SYDNEY TECHNICAL HIGH SCHOOL

TRIAL HIGHER SCHOOL CERTIFICATE

2008

MATHEMATICS

Time Allowed: 3 hours plus 5 minutes reading time

Instructions:

- Write your name and class at the top of this page, and at the top of each answer sheet
- At the end of the examination this examination paper must be attached to the front of your answers
- All questions are of equal value and may be attempted
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

(for Markers Use Only)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Total

Question 1 **(12 marks)**

a) Find $e^{-0.6}$ correct to 3 decimal places. 1

b) Expand and simplify $(\sqrt{2}\text{-}3)^2$ 2

c) Given $\frac{1}{P} = \frac{1}{Q} + \frac{1}{R}$ make Q the subject of the formula. 2

d) (i) Find $\int_1^2 \frac{dx}{x}$ 1

(ii) Evaluate $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \cos\left(\frac{x}{2}\right) dx$. Leave your answer as an exact value. 2

e) Solve the inequality $|2x - 3| \leq 7$ 2

f) Solve the following equations simultaneously

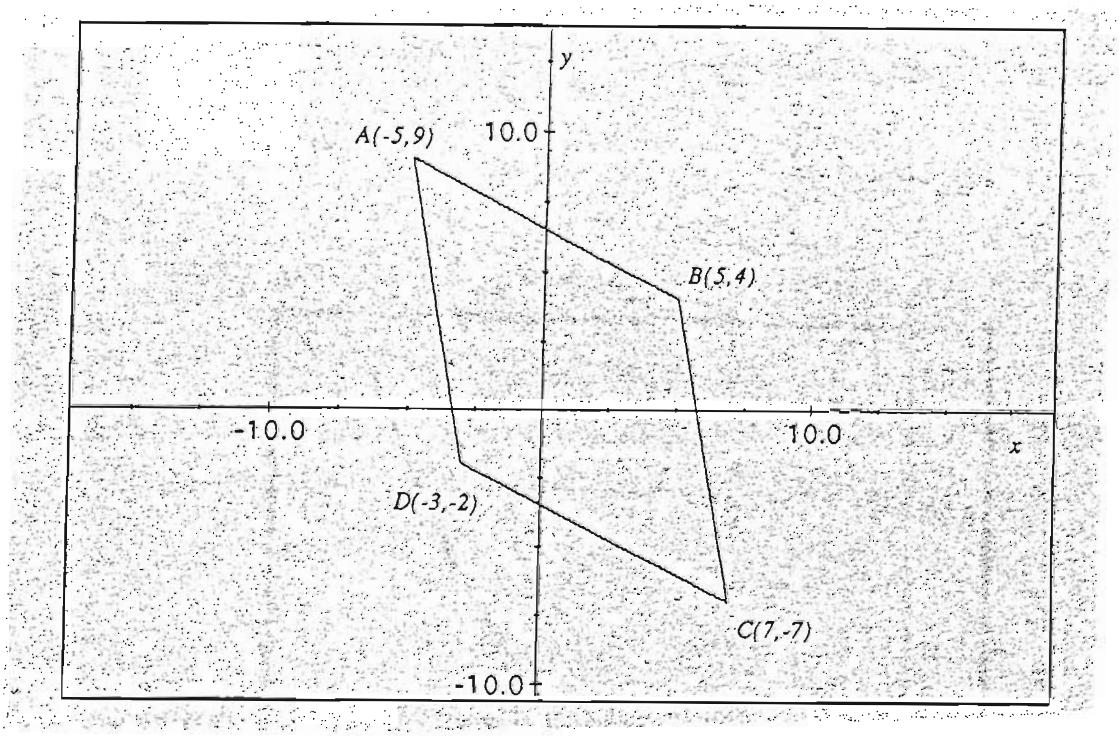
$2x + y = 4$

$5x + 2y = 9$ 2

Question 2 **(Use a separate sheet of paper)** **(12 marks)**

a) A rhombus is a parallelogram with four sides of equal length.

The figure shown below, with vertices $A(-5,9)$, $B(5,4)$, $C(7,-7)$ and $D(-3,-2)$ is a rhombus.



