

SYDNEY TECHNICAL HIGH SCHOOL



MATHEMATICS 2 UNIT

YEAR 11 PRELIMINARY COURSE

MAY 2014

NAME _____ TEACHER _____

Time allowed 75 minutes

Instructions * Begin each question on a new page.

* Marks shown are a guide and may be varied.

* Show necessary working.

* Full marks may not be awarded if your working is poorly set out or illegible.

* Leave all answers in simplest form.

* Use a ruler for all straight lines.

Question 1 (7 marks)

- a) Evaluate $\sqrt{\frac{\pi}{1.6^2}}$ correct to two significant figures. 1
- b) Simplify i) $\sqrt{8} + \sqrt{18}$ 1
- ii) $(3\sqrt{2} - 4)^2$ 1
- c) Evaluate $|-4| - |8 - 20|$ 1
- d) Find the values of p and q such that $\frac{\sqrt{5}}{\sqrt{5}-2} = p + q\sqrt{5}$ 2
- e) Simplify $2^x \div 2^{x-3}$ 1

Question 2 (7 marks) Start a new page.

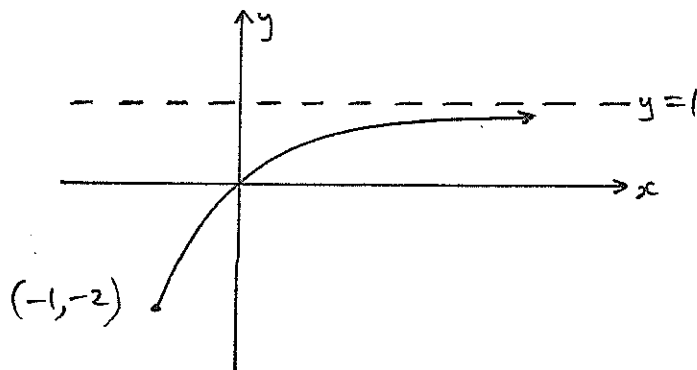
- a) Subtract $2x^2 - 3x - 1$ from $2x^2 - x + 5$ 1
- b) Simplify $\frac{(3m^2x^3)^2 \times 2mx^2}{4m^8x}$ 2
- c) Rewrite $m(m+2)^{-1}$ without a negative index. 1
- d) Fully factorise: i) $2x^2 - 18$ 1
- ii) $3x^2 + 11x - 4$ 1
- iii) $a^2 - ab - a + b$ 1

Question 3 (7 marks) Start a new page.

- a) Simplify $\frac{a}{a+\frac{1}{a}}$ 1
- b) Solve: i) $7(m-4) = 2(m+11)$ 1
- ii) $\frac{6-4y}{2} < \frac{y}{3} + 2$ 2
- c) Solve each quadratic equation, leaving answers in simplest exact form:
- i) $3x - 4x^2 = 0$ 1
- ii) $4x^2 - 6x - 1 = 0$ 2

Question 4 (7 marks) Start a new page.

- a) Solve $|2x + 1| = 7$ 2
- b) Given $H(x) = x^2 - 3x$, find and simplify: i) $H(-1)$ 1
- ii) $H(m + 4)$ 2
- c) Write the domain and range of the function graphed below: 2

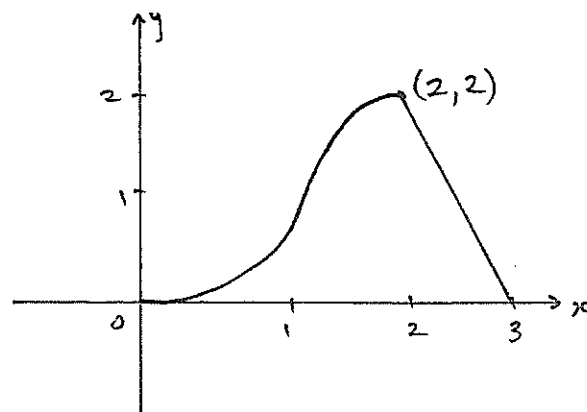


Question 5 (7 marks) Start a new page.

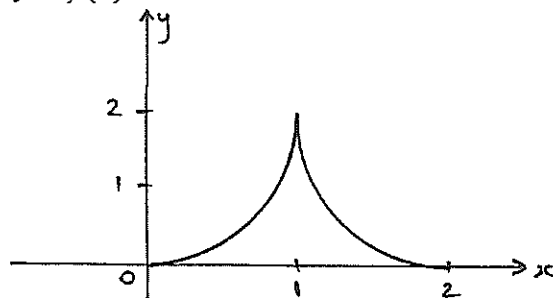
- a) Neatly sketch the parabola $y = x^2 - x - 6$. Clearly show intercepts on both axes and the coordinates of the vertex. 2
- b) Sketch each curve below on separate number planes. Use a ruler and clearly label asymptotes or other key features. i) $x^2 + (y - 2)^2 = 4$ 2
- ii) $y = \frac{1}{x+2} + 1$ 2
- c) Given $\frac{1}{x\sqrt{x}} = x^a$, find the value of a . 1

Question 6 (7 marks) Start a new page.

