Sydney Technical High School

Name: ....

Maths Class: ....

Year 12
Mathematics
HSC Course
Assessment 1
December, 2017

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided at the rear of this Question Booklet, and may be removed at any time.

Section 1 Multiple Choice

Questions 1-10

10 Marks

Section II Questions 11-14

60 Marks

SECTION I

10 Marks

Allow about 15 minutes for this section.

Use the multiple choice answer sheet provided in the answer booklet.

1. The quadratic equation $2x^2 - x + 5 = 0$ has:

   A. two real solutions
   B. one real solution
   C. no real solutions
   D. rational solutions

2. \$1200 is invested for two years at 10% per annum compounded annually.

   The amount of interest earned in the second year is

   A. \$120
   B. \$126
   C. \$132
   D. \$252

3. What are the coordinates of the focus of the parabola $4y = x^2 - 8$?

   A. (0, -8)
   B. (0, -7)
   C. (0, -2)
   D. (0, -1)

4. The parabola $y = x^2 - 6x + 13$ is

   A. positive definite
   B. negative definite
   C. indefinite
   D. none of the above

5. Which of the following does NOT represent the sum $6 + 8 + 10 + 12 + 14$?

A. $\sum_{n=3}^{7} 2n$

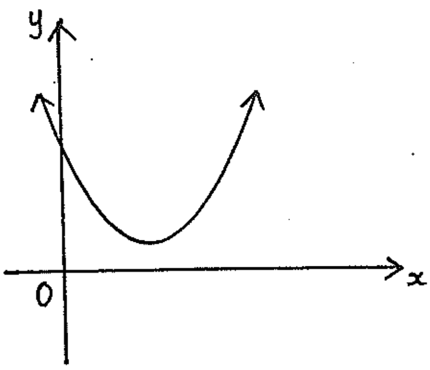
B. $\sum_{n=4}^{8} 2n - 1$

C. $\sum_{n=2}^{6} 2(n + 1)$

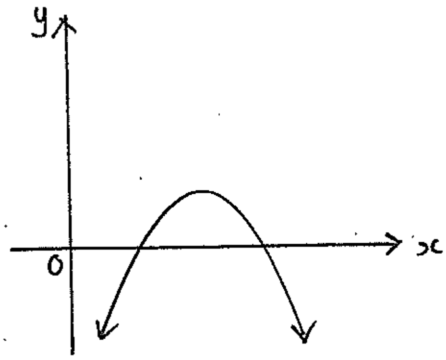
D. $\sum_{n=1}^{5} 2n + 4$

6. Which of the following could represent the graph of $2x - x^2 = 3 - y$?

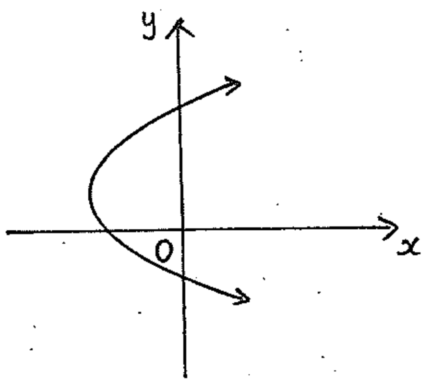
A.



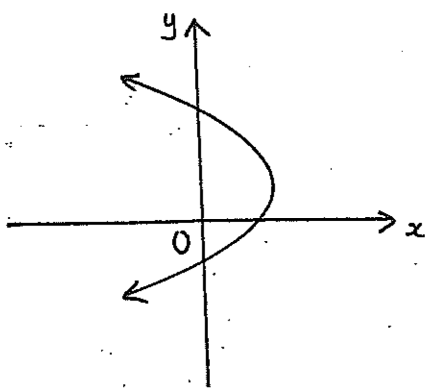
B.



C.



D.



7. Which of the following statements is true for the series: $(1-\sqrt{2})+\frac{1-\sqrt{2}}{2}+\frac{1-\sqrt{2}}{2^2}+\cdots$

A. It is geometric with common ratio of 2.

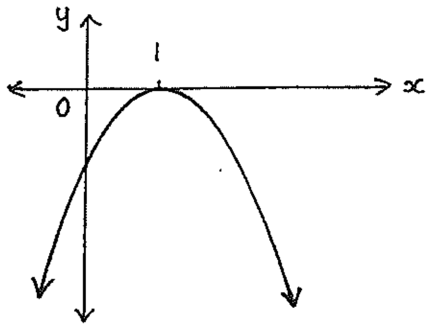
B. It is geometric with a limiting sum.

C. It has a common ratio of $(1-\sqrt{2})$

D. It is not geometric.

8. The parabola shown has its vertex at (1,0).

Find the equation of its directrix if the latus rectum is 12 units long.



