

Maths Class: _____

2007

Time Allowed: 3 hours plus 5 mins reading time

- Write your name and class at the top of this page, and at the top of each answer sheet
- At the end of the examination this examination paper must be attached to the front of your answers
- All questions are of equal value and may be attempted
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

[illegible]

Question 1 (12 Marks)**Marks**

- a) Find the value of $\frac{16.2^2}{14.7 - 8.1}$ correct to 3 significant figures 2
- b) Simplify $4\sqrt{32} - 2\sqrt{8}$ 2
- c) Write down the exact value of $\sin \frac{5\pi}{4}$ 2
- d) Simplify $4(2x + 1) - (x^2 + 2x - 3)$ 2
- e) Fully factorise $2x^3 - 2y^3$ 2
- f) Find the primitive of $x^2 - 2x + \frac{1}{x}$ 2

Question 2 (12 marks) Start a new page

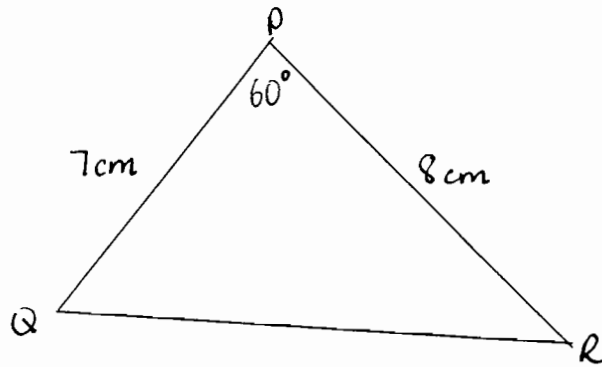
Marks

a) Solve $|1 - 2x| > 7$

2

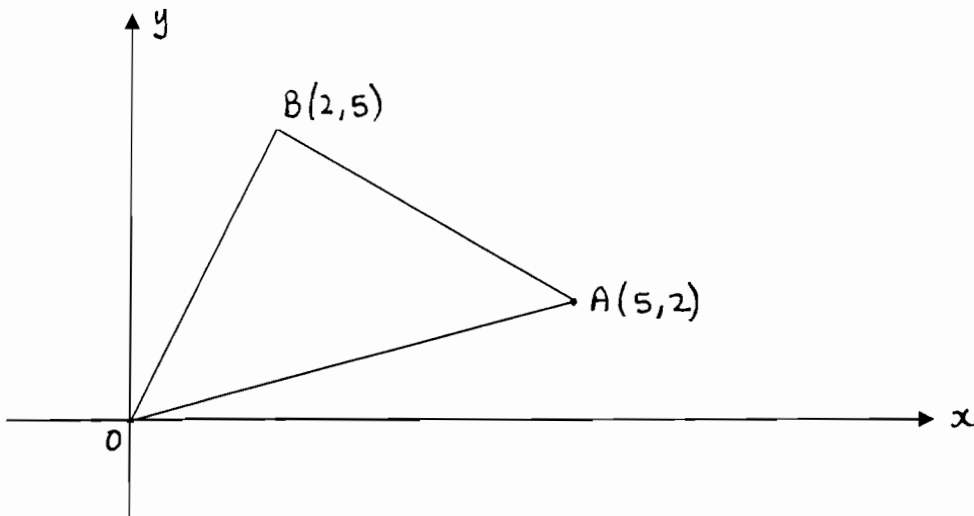
b) Find the exact area of $\triangle PQR$

2



Not to scale

c)



Not to scale

The points $O(0,0)$, $A(5, 2)$ and $B(2, 5)$ are the vertices of a triangle ABO .

(i) Find the distance OA and the distance OB

2

(ii) Show that the equation AB is $x + y - 7 = 0$

2

(iii) Calculate the perpendicular distance from O to AB

2

(iv) Find the midpoint, M , of AB

1

(v) Without any more calculations what is the distance of OM , give a reason for answer.

