SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

YEAR 11 COMMON TEST

MAY 2005

Time allowed: 70 minutes

Instructions:

- Show all necessary working in every question
- Start each question on a new page
- Attempt all questions
- All questions are not of equal value
- Marks shown are approximate & may be changed
- Full marks may not be awarded for careless or badly arranged work
- Your sketches must be neat. Use a ruler to draw axes.
- Approved calculators may be used
- These questions are to be handed in with your answers.

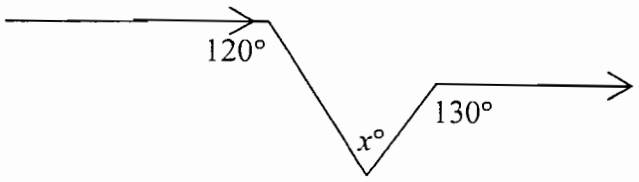
Name:_____

Class:_____

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Total
/10	/10	/10	/8	/8	/8	/54

Question 1 **Marks**

a)



Find the value of x (no reasons necessary). 1

b) Factorise $a^2 - b^2 - (a-b)^2$ 2

c) The hyperbola $y = \dfrac{3}{a-x}$ has a vertical asymptote at $x = 1$. What is the value of a? 1

d) If $\tan a = -\dfrac{1}{3}$ and $\cos a > 0$, find the exact value of $\sin a$. 2

e) Given that n is a positive number indicate 2

(i) the largest

(ii) the smallest

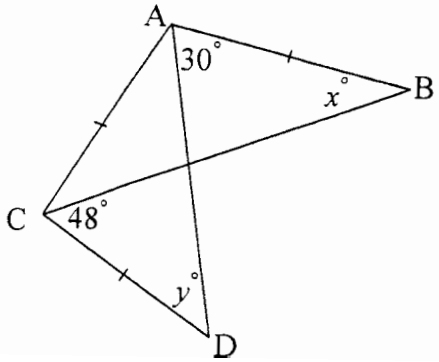
of the following numbers:

$3^{\frac{-n}{3}}$, $3^{\frac{n}{3}}$, 3^{n}, 3^{-n}

f) Solve $|5x-3| = |3x+1|$ 2

Question 2 (start a new page)

a)



ΔABC and ΔACD are isosceles. 3

By forming a pair of simultaneous equations or otherwise, find the value of x.

b) If $\frac{a^n + a^{n+2}}{a^n} = 10$ find a given that $a > 0$. 2

c) Solve $\frac{x-2}{x} \geq 1$ 3

d) There are two values of θ in the domain $0° \leq \theta \leq 360°$ where sin θ and cos θ are numerically equal. Find these two values. 2

Question 3 (start a new page)

a) If $p = \frac{\sqrt{3}}{4-\sqrt{3}}$ and $q = \frac{\sqrt{3}}{4+\sqrt{3}}$ evaluate $\frac{p+q}{1-pq}$ 3

b) How many solutions does the equation $(\cos x - 2)\ (\sin^2 x - 1) = 0$ 3
have in the domain $0° \leq x \leq 360°$?
There is no need to solve the equation.
Justify your answer.

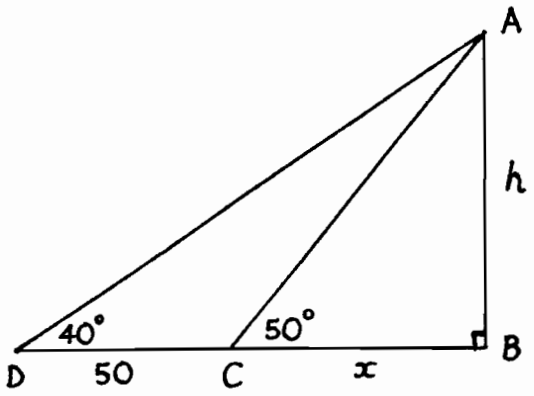
c) i) Sketch the graph of $y = x^2 + \frac{1}{2}$ 4

ii) On a separate diagram sketch $y = \frac{2}{2x^2 + 1.}$

iii) Use your diagram or otherwise to write down the range of $y = \frac{2}{2x^2 + 1}$

Question 4 (start a new page)

a)