A. $x=1$ B. $y=3$ C. $y=0$ D. $y=-3$

9. Helen planted a bed of gardenias in rows on her commercial property. Each row had to be fertilised before she started planting.

There were 13 gardenia plants in the first row, 19 gardenia plants in the second row, and so on. Each succeeding row had 6 more gardenia plants than the row before it.

If Helen wanted to plant 1533 gardenias, how many rows will she need to fertilise?

A. 20 B. 21 C. 23.75 D. 241

10. If $\sqrt{7}+\sqrt{28}+\sqrt{63}+\cdots=300\sqrt{7}$ how many terms are in the series?

A. 300 B. 298 C. 25 D. 24

SECTION II

60 Marks

Use the answer booklet provided.

Allow about 1 hour 15 minutes for this section.

Question 11 (15 Marks)

Marks

a. If $\propto$ and β are the roots of the equation $3x^2 - 7x - 1 = 0$, find the value of

i. $\propto + \beta$ **1**

ii. $\propto \beta$ **1**

iii. $\propto^2 \beta + \propto \beta^2$ **1**

iv. $\propto^2 + \beta^2$ **2**

b. Consider the sequence 48, 44, 40, ...

i. Write a formula for Tn **1**

ii. Which is the first negative term and find its value. **2**

iii. Find the sum of all its positive terms. **2**

c. Express $x^2 + 2x - 2$ in the form $ax(x+1) + bx^2 + c(x+1)$ **2**

d. Solve $2(x^2 + 1)^2 - 19(x^2 + 1) - 10 = 0$ **3**

Question 12 (Start a new page) (15 Marks) **Marks**

a. Find the equation of the parabola with vertex (-1, 3) and directrix $y = -1$ **2**

b. The roots of the quadratic equation $2x^2 + 3x - 2 = 0$ are α and β.

Find the quadratic equation whose roots are 2α and 2β. **2**

c. The equation of a parabola is given by $2y = x^2 - 4x + 6$. Find

i. the coordinates of the vertex **2**

ii. the coordinates of the focus **1**

iii the equation of the directrix **1**

d. A weight lifter in training tires with each lift such that he can only lift 90% of the preceding lift. If his first lift was 200kg:

i. What weight will he raise on his fifth lift? **2**

ii. Theoretically, what would be the total of the weights lifted by the time he was totally exhausted.? **2**

e. Find the value/s of k for which the roots of $x^2 - (k + 2)x + (k + 5) = 0$ are real. **3**

Question 13 (Start a new page) (15 marks)

Marks

a. If one root of the equation $mx^2 - px + 1 = 0$ is double the other, prove that $2p^2 = 9m$ **3**

b. A parabola has the equation $x^2 = -12y$

i. Find the equation of the tangent to the parabola at point T(6, -3). **2**

ii. Find the equation of the normal at T(6, -3). **1**

iii. Find the coordinates of M and N, the points where the tangent and normal respectively cut the y axis. **2**

iv. Find the area of $\Delta\, MNT$ **2**

c. $2000 is invested into a Credit Union account at the beginning of each year. Interest is paid at the end of each year, at a rate of 1.5% per annum, on the whole amount in the account at that time.

(i) What is the value of the investment at the end of 3 years? **2**

(ii) At the end of how many years, before the next $2000 is invested, would the accumulated amount in the account first exceed $100 000? **3**

Question 14 (Start a new page) (15 Marks) Marks

a. Let A and B be the fixed points (-1, 0) and (2, 0) respectively, and let P be the variable point (x, y).

i. Write down expressions for PA^2 and PB^2 in terms of x and y **2**

ii. Suppose that P moves so that PA=2PB. Find the locus of P. **2**

iii. Give a geometric description of this locus. **2**

b. The first term and common difference of an arithmetic sequence are both non-zero.

T_n represents the nth term where $T_n = a + (n-1)d$

S_n represents the sum to n terms.

T_6, T_4 and T_{10} (in that order) form a geometric sequence.

i. Find an expression for T_6 in terms of a and d. **1**

ii. Show that $2a + 9d = 0$ and hence that $S_{10} = 0$ **3**

iii. Show that $S_6 + S_{12} = 0$ **2**

iv. Deduce that $T_7 + T_8 + T_9 + T_{10} = T_{11} + T_{12}$ **3**

$\frac{T_4}{T_6} = \frac{T_{10}}{T_4}$

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SECTION I

1	C
2	C
3	D
4	A
5	B
6	A
7	B
8	B
9	B
10	D

SECTION II

Question 11

a. $3x^2 - 7x - 1 = 0$

i. $\alpha + \beta = -\frac{b}{a}$

$= \frac{7}{3}$

ii. $\alpha\beta = \frac{c}{a}$

$= -\frac{1}{3}$

iii. $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta)$

$= -\frac{1}{3} \times \frac{7}{3}$

$= -\frac{7}{9}$

iv. $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{7}{3}\right)^2 - 2 \times -\frac{1}{3}$