- a) Neatly copy the curve below into your answer booklet. Add a new section of the curve so that the total curve clearly represents an odd function. Label key points. 1



b) Given the curve $y = f(x)$ below:



i) Evaluate $f(2)$

1

ii) Sketch the graph of $y = f(x) - 1$

1

c) i) Sketch the graph of $y = |4 - x|$

1

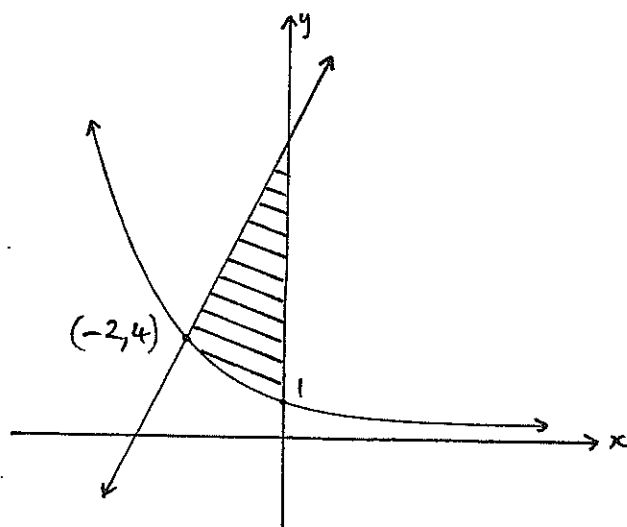
ii) Hence, or otherwise, solve $|4 - x| < 3$

1

d) Simplify $\frac{x^3-1}{x^2-1}$

2

Question 7 (7 marks) Start a new page.



The diagram, not to scale, shows the line
 $2x - y + 8 = 0$ and an exponential function
of the form $y = a^{-x}$.

a) i) Write the equation of the exponential function shown.

1

ii) The shaded area represents the intersection of three (3) regions. Write the inequalities for these regions.

3

b) Solve simultaneously to find the points of intersection of the graphs of

$$xy = 4 \text{ and } 2x - y - 2 = 0$$

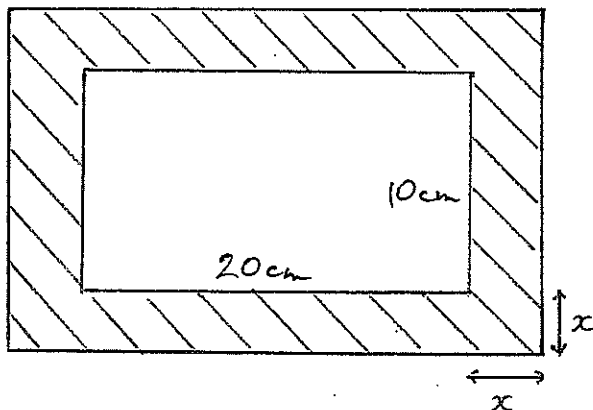
3

Question 8 (7 marks) Start a new page.

a) Find the domain and range of the function $f(x) = 10 + \frac{1}{\sqrt{x-4}}$

2

b)



A photograph measures 20 cm by 10 cm. It is surrounded by the shaded border of uniform width x cm. The total area of photo + border is 416 cm^2 .

Write and solve a quadratic equation in x and find the dimensions of the border. 3

c) Find a value x such that $\sqrt{x + \sqrt{x + \sqrt{x + \dots}}} = \frac{1 + \sqrt{17}}{2}$

2

(hint: you may want to start by squaring both sides)

END OF TEST

Solutions

① a) 1.1

b) i) $2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

ii) $18 - 24\sqrt{2} + 16 = 34 - 24\sqrt{2}$

c) $4 - 12 = -8$

d) $\frac{\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2} = \frac{5+2\sqrt{5}}{5-4}$
 $= 5+2\sqrt{5}$

