Name: .... Maths Teacher: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics

HSC TASK 1

DECEMBEER 2016

Time allowed: 90 min

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A BOSTES reference sheet is provided at the rear of this Question Booklet, and may be removed at any time.

Section 1 Multiple Choice
Questions 1-10
10 Marks (allow 15 minutes)

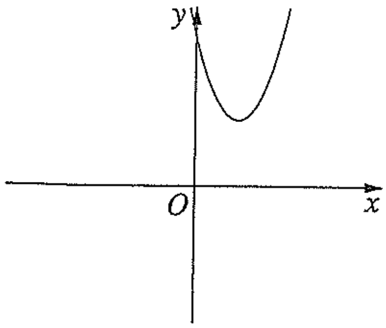
Section II Questions 11-14
60 Marks (allow 1 hour 15 min)

Total Marks 70

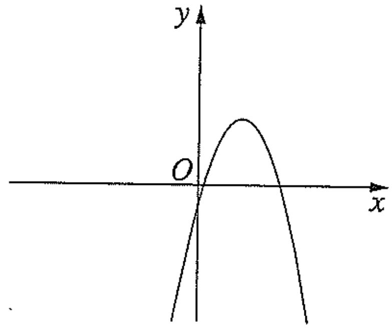
SECTION 1 (answers in answer booklet)

1. Which diagram best shows the graph of the parabola $y = 3 - (x-2)^2$?

(A)

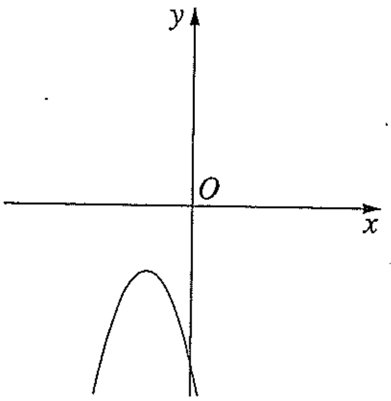


(B)

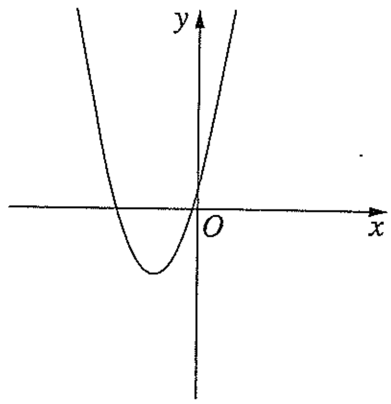


C

(C)



(D)



2. The quadratic equation $3x^2 - x - 4 = 0$ has roots α and β.

What is the value of $\alpha + \beta$?

C

(A) $-\frac{4}{3}$

(B) $-\frac{1}{3}$

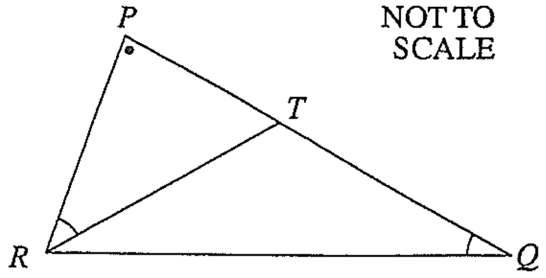
(C) $\frac{1}{3}$

(D) $\frac{4}{3}$

3. Find the values of m for which $24 + 2m - m^2 \le 0$

A) $m \le -4$ or $m \ge 6$ B) $m \le -6$ or $m \ge 4$ C) $-4 \le m \le 6$ D) $-6 \le m \le 4$

4.



ΔPQR is similar to ΔPRT where $\angle PQR = \angle PRT$.

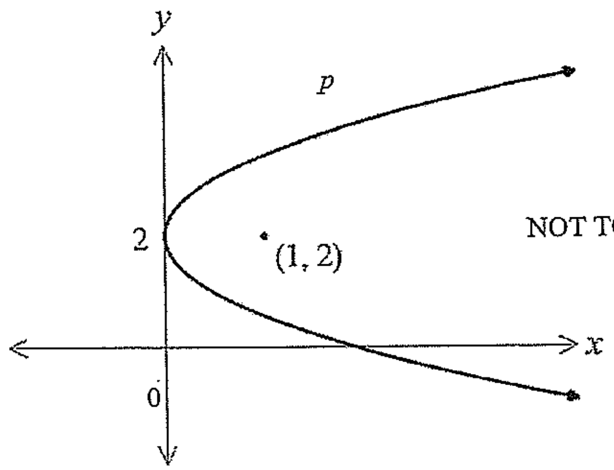
Then $\dfrac{QR}{RT} =$

(A) $\dfrac{PQ}{PT}$ (B) $\dfrac{PR}{PT}$ (C) $\dfrac{PT}{PR}$ (D) $\dfrac{RT}{PT}$

5.

