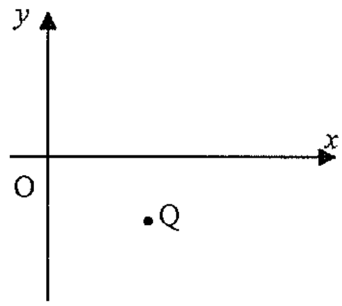
Multiple Choice		6
Question 7		16
Question 8		14
Question 9		14
Question 10		16
TOTAL		**66**

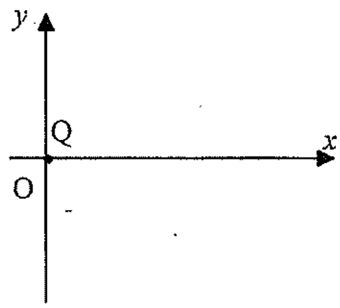
Section 1
Multiple Choice (6 marks)
Use the multiple choice answer sheet for Question 1-5

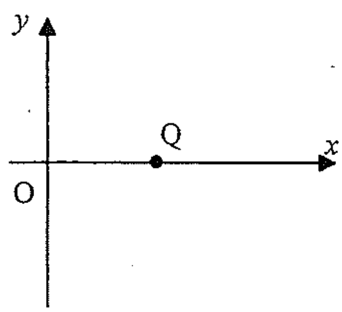
1. In the Argand Diagram below, P represents the complex number z.

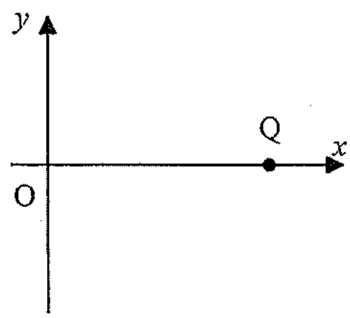


Which of the following Argand diagram shows the point Q representing $z + \overline{z}$?

(A)


(B)


(C)


(D)


2. What is the acute angle between the asymptotes of the hyperbola $\frac{x^2}{3} - y^2 = 1$

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

3. Which conic has eccentricity $\frac{\sqrt{13}}{3}$?

(A) $\frac{x^2}{3}+\frac{y^2}{2}=1$

(B) $\frac{x^2}{3^3}+\frac{y^2}{2^2}=1$

(C) $\frac{x^2}{3}-\frac{y^2}{2}=1$

(D) $\frac{x^2}{3^3}-\frac{y^2}{2^2}=1$

4. The equation $x^2-xy+y^2=3$ defines y implicitly in terms of x.
The expression for $\frac{dy}{dx}$ is:

(A) $\frac{y-2x+3}{2y-x}$

(B) $\frac{y-2x+3}{x-2y}$

(C) $\frac{y-2x}{2y-x}$

(D) $\frac{y-2x}{x-2y}$

5. Consider the polynomial $P(x)$ of degree 3.
Two real numbers $a < b$ are such that:

$$a < b$$
$$P(a) > P(b) > 0$$
$$P'(a) = P'(b) = 0$$

The polynomial has:

(A) 3 real zeros

(B) 1 real zero γ such that $\gamma < a$

(C) 1 real zero γ such that $a < \gamma < b$

(D) 1 real zero γ such that $\gamma > b$

6. $P(4,25)$ is a point on the rectangular hyperbola $xy = 100$.
The tangent at P cuts the hyperbola's asymptotes at Q and R.
The area of ΔOQR (where O is the origin) is:

(A) $200\sqrt{2}\, u^2$

(B) $2\sqrt{50}\, u^2$

(C) $100\, u^2$

(D) $200\, u^2$

Section II

Total Marks (64)
Attempt Questions 7 – 10.

Answer each question in your writing booklet.
In Questions 6-9, your responses should include relevant mathematical reasoning and/or calculations.