$= \frac{55}{9}$

b. $48, 44, 40, \ldots$

i. $a = 48$

$d = -4$

$T_n = a + (n-1)d$

$= 48 + (n-1) \times -4$

$= 48 - 4n + 4$

$\therefore T_n = 52 - 4n$

ii. $52 - 4n < 0$

$-4n < -52$

$n > 13$

$T_{14} = 52 - 4 \times 14$

$\therefore T_{14} = -4$

iii. $48 + 44 + 40 + \ldots + 4$

$S_{12} = \frac{12}{2}\left[48 + 4\right]$

$= 312$

c. $x^2 + 2x - 2 \equiv ax(x+1) + bx^2 + c(x+1)$

$\equiv ax^2 + ax + bx^2 + cx + c$

$\equiv (a+b)x^2 + (a+c)x + c$

Equating coefficient:

$c = -2$

$a + b = 1 \quad -①$

$a + c = 2 \quad -②$

sub c into ②

$a - 2 = 2$

$a = 4$

sub a into ①

$4 + b = 1$

$b = -3$

$\therefore x^2 + 2x - 2 \equiv 4x(x+1) - 3x^2 - 2(x+1)$

d. $2(x^2+1)^2 - 19(x^2+1) - 10 = 0$

let $a = x^2 + 1$

$2a^2 - 19a - 10 = 0$

$(a-10)(2a+1) = 0$

$a = 10 \qquad 2a + 1 = 0$

$a = -\frac{1}{2}$

$x^2 + 1 = 10 \qquad x^2 + 1 = -\frac{1}{2}$

$x^2 = 9 \qquad$ no soln

$\therefore x = \pm 3$

Question 12

a. vertex $(-1, 3)$

$a = 4$

$(x - -1)^2 = 4 \times 4(y-3)$

$(x+1)^2 = 16(y-3)$

b. $2x^2 + 3x - 2 = 0$

$(x+2)(2x-1) = 0$

So $\alpha = -2, \ \beta = \frac{1}{2}$

$2\alpha = -4, \ 2\beta = 1$

$\therefore (x+4)(x-1) = 0$

c. $2y = x^2 - 4x + 6$

$2y - 6 = x^2 - 4x$

$2y - 6 + 4 = x^2 - 4x + 4$

$2(y-1) = (x-2)^2$

i. vertex $(2, 1)$

ii. $4a = 2$

$a = \frac{1}{2}$

focus $\left(2, \frac{3}{2}\right)$

iii. $y = \frac{1}{2}$

d. i. $200, 180, 162, \ldots$

$a = 200$

$r = 0.9$

$T_5 = 200 \times 0.9^4$

$= 131.22$ kg

ii. $S = \frac{200}{1 - 0.9}$

$= 2000$ kg

e. $x^2 - (k+2)x + (k+5) = 0$

$\Delta = \left[-(k+2)\right]^2 - 4 \times 1(k+5)$

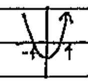
$= k^2 + 4k + 4 - 4k - 20$

$= k^2 - 16$

For real roots : $\Delta \geq 0$

$k^2 - 16 \geq 0$

$(k+4)(k-4) \geq 0$



$\therefore k \leq -4, \ k \geq 4$

Question 13

a. $mx^2 - px + 1 = 0$

let the roots be α and 2α

$\alpha + 2\alpha = \frac{p}{m}$

$3\alpha = \frac{p}{m}$

$\alpha = \frac{p}{3m} \quad -①$

$\alpha \times 2\alpha = \frac{1}{m}$

$2\alpha^2 = \frac{1}{m} \quad -②$

sub ① into ②

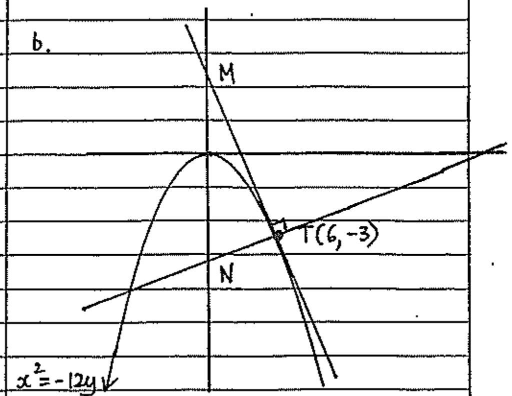
$2 \times \left(\frac{p}{3m}\right)^2 = \frac{1}{m}$

$\frac{2p^2}{9m^2} = \frac{1}{m}$

$2mp^2 = 9m^2$

$\therefore 2p^2 = 9m$

b.



