

# SYDNEY TECHNICAL HIGH SCHOOL



## PRELIMINARY HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK 1

MAY 2014

# Mathematics Extension 1

### General Instructions

- Working time - 70 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- In questions 6 to 11, show relevant mathematical reasoning and/or calculations
- Start each question in section 2 on a new page

Total marks - 53

### Section 1 - 5 marks

Attempt Questions 1 – 5.  
Allow about 7 minutes for this section.

### Section 2 - 48 marks

Attempt Questions 6 – 11.  
Allow about 63 minutes for this section.

Name : \_\_\_\_\_

Teacher : \_\_\_\_\_

## Section 1

5 marks

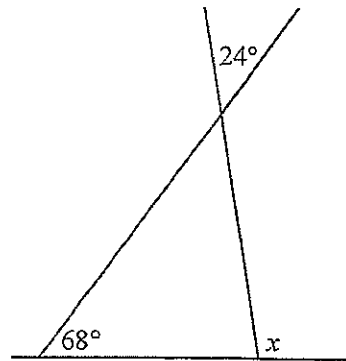
Attempt Questions 1 – 5

Allow about 7 minutes for this section

Use the multiple-choice answer sheet in your answer booklet for Questions 1 – 5.  
Do not remove the multiple-choice answer sheet from your answer booklet.

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1



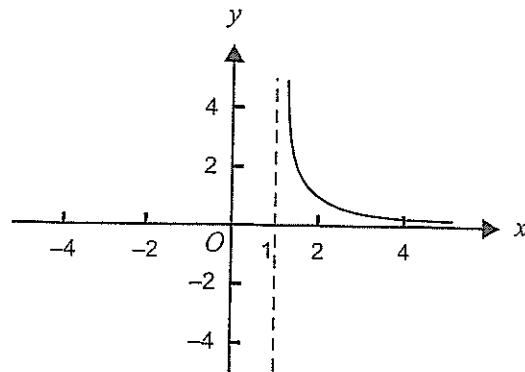
The size of angle  $x$  is

- A.  $88^\circ$
- B.  $92^\circ$
- C.  $112^\circ$
- D.  $116^\circ$

2. How many asymptotes does the graph of the function  $y = \frac{3x^2}{x(2-x)}$  have?

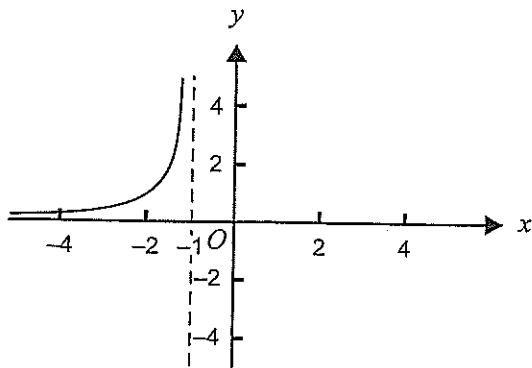
- A. 0
- B. 1
- C. 2
- D. 3

3. Part of the graph of the function with rule  $y = f(x)$  is shown below

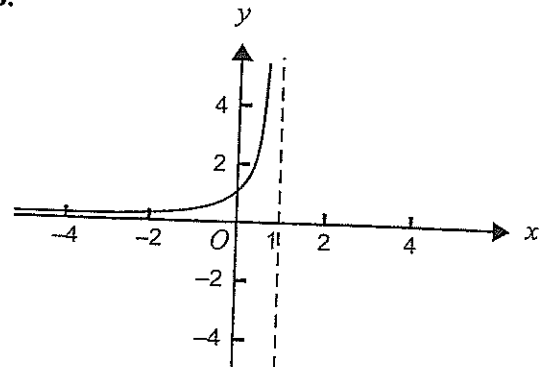


Which one of the following is most likely to be the corresponding part of the function with rule  $y = f(-x)$  ?