We wish to find the height AB of a vertical cliff. From a point D the angle of elevation of A is $40°$. From a point C 50m nearer the base of the cliff the angle of elevation is $50°$. 4

i) Show that $h = (50 + x)\ \tan\ 40°$

ii) Show that $h = x\ \tan\ 50°$

iii) Using simultaneous equations find h

b) i) Sketch the graph of $y = |x + 1|$. 4

ii) By using your graph or otherwise solve $\frac{2}{x} > |x + 1|$

Question 5 (start a new page).

a) i) Sketch the graph of $y = \cos\ x$ for $0° \le x \le 360°$ 4

ii) Hence solve $-\frac{\sqrt{3}}{2} \le \cos x \le \frac{\sqrt{3}}{2}$ for $0° \le x \le 360°$

b) i) If $xy = c^2$ prove that $\frac{1}{c+x} + \frac{1}{c+y} = \frac{1}{c}$. 4

ii) Hence or otherwise simplify $\frac{1}{6+\sqrt{51}+\sqrt{15}} + \frac{1}{6+\sqrt{51}-\sqrt{15}}$

Question 6 (start a new page)

a) Find the point/s of intersection for the graphs of $y = x^2 - 1$ and $y = \dfrac{1}{x^2 - 1}$ 3

b) i) Sketch the graph of $y = -\sqrt{2 - x^2}$ 5

ii) On the same diagram shade the region where $y \geq -\sqrt{2 - x^2}$, $|x| \leq 1$ and $y \leq 0$ hold simultaneously.

iii) Find the exact value for the area of the shaded region.

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Question 1

a) $x = 70$

b) $a^2 - b^2 - (a-b)^2$

$(a-b)(a+b) - (a-b)^2$

$(a-b)((a+b) - (a-b))$

$2b(a-b)$

c) $y = \frac{3}{a-x}$ $\quad a - x = 0$

$\therefore \quad a = 1$

d) $\tan\alpha = -\frac{1}{3} \quad \cos\alpha > 0$

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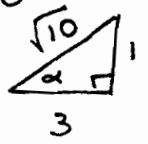


$\therefore \sin\alpha = -\frac{1}{\sqrt{10}}$

e) $3^{-\frac{n}{3}} \quad 3^{\frac{n}{3}} \quad 3^{n} \quad 3^{-n}$

i) largest 3^n

ii) smallest 3^{-n} } n a +ve integer

f) $|5x-3| = |3x+1|$

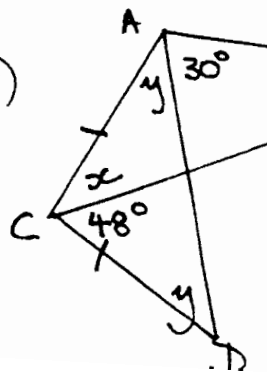
$5x - 3 = 3x + 1$ or $5x - 3 = -3x - 1$

$2x = 4$ $\qquad$ $8x = 2$

$x = 2$ or $x = \frac{1}{4}$

Question 2

a)



$2x + y + 30 = 180$

$2y + x + 48 = 180$

$\therefore 2x + y = 150 \quad -①$

$x + 2y = 132 \quad -②$

b) $\frac{a^n + a^{n+2}}{a^n} = 10$

$\frac{a^n(1 + a^2)}{a^n} = 10$

$\therefore a^2 = 9$

$a = 3$ only $\quad a > 0$

c) $\frac{x-2}{x} \geq 1$

$x(x-2) \geq x^2$

$x(x-2) - x^2 \geq 0$

$x(x - 2 - x) \geq 0$

$-2x \geq 0$

$x \leq 0$ but $x \neq 0$

$\therefore x < 0$

d) $\sin\theta = \cos\theta$

$\tan\theta = 1$

acute $\theta = 45$

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$\therefore \theta = 45°, 225°$

Question 3

a) $p = \frac{\sqrt{3}}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}} = \frac{4\sqrt{3}+3}{13}$

$q = \frac{\sqrt{3}}{4+\sqrt{3}} \times \frac{4-\sqrt{3}}{4-\sqrt{3}} = \frac{4\sqrt{3}-3}{13}$