(i) Find the side length of $ABCD$. Give your answer in simplified surd form. 1

(ii) Find the gradient of the longer diagonal. 1

(iii) Show that the diagonals of $ABCD$ are perpendicular. 2

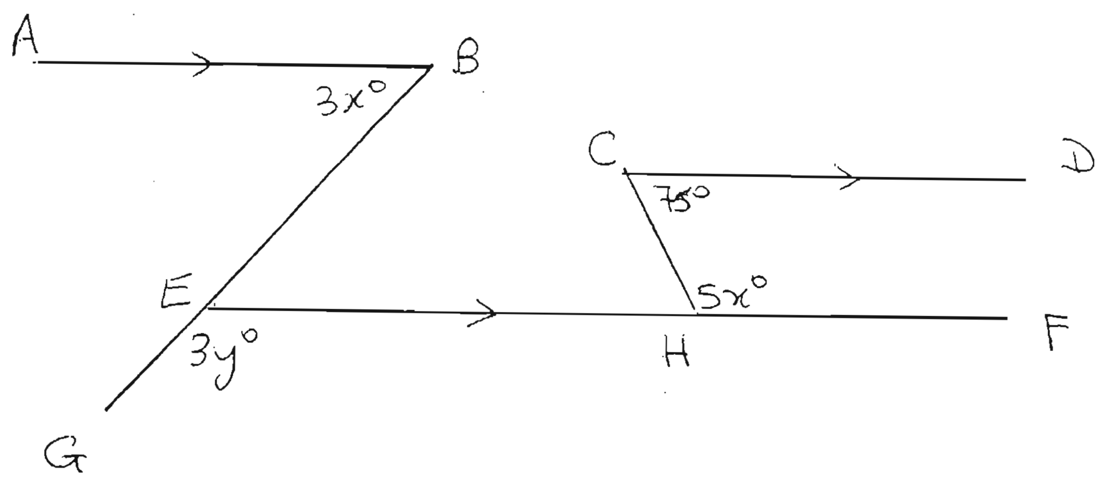
(iv) Find the coordinates of the midpoint of each diagonal. 1

(v) What does this result to part (d) say about the diagonals of this rhombus? 1

(vi) Find the equation of the line passing through AC. 2

(b) In the diagram below the lines AB, CD and EF are parallel. 4

Find the value of x and y. Give reasons for each answer.



Question 3 (12 marks) (Use a separate sheet of paper)

a) Differentiate

(i) x^2e^x 2

(ii) $\ln\left(\frac{x-5}{x+3}\right)$ 2

b) (i) Find $\int \frac{dx}{3x-1}$ 1

(ii) Evaluate $\int_0^1 e^{4x}\,dx,$ leaving your answer in exact form 2

c) For what values of m does the equation $4x^2 + (1+m)x + 1 = 0$ have equal roots. 2

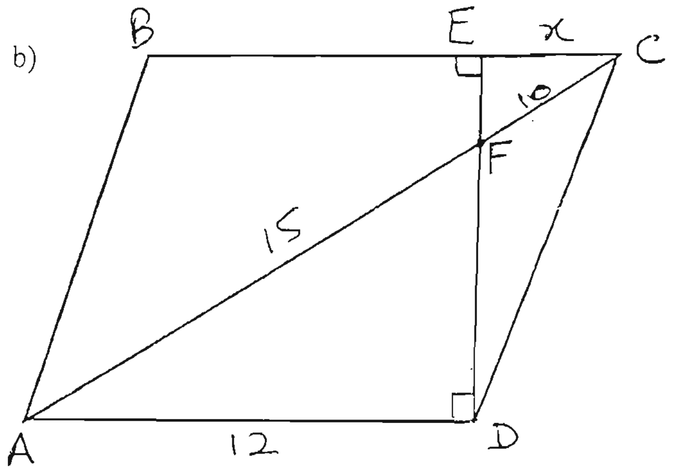
d) For acute angles A and B it is given that $sinA = \frac{12}{13}$ and $cosB = \frac{15}{17}$

Find the exact value of $\sec A + \tan B$. 3

Question 4 (12 marks) (Use a separate sheet of paper)

a) The sum of the first 4 terms of a geometric progression is 30, and the limiting sum is 32. If the common ratio is negative find the first three terms. 3

b)



$ABCD$ is a parallelogram.

(i) Prove that ΔEFC and ΔDFA are similar.

(ii) Find the value of x. 4

Not to Scale

c) Solve $\sin\left(x+\frac{\pi}{3}\right)=0$ for $0 \leq x \leq \pi$ 2

d) $\propto$ and β are the roots of $2x^2-5x+5=0$. Write down the value of

(i) $\propto + \beta$

(ii) $\propto \beta$

(iii) $\frac{1}{\propto}+\frac{1}{\beta}$ 3

Question 5 (12 marks) (Use a separate sheet of paper)

a) A function is defined by $f(x)=3x^2-2x^3$

(i) Find the coordinates of any turning points and determine their nature 3

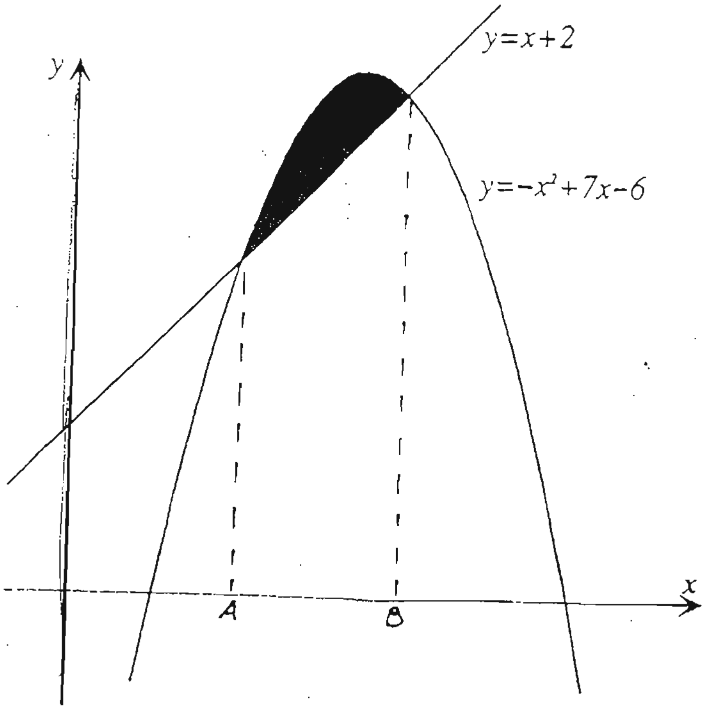
(ii) Sketch the curve, indicating all intercepts and turning points. 2