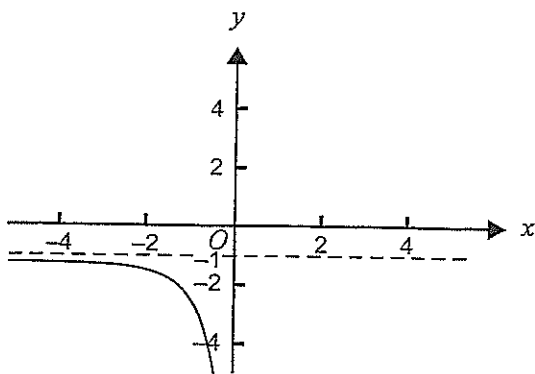
A.



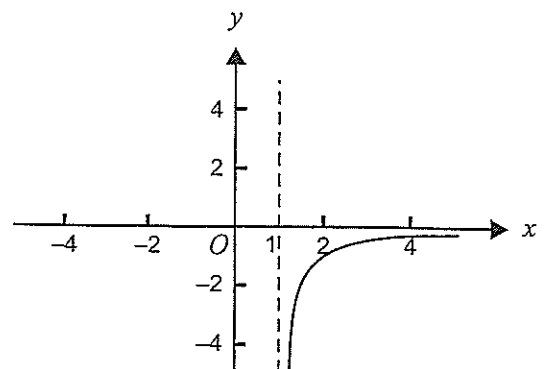
B.



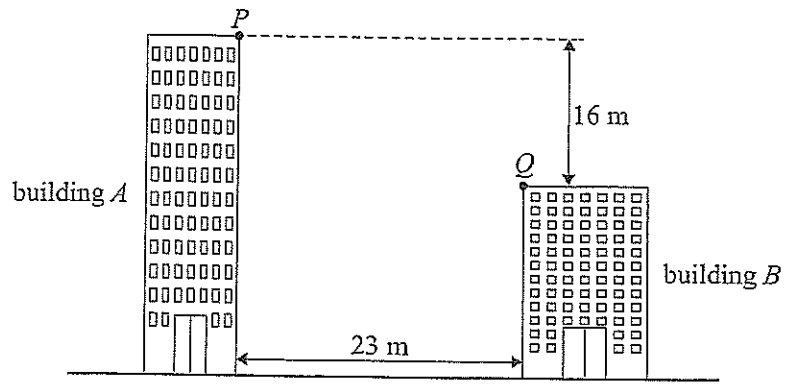
C.



D.



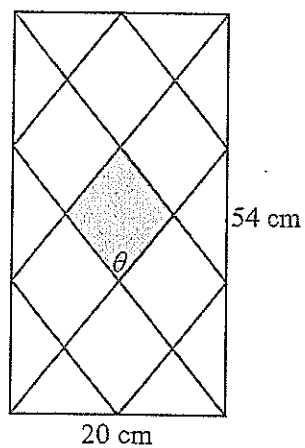
4.



In the diagram above, the angle of depression of point Q from point P is closest to

- A.  $35^\circ$
- B.  $41^\circ$
- C.  $46^\circ$
- D.  $55^\circ$

5. The rectangle shown below is 54 cm high and 20 cm wide.  
The rhombuses drawn inside the rectangle are all the same size and shape.



The size of the angle  $\theta$ , in the shaded rhombus, is closest to

- A.  $34^\circ$
- B.  $56^\circ$
- C.  $58^\circ$
- D.  $67^\circ$

## Section 2

48 marks

Attempt Questions 6 – 11

Allow about 63 minutes for this section

Answer each question in your answer booklet. Start each question on a new page.

In Questions 6 – 11, your response should include relevant mathematical reasoning and/or calculations.

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### Question 6 (8 marks)

- a) Fully factorise  $2x^4 + 16x$  1
- b) If  $\sec \theta = 3$  and  $\tan \theta < 0$ , find the exact value of  $\sin \theta$ . 2
- c) Simplify  $\left(\frac{x^{p+q}}{x^q}\right)^p \div \left(\frac{x^q}{x^{q-p}}\right)^{p-q}$  2
- d) If  $g(5x) = 50x^2 + 10x + 1$ , find an expression for  $g(x)$ . 1
- e) Draw a neat sketch of  $y = \frac{x-2}{x+2}$  2