Question 7 **(16 Marks)** Use a Separate Sheet of paper

a) Consider the complex numbers $z = -\sqrt{3} + i$ and $w = 3\left(\cos\frac{\pi}{7} + i\sin\frac{\pi}{7}\right)$

i) Evaluate $|z|$ **1**

ii) Evaluate $\arg(z)$ **1**

iii) Find the argument of $\frac{z}{w}$ **1**

b) Find the value of $\frac{dy}{dx}$ at the point $(2,-1)$ on the curve $x + x^2y^3 = -2$ **3**

c) Find the Cartesian equation of the locus of a point P which represents the complex number z where $|z - 2i| = |z|$. **2**

d) Let $w = 3 - 4i$ and $z = 2 + 2i$

i) Find $w\bar{z}$ in the form $x + iy$ **1**

ii) Find $Im\left(\frac{1}{2 - w}\right)$ **2**

iii) The point representing the complex number z is rotated $270°$ in an anticlockwise direction about the origin in an Argand diagram.

What is the complex number represented by the new position of the point? **1**

e) i) Sketch and shade on an Argand diagram the region R in which $|z - 4i| \leq 3$ and $0 \leq \arg(z + 1) \leq \frac{\pi}{4}$ hold simultaneously. **2**

ii) Find the value of $\arg(z + 1)$ at the point in the region R where $\arg(z + 1)$ is a minimum. Answer to the nearest degree. **2**

End of Question 7

Question 8 **(14 Marks)** Use a Separate Sheet of paper

a) Express $\frac{7+2i}{5-i}$ in the form $x+iy$ where x and y are real **2**

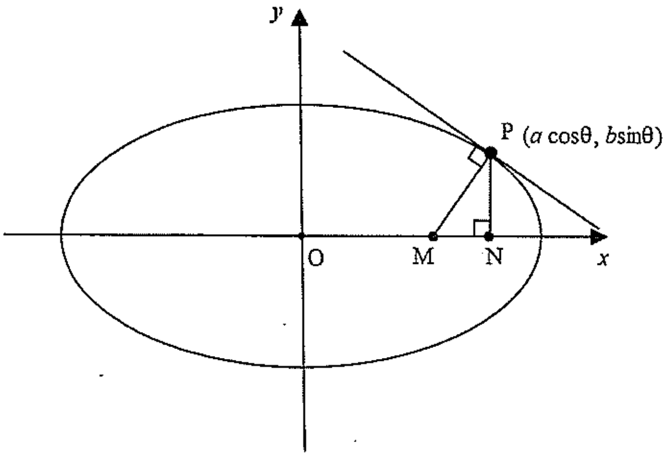
b) If $z=(1-i)^{-1}$

i) Express $\bar{z}$ in modulus argument form **2**

ii) If $(\bar{z})^{13}=a+ib$ where a and b are real numbers, find the values of a and b **2**

c) $P(a\cos\theta, b\sin\theta)$ is a point on the ellipse with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

The normal at P meets the x-axis at M and PN is perpendicular to the x-axis.



i) Show that the equation of the normal at P is given by
$ax\sin\theta - by\cos\theta = (a^2-b^2)\sin\theta\cos\theta$ **2**

ii) Hence show that $MN = \left|\frac{b^2\cos\theta}{a}\right|$. **2**

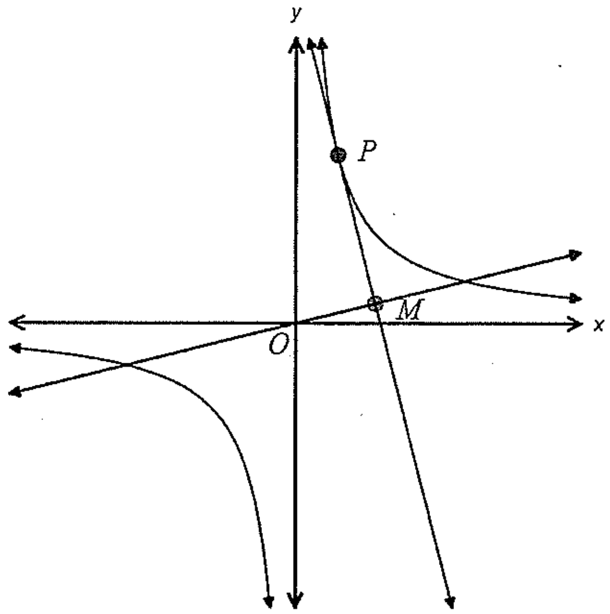
d) i) If a is a root of $P(x)$ with multiplicity n, show that a is also a root of $P'(x)$ with multiplicity $n-1$. **1**

ii) Given $P(x)=2x^4+9x^3+6x^2-20x-24$ has a triple root, factorise $P(x)$ into its linear factors. **3**