1

Question 3 (12 marks) Start a new page**Marks**a) Differentiate with respect to x :

i) $y = x^2 - 4x + 1$ 1

ii) $y = (e^{2x} + 1)^2$ 2

iii) $y = x^2 \cos 2x$ 2

b) i) Find $\int \frac{4}{4x+1} dx$ 1

ii) Evaluate $\int_0^{\frac{\pi}{4}} 2 \sec^2 x \, dx$ 2

c) The roots of the equation $x^2 + 5x = 7$ are α and β
Find the value of

i) $\alpha + \beta$ 1

ii) $\alpha\beta$ 1

iii) $\alpha^2 + \beta^2$ 2

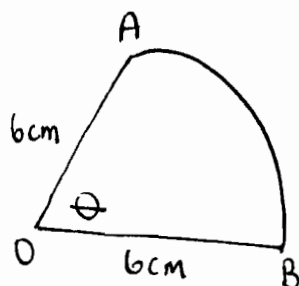
Question 4 (12 marks) Start a new page

Marks

- a) A ship sails from Port A 70 nautical miles due west to Port B. It then proceeds 40 nautical miles on a bearing of 120°T to Port C.

- i) Find the distance of Port C from Port A (correct to 2 decimal places) 2
- ii) Find the bearing of Port C from Port A (correct to the nearest degree). 2

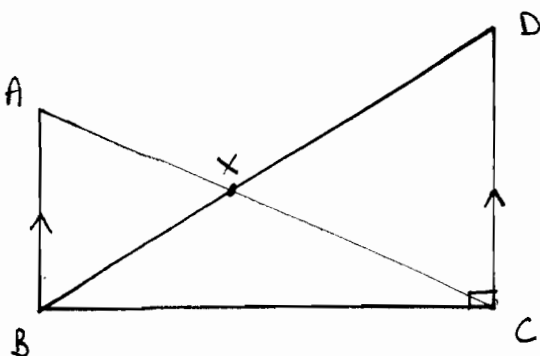
b)



The perimeter of sector AOB is 13.5cm

- i) Find the size of $\angle AOB$, correct to the nearest minute 2
- ii) Find the area of sector AOB 2

c)



In the diagram AB is parallel to CD
and $CD \perp BC$ 2

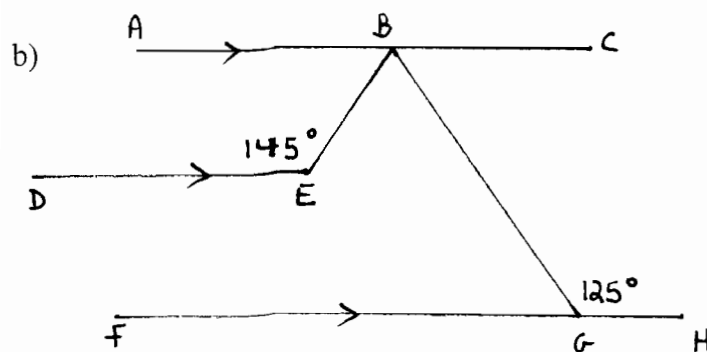
- i) Show that triangle AXB is similar to triangle CXD 2
- ii) Given $AB:DC=2:3$ Show that $9(BX)^2 = 4(XD)^2$ 2

Question 5 (12 marks) Start a new page

Marks

a) For the sequence 95, 91, 87, find,

- i) An expression for the n th term, T_n , in its simplest form 2
- ii) Which term is the first term less than zero 2
- iii) What is the sum of all the terms greater than zero 2



In the diagram given

$AC \parallel DE$ and $AC \parallel FH$

$\angle DEB = 145^\circ$ and $\angle BGH = 125^\circ$

Find the size of $\angle EBG$, giving reasons

2

c) i) For what values of x will a limiting sum exist for the geometric series,
 $3 - 12x + 48x^2 - \dots$?

2

ii) Find the value of x for which the limiting sum is 9.

2

Question 6 (12 marks) Start a new page

Marks

a) Find the equation of the *normal* to the curve $y = \ln(2x + 3)$ at the point where $x = -1$.

3

b) The function $f(x)$ is given by $f(x) = 2x(x - 3)^2$

i) Find the coordinates of the points where the curve $y = f(x)$ cuts the x -axis

2

ii) Find the coordinates of any turning points on the curve $y = f(x)$, and determine their nature

4

iii) Sketch the curve $y = f(x)$ in the domain $-1 \leq x \leq 4$

2

iv) Hence solve $2x^3 - 12x^2 + 18x - 8 = 0$

1

Question 7 (12 marks) Start a new page

Marks

a) What is the value of $\log_2 \sqrt{8}$ 1

b) Given $3x^2 + 4x + 5 \equiv A(x+1)^2 + B(x+1) + C$
Find the value of the constants A , B and C 3

c) Consider the function $f(x) = x \sin^2 x$

i) Copy and complete the table below in your writing booklet. Values of $f(x)$ are given to 3 decimal places where appropriate.

