

SYDNEY TECHNICAL HIGH SCHOOL**HSC ASSESSMENT TASK 1****DECEMBER 2014****Mathematics****General Instructions**

- Working time - 70 minutes
- Write using black or blue pen
- Begin each question on a new page
- Board-approved calculators may be used
- All working should be shown in every question in section 2

Total marks - 60**Section 1****6 marks**

- Attempt Questions 1 – 6
- Allow about 8 minutes for this section

Section 2**54 marks**

- Attempt Questions 7 – 15
- Allow about 62 minutes for this section

Name : _____**Teacher :** _____

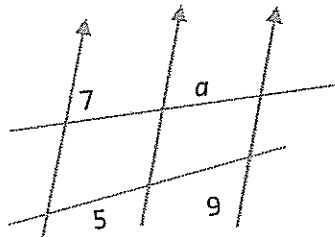
Section 1 – Multiple Choice

For Questions 1 - 6

Indicate which of the answers A, B, C or D is the correct answer.

Record this on your answer sheet

Question 1



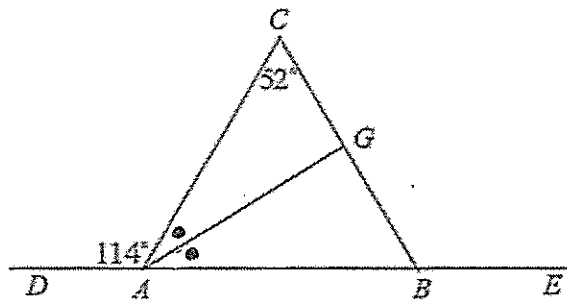
Not to scale

The value of a in the above diagram is

- (A) 9
- (B) 11
- (C) 12
- (D) 12.6

Question 2

In the diagram, $\angle CAD = 114^\circ$ & $\angle ACB = 52^\circ$. DE is a straight line. AG bisects $\angle CAB$.



What is the value of $\angle AGB$?

- (A) 33°
- (B) 52°
- (C) 62°
- (D) 85°

Question 3

What are the co-ordinates of the focus of the parabola, $(x - 2)^2 = 8(2 - y)$?

- (A) (2,4)
- (B) (2,2)
- (C) (2,0)
- (D) (2,-2)

Question 4

What is the value of k if the sum of the roots of $x^2 - (k - 1)x + 2k = 0$ is equal to the product of the roots?

- (A) -3
- (B) -2
- (C) -1
- (D) 1

Question 5

The first term of a geometric series is 81 and the fourth term is $\frac{1}{9}$. What is the value of the common ratio?

- (A) $\frac{1}{3}$
- (B) $\frac{1}{9}$
- (C) $\frac{1}{27}$
- (D) $\frac{1}{81}$

Question 6

The centre and radius of the circle with equation $x^2 + y^2 - 6y = 4x$ are

- (A) Centre (0,3) radius 2
- (B) Centre (0,9) radius 2
- (C) Centre (2,3) radius $\sqrt{13}$
- (D) Centre (2,3) radius 13

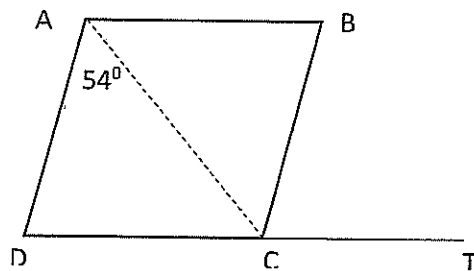
Section 2 :

Question 7: Start this question at the top of a new page

a. Solve $6x^2 - 23x - 4 < 0$ 2

b. Evaluate $\sum_{n=3}^9 3^{n-2}$ 2

c. In the diagram ABCD is a rhombus where $\angle DAC = 54^\circ$ and DC is produced to T.



Copy the diagram into your answer booklet

- i. What is the value of $\angle DAB$? 1
- ii. What is the value of $\angle BCT$? Give reasons. 1

Question 8: Start this question at the top of a new page

The first three terms of an Arithmetic sequence are given as $x, 2x + 2, 4x - 1$,

- a. Find the value of x . 2
- b. Find the value of the 12th term in this sequence. 2
- c. Find the sum of the first 20 terms in this sequence. 2

Question 9: Start this question at the top of a new page

- a. If α and β the roots of the equation $4x^2 - 3x - 6 = 0$, find the value of
 - i. $\alpha + \beta$ 1
 - ii. $(\alpha - \beta)^2$ 3
- b. Write down the locus of a parabola with vertex $(-1, 3)$ and directrix $y = 5$. 2