End of Question 8

Question 9 **(14 Marks)** Use a Separate Sheet of paper

a) $P\left(t, \frac{1}{t}\right)$ is a variable point on the rectangular hyperbola $xy = 1$. M is the foot of the perpendicular from the origin to the tangent to the hyperbola at P.



i) Show that the tangent to the hyperbola at P has equation $x + t^2 y = 2t$ **2**

ii) Find the equation of OM **1**

iii) Show that the equation of the locus of M as P varies is $x^4 + 2x^2y^2 - 4xy + y^4 = 0$ and indicate any restrictions on the values of x and y. **3**

b) The polynomial $p(x) = x^5 + 2x^2 + mx + n$ has a double zero at $x = -2$. Find the value of m and n, and find the product of the other three zeros. **3**

c) Sketch the hyperbola with parametric equations

$$x = 3\sec\theta$$
$$y = 4\tan\theta$$

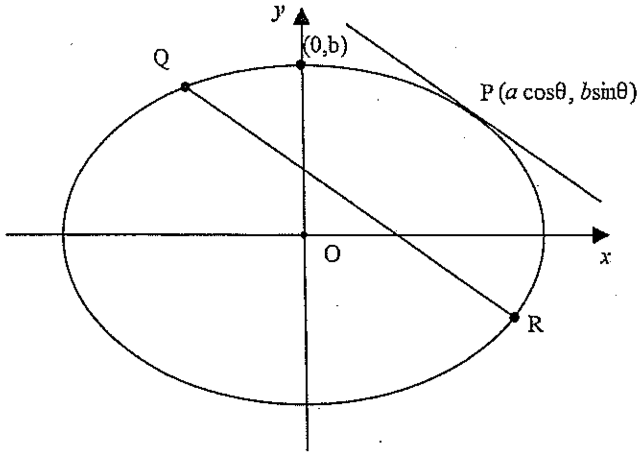
Indicate the vertices, the foci, and the equations of the directrices and asymptotes **5**

End of Question 9

Question 10 (16 Marks) Use a Separate Sheet of paper

a) Sketch the region in the complex plane where $Re[(2-3i)z] < 12$ **2**

b) Consider the ellipse $\mathcal{E}$, with equation $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the points $P(a\cos\theta, b\sin\theta)$, $Q(a\cos(\theta+\varphi), b\sin(\theta+\varphi))$ and $R(a\cos(\theta-\varphi), b\sin(\theta-\varphi))$ on $\mathcal{E}$.



i) Show that the equation of the tangent to $\mathcal{E}$ at the point P is $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$. **2**

ii) Show that the chord QR is parallel to the tangent at P. **2**

iii) Show that OP bisects the chord QR. **3**

c) i) Using De Moives's theorem show that the solution of the equation $z^3 = 1$ in the complex number system are:
$z = \cos\theta + i\sin\theta$ for $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ **2**

ii) If $\omega = \text{cis}\frac{2\pi}{3}$ show that $\omega^2+\omega+1=0$ and $\omega^3-\omega^2-\omega-2=0$ **2**

iii) Hence or otherwise solve the cubic equation $z^3-z^2-z-2=0$ over the complex field **3**

End of Examination

☺ ☺ ☺

Extension 2 Mathematics
Assessment Task 2
March 2016

Multiple Choice

1. D 2. C 3. D 4. C 5. B 6. D

Question 7

a i) $|z| = -\sqrt{3} + i$ $w = 3(\cos\frac{\pi}{7} + i\sin\frac{\pi}{7})$

$|z| = |\sqrt{3} + i|$

$= \sqrt{(\sqrt{3})^2 + 1}$

$= 2$

ii) $\arg(z) = \alpha$ (z is in 2^{nd} Quad)