x	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
$f(x)$	0	0.393	1.571		0

1

ii) Using Simpson's Rule with five function values, evaluate

$$\int_0^{\pi} x \sin^2 x dx, \text{ correct to 2 decimal places.}$$

3

d) i) Sketch the curve $y = 1 - \cos 2x$, $0 \leq x \leq 2\pi$ 2

ii) Find the area bounded by the curve, $y = 1 - \cos 2x$, the x -axis and the lines $x = 0$ and $x = \pi$ 2

Question 8 (12 marks) Start a new page**Marks**

a) Given $\log_a x = 0.417$ and $\log_a y = 0.609$ find the value of

i) $\log_a(ax)$ 2

ii) $\log_a \frac{x^2}{y}$ 2

b) The region beneath the curve $y = 3e^{-2x} + 1$ which is above the x – axis and between the lines $x = 0$ and $x = 1$ is rotated about the x – axis

i) Sketch the region 2

ii) Find the volume of the solid revolution 4

c) The price of one gram of gold, \$P, was studied over the period of t days.

i) Throughout the period of study $\frac{dP}{dt} > 0$
What does this say about the price of gold? 1

ii) If it was noted over this time that the rate of change in the price of gold increased. What does this statement imply about $\frac{d^2P}{dt^2}$? 1

Question 9 (12 marks) Start a new page**Marks**

a) For what values of k does the equation $x^2 - (k + 2)x + 1 = 0$ have;

i) Equal roots 2

ii) No real roots 1

b) The population of a town at the end of t years is given by $P = Ae^{kt}$, where A and k are constants.

After 1 year the population is 1060

i) Find the value of A if the population was initially 1020 1

ii) Find the value of k 2

iii) Calculate the population after 12 years 2

iv) What is the **rate** of increase in the population after 12 years 2

v) How many years will it take the population to double? 2

Question 10 (12 marks) Start a new page**Marks**

- a) Shrek borrows \$1 000 000 from the Muffin man, at 7.8% p.a. monthly reducible interest to buy a new swamp in Far-Far away land.

He repays the loan in equal monthly repayments of \$8000.

- i) Write an expression for the amount Shrek owes immediately **before** the 1st repayment 1

- ii) Show that Shrek owes the Muffin man after n months:

$$An = 1000\,000(1.0065)^n - 8000 \left[\frac{1.0065^n - 1}{0.0065} \right] \quad 3$$

- iii) How many months does Shrek take to repay half the loan to the Muffin man? 2

- b) A new grain silo with a capacity of $4000m^3$ is to be constructed on a farm. The silo is a fully enclosed cylinder and is to be constructed from concrete.

To Save costs, the farmer wants to minimise the surface area of the silo.

- i) Write an expression for the volume of the silo in terms of radius (r) and height (h) 1

- ii) Write an expression for the surface area (A) of the concrete silo in terms of r 2

- iii) Show that $\frac{dA}{dr} = \frac{4\pi r^3 - 8000}{r^2}$ 1

- iv) Hence, find the dimensions of the silo to minimise the surface area of the silo. Express your dimensions to 1 decimal place. 2

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

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Answers (2u)

Question 1

a) $39.7636 \dots$ ①
 $= 39.8$ ①

b) $4\sqrt{32} - 2\sqrt{8} = 16\sqrt{2} - 4\sqrt{2}$ ①
 $= 12\sqrt{2}$

c) $\sin \frac{5\pi}{4} = -\sin \pi/4$ ①
 $= -1/\sqrt{2}$ ①

d) $4(2x+1) - (x^2+2x-3)$
 $= 8x + 4 - x^2 - 2x + 3$ ①
 $= 6x - x^2 + 7$ ①

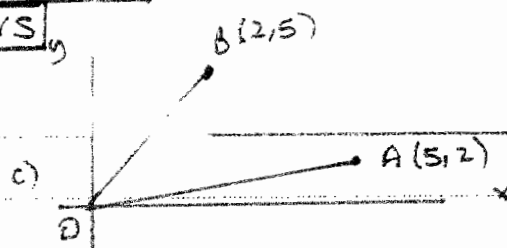
e) $2x^3 - 2y^3$
 $= 2(x^3 - y^3)$ ①
 $= 2(x-y)(x^2+xy+y^2)$ ①