**Question 7** (8 marks) Start a new page

- a) Solve  $\sin(\theta - 75^\circ) = \frac{-\sqrt{3}}{2}$  for  $0^\circ \leq \theta \leq 360^\circ$  2
- b) If  $A$  is an acute angle, simplify  $\frac{\tan A}{\sqrt{1+\tan^2 A}}$  2
- c) In pentagon  $ABCDE$ , angle  $A = 120^\circ$ , angle  $E = 140^\circ$ ,  
 $AB$  is parallel to  $DC$ , and  $BC$  is parallel to  $AE$ .
- i) Draw a neat sketch clearly showing this information. 1
- ii) Find the size of angle  $B$ , giving reasons. 1
- iii) Find the size of angle  $D$ , giving reasons. 2

**Question 8** (8 marks) Start a new page

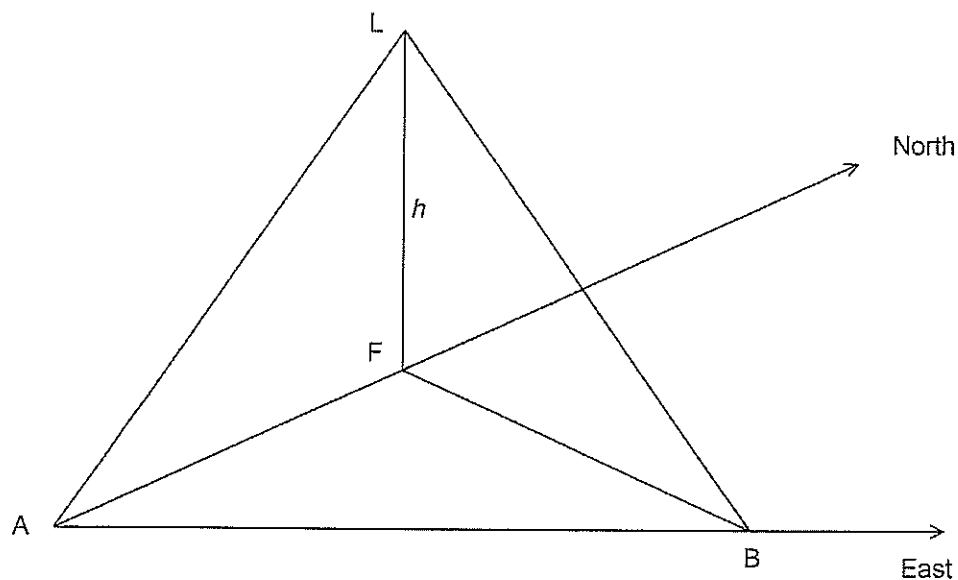
- a) Solve  $2\cos^2 x = \sin x + 1$ , for  $0^\circ \leq x \leq 360^\circ$ . 3
- b) Simplify  $\frac{5^{-n} \times 25^{2n-2}}{5^{3n-2} \times 10^{-1}}$  2
- c) Solve  $\frac{5}{4-x} \geq 1$  3

**Question 9** (8 marks) Start a new page

a) Solve  $|2x - 1| = 3x + 6$  2

b) Show that  $\sec \alpha - \cos \alpha = \sin \alpha \tan \alpha$  2

c)



A vertical flagpole,  $FL$ , of height  $h$  metres stands in the middle of a park. From point  $A$ , due South of the flagpole, the angle of elevation to the top of the flagpole is  $35^\circ$ . From point  $B$ , which is 45 metres due East of point  $A$ , the angle of elevation to the top of the flagpole is  $28^\circ$ .

- i) Find an expression for the length of  $AF$  in terms of  $h$ . 1
- ii) Find the height of the flagpole, in metres correct to 1 decimal place. 3