$p + q = \frac{4\sqrt{3} + 3 + 4\sqrt{3} - 3}{13}$

$= \frac{8\sqrt{3}}{13}$

$1 - pq = 1 - \frac{\sqrt{3}}{4-\sqrt{3}} \times \frac{\sqrt{3}}{4+\sqrt{3}}$

$$\frac{p+q}{1-pq} = \frac{8\sqrt{3}}{13} \div \frac{10}{13}$$

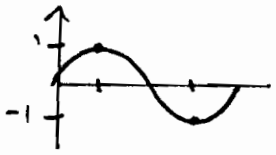
$$= \frac{8\sqrt{3}}{10}$$

$$= \frac{4\sqrt{3}}{5}$$

b) $(\cos x - 2)(\sin^2 x - 1) = 0$

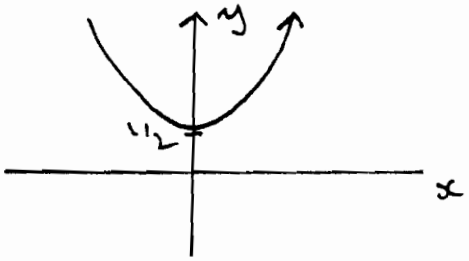
$\cos x = 2$ $\quad$ $\sin^2 x = 1$

no solutions $\quad$ $\sin x = \pm 1$

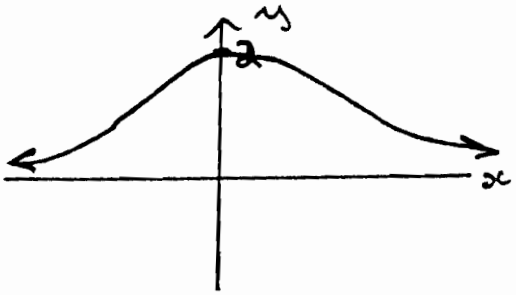


2 solutions

c) i) $y = x^2 + \frac{1}{2}$



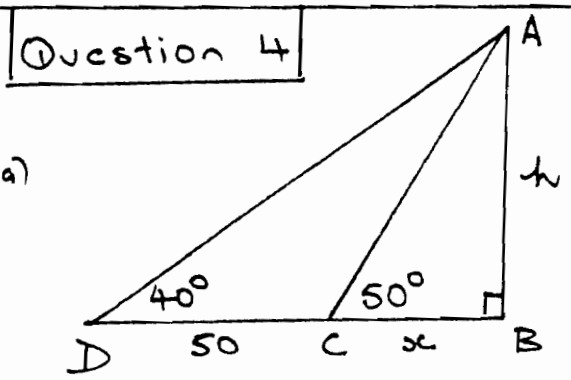
ii) $y = \dfrac{2}{2x^2+1}$



iii) Range $\quad 0 < y \leq 2$

Question 4

a)



ii) $\tan 50^\circ = \dfrac{h}{x}$

$\therefore \quad \therefore \quad h = x \tan 50^\circ$

i) $\tan 40 = \dfrac{h}{50+x}$

$(50+x)\tan 40^\circ = h$

iii) $x \tan 50^\circ = (50+x)\tan 40^\circ$

$x \tan 50^\circ = 50 \tan 40^\circ + x \tan 40^\circ$

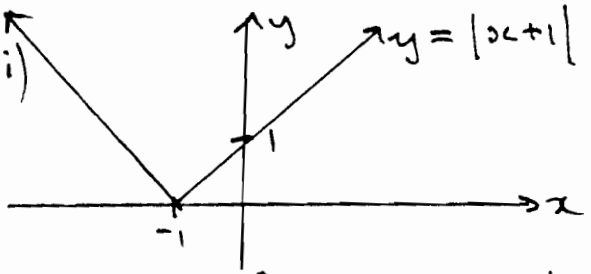
$x \tan 50^\circ - x \tan 40^\circ = 50 \tan 40^\circ$

$x(\tan 50 - \tan 40) = 50 \tan 40^\circ$

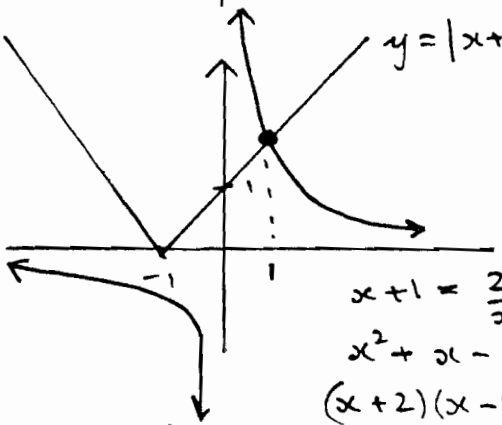
$$x = \frac{50 \tan 40^\circ}{\tan 50 - \tan 40}$$