$\tan\alpha = \frac{-1}{\sqrt{3}}$

$\arg(z) = \tan^{-1}(-\frac{1}{\sqrt{3}})$

$= \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

iii) $\arg(\frac{z}{w}) = \frac{5\pi}{6} - \frac{\pi}{7}$

$= \frac{29\pi}{42}$

b) $x^2 + x^2y^3 = -2$

$1 + 2xy^3 + x^2 . 3y^2\frac{dy}{dx} = 0$

$3x^2y^2\frac{dy}{dx} = -1 - 2xy^3$

$\frac{dy}{dx} = \frac{-1 - 2xy^3}{3x^2y^2}$

At $(2,-1)$ $\frac{dy}{dx} = \frac{1}{4}$

b) $x + x^2y^3 = -2$

$1 + 2xy^3 + x^2 . 3y^2\frac{dy}{dx} = 0$

$3x^2y^2\frac{dy}{dx} = -1 - 2xy^3$

$\frac{dy}{dx} = \frac{-1 - 2xy^3}{3x^2y^2}$

At $(2,-1)$ $\frac{dy}{dx} = \frac{1}{4}$

c) $|z - 2i| = |z|$

perpendicular bisector of $(0,0)$ and $(0,2)$

$\therefore$ The locus is the line $y = 1$

d i) $w = 3 - 4i$ $z = 2 + 2i$

$w\bar{z} = (3-4i)(2-2i)$

$= 6 - 6i - 8i + 8i^2$

$= -2 - 14i$

ii) $2 - w = 2 - (3 - 4i)$

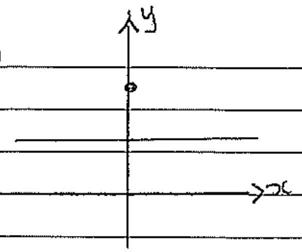
$= -1 + 4i$

$\frac{1}{2-w} = \frac{1}{-1+4i} \times \frac{-1-4i}{-1-4i}$

$= \frac{-1-4i}{1-16i^2}$

$= \frac{-1-4i}{17}$ $\therefore \mathrm{Im}\left(\frac{1}{2-w}\right) = \frac{-4}{17}$

c)