(iii) State the domain over which both $f(x)>0$ and $f'(x)>0$ 1

(iv) On the same set of axes sketch the line $f(x)=\frac{1}{2}$ 1

(v) Hence find the <u>number</u> of solutions to the equation $6x^2-4x^3=1$ 1 b)

4



The diagram shows the graphs of the functions $y = -x^2 + 7x - 6$ and $y = x + 2$.

(i) Show that the value of A and B is 2 and 4 respectively

(ii) Calculate the area of the shaded region.

Question 6 (12 marks) (Use a separate sheet of paper)

a) Evaluate $\sum_{r=1}^{4} 3^{1-r}$ 1

b) For the arithmetic progression 32, 25, 18, ...

find the (i) the 15^{th} term 1

(ii) S_{15} 1

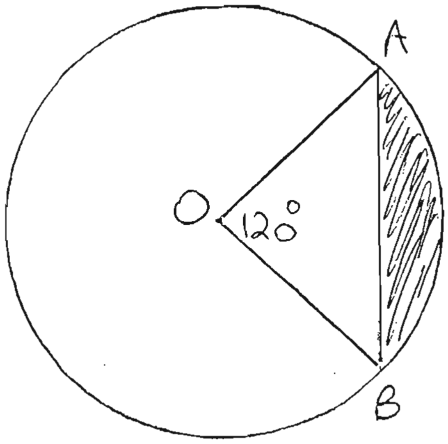
(iii) the sum of the <u>next</u> 20 terms 2

c) The area under the curve $y = 4^x$ between $x = 0$ and $x = 2$ is rotated about the $x-$axis. Copy and complete the table.

x	0	0.5	1	1.5	2
4^{2x}					

Use your results with Simpson's rule to find an approximate value for the volume of revolution. Use 5 function values and answer correct to 1 decimal place. 3

d)



The circle has a radius of 2cm

(i) Find arc length AB

(ii) Find the shaded area 4

(correct to 1 decimal place)

Question 7 (12 marks) (Use a separate sheet of paper)

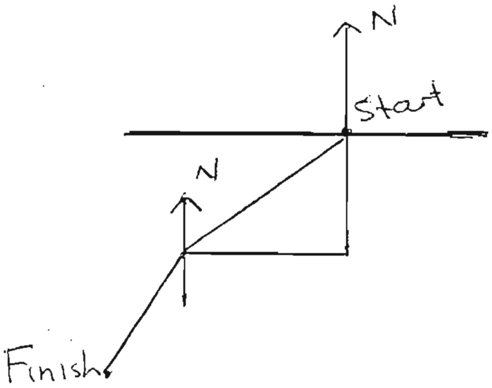
a) $f'(x) = 3x^2 - 4$.

Find $y = f(x)$ if the function passes through (3, 8). 2

b) A boat travels 5km on a bearing of 207° T, then travels 8km on a bearing of 200°T.

Find the straight line distance between the start and finish to 3 significant figures. 4

Copy and complete the given diagram to assist your working.



c) $30 000 is borrowed to buy a car. Interest is charged at 12% pa, compounding <u>monthly</u>.

The loan is repaid in equal monthly repayments over 4 years. Let A_n be the amount owing after n months.

(i) If M is the monthly payment write an expression for the amount owing

after $\propto$) 1 month

β) 3 months

(ii) Find M

(iii) Find the total amount paid over the 4 years. 6

Question 8 (12 marks) (Use a separate sheet of paper)

a) Evaluate $\lim_{x \to 0} \frac{sin2x}{x}$ 2

b) Evaluate $log_5 100 - log_5 4$ 2

c) A particle moves in such a way that its distance, x metres, from the origin after t seconds is given by

$$x = 2 + 3t - t^3 \text{ for } t > 0$$

(i) Find an equation for its velocity after t seconds. 1

(ii) At what time does the particle stop? 1

(iii) Where is the particle initially? 1

(iv) Find the velocity after 2 seconds. 1

(v) How far has the particle travelled in the first 2 seconds. 2

d) Find the volume of the solid formed when the curve $y = \sqrt{x}$ is rotated about the x axis between $x = 1$ and $x = 5$. (leave the answer in terms of π). 2

Question 9 **(12 marks)** **(Use a separate sheet of paper)**

a) If $F(x) = \begin{cases} x^2 - 2 & x \leq -1 \\ 2^x & -1 < x < 2 \\ log_{10}x & x \geq 2 \end{cases}$

evaluate $f(-1) + f(1) + f(10)$. 2

b) Draw a neat sketch of $y = 3sin2x$ within the domain $0 \leq x \leq 2\pi$.