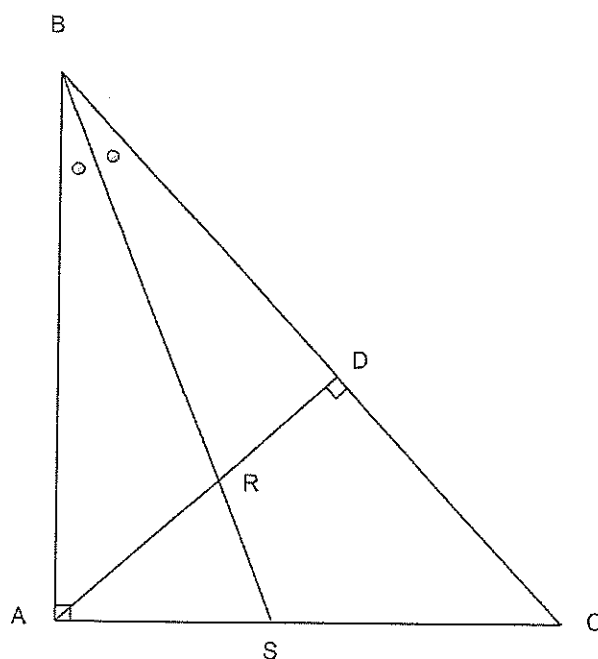
**Question 10** (8 marks) Start a new page

a) Solve  $(2x - 1)^2 = 5$  1

b) i) Draw a neat sketch of  $y = x^2 - 6x + 8$ , 2  
clearly showing all intercepts and the vertex.

ii) On a separate diagram draw a neat sketch of  $y = \frac{1}{x^2 - 6x + 8}$  2  
clearly showing all important features.

c)



In triangle ABC, angle  $A = 90^\circ$ ,  $SB$  bisects angle  $B$  and  $AD$  is perpendicular 3  
to  $BC$  and meets  $SB$  at  $R$ .

By letting angle  $SBC = x$ , or otherwise, prove that triangle  $ASR$  is isosceles.

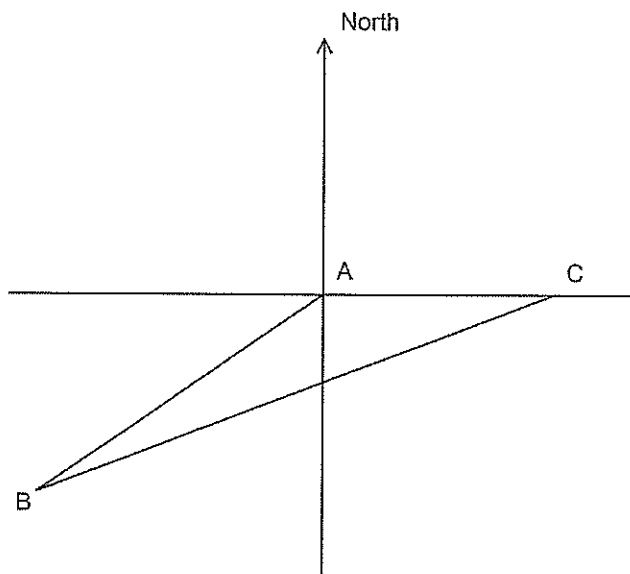
**Question 11** (8 marks) Start a new page

- a) Solve simultaneously for  $x$  and  $y$ ,

2

$$y = x^2 - 2x - 1 \quad \text{and} \quad 2x - y - 1 = 0$$

- b)



A surveyor standing at point  $A$  notes that, point  $B$  is on a bearing of  $228^\circ T$  and point  $C$  is due East of point  $A$ . The surveyor then walks 85 metres to point  $B$  where he notes that the bearing of point  $C$  from point  $B$  is  $070^\circ T$ .

3

Find the distance from point  $B$  to point  $C$ .  
(Give answer in metres correct to 1 decimal place)