This graph shows the parabola, p, with vertex at 2 on the y-axis and focus $(1, 2)$

C



Which equation represents the parabola, p?

C

(A) $y^2 = 8(x-2)$

(B) $(y-2)^2 = 8x$

(C) $y^2 = 4(x-2)$

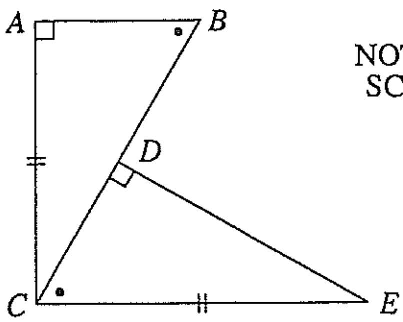
(D) $(y-2)^2 = 4x$

6.

The equation of the directrix of the parabola $y^2 = -8x$ is

A. $x = 2$ B. $y = 2$ C. $x = -2$ D. $y = -2$

7.



NOT TO SCALE

$\angle BAC = \angle CDE = 90°$

$AC = CE$

$\angle ABC = \angle DCE$

Consider the two statements:

I. $\Delta ABC \ ||| \ \Delta DCE$

II. $\Delta ABC \equiv \Delta DCE$

C

Which of the above statements are true?

(A) I only

(B) II only

(C) Both I and II

(D) Neither I nor II

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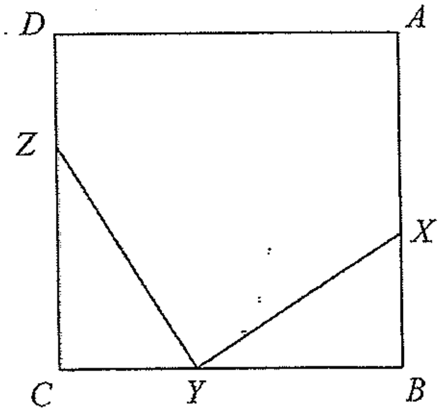
8. In the diagram below, $ABCD$ is a square. X, Y and Z are points on sides AB, BC and CD respectively such that $XB = YC = ZD$.



Which test proves $\Delta BXY \equiv \Delta CYZ$?

(A) AAA

(B) AAS

C

(C) SAS

(D) RHS

---

9. What is the focus of $(x-3)^2 = 8y$?

(A) (0,3)

(B) (3,2)

(C) (2,3)

(D) (3,0)

---

10. For what values of k does the equation $x^2 - 6x - 3k = 0$ have real roots?

A $k \leq -3$ B $k \geq -3$ C $k \leq 3$ D $k \geq 3$

Section II

Question 11 (15 Marks)

Mark

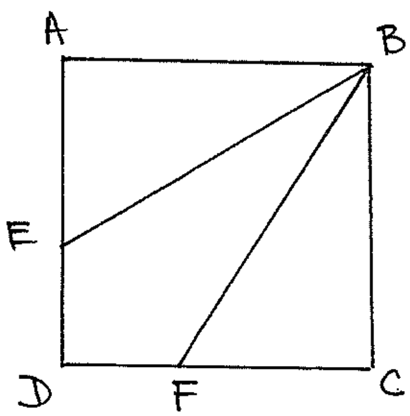
a) Solve $3x^2 - 4x > 0$ — 2

b) One of the roots of $4kx^2 + x - 20k = 0$ is $x = 2$. Find the other root. — 2

c) Form the quadratic equation whose roots are $\frac{-2}{3}$ and $\frac{3}{4}$, expressing your answer without fractions. — 2

d) Find the least value of $3(1-4x)^2 + 5$. — 1

e)



ABCD is a square. E and F are points on AD and DC respectively chosen so that ED = FD.

i) Copy diagram into answer book.