f) $\int x^2 - 2x + 1/x \, dx$
 $= \frac{x^3}{3} - x^2 + \ln x + C$ ②

Question 2

a) $|1-2x| > 7$
 $1-2x > 7 \quad -1+2x > 7$
 $-2x > 6 \quad 2x > 8$
 $x < -3$ ① $x > 4$ ①

b) $A = \frac{1}{2} ab \sin C$
 $= \frac{1}{2} \times 7 \times 8 \times \sin 60^\circ$
 $= \frac{1}{2} \times 7 \times 8 \times \sqrt{3}/2$ ①
 $= 14\sqrt{3} \text{ cm}^2$ ①



c) (i) $d_{OA} = \sqrt{29}$ ①
 $d_{OB} = \sqrt{29}$ ①

(ii) $m_{AB} = \frac{2-5}{5-2}$
 $= -1$

$\therefore y - 5 = -1(x - 2)$
 $y - 5 = -x + 2$
 $x + y - 7 = 0$

(iii) pt(0,0) line $x + y - 7 = 0$

$d_{\perp} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$ ①
 $= \frac{|0 + 0 - 7|}{\sqrt{1^2 + 1^2}}$
 $= \frac{7}{\sqrt{2}}$ ①

iv) Midpt $m(3.5, 3.5)$ ①

v) $\text{dist } Om = \frac{7}{\sqrt{2}}$ as $\triangle AOB$ is

Isosceles $\therefore Om$ is \perp bisector of AB .

① \rightarrow must have a suitable reason.

Question 3

a) i. $\frac{dy}{dx} = 2x - 4$ ①

ii. $\frac{dy}{dx} = 2(e^{2x} + 1) \cdot 2e^{2x}$
 $= 4e^{2x}(e^{2x} + 1)$ ②

iii. $\frac{dy}{dx} = \cos 2x (2x) + x^2 (-2\sin 2x)$

$= 2x \cos 2x - 2x^2 \sin 2x$ ②

b) i. $\int \frac{4}{4x+1} dx = \ln(4x+1) + c$ ①

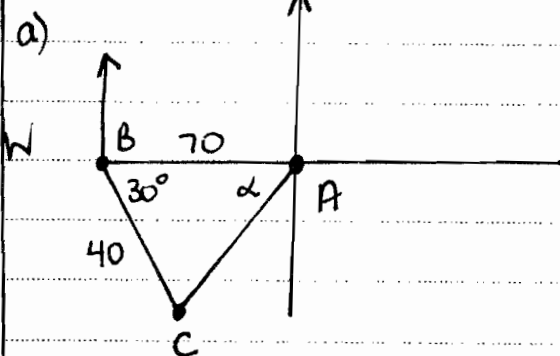
ii. $\int_0^{\pi/4} 2 \sec^2 x dx = 2 \tan x \Big|_0^{\pi/4}$
 $= 2 [\tan \frac{\pi}{4} - \tan 0]$
 $= 2 [1 - 0]$
 $= 2$ ①

c) $x^2 + 5x - 7 = 0$ $a=1, b=5, c=-7$

(i) $\alpha + \beta = -b/a$ (ii) $\alpha\beta = c/a$
 $= -5$ ① $= -7$ ①

(iii) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ①
 $= (-5)^2 - 2(-7)$
 $= 39$ ①

Question 4

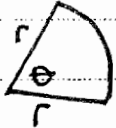


ii. $\frac{\sin \alpha}{40} = \frac{\sin 30^\circ}{70}$

$\sin \alpha = 0.49236 \dots$

$\alpha = 29^\circ 30'$

Bearing $= 270^\circ - 29^\circ 30'$
 $= 240^\circ 30'$

b)  $l = r\theta$ $13.5 = 2r + r\theta$
 $13.5 = 12 + 6\theta$ ①
 $r = 6$ $0.25 = \theta$ (rad)
 $\theta = 0.25 \times \frac{180^\circ}{\pi} = 14^\circ 19'$ ①

ii) $A = \frac{1}{2} r^2 \theta$
 $= \frac{1}{2} \times 6^2 \times 0.25$
 $= 4.5 \text{ cm}^2$

c) In $\triangle AXB$ and $\triangle CXD$
 ① $\angle BAX = \angle DCX$ (alternate angles) $AB \parallel CD$
 $\angle AXB = \angle CXD$ (vertically opposite)
 $\therefore \triangle AXB \parallel \triangle CXD$ (equiangular) ①

ii) $\frac{AB}{CD} = \frac{XB}{XD}$ corresponding sides of $\parallel \Delta$'s in proportion ①