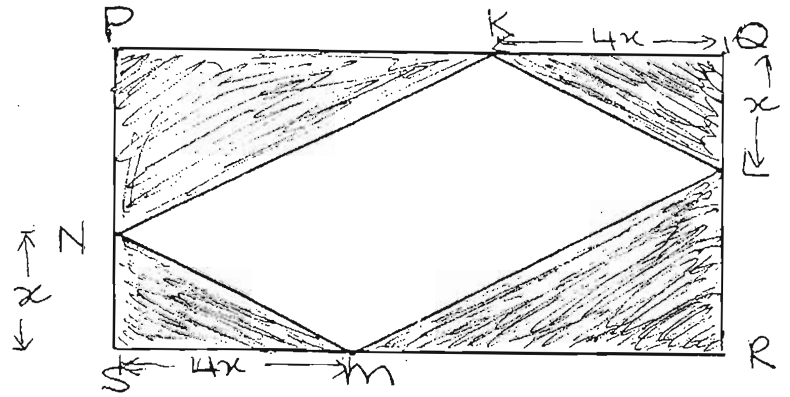
State the (i) period

(ii) amplitude. 4

c) In the diagram, PQRS is a rectangle with PQ=40cm, SP=10cm.

The shaded portions are cut away, leaving the parallelogram KLMN.

QL=SN=x cm and QK=SM = $4x$ cm.



(i) Show that the area of the parallelogram KLMN is given by

$A = 80x - 8x^2$. 3

(ii) Find the allowable values of x 1

(iii) Find the value of x for which A is a maximum 2

Question 10 **(12 marks)** **(Use a separate sheet of paper)**

a) For all values of x in the domain of $0 \leq x \leq 6$, a function $f(x)$ satisfies

$f'(x) > 0$ and $f''(x) > 0$.

Sketch a possible graph of $y = f(x)$ in this domain. 2

b) (i) Find the points of intersection of the curve $y = 4 - \sqrt{2x}$ with the x and y axes. 2

(ii) The area enclosed by the curve $y = 4 - \sqrt{2x}$, the x axis and the y axis is rotated about the y axis. Find the volume of the solid of revolution so formed (leave your answer in terms of π) 4

c) The line x = m, cuts the curves $y = log_e x$ and $y = log_e 5x$ at R and S respectively.

Show that the tangents to the curves at R and S are parallel. Also show that the distance RS remains constant for all values of M (ie the distance is independent of m). 4

END OF PAPER

Mathematics 2008

HSC Trial Exam

Question 1

a) $e^{-0.6} = 0.549$ (3 dp)

b) $(\sqrt{2} - 3)^2 = 2 - 6\sqrt{2} + 9$

$= 11 - 6\sqrt{2}$

c) $\frac{1}{P} = \frac{1}{Q} + \frac{1}{R}$

$\therefore \frac{1}{Q} = \frac{1}{P} - \frac{1}{R}$

$= \frac{R - P}{PR}$

$\therefore Q = \frac{PR}{R - P}$

d) (i) $\int_1^2 \frac{dx}{x} = \left[\ln x\right]_1^2$

$= \ln 2 - \ln 1$

$= \ln 2$

(ii) $\int_{\pi/3}^{\pi/2} \cos\left(\frac{x}{2}\right) dx = 2\left[\sin\left(\frac{x}{2}\right)\right]_{\pi/3}^{\pi/2}$

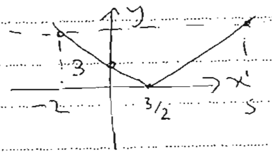
$= 2\left[\sin\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{6}\right)\right]$

$= 2\left[\frac{1}{\sqrt{2}} - \frac{1}{2}\right]$

$= 2\left[\frac{\sqrt{2}}{2} - \frac{1}{2}\right]$

$= \sqrt{2} - 1$

e) $|2x - 3| \le 7$



$2x - 3 = 7$ $\quad$ $2x - 3 = -7$

$2x = 10$ $\quad$ $2x = -4$

$x = 5$ $\quad$ $x = -2$

$\therefore -2 \le x \le 5$

f) $2x + y = 4$ (1)

$5x + 2y = 9$ (2)

(1) $\times 5$ $\quad 10x + 5y = 20$ (3)

(2) $\times 2$ $\quad 10x + 4y = 18$ (4)

(3) − (4)

$y = 2$

In (1) $\quad 2x + 2 = 4$

$2x = 2$

$x = 1$

Question 2

a) (i) Using A and B

side length $= \sqrt{(-5-5)^2 + (9-4)^2}$

$= \sqrt{(10)^2 + (5)^2}$

$= \sqrt{125}$

$= 5\sqrt{5}$ units.