$|z-2i| = |z|$

perpendicular bisector of $(0,0)$ and $(0,2)$

$\therefore$ The locus is the line $y=1$

d) $w = 3-4i \qquad z = 2+2i$

i) $w\bar{z} = (3-4i)(2-2i)$

$= 6-6i-8i+8i^2$

$= -2-14i$

ii) $2-w = 2-(3-4i)$

$= -1+4i$

$$\frac{1}{2-w} = \frac{1}{-1+4i} \times \frac{-1-4i}{-1-4i}$$

$$= \frac{-1-4i}{1-16i^2}$$

$$= \frac{-1-4i}{17}$$

$\therefore \operatorname{Im}\left(\frac{1}{2-w}\right) = \frac{-4}{17}$

iii) Multiplying $\qquad i^3 = 270^\circ$

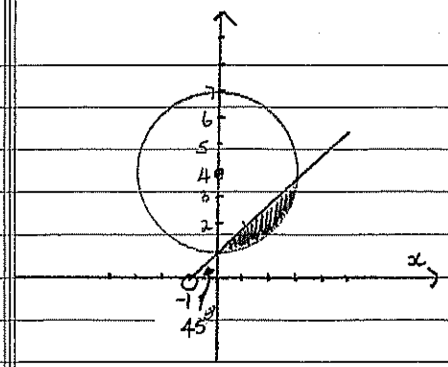
$i^3 z$

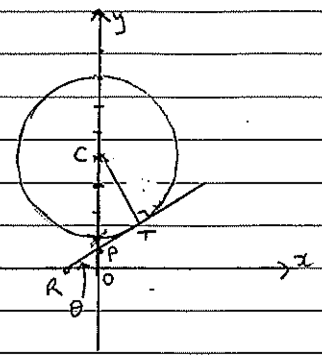
$= i^3(z+2i)$

$= -i(2+2i)$

$= -2i - 2i^2$

$= -2i + 2$





$RO = 1$

$CT = 3$

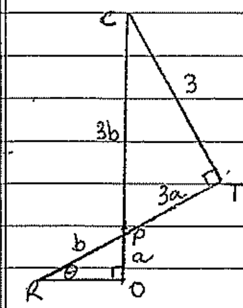
$PO = a$

$PR = b$

$CO = 4$ units

$\therefore \triangle CTP \parallel\!\mid \triangle OPR$

$1 : 3$



$1^2 + a^2 = b^2$

$a + 3b = 4$

$a = 4-3b$

$1^2 - a^2 = b^2$

$1^2 + (4-3b)^2 = b^2$

$1+16-24b+9b^2 = b^2$

$8b^2 - 24b + 17 = 0$

$$b = \frac{24 \pm \sqrt{24 - 4\times8\times17}}{2\times8}$$

$$b = \frac{24 \pm \sqrt{32}}{16} \Rightarrow b = \frac{6 \pm \sqrt{2}}{4}$$

$\cos\theta = 1 \div \left(\frac{6\pm\sqrt{2}}{4}\right)$

$\cos\theta = 0.87, 0.539$

$\theta = 29^\circ, 57^\circ$

Minimum value of $\arg(z+1)$ is 29°

Question 8

a) $\frac{7+2i}{5-i} \times \frac{5+i}{5+i} = \frac{35+7i+10i+2i^2}{25+1}$

$= \frac{33+17i}{26}$

$= \frac{33}{26} + \frac{17}{26} i$

b) i) $z = \frac{1}{1-i} \times \frac{1+i}{1+i} = \frac{1+i}{1-i^2}$

$\bar{z} = \frac{1}{2} - \frac{1}{2} i$

$|\bar{z}| = \sqrt{(\frac{1}{2})^2 + (-\frac{1}{2})^2} = \frac{1}{\sqrt{2}}$

$\arg(\bar{z}) = -\frac{\pi}{4}$

$\therefore \bar{z} = \frac{1}{\sqrt{2}} \text{cis}\left(-\frac{\pi}{4}\right)$

ii) $(\bar{z})^{13} = \left[\frac{1}{\sqrt{2}} \text{cis}\left(-\frac{\pi}{4}\right)\right]^{13}$

$= \left(\frac{1}{\sqrt{2}}\right)^{13} \text{cis}\left(\frac{-13\pi}{4}\right)$

$= \frac{1}{64\sqrt{2}} \text{cis}\left(\frac{3\pi}{4}\right)$

$= \frac{1}{64\sqrt{2}}\left(-\frac{1}{2} + \frac{1}{\sqrt{2}} i\right)$

$a = -\frac{1}{128}$ $b = \frac{1}{128}$

c) i) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\frac{2x}{a^2} + \frac{2y}{b^2} \times \frac{dy}{dx} = 0$

$\frac{dy}{dx} = -\frac{2x}{a^2} \times \frac{b^2}{2y}$

$= \frac{-xb^2}{a^2 y}$

At $P(a\cos\theta, b\sin\theta)$ $\frac{dy}{dx} = \frac{-b^2 a\cos\theta}{a^2(b\sin\theta)}$

Gradient of tangent at P $= \frac{-b\cos\theta}{a\sin\theta}$

Gradient of normal at P $= \frac{a\sin\theta}{b\cos\theta}$

Equation of normal at P

$y - b\sin\theta = \frac{a\sin\theta}{b\cos\theta}(x - a\cos\theta)$

$yb\cos\theta - b^2\sin\theta\cos\theta = a\sin\theta x - a^2\sin\theta\cos\theta$