- c) Solve  $|x + 1| > \sqrt{25 - x^2}$

3

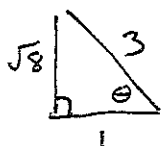
**End of paper**

# Ext 1 SOLUTIONS MAY 2014

1. B
2. D
3. A
4. A
5. C

6. a)  $2x(x+2)(x^2-2x+4)$

b) 4th quad



$$\sin \theta = -\frac{\sqrt{8}}{3}$$

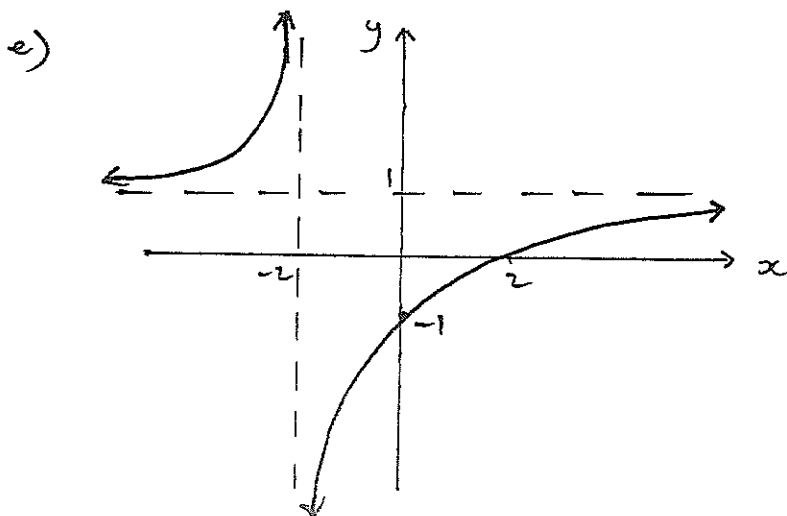
c)  $\left(\frac{x^{p+q}}{x^q}\right)^p \div \left(\frac{x^q}{x^{q-p}}\right)^{p-q}$

$$= (x^p)^p \div (x^p)^{p-q}$$

$$= x^{pq}$$

d)  $g(5x) = 2(5x)^2 + 2(5x) + 1$

$$\therefore g(x) = 2x^2 + 2x + 1$$



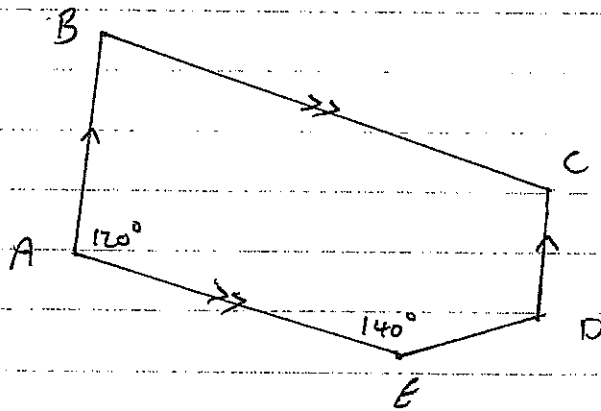
7.

$$\begin{aligned} \theta - 75^\circ &= 240^\circ, 300^\circ \\ \theta &= 315^\circ, 375^\circ \end{aligned}$$

$$\therefore \theta = 315^\circ, 15^\circ$$

$$\begin{aligned} b. \quad & \frac{\tan A}{\sqrt{1+\tan^2 A}} \\ &= \frac{\tan A}{\sqrt{\sec^2 A}} \\ &= \tan A \cos A \\ &= \sin A \end{aligned}$$

c. i)



$$ii) \quad \angle B = 60^\circ \text{ (co-interior angles, } BC \parallel AE \text{)}$$

$$iii) \quad \angle C = 120^\circ \text{ (co-interior angles, } AB \parallel CE \text{)}$$

$$\text{angle sum of pentagon} = 540^\circ$$

$$\begin{aligned} \therefore \angle D &= 540^\circ - 120^\circ - 140^\circ - 60^\circ - 120^\circ \\ &= 100^\circ \end{aligned}$$

8.

$$\begin{aligned}
 \text{a.} \quad & 2 \cos^2 x = \sin x + 1 \\
 & 2(1 - \sin^2 x) = \sin x + 1 \\
 & 2 \sin^2 x + \sin x - 1 = 0 \\
 & (2 \sin x - 1)(\sin x + 1) = 0 \\
 & \sin x = \frac{1}{2}, -1
 \end{aligned}$$

$$x = 30^\circ, 150^\circ, 270^\circ$$

$$\begin{aligned}
 \text{b.} \quad & \frac{5^{-n} \times 25^{2n-2}}{5^{3n-2} \times 10^{-1}} \\
 & = \frac{5^{-n} \times (5^2)^{2n-2}}{5^{3n-2} \times 5^{-1} \times 2^{-1}} \\
 & = \frac{5^{-n} \times 5^{4n-4}}{5^{3n-2} \times 5^{-1} \times 2^{-1}} \\
 & = \frac{5^{3n-4}}{5^{3n-3} \times 2^{-1}} \\
 & = 5^{-1} \times 2 \\
 & = \frac{2}{5}
 \end{aligned}$$

$$\text{c.} \quad \frac{5}{4-x} \geq 1$$

$$\frac{5(4-x)^2}{4-x} \geq (4-x)^2$$

$$5(4-x) \geq (4-x)^2$$

$$(4-x)^2 - 5(4-x) \leq 0$$

$$(4-x)(-x-1) \leq 0$$

$$-1 \leq x < 4$$

9.