$x = 118.97$ units (2 dec pl)

b) i)



ii)



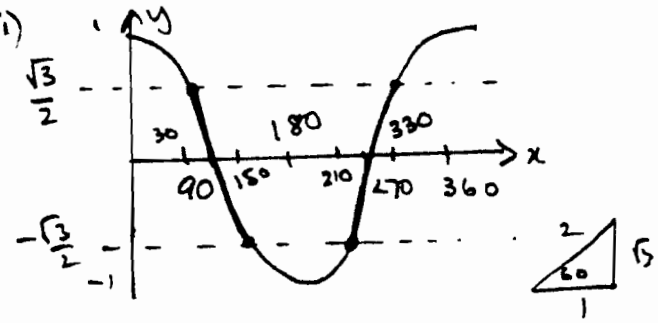
$x + 1 = \dfrac{2}{x}$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

Question 5

a) i)



ii)

$\cos x = \frac{\sqrt{3}}{2}$, $\cos x = -\frac{\sqrt{3}}{2}$

acute $x = 30^\circ$

$x = 30^\circ, \quad 150^\circ$ $\qquad$ $x = 210^\circ, 330^\circ$

$\therefore$ Solution: $30^\circ \leq x \leq 150^\circ$

and $210^\circ \leq x \leq 330^\circ$

b) $\frac{1}{c+x} + \frac{1}{c+y}$

$= \frac{c+y+c+x}{(c+x)(c+y)}$

$= \frac{2c+x+y}{c^2+cx+cy+xy}$ $\quad c^2$

$= \frac{2c+x+y}{2c^2+cx+cy}$

$= \frac{(2c+x+y)}{c(2c+x+y)}$

$= \frac{1}{c}$

$\therefore \frac{1}{c+x} + \frac{1}{c+y} = \frac{1}{c}$

ii) If $x = \sqrt{51} + \sqrt{15}$

$y = \sqrt{51} - \sqrt{15}$

$\frac{1}{6+x} + \frac{1}{6+y} = \frac{1}{6}$

Question 6

a) $y = x^2 - 1$ $\qquad$ $y = \frac{1}{x^2-1}$

$x^2 - 1 = \frac{1}{x^2-1}$

$(x^2-1)^2 = 1$

$x^2 - 1 = 1$ or $x^2 - 1 = -1$

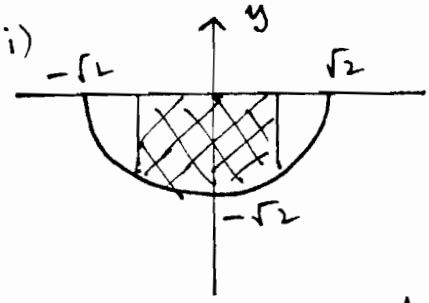
$x^2 = 2$ $\qquad$ $x^2 = 0$

$x = \pm\sqrt{2}$ $\qquad$ $x = 0$

$\therefore$ pts intersection

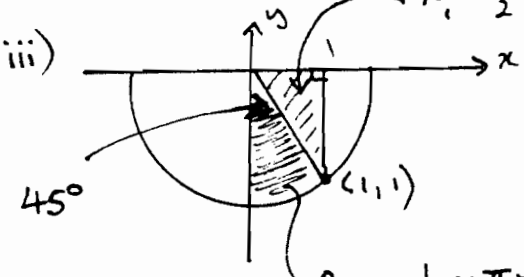
$(\sqrt{2}, 1)$ $\quad$ $(-\sqrt{2}, 1)$ $\quad$ $(0, -1)$

b) i)



$y = -\sqrt{2 - x^2}$

iii)



$A_1 = \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

$A_2 = \frac{1}{8} \times \pi \times \sqrt{2}^2 = \frac{\pi}{4}$

$\therefore$ Required area $= 2\left(\frac{1}{2} + \frac{\pi}{4}\right)$