Question 10: Start this question at the top of a new page

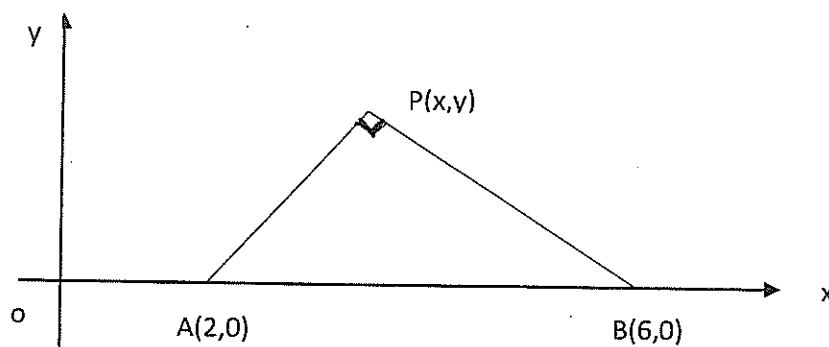
- a. Find the value of A, B and C given,

$$3x^2 - 4 \equiv A(x + 1)^2 - B(x + 2) + C \quad 3$$

- b. Find the sum of this series

$$48 + 24 + 12 + \dots + \frac{3}{32} \quad 3$$

Question 11: Start this question at the top of a new page



- i. Write down the gradient of PA in terms of x and y . 1
- ii. Find the locus of all points P such that angle APB is a right angle. 2
- iii. Deduce that the locus of all points P such that angle APB is a right angle is a circle. Write down the centre and radius of this circle. 3

Question 12: Start this question at the top of a new page

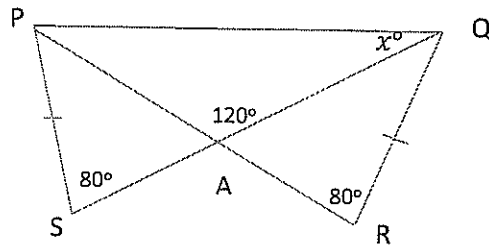
- a. A parabola P has equation $y = x^2 - 6x + 12$

Write down

- i. The co-ordinates of the vertex of P. 2
- ii. The co-ordinates of the focus of P. 1
- iii. The equation of the directrix of P. 1

- b. Find the equation of the tangent to the parabola $x^2 = -8(y - 4)$ at the point $(4, 2)$. 2

Question 13: Start this question at the top of a new page



NOT TO SCALE

PR and QS are straight lines intersecting at a point A. Also $PS = QR$,

$\angle PSA = \angle QRA = 80^\circ$, $\angle PAQ = 120^\circ$ and $\angle PQA = x$.

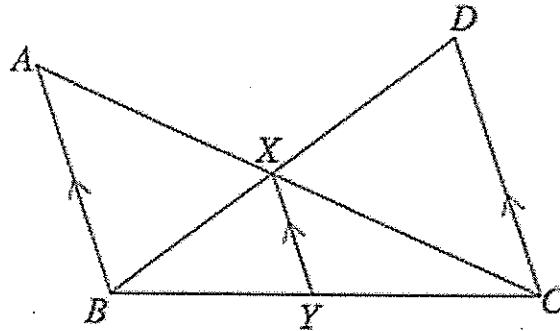
- a. Copy the diagram into your answer page
- b. Prove that $\triangle PSA$ is congruent to $\triangle QRA$ 3
- c. Hence, prove that $\triangle PAQ$ is isosceles and find the value of x . 3

Question 14: Start this question at the top of a new page

- a. A seedling with height 46 mm is planted and its growth tracked. Its height after one week was 59.8 mm and after a further week its height was recorded as 77.74 mm. If the seedling continues to grow at the same rate, find,
 - i. Its height after 6 weeks 1
 - ii. The week in which its height is first recorded as over 1 metre 2
- b. Find the value/s of m required for the line $y = mx - 12$ to be a tangent to the parabola $y = 2x^2 - x - 10$ 3

Question 15: Start this question at the top of a new page

In the diagram below $AB \parallel XY \parallel DC$.