(ii) longer diagonal is AC

gradient $AC = \frac{9 - -7}{-5-7}$

$= \frac{16}{-12}$

$= -4/3 \quad = m_1$

(iii) shorter diagonal is DB

gradient $DB = \frac{-2-4}{-3-5}$

$= \frac{-6}{-8}$

$= 3/4 \quad = m_2$

Now $m_1 m_2 = -\frac{4}{3} \times \frac{3}{4}$

$= -1$

∴ Satisfies condition for perpendicular lines

∴ diagonals perpendicular

(iv)

$M_{AB} = \left(\frac{-5+7}{2}, \frac{9+-7}{2}\right) \qquad M_{BD} = \left(\frac{-3+5}{2}, \frac{-2+}{2}\right)$

$= (1, 1) \qquad\qquad = (1, 1)$

(v) Result confirms diagonals bisect, at (1, 1)

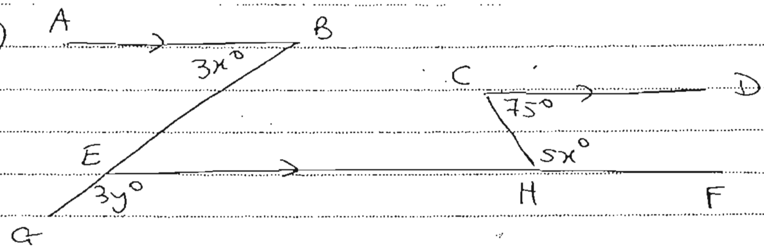
(vi) Gradient $AC = -4/3$

∴ Eqn AC : $y - 9 = -4/3(x + +5)$

$3y - 27 = -4x - 20.$

$4x + 3y - 7 = 0$

b)



Since CD ∥ HF $\quad 75° + 5x° = 180°$

ie cointerior angles supplementary

∴ $5x° = 105°$

$x = 21.$

Since AB ∥ EH, ∠BEH $= 3x°$ (alternate angles equal)

Then $3x + 3y = 180$ (straight angle is 180°)

But $x = 21$

∴ $3y = 180 - 63$

$= 117$

$y = 39$

Question 3

a) (i) $y = x^2 e^x$ $\qquad u = x^2 \quad v = e^x$

$y' = x^2 e^x + 2x(e^x)$ $\qquad u' = 2x \quad v' = e^x$

$= x e^x (x + 2)$

(ii) $y = \ln\left(\frac{x-5}{x+3}\right)$

$= \ln(x-5) - \ln(x+3)$

$y' = \frac{1}{x-5} - \frac{1}{x+3}$

$= \frac{\cancel{x}+3-(\cancel{x}-5)}{(x-5)(x+3)}$

$= \frac{8}{(x-5)(x+3)}$

b) (i) $\int \frac{dx}{3x-1} = \frac{1}{3}\ln(3x-1) + c$

(ii) $\int_0^1 e^{4x}\,dx = \left[\frac{1}{4}e^{4x}\right]_0^1$

$= \frac{1}{4}e^4 - \frac{1}{4}e^0$

$= \frac{1}{4}e^4 - \frac{1}{4} = \frac{1}{4}(e^4 - 1)$

c) $4x^2 + (1+m)x + 1 = 0$

Equal roots when $\Delta = 0$.

$\Delta = b^2 - 4ac$

$= (1+m)^2 - 4(4)(1)$

$= 1 + 2m + m^2 - 16$

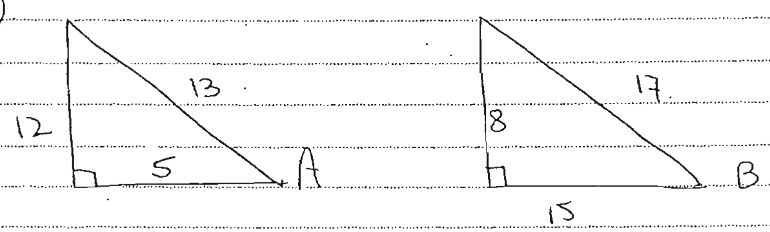
$= m^2 + 2m - 15$

Solve $m^2 + 2m - 15 = 0$

$(m+5)(m-3) = 0$

$m = -5$ or $m = 3$

d)



Complete each triangle

$\sec A + \tan B = \frac{13}{5} + \frac{8}{15}$

$= \frac{39+8}{15}$

$= \frac{47}{15}$

Question 4

a) $S_n = \dfrac{a(1-r^n)}{1-r}$

$\therefore S_4 = \dfrac{a(1-r^4)}{1-r} = 30.$

$S_\infty = \dfrac{a}{1-r} = 32$

$\therefore$ In S_4 $\quad 32(1-r^4) = 30$

$1-r^4 = \dfrac{30}{32}$

$r^4 = \dfrac{2}{32}$

$= \dfrac{1}{16}$

$r = \pm\frac{1}{2}$

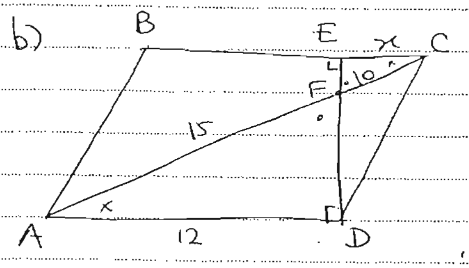
But $r < 0$ $\quad \therefore r = -\frac{1}{2}$ and $a = 48$

$\therefore T_1 = 48$

$T_2 = -24$

$T_3 = 12.$

b)



(i) $\lfloor FEC = \lfloor FDA = 90^\circ$ (given)

$\lfloor EFC = \lfloor AFD$ (vertically opposite angles equal)

$\therefore \Delta EFC$ and ΔDFA are equiangular.