ii) Prove that the triangles BAE and BCF are congruent. — 3

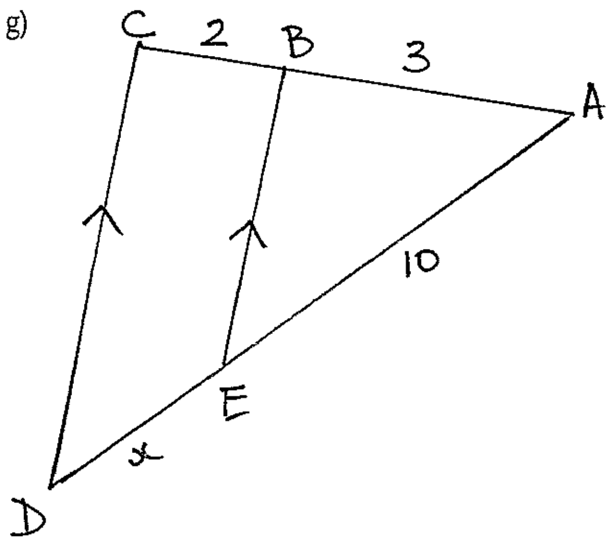
iii) If $\frac{BE}{BA} = \frac{3}{2}$, find the exact value of tan AEB. — 1

f) For the parabola $y = 3x^2 - 12x + 2$, find — 2

i) the equation of its axis of symmetry.

ii) the co-ordinates of its vertex.

g)



Find x (reason required) — 2

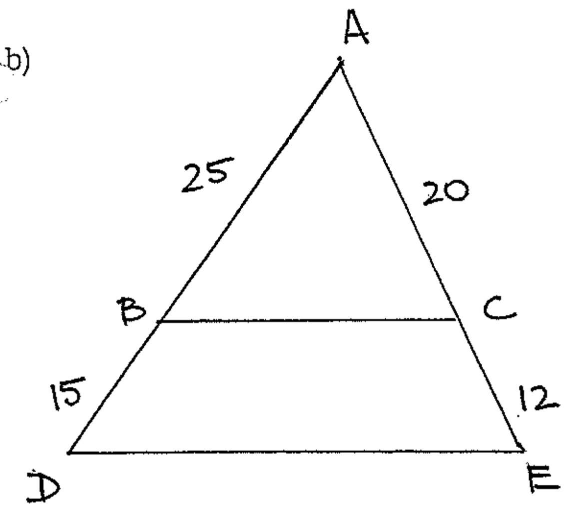
Question 12 (start a new page) (15 Marks)

Mark

a) i) Show that the x-coordinates of the points of intersection of the circle $x^2 + y^2 = 4$ and the line $y = x + 1$ satisfy the equation $2x^2 + 2x - 3 = 0$. 2

ii) Evaluate the discriminant Δ and explain why this shows that there are two points of intersection.

C b)



The diagram shows triangles ABC and ADE.

AB = 25, BD = 15, AC = 20 and CE = 12,

Copy the diagram into your answer booklet.

i) Prove that the triangles are similar . 3

ii) Prove that BC is parallel to DE . 1

iii) If DE = 20, find the length of BC . 1

c) The roots of $4x^2 + 9x + 1 = 0$ are α, β. Write down the value of :

C

i) $\alpha + \beta$ 1

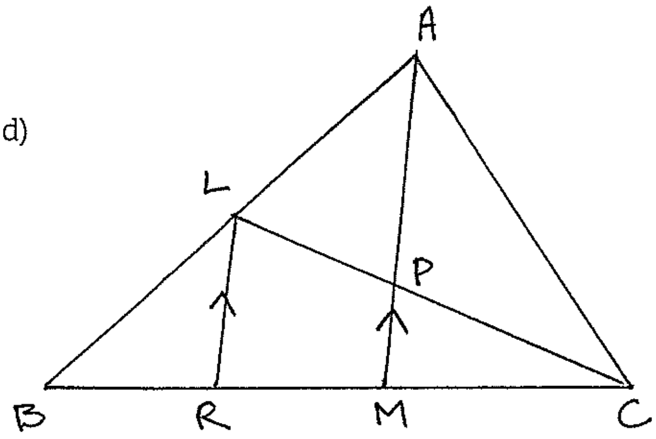
ii) $\alpha\beta$ 1

iii) $\alpha^2 + \beta^2$ 2

iv) $(\alpha - \beta)^2$ 2

Mark

d)