NOT TO SCALE

Copy the diagram into your answer page

- | | |
|--|---|
| a. Prove that $\triangle AXB$ is similar to $\triangle CXD$ | 2 |
| b. If $XB = 12$ cm, $XC = 30$ cm, $BY = 8$ cm and $YC = 24$ cm, find the length of AX and DX , giving reasons. | 3 |
| c. Hence, find $AB : DC$ | 1 |

END OF ASSESSMENT TASK

Section 1

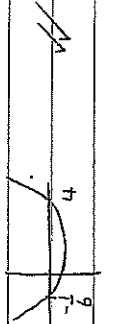
Question 1

- D
2 D
3 C
4 C
5 B
6 C

Section 2

Question 7

a) $6x^2 - 23x - 4 < 0$
 $(6x+1)(x-4) < 0$



$-\frac{1}{6} < x < 4$

Question 8

b) $\sum_{r=3}^9 3^{n-2} = 3 + 3^2 + 3^3 + 3^4 + 3^5 + 3^6 + 3^7$

$a(r^n-1) = 3(3^7-1)$

$r-1 = 2$

$= 3279$ (or add using your calculator)

c) i. $\angle DAB = 2 \angle DAC$

$= 108^\circ$ (diagonal of a rhombus bisects interior angles).
 (Needs reason)

ii. $\angle BCD = \angle DAB$ (opposite angles in a rhombus are equal)
 $= 108^\circ$

$\angle BCT + \angle BCD = 180^\circ$ (straight line)
 $\angle BCT = 72^\circ$

Question 8

a. AP $T_2 - T_1 = T_3 - T_2$
 $(2x+2) - x = (4x-1) - (2x+2)$
 $x+2 = 2x-3$
 $x = 5$

b. 5, 12, 19, ...

$T_n = a + (n-1)d$
 $= 5 + 11 \times 7$
 $= 82$

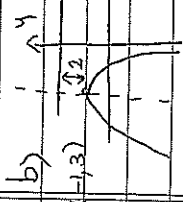
c. $S_n = \frac{n}{2}(2a + (n-1)d)$
 $= \frac{20}{2}(10 + 19 \times 7)$
 $= 1430$

Question 9

a. $4x^2 - 3x - 6 = 0$

i. $\alpha + \beta = -\frac{b}{a}$
 $= \frac{3}{4}$

ii. $(\alpha - \beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$
 $= \alpha^2 + \beta^2 - 2\alpha\beta$
 $= (\alpha + \beta)^2 - 4\alpha\beta$
 $= \left(\frac{3}{4}\right)^2 - 4\left(-\frac{3}{2}\right)$
 $= \frac{9}{16} + 6$
 $= \frac{105}{16}$



$(x+1)^2 = -8(y-3)$

one here

Question 10

a. $3x^2 - 4 \equiv A(x+1)^2 - B(x+2) + C$

RHS = $A(x^2 + 2x + 1) - B(x + 2) + C$

Equating coefficients

$3 = A$

$\therefore A = 3$ ✓

$0 = 2A - B$

$\therefore B = 6$ ✓

$-4 = A - 2B + C$

$-4 = 3 - 2(6) + C$

$C = 5$

$\therefore C = 5$ ✓

b. $48 + 24 + 12 + \dots + \frac{3}{32}$ ✓

C.P. $a = 48$ $r = \frac{1}{2}$ $T_n = \frac{3}{32}$ ✓

$a r^{n-1} = \frac{3}{32}$

$48 \left(\frac{1}{2}\right)^{n-1} = \frac{3}{32}$

$\left(\frac{1}{2}\right)^{n-1} = \frac{1}{512}$

$2^{n-1} = 2^9$

$n = 10$ ✓

$\therefore S_n = \frac{a(r^n - 1)}{r - 1}$ $\text{or } a \frac{(1 - r^n)}{1 - r}$

$= \frac{48 \left(\frac{1}{2}^{10} - 1\right)}{\frac{1}{2} - 1}$ $\frac{48(1 - \frac{1}{2}^{10})}{1 - \frac{1}{2}}$

$= \frac{3069}{32}$ ✓

Question 11.

a. $M_{PT} = \frac{y - 0}{x - 2}$

$= \frac{y}{x - 2}$ ✓

$= \frac{y}{x - 2}$

$= \frac{y}{x - 2}$

b. $M_{PA} \times M_{PB} = -1$ ✓

$\frac{y}{x - 2} \times \frac{y}{x - 6} = -1$

$y^2 = -1(x^2 - 8x + 12)$

$y^2 = -x^2 + 8x - 12$

$x^2 - 8x + y^2 = -12$ ✓

$+16$

$(x - 4)^2 + y^2 = 4$ ✓

$C = (4, 0)$ ✓

$R = 2$ ✓

this maybe done in part (b)

Needs to be in this format.