$\therefore$ Similar

(ii) Corresponding sides are in the same ratio

$\therefore \dfrac{x}{12} = \dfrac{\cancel{10}\,2}{\cancel{15}\,3}$

$x = \dfrac{24}{3}$

$= 8$

c) $\sin(x + \pi/3) = 0. \qquad 0 \le x \le \pi$

$x + \pi/3 = 0, \pi, 2\pi, 3\pi, \ldots$

$x = -\pi/3, 2\pi/3, 5\pi/3, 8\pi/3, \ldots$

For given domain:

$x = 2\pi/3$

d) $2x^2 - 5x + 5 = 0$

(i) $\alpha + \beta = 5/2$

(ii) $\alpha\beta = 5/2$

(iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{\beta+\alpha}{\alpha\beta}$

$= \dfrac{5/2}{5/2}$

$= 1$

Question 5

a) $f(x) = 3x^2 - 2x^3$

(i) $f'(x) = 6x - 6x^2$

for turning points (stationary) $f'(x) = 0$

$\therefore$ Solve $6x(1-x) = 0$

$x = 0$, $x = 1$

$f''(x) = 6 - 12x$

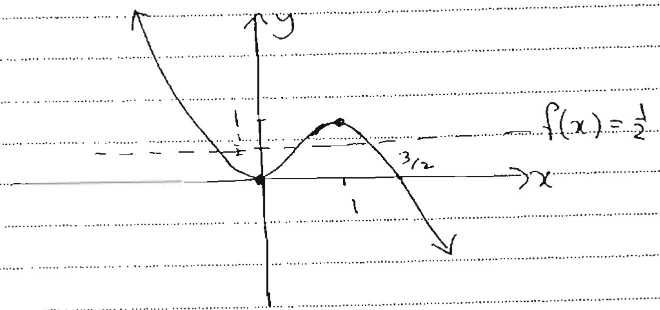
$f''(0) = 6 > 0 \Rightarrow$ min

$f''(1) = 6 - 12 < 0 \Rightarrow$ max

$\therefore$ min at $(0, 0)$

max at $(1, 1)$

(ii)



$f(x) = 0$ when $x^2(3 - 2x) = 0$

ie $x = 0$ or $x = 3/2$

(iii) $f(x) > 0$ above y axis
$f'(x) > 0$ increasing
} Both hold for $0 < x < 1$

(iv) $f(x) = \frac{1}{2}$ (above)

(v) $6x^2 - 4x^3 = 1 \Rightarrow 3x^2 - 2x^3 = \frac{1}{2}$

Since $f(x) = 3x^2 - 2x^3$ and $f(x) = \frac{1}{2}$ intersect 3 times, there will be 3 solutions.

b) $y = -x^2 + 7x - 6$, $y = x + 2$

(i) Intersect when

$-x^2 + 7x - 6 = x + 2$

ie $x^2 - 6x + 8 = 0$

$(x - 4)(x - 2) = 0$.

$x = 2$ or $x = 4$

From graph $A = 2$

$B = 4$

(ii) Area $= \int_2^4 (-x^2 + 7x - 6) - (x + 2)\, dx$

$= \int_2^4 (-x^2 + 6x - 8)\, dx$

$= \left[-\frac{1}{3}x^3 + 3x^2 - 8x\right]_2^4$

$= -\frac{1}{3}(64) + 3(16) - 32 - \left(-\frac{8}{3} + 12 - 16\right)$

$= -\frac{64}{3} + 48 - 32 + 8/3 - 12 + 16$

$= -\frac{56}{3} + 20$

$= 1\frac{1}{3}\ u^2$

Question 6

a) $\sum_{r=1}^{4} 3^{1-r} = 3^0 + 3^{-1} + 3^{-2} + 3^{-3}$

$= 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27}$

$= 1\frac{13}{27}$

b) $32, 25, 18 \ldots$ $\quad a = 32, \quad d = -7$

(i) $T_{15} = a + 14d$

$= 32 + 14 \times (-7)$

$= 32 - 98$

$= -66$

(ii) $S_{15} = \frac{15}{2}\left[2a + 14d\right]$

$= \frac{15}{2}\left[64 + 14(-7)\right]$

$= 15\left[32 - 49\right]$

$= 15 \times -17$

$= -255$

(iii) Sum next 20 terms

$= S_{35} - S_{15}$

$= \frac{35}{2}\left[64 + 34 \times (-7)\right] - (-255)$

$= 35\left[32 + 17 \times (-7)\right] + 255$

$= -3045 + 255$

$= -2790$

c)

x	0	0.5	1	1.5	2
4^{2x}	1	4	16	64	256
	y_0	y_1	y_2	y_3	y_4

$\text{Vol} = \pi \int_0^2 4^{2x}\, dx$

$\doteqdot \pi\left[\frac{h}{3}\left(y_0 + y_4 + 4 \times (y_1 + y_3) + 2(y_2)\right)\right]$