i. $y = -\frac{x^2}{12}$

$\frac{dy}{dx} = -\frac{x}{6}$

at T, $x = 6$, $m_{tangent} = -\frac{6}{6}$

$= -1$

$y - (-3) = -1(x-6)$

$y + 3 = -x + 6$

$\therefore y = -x + 3$

ii. $m_{normal} = 1$

$y - (-3) = 1(x-6)$

$y + 3 = x - 6$

$\therefore y = x - 9$

iii. $M(0, 3)$ and $N(0, -9)$

iv. $A_{\Delta MNT} = \frac{1}{2} \times MN \times h$

$= \frac{1}{2} \times 12 \times 6$

$= 36 \ u^2$

c. $2000

$r = 1.5\%$ p.a

i. $A_1 = 2000(1+0.015)^3$

$= 2000(1.015)^3$

$A_2 = 2000(1.015)^2$

$A_3 = 2000(1.015)^1$

Value $= 2000(1.015 + 1.015^2 + 1.015^3)$

$= \$6\,181.81$

ii. $A_n = 2000(1.015 + \underbrace{1.015^2 + 1.015^3}_{} + \ldots + 1.015^n)$

GP where $a = 1.015$, $r = 1.015$, $n = ?$

$= 2000 \times \dfrac{1.015(1.015^n - 1)}{1.015 - 1}$

$= \dfrac{2030(1.015^n - 1)}{0.015}$

$A_n > 100\,000$

$\dfrac{2030(1.015^n - 1)}{0.015} > 100\,000$

$2030(1.015^n - 1) > 1500$

$1.015^n - 1 > \dfrac{1500}{2030}$

$1.015^n > \dfrac{353}{203}$

$\log_{10} 1.015^n > \log_{10} \dfrac{353}{203}$

$n \log_{10} 1.015 > \log_{10} \dfrac{353}{203}$

$n > \dfrac{\log_{10} \frac{353}{203}}{\log_{10} 1.015}$

$n > 37.16\ldots$

$\therefore$ investment exceeds $100 000 after 38 years

Question 14

a. $A(-1, 0)$

$B(2, 0)$

$P(x, y)$

i. $PA^2 = (x - -1)^2 + (y-0)^2$

$PA^2 = (x+1)^2 + y^2$

$PB^2 = (x-2)^2 + (y-0)^2$

$PB^2 = (x-2)^2 + y^2$

ii. Given $PA = 2PB$

$PA^2 = 4PB^2$

$(x+1)^2 + y^2 = 4\left[(x-2)^2 + y^2\right]$

$x^2 + 2x + 1 + y^2 = 4\left[x^2 - 4x + 4 + y^2\right]$

$x^2 + 2x + 1 + y^2 = 4x^2 - 16x + 16 + 4y^2$

$0 = 3x^2 - 18x + 15 + 3y^2$

$\therefore$ locus of P is

$0 = x^2 - 6x + 5 + y^2$

iii. $-5 = x^2 - 6x + y^2$

$-5 + 9 = x^2 - 6x + 9 + y^2$

$4 = (x-3)^2 + y^2$

$\therefore$ locus is a circle with centre $(3, 0)$ and radius 2.

b. i. $T_6 = a + 5d$

ii. $T_4 = a + 3d$

$T_{10} = a + 9d$

Given T_6, T_4, T_{10} GP

$\dfrac{T_4}{T_6} = \dfrac{T_{10}}{T_4}$

$\dfrac{a+3d}{a+5d} = \dfrac{a+9d}{a+3d}$

$(a+3d)^2 = (a+5d)(a+9d)$

$a^2 + 6ad + 9d^2 = a^2 + 14ad + 45d^2$

$0 = 8ad + 36d^2$

$0 = 4d(2a+9d)$

$\therefore 2a + 9d = 0$ (since $4d \neq 0$)

Now $S_{10} = \dfrac{10}{2}\left[a + a + 9d\right]$

$= 5(2a + 9d)$

$= 5 \times 0$

$= 0$

iii. $S_6 + S_{12}$

$= \dfrac{6}{2}\left[a + a + 5d\right] + \dfrac{12}{2}\left[a + 11d\right]$

$= 3(2a + 5d) + 6(2a + 11d)$

$= 6a + 15d + 12a + 66d$

$= 18a + 81d$

$= 9(2a + 9d)$

$= 9 \times 0$

$= 0$

iv. $T_7 + T_8 + T_9 + T_{10}$

$= a + 6d + a + 7d + a + 8d + a + 9d$

$= 4a + 30d$

$= 2a + 9d + 2a + 21d$

$= 0 + 2a + 21d$

$= a + 10d + a + 11d$

$= \underbrace{a+10d}_{T_{11}} + \underbrace{a+11d}_{T_{12}}$