$\frac{2}{3} = \frac{XB}{XD}$

$2XD = 3BX$

$4(XD)^2 = 9(BX)^2$

both = ①

Question 5

a) 95, 91, 87, ... $a = 95$ $d = -4$ AP

i) $T_n = a + (n-1)d$ ①
 $= 95 + (n-1)(-4)$
 $= 95 - 4n + 4$ ①
 $= 99 - 4n$

ii) $T_n < 0$
 $99 - 4n < 0$
 $4n > 99$
 $n > 24.75$ ①
 \therefore 25th term is 1st negative. ①

iii) ie 24 terms > 0 $n = 24$ $a = 95$
 $S_n = \frac{n}{2}(2a + (n-1)d)$ $d = -4$ ①
 $= \frac{24}{2}(2(95) + 23(-4))$ ①
 $= 1176$ ①

b) $\angle C B G + 125^\circ = 180^\circ$ (co-interior angles AC || FH) ①
 $\angle C B G = 55^\circ$
 $\angle A B E + 145^\circ = 180^\circ$ (co-interior angles AC || DE) ①
 $\angle A B E = 35^\circ$
 $35 + 55 + \angle E B G = 180^\circ$ (straight) ①
 $\angle E B G = 90^\circ$

c) $a = 3$ $r = -4x$
 i) $\therefore -1 < r < 1$ ①
 $-1 < -4x < 1$
 $1/4 > x > -1/4$
 $\therefore -1/4 < x < 1/4$ ①

ii) $S_\infty = \frac{a}{1-r}$ $q = \frac{3}{1+4x}$ ①

Question 6

a) $\frac{dy}{dx} = \frac{2}{2x+3}$ at $x = -1$ $y = 0$

$M_T = 2$
 $M_N = -1/2$ ①

eq: $4 - 0 = -1/2(x+1)$
 $2y = -x - 1$ ①
 $x + 2y + 1 = 0$ ①

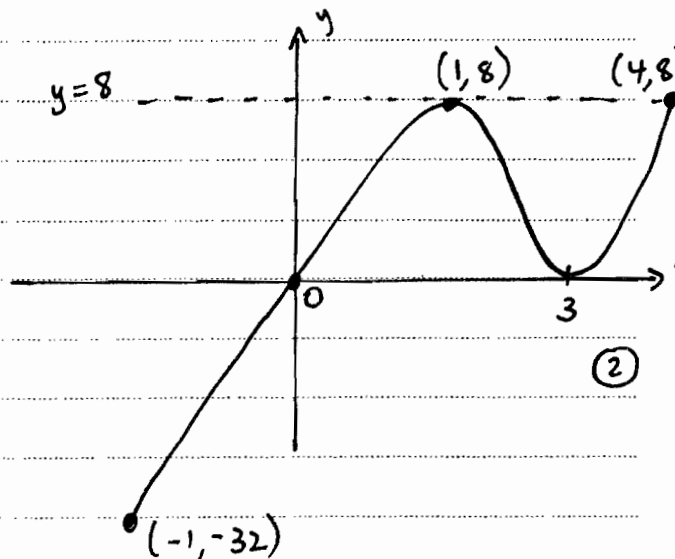
b) $f(x) = 2x(x-3)^2 = 2x^3 - 12x^2 + 18x$
 i) x-int $y = 0$
 $(0, 0)$ ① and $(3, 0)$ ①

ii) Stat pts $f'(x) = 0$
 $f'(x) = 6x^2 - 24x + 18 = 0$
 $6(x-3)(x-1) = 0$
 $x = 3$ $x = 1$
 $y = 0$ $y = 8$

test	x	0	1	2	x	2	3	4
	y'	18	0	-12		-12	0	1
		/	\	/		/	\	/

MAX (1, 8) MIN (3, 0)