Question 12

a. $y = x^2 - 6x + 12$

$x^2 - 6x + 9 = y - 12 + 9$

$(x - 3)^2 = 1(y - 3)$ $a = \frac{1}{4}$

i. vertex (3, 3)

ii. focus $(3, 3\frac{1}{4})$

iii. $y = 2\frac{3}{4}$

b. $x^2 = 8(4 - y) \rightarrow x^2 = 4 - y$

$x^2 = -8(y - 4)$

$y = 4 - \frac{x^2}{8}$ ✓

$\frac{dy}{dx} = -\frac{2x}{8}$ at $x = 4$

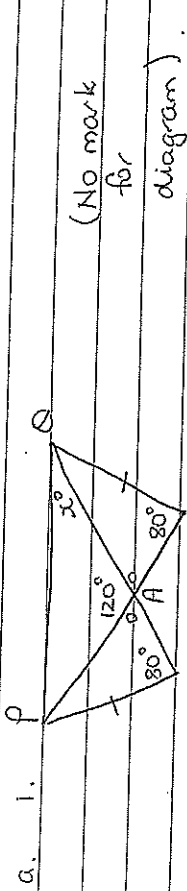
$\therefore y - y_1 = m(x - x_1)$ any

$y - 2 = -1(x - 4)$ $M_T = -1$

$y - 2 = -x + 4$

$x + y - 6 = 0$ ✓

Question 13.



ii. In $\triangle PSA$ and $\triangle QRA$

$$PS = QR \text{ given}$$

$$\angle PSA = \angle QRA \text{ (given } 80^\circ)$$

$$\angle PAS = \angle QAR \text{ (vertically opposite)}$$

$$\therefore \triangle PSA \cong \triangle QRA \text{ (AAS)}$$

iii. $PA = QA$ ✓ corresponding sides of congruent triangles ✓

$\therefore \triangle PAQ$ is isosceles, 2 sides equal.

$$\angle APQ = \angle AQP \text{ (equal angles opposite equal sides)}$$

$$120 + 2x = 180$$

$$2x = 60$$

$$x = 30$$

Question 14

a) 46mm 59.8mm 77.74mm

planted week 1 week 2.

C.P

Term 1 T_2 T_3

$$a = 46$$

$$r = 1.3$$

1. Week 6 \rightarrow Term 7 $a r^{n-1}$

$$= 46(1.3)^6$$

$$= 222.033214 \text{ mm}$$

11. Week W \rightarrow Term (W+1)

$$46(1.3)^{W+1-1} > 1000$$

$$1.3^W > 21.73913 \dots$$

$$1.3^{12} = 23.298$$

$$1.3^{12} > 21.73913$$

ie 12 weeks needed.

$$b) 2x^2 - x - 10 = mx - 12$$

$$2x^2 - x - mx + 2 = 0$$

$$2x^2 - (1+m)x + 2 = 0$$

for tangent $\Delta = 0$

$$b^2 - 4ac = 0$$

$$(1+m)^2 - 4(2)(2) = 0$$

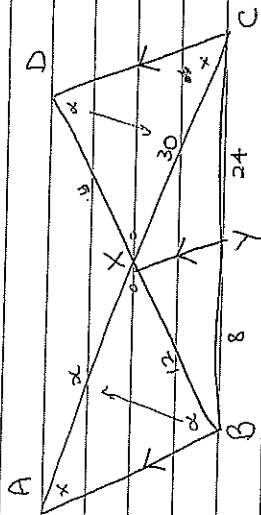
$$1 + 2m + m^2 - 16 = 0$$

$$m^2 + 2m - 15 = 0$$

$$(m+5)(m-3) = 0$$

$$m = -5, m = 3$$

Question 15



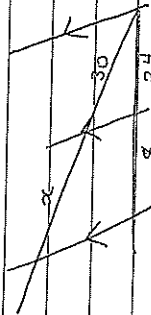
i. In $\triangle AXB$ and $\triangle CXD$

$\angle AXB = \angle CXD$ (vertically opposite)

$\angle BAX = \angle DCX$ (alternate angles $AB \parallel CD$)

$\therefore \triangle AXB \sim \triangle CXD$ (equiangular).

ii.



$$\frac{\alpha}{30} = \frac{8}{24}$$

ratio of intercepts on parallel lines

$$24\alpha = 240$$

$$\alpha = 10$$

$$AX = 10 \text{ cm}$$

Likewise

$$\frac{y}{12} = \frac{24}{8}$$

$$8y = 288$$

$$y = 36 \quad XD = 36 \text{ cm}$$

iii. Ratio $AB : CD = AX : XC$ ratio of corresponding

sides in similar triangles,

$$= 1 : 3.$$