$= \pi\left[\frac{1}{6}\left(1 + 256 + 4(68) + 2(16)\right)\right]$

$\text{Vol} \doteqdot \pi\left[\frac{1}{6}(561)\right]$

$= 293.7\ u^3$ (1 dp)

d) (i) $120^\circ = \frac{2\pi}{3}^c$

$l = r\theta^c$

$= 2\left(\frac{2\pi}{3}\right)$

$= \frac{4\pi}{3}$ cm

(ii) $\text{Area} = \frac{1}{2}r^2(\theta^c - \sin\theta^c)$

$= \frac{1}{2}(4)\left[\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right]$

$= 2\left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2}\right)$ cm^2

Question 7

a) $f'(x) = 3x^2 - 4$

$f(x) = x^3 - 4x + c$

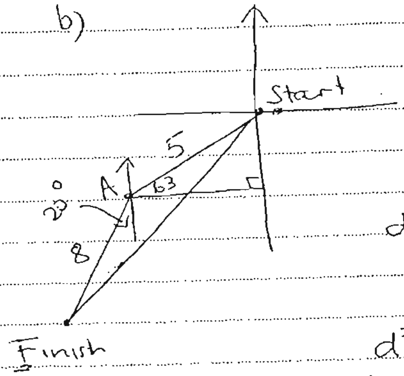
$(3,8)$ satisfies

$\therefore 8 = 3^3 - 4(3) + c$

$8 = 27 - 12 + c \Rightarrow c = -7$

$\therefore y = x^3 - 4x - 7$

b)



Angle at $A = 63 + 90 + 20$

$= 173^\circ$

d = distance $S \to F$

$\therefore$ By cosine rule

$d^2 = 5^2 + 8^2 - 2 \times 5 \times 8 \cos 173^\circ$

$= 25 + 64 - 80 \cos 173^\circ$

$= 89 - 80 \cos 173^\circ$

$d^2 = 168.4036921$

$\therefore d = 12.97704482$

$= 13.0$ km (3 sig figs)

c) \$30000 $\quad$ 12% pa = 1% per month

48 repayments

(i) α) $A_1 = 30000(1.01) - M$

β) $A_2 = [30000(1.01) - M](1.01) - M$

$= 30000(1.01)^2 - M(1.01 + 1)$

Similarly

$A_3 = 30000(1.01)^3 - M(1.01^2 + 1.01 + 1$

(ii) $A_{48} = 0$ since fully repaid

$0 = A_{48} = 30000(1.01)^{48} - M(1.01^{47} + 1.01^{46} + \dots + 1)$

ie $30000(1.01)^{48} = M(1 + 1.01 + \dots + 1.01^{47})$

GP with $a = 1$ $\quad r = 1.1$

$n = 48$

$$\therefore M = \frac{30000(1.01)^{48}}{\left(\frac{1 - 1.01^{48}}{1 - 1.01}\right)}$$

$$= \frac{30000(1.01)^{48}(0.01)}{1.01^{48} - 1}$$

$= \$790.02$ (nearest cent)

(iii) Total repaid $= M \times 48$

$= \$37920.72$ (nearest cent)

Question 8

a) $\lim\limits_{x \to 0} \dfrac{\sin 2x}{x} = \lim\limits_{x \to 0} \left(\dfrac{\sin 2x}{2x}\right) \times 2$

$= 2$

b) $\log_5 100 - \log_5 4 = \log_5 \left(\frac{100}{4}\right)$

$= \log_5 25$

$= \log_5 5^2$

$= 2$

c) $x = 2 + 3t - t^3, \quad t > 0$

(i) $\dfrac{dx}{dt} = 3 - 3t^2$

vel $= 3 - 3t^2$

(ii) Stops when $v = 0$

ie solve $3 - 3t^2 = 0$

$t = 1 \quad (t > 0)$

Stops after 1 second.

(iii) $t = 0$ in $x = 2 + 3t - t^3$

$= 2$

$\therefore$ Initially 2 m to the right of 0.

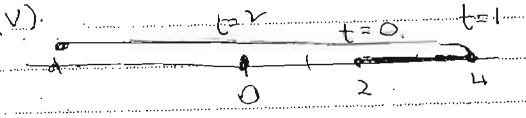
(iv) When $t = 2$

$v = 3 - 3(2)^2$

$= -9$

ie $v = -9$ m/sec (travelling to the left).

(v)



When $t = 1$, $x = 2 + 3 - 1$

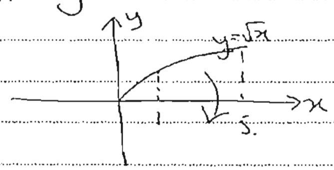
$= 4$

$t = 2$, $x = 2 + 6 - 8$

$= 0$

$\therefore$ Has travelled $2 + 4 = 6$ m.