iii) End pts $(-1, -32)$ & $(4, 8)$



iv) $2x^3 - 12x^2 + 18x = 8$

Question 7

a) $\log_2 \sqrt{8} = \frac{1}{2} \log_2 8$

$$= \frac{1}{2} \times 3 \log_2 2$$

$$= 1.5$$

①

b) $3x^2 + 4x + 5 \equiv A(x^2 + 2x + 1) + Bx + C$

equating

$$3 = A$$

$$4 = 2A + B$$

$$4 = 6 + B$$

$$B = -2$$

$$5 = A + B + C$$

$$5 = 3 - 2 + C$$

$$C = 4$$

$\therefore A = 3, B = -2, C = 4$

①

①

①

c) $\frac{3\pi}{4}$
1.178

①

ii. $\frac{h}{3} [F + L + 4m]$

①

$$\frac{\pi/4}{3} [0 + 1.571 + 4 \times 0.393]$$

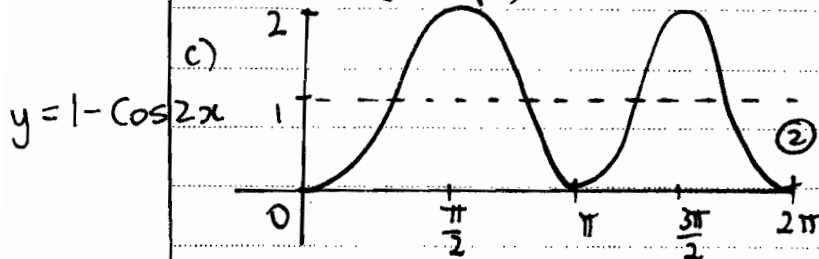
①

$$+ \frac{\pi}{12} [1.571 + 0 + 4 \times 1.178]$$

$$= 2.46772 \dots$$

①

$$= 2.47 \text{ (2dp)}$$



ii. $\int_0^\pi 1 - \cos 2x \, dx$

①

Question 8

a) $\log_a(ax) = \log_a a + \log_a x$

$$= 1 + 0.417$$

$$= 1.417$$

①

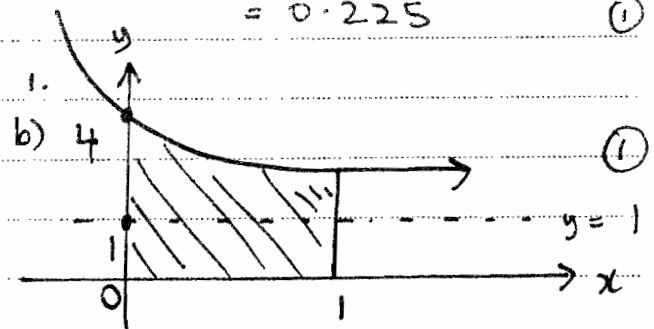
ii) $\log_a \frac{x^2}{y} = 2 \log_a x - \log_a y$

①

$$= 2(0.417) - 0.609$$

$$= 0.225$$

①



①

ii.

$$V_x = \pi \int y^2 \, dx$$

$$= \pi \int_0^1 (3e^{-2x} + 1)^2 \, dx$$

①

$$= \pi \int_0^1 9e^{-4x} + 6e^{-2x} + 1 \, dx$$

①

$$= \pi \left[-\frac{9}{4} e^{-4x} + \frac{6e^{-2x}}{-2} + x \right]_0^1$$

①

$$= \pi \left[-\frac{9}{4} e^{-4} - 3e^{-2} + 1 - \left(-\frac{9}{4} - 3 \right) \right]$$