If BM : MC = 10 : 9

and BL : LA = 3 : 2

Redraw this diagram into your answer booklet.

find i) BR : RM 1

ii) MP : RL 1

(reasons <u>not</u> required)

Question 13 (start a new page) (15 Marks)

Mark

a) Use the substitution $u = 3^x$ *to* solve :

$$3^{2x} - 10.3^x + 9 = 0$$ 2

b) Find the equation of the parabola with focus (1, 0) and directrix $x = -1$. 2

c) i) Write down the co-ordinates of the focus of x^2=$8y$. 1

ii) Write down the equation of the directrix of this parabola. 1

iii) Sketch the parabola showing the focus and the directrix. 1

iv) Find the equation of the tangent to this parabola at the point (-8, 8). 2

d) Find the value of k if the roots of the equation $3x^2 + kx + k = 0$ are

i) equal 2

ii) real and different 2

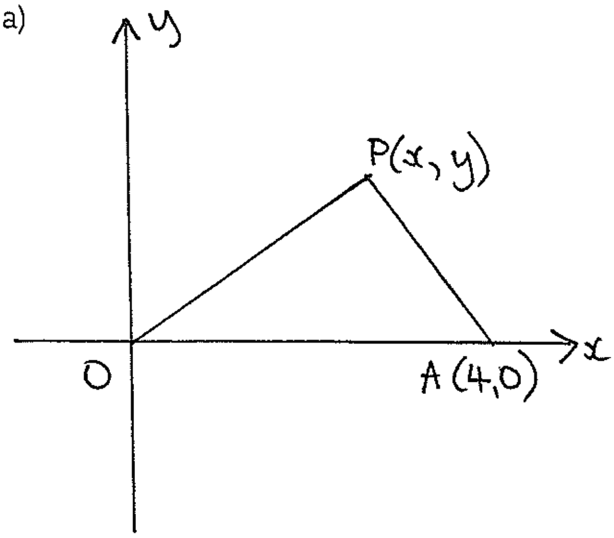
e) For what value of k is

$$3x^2 + (12 - k)x + 12 \text{ always positive?}$$ 2

Question 14 (start a new page) (15 Marks)

Mark

a)



i) In terms of x and y, write down the values of OP^2 and AP^2 1

ii) Using Pythagoras' theorem, or otherwise, show that the locus of all points P which move so that angle OPA is a right angle is $x^2 + y^2 - 4x = 0$ 2

iii) Deduce that the locus of P is a circle and find its centre and radius. 2

b) If $x^2 - 2x + 9 \equiv Ax(x-1) + B(x-1) + C$ 2

find A, B and C.

c) Prove that the roots of $x^2 + (k+1)x + k = 0$ are always real.

Explain your solution fully. 3

d) i) Express the parabola $y^2 + 6y - 8x - 23 = 0$ in the form

$(y-k)^2 = 4a(x-h)$ 2

ii) Sketch the parabola and clearly label the focus, vertex and directrix. 3

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REFERENCE SHEET

– Mathematics –

– Mathematics Extension 1 –

– Mathematics Extension 2 –

Mathematics

Factorisation

$a^2-b^2=(a+b)(a-b)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

Angle sum of a polygon

$S=(n-2)\times 180°$

Equation of a circle

$(x-h)^2+(y-k)^2=r^2$

Trigonometric ratios and identities

$\sin\theta=\dfrac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta=\dfrac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta=\dfrac{\text{opposite side}}{\text{adjacent side}}$

$\operatorname{cosec}\theta=\dfrac{1}{\sin\theta}$

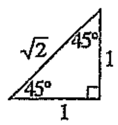
$\sec\theta=\dfrac{1}{\cos\theta}$

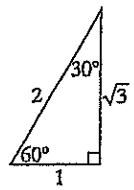
$\tan\theta=\dfrac{\sin\theta}{\cos\theta}$

$\cot\theta=\dfrac{\cos\theta}{\sin\theta}$

$\sin^2\theta+\cos^2\theta=1$

Exact ratios





Sine rule

$\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

Cosine rule

$c^2=a^2+b^2-2ab\cos C$

Area of a triangle

$\text{Area}=\dfrac{1}{2}ab\sin C$

Distance between two points

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Perpendicular distance of a point from a line

$d=\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Slope (gradient) of a line