a.  $|2x-1| = 3x+6$

$$2x-1 = 3x+6$$

$$x = -7$$

test  $x = -7$  ✗

$$2x-1 = -3x-6$$

$$5x = -5$$

$$x = -1$$

$x = -1$  ✓

$\therefore x = -1$

b. LHS =  $\sec \alpha - \cos \alpha$

$$= \frac{1}{\cos \alpha} - \cos \alpha$$

$$= \frac{1 - \cos^2 \alpha}{\cos \alpha}$$

$$= \frac{\sin^2 \alpha}{\cos \alpha}$$

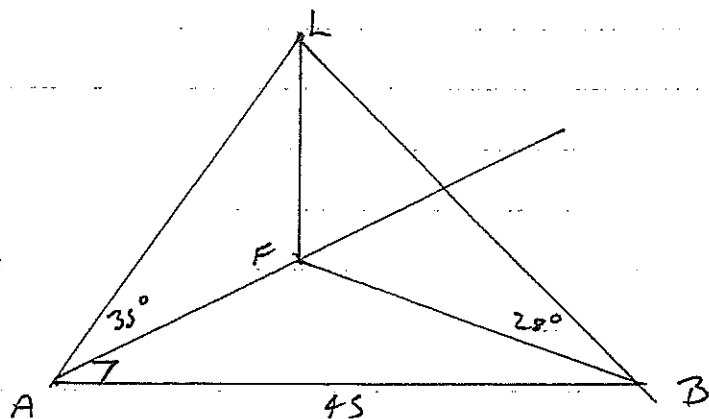
$$= \sin \alpha \tan \alpha$$

$$= \text{RHS}$$

c. i.  $\tan 35^\circ = \frac{h}{AF}$

$$AF = \frac{h}{\tan 35^\circ}$$

$$= h \tan 55^\circ$$



ii.  $BF = h \tan 62^\circ$

$$AB^2 + AF^2 = BF^2$$

$$45^2 = h^2 \tan^2 62^\circ - h^2 \tan^2 55^\circ$$

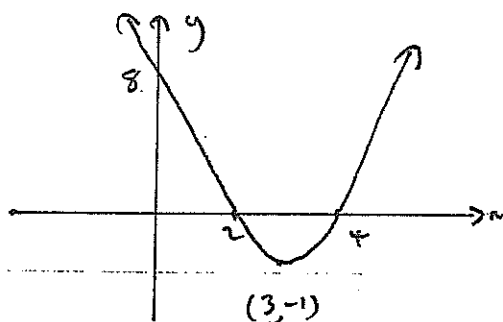
$$h = \frac{45}{\sqrt{\tan^2 62^\circ - \tan^2 55^\circ}}$$

$$= 36.8 \text{ m.}$$

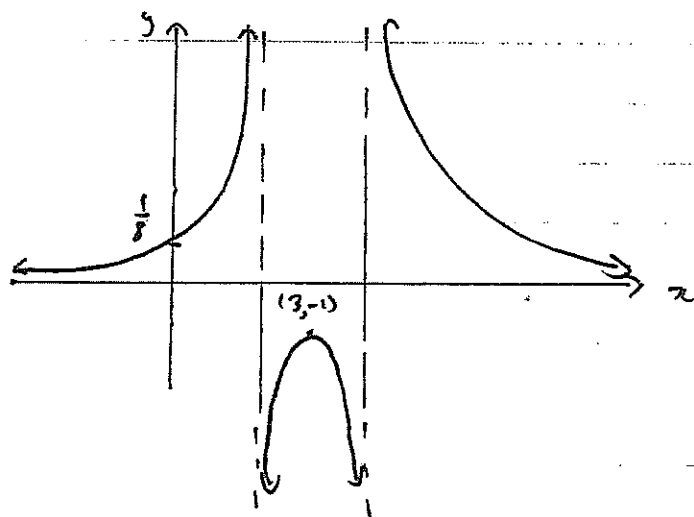
10.