①

$$= \pi \left[-\frac{9}{4} e^{-4} - 3e^{-2} + \frac{25}{4} \right]$$

c) $\frac{dp}{dt} > 0$ price of gold increasing

①

ii) $\frac{d^2p}{dt^2} > 0$

①

Question 9

$$a) x^2 - (k+2)x + 1 = 0$$

i) Equal roots $\Delta = 0$

$$b^2 - 4ac = 0$$

$$(k+2)^2 - 4(1)(1) = 0 \quad] \textcircled{1}$$

$$k^2 + 4k + 4 - 4 = 0$$

$$k^2 + 4k = 0$$

$$k(k+4) = 0$$

$$k = 0, k = -4 \quad \textcircled{1}$$

ii) $\Delta < 0, -4 < k < 0 \quad \textcircled{1}$

b) $t = 0 \quad P = 1020$

$$\therefore A = 1020 \quad \textcircled{1}$$

ii. $t = 1 \quad P = 1060$

$$1060 = 1020 e^{k(1)}$$

$$\frac{1060}{1020} = e^k \quad \textcircled{1}$$

$$\ln\left(\frac{106}{102}\right) = k$$

$$k = \ln\left(\frac{106}{102}\right) \quad \textcircled{1}$$

$$\div 0.038466 \dots$$

iii) $t = 12 \quad P = ? \quad \textcircled{1}$

$$P = 1020 e^{k \cdot 12} \quad k = \ln\left(\frac{106}{102}\right)$$

$$= 1618.335 \dots$$

$$\div 1618 \quad \textcircled{1}$$

iv) rate = $\frac{d}{dt}$

$$\frac{dP}{dt} = k \cdot (1020 e^{kt}) \quad k = \ln\left(\frac{106}{102}\right)$$

$$t = 12$$

$$= 62.2513 \dots$$

$$= 62.25 \text{ people/yr.} \quad \textcircled{1}$$

v) $t = ? \quad P = 2A$

$$2A = A e^{kt} \quad k = \ln\left(\frac{106}{102}\right)$$

$$2 = e^{kt} \quad \textcircled{1}$$

$$\ln 2 = \ln e^{kt}$$

$$\ln 2 = k \cdot t$$

$$t = \ln 2 \div k$$

$$= 18.0196 \dots \quad \textcircled{1}$$

$$\div 18 \text{ years.}$$

Question 10

a) Monthly repayment = 8000
Principal = 1 000 000
rate = $7.8\% \div 12$ (monthly)
= 0.0065

$$(i) 1000\ 000 (1.0065) \quad (1)$$

$$(ii) A_1 = 1000\ 000 (1.0065) - 8000$$

$$A_2 = A_1 (1.0065) - 8000$$

$$= 1000\ 000 (1.0065)^2 - 8000 (1.0065) - 8000 \quad (1)$$

$$A_n = 1000\ 000 (1.0065)^n - 8000 \left[1.0065^{n-1} + 1.0065^{n-2} + \dots + 1 \right] \quad (1)$$

$$= 1000\ 000 (1.0065)^n - 8000 \left[\frac{a(r^n - 1)}{r - 1} \right] \quad (1)$$

$$a = 1 \quad r = 1.0065 \quad n = n$$

$$= 1000\ 000 (1.0065)^n - 8000 \left[\frac{1.0065^n - 1}{0.0065} \right]$$

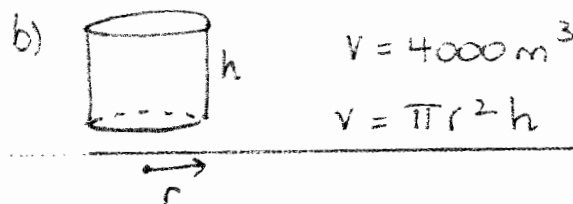
$$(iii) 500\ 000 = 100\ 000 (1.0065)^n - 1230769 [1.0065^n - 1]$$

$$(1) 500\ 000 = 100\ 000 (1.0065)^n - 1230769 (1.0065)^n + 1230769$$

$$230769 (1.0065)^n = 730769$$

$$1.0065^n = 3.1666 \dots$$

$$\log 1.0065^n = \log 3.1666 \dots$$



$$(i) 4000 = \pi r^2 h \quad (1)$$

$$(ii) A = 2\pi r^2 + 2\pi r h \quad h = \frac{4000}{\pi r^2} \quad (1)$$

$$A = 2\pi r^2 + 2\pi r \left[\frac{4000}{\pi r^2} \right]$$

$$= 2\pi r^2 + \frac{8000}{r} \quad (1)$$

$$= 2\pi r^2 + 8000 r^{-1}$$

$$(iii) \frac{dA}{dr} = 4\pi r - 8000 r^{-2} \quad (1)$$

$$= \frac{4\pi r^3 - 8000}{r^2}$$

iv) Min Surface Area $dA/dr = 0$

$$\frac{4\pi r^3 - 8000}{r^2} = 0$$

$$4\pi r^3 = 8000$$

$$r^3 = \frac{8000}{4\pi}$$

$$r = \sqrt[3]{\frac{2000}{\pi}} \quad (1)$$

$$\div 8.6025 \dots$$

$$\div 8.6 \text{ (1dp)}$$

test

r	8	8.6025...	9
$\frac{dA}{dr}$	1	—	1

Must test

% dimensions are (1)