$\therefore p=5, q=2$

e) 2^3 or 8

② a) $2x^2 - x + 5 - 2x^2 + 3x + 1 = 2x + 6$

b) $\frac{9m^4x^6 \times 2m^2x^2}{4m^8x} = \frac{9x^7}{2m^3}$

c) $\frac{m}{m+2}$

d) i) $2(x+3)(x-3)$

ii) $(3x-1)(x+4)$

iii) $a(a-b) - (a-b)$
 $= (a-b)(a-1)$

③ a) $\frac{a}{\frac{a^2+1}{a}} = a \times \frac{a}{a^2+1}$
 $= \frac{a^2}{a^2+1}$

b) i) $7m - 28 = 2m + 22$

$5m = 50$

$m = 10$

ii) $\frac{6-4y}{2} < \frac{y}{2} + 2$

$18 - 12y < 2y + 12$

$-14y < -6$

$y > \frac{3}{7}$

c) i) $x(3-4x) = 0$
 $x = 0, \frac{3}{4}$

ii) $x = \frac{6 \pm \sqrt{36 - 4 \times 4 \times -1}}{8}$

$= \frac{6 \pm \sqrt{52}}{8}$

$= \frac{6 \pm 2\sqrt{13}}{8}$

$= \frac{3 \pm \sqrt{13}}{4}$

4) a) $2x+1=7$ or $-2x-1=7$
 $x=3$ $-2x=8$
 $x=-4$

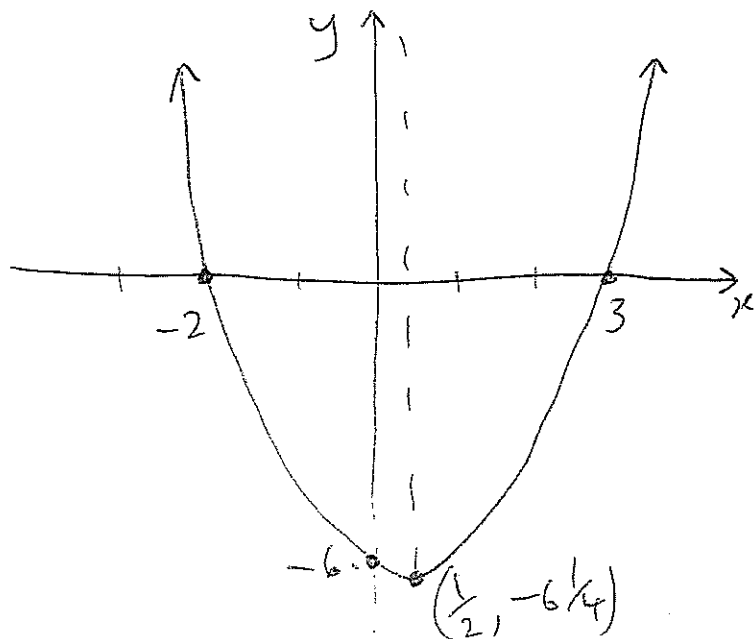
$\therefore x = 3, -4$

b) i) $H(-1) = (-1)^2 + 3$
 $= 4$

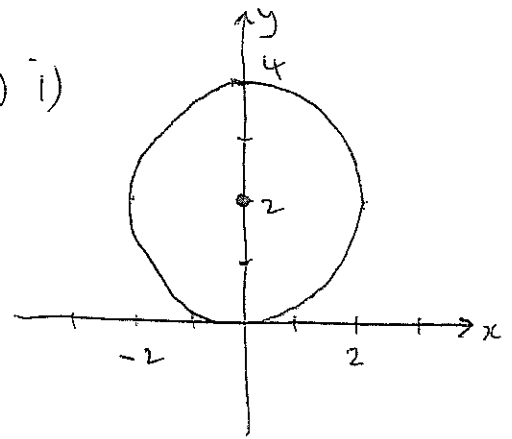
ii) $H(m+4) = (m+4)^2 - 3(m+4)$
 $= m^2 + 8m + 16 - 3m - 12$
 $= m^2 + 5m + 4$

c) $D: x \geq -1$
 $R: -2 \leq y < 1$

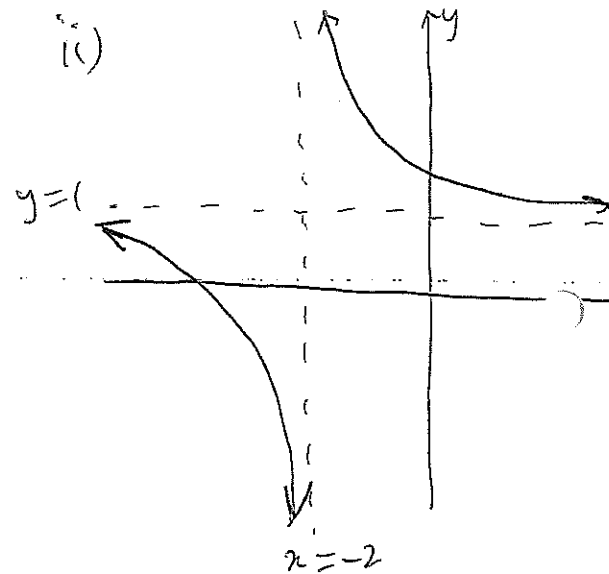
5) a) $y = (x-3)(x+2)$



b) i)