$ax\sin\theta - by\cos\theta = (a^2-b^2)\sin\theta\cos\theta$

cii) $N(a\cos\theta, 0)$

let $y=0$ equation of normal

$ax\sin\theta = (a^2-b^2)\sin\theta\cos\theta$

$x = \frac{(a^2-b^2)\sin\theta\cos\theta}{a\sin\theta}$

$\therefore M = \left[\frac{(a^2-b^2)\cos\theta}{a}, 0\right]$

$$MN = \left|\frac{(a^2-b^2)\cos\theta}{a} - a\cos\theta\right|$$

$$= \left|\frac{a^2\cos\theta - b^2\cos\theta - a^2\cos\theta}{a}\right|$$

$$= \left|\frac{b^2\cos\theta}{a}\right|$$

d) i) $P(x) = (x-\alpha)^n Q(x) \qquad Q(\alpha) \neq 0$

$P'(x) = n(x-\alpha)^{n-1} Q(x) + (x+\alpha)^n \cdot Q'(x)$

$= (x-\alpha)^{n-1}\left[n Q(x) + (x+\alpha)^n \cdot Q'(x)\right]$

$= (x-\alpha)^{n-1} \cdot Q(x)$ where $Q(\alpha) \neq 0$

$\therefore$ α is a root of $P'(x)$ of multiplicity $n-1$

d ii) $P(x) = 2x^4 + 9x^3 + 6x^2 - 20x - 24$

$P'(x) = 8x^3 + 27x^2 + 12x - 20$

$P''(x) = 24x^2 + 54x + 12$

$\therefore$ $24x^2 + 54x + 12 = 0$

$4x^2 + 9x + 2 = 0$

$(4x+1)(x+2) = 0$

$x = -\frac{1}{4} \qquad x = -2$

Sub $x = -2 \qquad P(-2) = 2(-2)^4 + 9(-2)^3 + 6(-2)^2 - 20(-2) - 24 = 0$

$\therefore$ $x = -2$ is a triple root

$\therefore$ $P(x) = (x-2)^3(2x-3)$

Question 9

a i) $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$

$= -\frac{1}{t^2} \times 1$

$= -\frac{1}{t^2}$

$y - \frac{1}{t} = -\frac{1}{t^2}(x - t)$

$t^2 y - t = -x + t$

$\therefore$ $x + t^2 y = 2t$

ii) $y = t^2 x$ Gradient of OM is perpendicular to tangent at P.

iii) $t^2 = \frac{y}{x}$

$t = \pm\sqrt{\frac{y}{x}}$

$x + t^2 y = 2t$

$x + \frac{y}{x} \times y = 2 \pm \sqrt{\frac{y}{x}}$

$x + \frac{y^2}{x} = 2 \times \left(\pm\sqrt{\frac{y}{x}}\right)$

$\left(x + \frac{y^2}{x}\right)^2 = 4 \times \left(\pm\frac{y}{x}\right)$

$x^2 + 2y^2 + \frac{y^4}{x^2} = \frac{4y}{x}$

$x^4 + 2x^2y^2 + y^4 = 4xy$

restriction $\frac{y}{x} > 0$

The point M cannot lie on the hyperbola

(Alternate Method)

$M\left(\frac{2t}{1+t^4}, \frac{2t^3}{1+t^4}\right)$

$x = \frac{2t}{1+t^4} \qquad y = \frac{2t^3}{1+t^4}$

$x^4 + 2x^2y^2 - 4xy + y^4 = 0$

$\left(\frac{2t}{1+t^4}\right)^4 + 2\left(\frac{2t}{1+t^4}\right)^2\left(\frac{2t^3}{1+t^4}\right)^2 - 4\left(\frac{2t}{1+t^4}\right)\left(\frac{2t^3}{1+t^4}\right) + \left(\frac{2t^3}{1+t^4}\right)^4 = 0$

$\frac{16t^4}{(1+t^4)^4} + \frac{32t^8}{(1+t^4)^4} - \frac{16t^4}{(1+t^4)^2} + \frac{16t^{12}}{(1+t^4)^4}$

$\frac{16}{(1+t^4)^4}\left[t^4 + 2t^8 + t^{12}\right] - \frac{16t^4}{(1+t^4)^2}$

$\frac{16}{(1+t^4)^4}(t^4 + 2t^8 + t^{12}) - \frac{16t^4(1+2t^4+t^8)}{(1+t^4)^4}$

$\frac{16}{(1+t^4)^4}\left[t^4 + 2t^8 + t^{12} - (t^4 + 2t^8 + t^{12})\right]$

$\frac{16}{(1+t^4)^4} \times 0$

$= 0$

$\therefore$ Locus of P is $x^4 + 2x^2y^2 - 4xy + y^4 = 0$.