$m=\dfrac{y_2-y_1}{x_2-x_1}$

Point-gradient form of the equation of a line

$y-y_1=m(x-x_1)$

***n*th term of an arithmetic series**

$T_n=a+(n-1)d$

Sum to *n* terms of an arithmetic series

$S_n=\dfrac{n}{2}[2a+(n-1)d]$ or $S_n=\dfrac{n}{2}(a+l)$

***n*th term of a geometric series**

$T_n=ar^{n-1}$

Sum to *n* terms of a geometric series

$S_n=\dfrac{a(r^n-1)}{r-1}$ or $S_n=\dfrac{a(1-r^n)}{1-r}$

Limiting sum of a geometric series

$S=\dfrac{a}{1-r}$

Compound interest

$A_n=P\left(1+\dfrac{r}{100}\right)^n$

Mathematics (continued)

Differentiation from first principles

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Derivatives

If $y=x^n$, then $\frac{dy}{dx}=nx^{n-1}$

If $y=uv$, then $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$

If $y=\frac{u}{v}$, then $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

If $y=F(u)$, then $\frac{dy}{dx}=F'(u)\frac{du}{dx}$

If $y=e^{f(x)}$, then $\frac{dy}{dx}=f'(x)e^{f(x)}$

If $y=\log_e f(x)=\ln f(x)$, then $\frac{dy}{dx}=\frac{f'(x)}{f(x)}$

If $y=\sin f(x)$, then $\frac{dy}{dx}=f'(x)\cos f(x)$

If $y=\cos f(x)$, then $\frac{dy}{dx}=-f'(x)\sin f(x)$

If $y=\tan f(x)$, then $\frac{dy}{dx}=f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha+\beta=-\frac{b}{a}$ $\qquad$ $\alpha\beta=\frac{c}{a}$

Equation of a parabola

$(x-h)^2=\pm 4a(y-k)$

Integrals

$$\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+C$$

$$\int e^{ax+b}\,dx=\frac{1}{a}e^{ax+b}+C$$

$$\int\frac{f'(x)}{f(x)}dx=\ln|f(x)|+C$$

$$\int\sin(ax+b)\,dx=-\frac{1}{a}\cos(ax+b)+C$$

$$\int\cos(ax+b)\,dx=\frac{1}{a}\sin(ax+b)+C$$

$$\int\sec^2(ax+b)\,dx=\frac{1}{a}\tan(ax+b)+C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)\,dx\approx\frac{b-a}{2}\left[f(a)+f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)\,dx\approx\frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

Logarithms – change of base

$$\log_a x=\frac{\log_b x}{\log_b a}$$

Angle measure

$180°=\pi$ radians

Length of an arc

$l=r\theta$

Area of a sector

$\text{Area}=\frac{1}{2}r^2\theta$

Mathematics Extension 1

Angle sum identities

$\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$

$\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$

$\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

***t* formulae**

If $t=\tan\frac{\theta}{2}$, then

$$\sin\theta=\frac{2t}{1+t^2}$$

$$\cos\theta=\frac{1-t^2}{1+t^2}$$

$$\tan\theta=\frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta=a,\quad \theta=n\pi+(-1)^n\sin^{-1}a$

$\cos\theta=a,\quad \theta=2n\pi\pm\cos^{-1}a$

$\tan\theta=a,\quad \theta=n\pi+\tan^{-1}a$

Division of an interval in a given ratio

$$\left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$

Parametric representation of a parabola

For $x^2=4ay$,

$$x=2at,\quad y=at^2$$

At $(2at,\ at^2)$,

tangent: $y=tx-at^2$

normal: $x+ty=at^3+2at$

At $(x_1,\ y_1)$,

tangent: $xx_1=2a(y+y_1)$

normal: $y-y_1=-\frac{2a}{x_1}(x-x_1)$

Chord of contact from $(x_0,\ y_0)$: $xx_0=2a(y+y_0)$

Acceleration

$$\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Simple harmonic motion

$x=b+a\cos(nt+\alpha)$

$\ddot{x}=-n^2(x-b)$

Further integrals

$$\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\frac{x}{a}+C$$

$$\int\frac{1}{a^2+x^2}dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

Sum and product of roots of a cubic equation

$\alpha+\beta+\gamma=-\frac{b}{a}$

$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}$

$\alpha\beta\gamma=-\frac{d}{a}$

Estimation of roots of a polynomial equation

Newton's method

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

Binomial theorem

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^kb^{n-k}=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