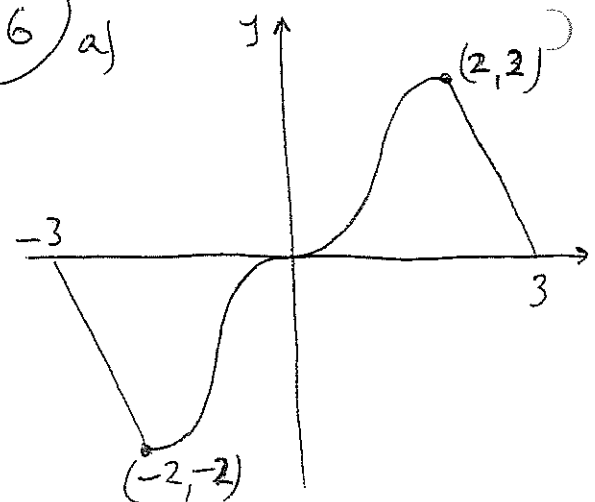


ii)

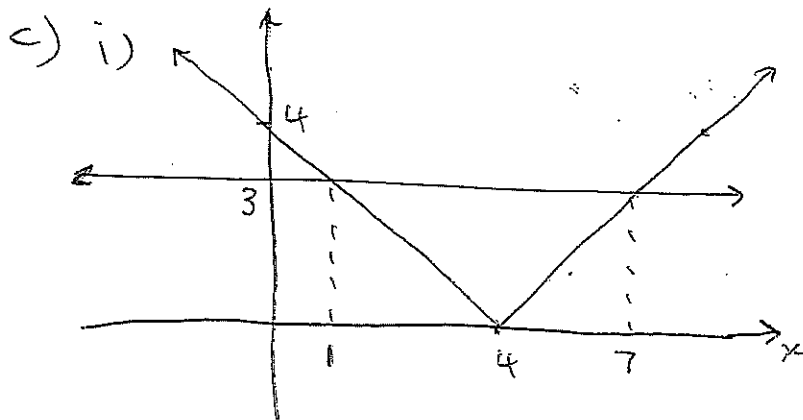
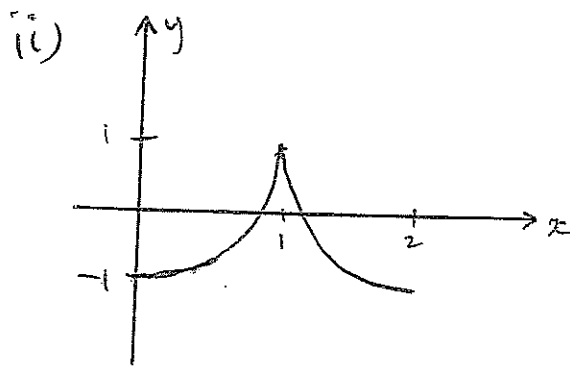


c) $\frac{1}{x^{3/2}} = x^{-3/2}$
 $\therefore a = -\frac{3}{2}$

6) a)



⑥ b) i) $f(z) = 0$



ii) $1 < x < 7$

d)

$$\frac{(x-1)(x^2+x+1)}{(x-1)(x+1)}$$

$$= \frac{x^2+x+1}{x+1}$$

⑦ a) i) $y = 2^{-x}$

ii) $x \leq 0$

$$y \geq 2^{-x}$$

$$2x - y + 8 \geq 0$$

b) $2x - y - 2 = 0$ — ①

$$xy = 4$$
 — ②

From ①: $y = 2x - 2$

Sub ②: $x(2x - 2) = 4$

$$2x^2 - 2x - 4 = 0$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$\therefore x = 2, -1$$

\therefore pts. of intersection are $(2, 2)$ and $(-1, -4)$

⑧ a) $x - 4 > 0$

$$D: x > 4$$

$$R: y > 10$$

b) $(20 + 2x)(10 + 2x) = 416$

$$200 + 40x + 20x + 4x^2 = 416$$

$$4x^2 + 60x - 216 = 0$$

$$x^2 + 15x - 54 = 0$$

$$(x - 3)(x + 18) = 0$$

$$\therefore x = 3, \text{ } \cancel{x = -18} \text{ no solution}$$

\therefore border is 26 cm long
16 cm wide

$$(8) \text{ c) (squaring) } x + \sqrt{x + \sqrt{x + \dots}} = \left(\frac{1 + \sqrt{17}}{2} \right)^2$$

$$x + \frac{1 + \sqrt{17}}{2} = \frac{1 + 2\sqrt{17} + 17}{4}$$

$$\therefore x = \frac{18 + 2\sqrt{17}}{4} - \frac{2 + 2\sqrt{17}}{4}$$

$$= \frac{18 + \cancel{2\sqrt{17}} - 2 - \cancel{2\sqrt{17}}}{4}$$

$$= \frac{16}{4}$$

$$= 4.$$