$P(t, \frac{1}{t}) \quad \therefore t \neq 0$

M cannot be at the origin

b) $P(x) = x^5 + 2x^2 + mx + n$

$P'(x) = 5x^4 + 4x + m$

$P(x)$ has a double root at $x = -2$

$P'(-2) = 0$

$80 - 8 + m = 0$

$m = -72$

$P(-2) = 0$

$-32 + 8 + 144 + n = 0$

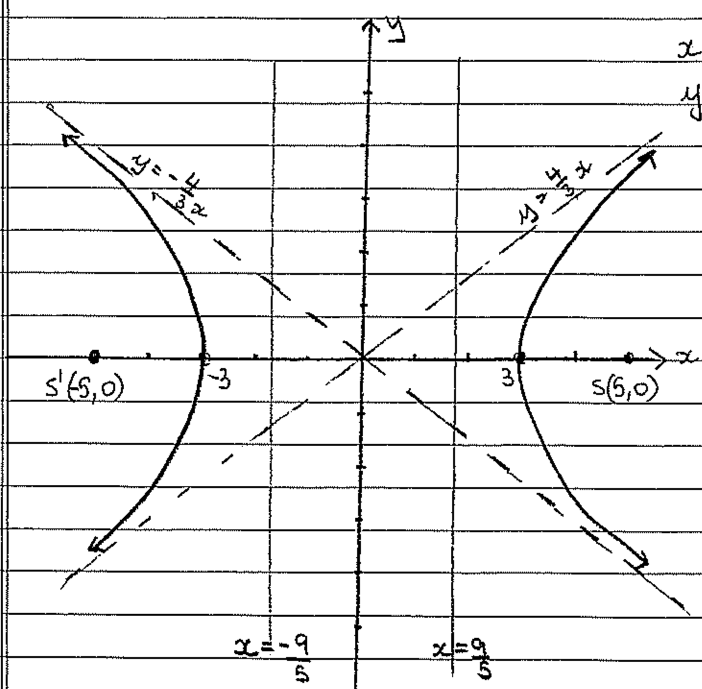
$n = -120$

Roots : $-2, -2, \alpha, \beta, \gamma$

Product of roots $\quad 4\alpha\beta\gamma = 120$

$\alpha\beta\gamma = 30$

c)



$x = 3\sec\theta$

$y = 4\tan\theta$

$a = 3$

$b = 4$

$4^2 = 3^2(e^2 - 1)$

$e = \frac{5}{3}$

Directrices $\frac{a}{e} = \frac{9}{5}$

Asymptotes $\frac{b}{a} = \frac{4}{3}$

foci $(-5,0)\ (5,0)$

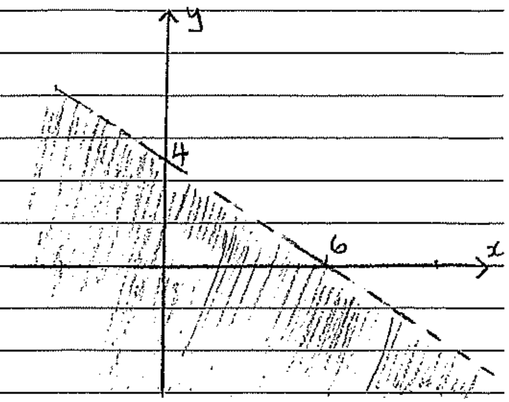
Question 10

a) $\left[(2-3i)z\right] < 12$

$Re\left[(2-3i)(x+iy)\right] < 12$

$Re\left[2x+3y+i(2y-3x)\right] < 12$

ie $2x+3y<12$



b) $x = a\cos\theta$ $\quad \frac{dx}{d\theta} = -a\sin\theta$

$y = b\sin\theta$ $\quad \frac{dy}{d\theta} = b\cos\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \div \frac{dx}{d\theta}$$

$$= b\cos\theta \times \frac{1}{-a\sin\theta}$$

$$= \frac{-b\cos\theta}{a\sin\theta}$$

Equation of tangent at P

$$y - b\sin\theta = \frac{-b\cos\theta}{a\sin\theta}(x - a\cos\theta)$$

$$ya\sin\theta - ab\sin^2\theta = -xb\cos\theta + ab\cos^2\theta$$

$$xb\cos\theta + ya\sin\theta = ab(\cos^2\theta + \sin^2\theta)$$

$$xb\cos\theta + ya\sin\theta = ab$$

$$\frac{xb\cos\theta}{ab} + \frac{ya\sin\theta}{ab} = 1$$

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$

bii) Gradient of QR

$Q\left[a\cos(\theta+\varphi), b\sin(\theta+\varphi)\right]$ $\quad R\left[a\cos(\theta-\varphi), b\sin(\theta-\varphi)\right]$