YEAR 11 H.S.C TASK 1 DECEMBER 2016

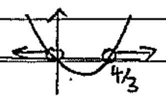
Section I MULTIPLE CHOICE

1	B	6	A
2	C	7	A
3	A	8	C
4	B	9	B
5	D	10	B

Section II QUESTION 11

a) $3x^2-4x>0$

$x(3x-4)>0$



$\therefore x>\frac{4}{3}$ $x<0$

b) sub $x=2$ into $4kx^2+x-20k=0$

$16k+2-20k=0$

$4k=2$

$k=\frac{1}{2}$

$\therefore 2x^2+x-10=0$

$(2x+5)(x-2)=0$

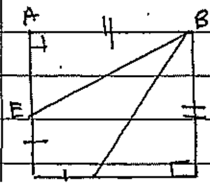
$\therefore$ roots $x=2$ and $x=\frac{-5}{2}$

c) $x^2-x(\text{sum})+(\text{product})=0$

$x^2-\frac{1}{12}x-\frac{1}{2}=0$

$\therefore$ equation $12x^2-x-6=0$

d) Least value 5

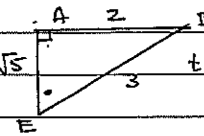
e) 

ii) In Δs BAE, BCF

$\hat{BAE}=\hat{BCF}$ (90°, angles of square)

$AB=BC$ (sides of square)

$AE=FC$ (since $ED=DF$ taken from equal sides)

iii) 

$\tan AEB=\frac{2}{\sqrt{5}}$

f) $y=3x^2-12x+2$

i) $x=\frac{12}{6}$ $\therefore x=2$

ii) $V(2,-10)$

g) $\frac{3}{2}=\frac{10}{x}$ (ratio of intercepts parallel lines)

$3x=20$

$\therefore x=6\frac{2}{3}$

Question 12

a) i) $x^2+y^2=4$ $y=x+1$

$x^2+(x+1)^2=4$

$x^2+x^2+2x+1=4$

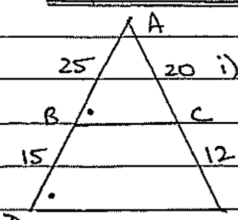
$2x^2+2x-3=0$

ii) $\Delta=4-4.2\times-3$

$\Delta=28$

since $\Delta>0$ $\therefore$ 2 real solutions

$\therefore$ 2 pts of intersection

b) 

i) In Δ's ABC, ADE

$\hat{A}$ is common

$\frac{AB}{AD}=\frac{25}{40}=\frac{5}{8}$

$\frac{AC}{AE}=\frac{20}{32}=\frac{5}{8}$

$\therefore \Delta ABC \parallel\!\!\!| \Delta ADE$ (included angle equal and ...)

ii) $\therefore \hat{ABC}=\hat{ADE}$ (corresponding angles in similar triangles)

$\therefore BC \parallel DE$ (corresponding angles are equal)

iii) $\frac{BC}{20}=\frac{25}{40}$

$\therefore BC=12.5$

c) $4x^2+9x+1=0$

i) $\alpha+\beta=-\frac{9}{4}$

ii) $\alpha\beta=\frac{1}{4}$

iii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\frac{81}{16}-\frac{2}{4}$

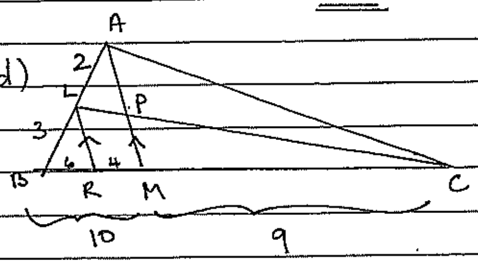
$=\frac{73}{16}$

iv) $(\alpha-\beta)^2=\alpha^2-2\alpha\beta+\beta^2$

$=(\alpha^2+\beta^2)-2\alpha\beta$

$=\frac{73}{16}-2\cdot\frac{1}{4}$

$=\frac{65}{16}$

d) 