d) $y = \sqrt{x}$



$\text{Vol} = \pi \int_1^5 x \, dx$

$= \pi \left[\frac{1}{2}x^2\right]_1^5$

$= \frac{\pi}{2}\left[25 - 1\right]$

$= 12\pi \ u^3$

Question 9

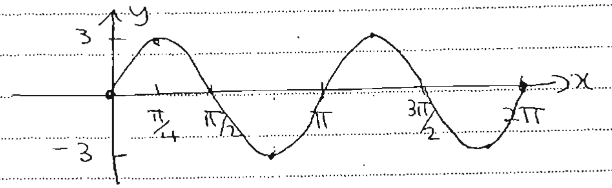
a) $f(-1) = (-1)^2 - 2 = -1$

$f(1) = 2^1 = 2$

$f(10) = \log_{10} 10 = 1$

$\therefore f(-1) + f(1) + f(10) = 2$

b) $y = 3\sin 2x \qquad 0 \le x \le 2\pi$

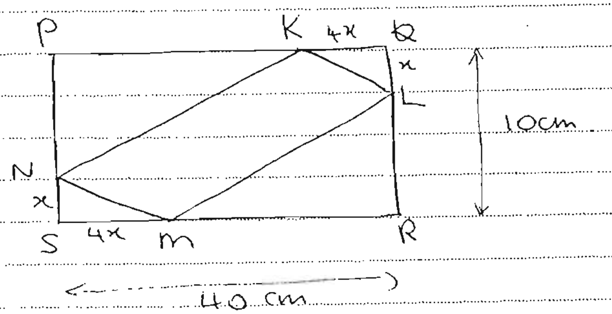


(i) Period $= \frac{2\pi}{2}$

$= \pi$

(ii) Amplitude $= 3$

c)



(i) Area parallelogram KMN

$= 40 \times 10 - \cancel{2} \times \frac{1}{\cancel{2}}(4x)(x) - \cancel{2} \times \frac{1}{\cancel{2}}(40 - 4x)(10 - x)$

$= \cancel{400} - 4x^2 - (\cancel{400} - 40x - 40x + 4x^2)$

$= 80x - 8x^2$

(ii) $0 < x < 10$

(iii) $\frac{dA}{dx} = 80 - 16x$

$\frac{dA}{dx} = 0$ when $16x = 80$

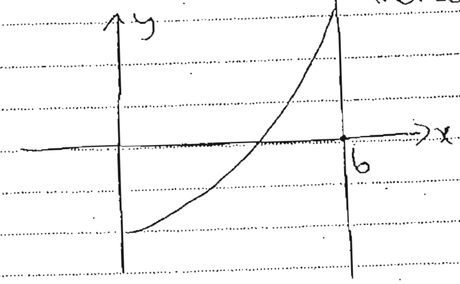
$x = 5$

$\frac{d^2A}{dx^2} = -16 < 0 \Rightarrow \max$

$\therefore$ Area max when $x = 5$.

Question 10

a) $0 \leq x \leq 6$. $\quad f'(x) > 0$ increasing $\quad f''(x) > 0$. concave up.



b) (i) $y = 4 - \sqrt{2x}$.

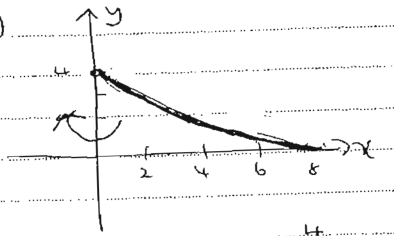
x axis: $y = 0$ $\quad$ ie $\sqrt{2x} = 4$

$2x = 16$

$x = 8$.

y axis: $x = 0$ $\quad$ ie $y = 4$

(ii)



$y = 4 - \sqrt{2x}$

$\sqrt{2x} = 4 - y$

$2x = (4-y)^2$

$x = \frac{(4-y)^2}{2}$

$\text{Vol} = \pi \int_0^4 x^2 \, dy$

$= \pi \int_0^4 \frac{(4-y)^4}{4} \, dy$

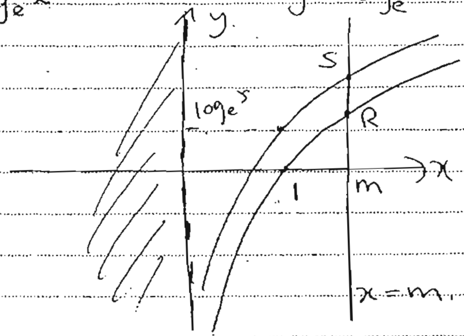
$= \frac{\pi}{4} \left[\frac{(4-y)^5}{-5} \right]_0^4$

$= -\frac{\pi}{20} \left[\cancel{(4-4)^5} - (4-0)^5 \right]$

$= -\frac{\pi}{20} (-4)^5$

$= \frac{\pi}{\cancel{20}_5} \times \cancel{4^5}\,4^4 = \frac{256\pi}{5} u^3$

c) $y = \log_e x$ $\qquad y = \log_e 5 + \log_e x$



$y = \log_e x$

$y' = \frac{1}{x}$

At R, $x = m$

$\therefore$ grad $= \frac{1}{m}$

$y = \log_e 5 + \log_e x$

$y' = 0 + \frac{1}{x}$

$= \frac{1}{x}$

At S, $x = m$

$\therefore$ grad $= \frac{1}{m}$

$\therefore$ They have the same gradient.

Tangents are parallel.

$R = (m, \log_e m)$ $\qquad S = (m, \log_e 5 + \log_e m)$

$RS = \sqrt{\cancel{(m-m)^2} + (\cancel{\log_e m} - (\log_e 5 + \cancel{\log_e m}))^2}$

$= \sqrt{(\log_e 5)^2}$

$= \log_e 5$ $\qquad \therefore$ RS remains constant

END