a.  $(2x-1)^2 = 5$   
 $2x-1 = \pm\sqrt{5}$   
 $x = \frac{1 \pm \sqrt{5}}{2}$

b. i.  $y = (x-4)(x-2)$



ii.



c. let  $\angle SBC = x$

$\angle BRD = 90 - x$  (angle sum of  $\triangle BDR$ )

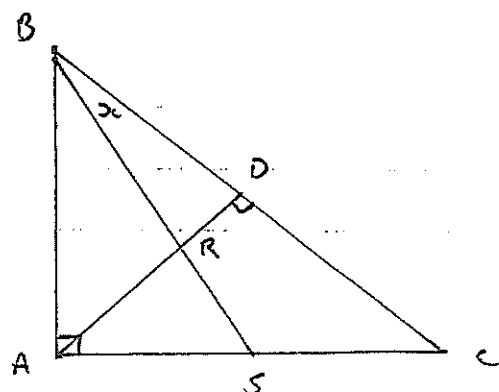
$\angle ARS = 90 - x$  (vertically opposite)

$\angle ABS = x$  (equal to  $\angle SBC$ )

$\therefore \angle ASB = 90 - x$  (angle sum of  $\triangle ABS$ )

$\therefore \angle ARS = \angle ASB$

$\therefore ASR$  is isosceles



11.

$$a. \quad y = x^2 - 2x - 1$$

$$2x - y - 1 = 0 \rightarrow y = 2x - 1$$

sub

$$2x - 1 = x^2 - 2x - 1$$

$$x^2 - 4x = 0$$

$$x(x - 4) = 0$$

$$x = 0, 4$$

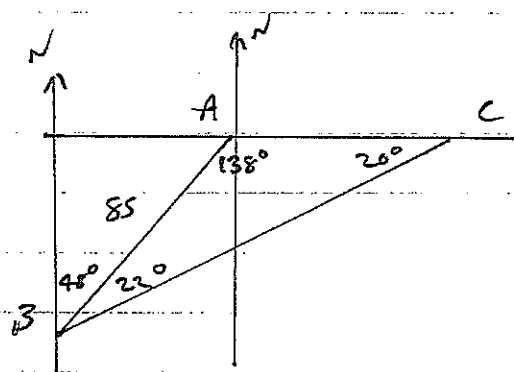
$$\therefore y = -1, 7$$

$\therefore$  solutions  $x = 0, y = -1$  and  $x = 4, y = 7$

$$b. \quad \frac{BC}{\sin 138^\circ} = \frac{85}{\sin 20^\circ}$$

$$BC = \frac{85 \times \sin 138^\circ}{\sin 20^\circ}$$

$$= 166.3 \text{ m.}$$



$$c. \quad y = |x+1|$$

$$y = \sqrt{25-x^2}$$

$$x+1 = \sqrt{25-x^2}$$

$$(x+1)^2 = 25-x^2$$

$$2x^2 + 2x - 24 = 0$$

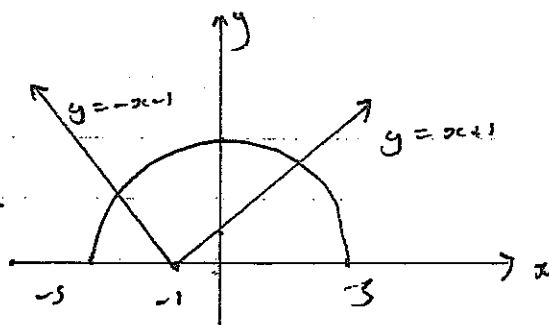
$$x^2 + x - 12 = 0$$

$$(x+4)(x-3) = 0$$

$$x = -4, 3$$

$$-x+1 = \sqrt{25-x^2}$$

same



$$\therefore -5 \leq x < -4, \quad 3 < x \leq 5$$