$$M_{QR} = \frac{b\sin(\theta-\varphi) - b\sin(\theta+\varphi)}{a\cos(\theta-\varphi) - a\cos(\theta+\varphi)}$$

$$= \frac{b(\sin\theta\cos\varphi - \cos\theta\sin\varphi - \sin\theta\cos\varphi - \cos\theta\sin\varphi)}{a(\cos\theta\cos\varphi + \sin\theta\sin\varphi - \cos\theta\cos\varphi + \sin\theta\sin\varphi)}$$

$$= \frac{-2b\cos\theta\sin\varphi}{2a\sin\theta\sin\varphi}$$

$$M_{QR} = \frac{-b\cos\theta}{a\sin\theta}$$

Gradient of tangent at $P = \frac{-b\cos\theta}{a\sin\theta}$

$\therefore$ chord QR is parallel to the tangent at P

biii) Let M be the midpoint of QR

$$\left[\frac{a\cos(\theta+\varphi) + a\cos(\theta-\varphi)}{2}, \frac{b\sin(\theta+\varphi) + b\sin(\theta-\varphi)}{2}\right]$$

$$x = \frac{a(\cos\theta\cos\varphi - \sin\theta\sin\varphi + \cos\theta\cos\varphi + \sin\theta\sin\varphi)}{2} = a\cos\theta\cos\varphi$$

$$y = \frac{b(\sin\theta\cos\varphi + \cancel{\cos\theta\sin\varphi} + \sin\theta\cos\varphi - \cancel{\cos\theta\sin\varphi})}{2} = b\sin\theta\cos\varphi$$

$$M(a\cos\theta\cos\varphi, b\sin\theta\cos\varphi)$$

Equation of OP is $y = \frac{b\sin\theta}{a\cos\theta}x$

$$y = \frac{b\sin\theta}{a\cos\theta} \times a\cos\theta\cos\varphi$$

$$y = b\sin\theta\cos\varphi$$

M lies on OP

$\therefore$ OP bisects QR

ci) $z = \cos\theta + i\sin\theta$

$= r^3(\cos\theta + i\sin\theta)^3$

$= r^3(\cos 3\theta + i\sin 3\theta)$

$r^3 = 1$

$\cos 3\theta + i\sin\theta = \cos 0 + i\sin 0$

$\cos 3\theta = \cos 0 \qquad \sin 3\theta = \sin 0$

$\cos 3\theta = 1$

$3\theta = 0 + 2k\pi$

$\theta = \frac{0 + 2k\pi}{3} \qquad k = 0, 1, 2.$

$\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$

cii) $\omega = cis\,\frac{2\pi}{3}$ If ω is a root then $\omega^3 = 1$

$\omega^3 - 1 = 0$

$(\omega - 1)(\omega^2 + \omega + 1) = 0$

$\omega^2 + \omega + 1 = 0$

$-\omega^2 - \omega - 1 = 0$

$\omega^3 - \omega^2 - \omega - 1 = 1$

$\omega^3 - \omega^2 - \omega - 2 = 0$

ciii) $z^3 - z^2 - z - 2 = 0$

$P(2) = (2)^3 - (2)^2 - (2) - 2$

$P(2) = 0$

$\therefore z = 2$ is a root

$$\begin{array}{r|l} & z^2 + z + 1 \\ z - 2 & z^3 - z^2 - z - 2 \\ & \underline{z^3 - 2z^2} \\ & \quad z^2 - z \\ & \quad \underline{z^2 - 2z} \\ & \qquad z - 2 \\ & \qquad \underline{z - 2} \\ & \qquad\quad 0. \end{array}$$

$P(2) = (z-2)(z^2+z+1)$

$z^2 + z + 1 = 0$

$z = \frac{-1 \pm \sqrt{1-4}}{2}$

$z = -\frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$z = -\frac{1}{2} + \frac{\sqrt{3}}{2}i \qquad z = -\frac{1}{2} - \frac{\sqrt{3}i}{2}$