i) $BR:RM=3:2$

ii) $MP:RL=9:13$

Question 13

a) $(3^x)^2-10.3^x+9=0$

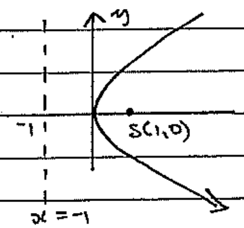
$u^2-10u+9=0$

$(u-9)(u-1)=0$

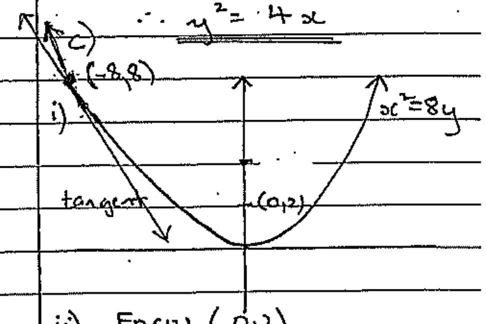
$\therefore u=9, 1$

$\therefore 3^x=9$ $3^x=1$

$\therefore x=2$ $x=0$

b) 

$\therefore y^2=4x$

c) 

ii) Focus $(0,2)$

iii) Directrix $y=-2$

iv) $y=\frac{x^2}{8}$

$\frac{dy}{dx}=\frac{2x}{8}=\frac{x}{4}$

$\therefore m_T=\frac{-8}{4}=-2$

$\therefore$ eqn of tangent

$y-8=-2(x--8)$

$y-8=-2x-16$

$2x+y+8=0$

d) i) roots equal

$\therefore \Delta = 0$

$k^2 - 4.3.k = 0$

$k^2 - 12k = 0$

$k(k-12) = 0$

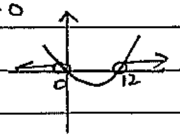
$\therefore k = 0 \quad k = 12$

ii) real, different

$\Delta > 0$

$k(k-12) > 0$

$\therefore k < 0, \ k > 12$



c)

$3x^2 + (12-k)x + 12$

$\therefore$ find k if +ve definite

$\therefore a > 0 \quad \Delta < 0$

$a = 3 \quad \therefore a > 0$

$\Delta = (12-k)^2 - 4.3.12$

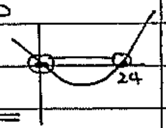
$\Delta = \cancel{144} - 24k + k^2 - \cancel{144}$

$\Delta = k^2 - 24k$

$k^2 - 24k < 0$

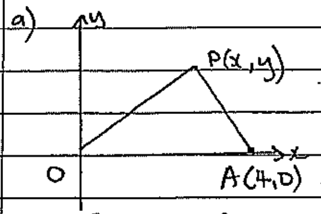
$k(k-24) < 0$

$\therefore 0 < k < 24$



Question 14

a)



i) $OP^2 = x^2 + y^2$

$AP^2 = (x-4)^2 + y^2$

ii) $x^2 + y^2 + (x-4)^2 + y^2 = 4^2$

$x^2 + y^2 + x^2 - 8x + 16 + y^2 = 16$

$2x^2 + 2y^2 - 8x = 0$

$x^2 + y^2 - 4x = 0$

iii) $x^2 - 4x + y^2 = 0$

$x^2 - 4x + 4 + y^2 = 4$

$(x-2)^2 + y^2 = 4$

$\therefore$ circle centre $(2,0)$

Radius 2

b) $x^2 - 2x + 9 \equiv Ax(x-1) + B(x-1)$

$x^2 - 2x + 9 = Ax^2 - Ax + Bx - B + C$

$= Ax^2 + x(-A+B) - B + C$

$\therefore A = 1$

$-A + B = -2$

$-1 + B = -2$

$\therefore B = -1$

$-B + C = 9$

$1 + C = 9$

$C = 8$

$\therefore A = 1 \quad B = -1 \quad C = 8$

c) $x^2 + (k+1)x + k = 0$

$\Delta = (k+1)^2 - 4.1.k$

$= k^2 + 2k + 1 - 4k$

$= k^2 - 2k + 1$

$\Delta = (k-1)^2$

since $(k-1)^2 \geq 0$ for all value of k

$\therefore \Delta \geq 0$ roots always real

d) $y^2 + 6y - 8x - 23 = 0$

i)

$(y^2 + 6y + 9) = 8x + 23 + 9$

$(y+3)^2 = 8x + 32$

$(y+3)^2 = 8(x+4)$

ii) Vertex $(-4,-3)$

Focal length $a = 2$