

Name: _____

Teacher/Class: _____

SYDNEY TECHNICAL HIGH SCHOOL



HSC ASSESSMENT TASK 1

DECEMBER 2008

MATHEMATICS

Time Allowed: 70 minutes

Instructions:

- Write your name and class at the top of each page.
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start **each** question on a **new** page.
- Diagrams unless otherwise stated are not to scale.

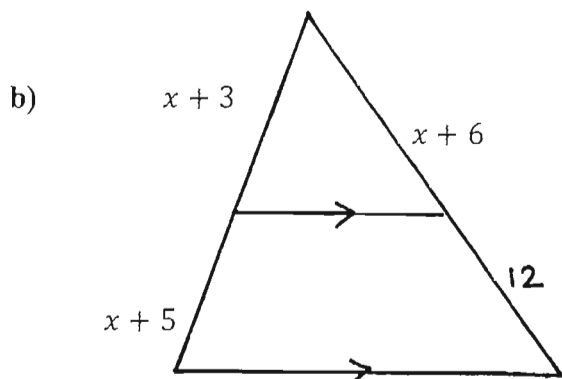
Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	Question 8	TOTAL
/6	/6	/6	/6	/7	/7	/6	/7	/51

QUESTION 1

(6 Marks)

MARKS

- a) i) Sketch $y = x^2 - 4x - 5$ 1
 ii) Hence or otherwise solve $x^2 - 4x - 5 > 0$ 1



Find 2 possible values for x
 (no reason required)

2

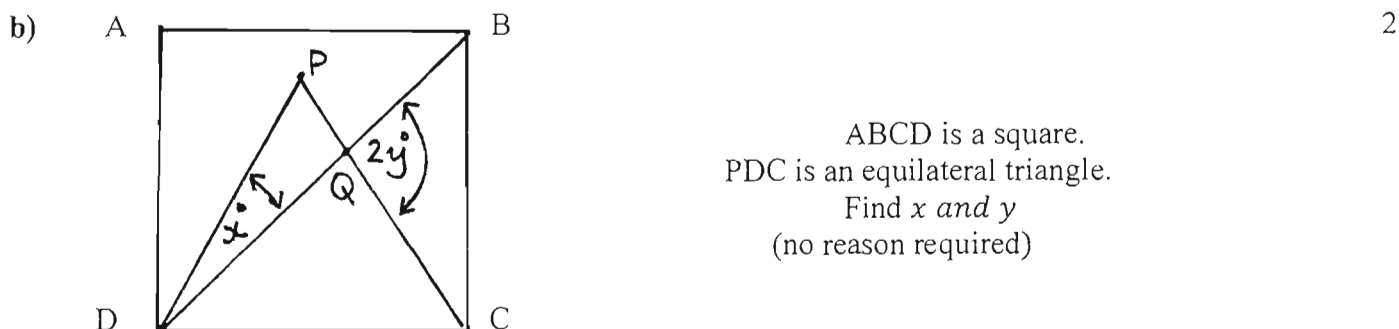
- c) Find k if the equation
 $x^2 - (k - 5)x + (k - 7) = 0$ has one root equal to -2 . 2

QUESTION 2

(Start a new page)

(6 marks)

- a) Solve $x^4 - 5x^2 + 4 = 0$ 2



- c) Find the centre and the radius of the circle $x^2 + y^2 + 8x - 2y - 8 = 0$ 2

QUESTION 3

(Start a new page)

(6 marks)

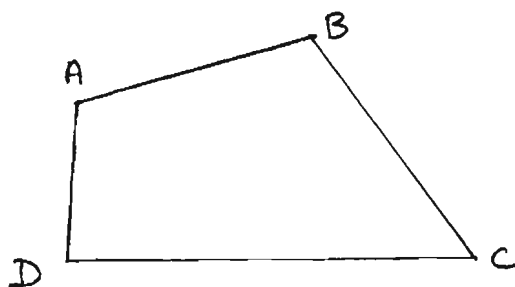
- a) The first 3 terms of a sequence are 5, 9 and 13 1
- i) Find the 46th term
- ii) Is 147 a term of the sequence? 2
- Use appropriate working to explain your answer
- b) Evaluate $\sum_{k=3}^{12} (3k-1)$ 2
- c) Find k if the product of the roots of $x^2 - (k-4)x + 3k = 0$ is 18. 1

QUESTION 4

(Start a new page)

(6 marks)

- a) ABCD is a quadrilateral in which the diagonal BD bisects the angles at B and D.



- i) Copy this quadrilateral onto your answer sheet and add the additional information. 3
- ii) By completing an appropriate congruence proof explain why $AB = BC$ and $AD = DC$. 1
- iii) What type of quadrilateral is ABCD? 1

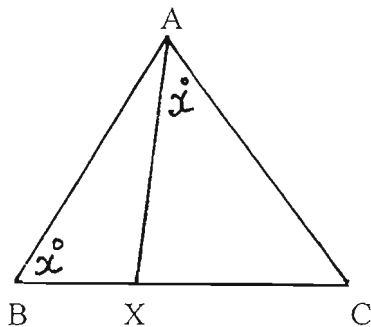
- b) Find the equation of the parabola with vertex $(-2, 3)$, axis parallel to the y axis and passing through the point $(-6, -5)$. 2

QUESTION 5

(Start a new page) (7 marks)

- a) The third term of a geometric series is $\frac{3}{4}$ and the seventh term is 12. Find the first three terms. 3

- b) 3



If $\angle CAX = \angle ABC = x^\circ$

i) Prove $\triangle CAX \parallel \triangle CBA$

ii) Hence explain why $CA^2 = CX \cdot CB$ 1

QUESTION 6

(Start a new page)

(7 marks)

- a) i) Find the first and last multiples of seven between 400 and 600. 1
 ii) Hence find the sum of all the multiples of seven between 400 and 600. 2
- b) i) Find the equation of the directrix of the parabola $x^2 = -12y$ 1
 ii) Find the equation of the tangent at the point $(-6, -3)$ on the parabola $x^2 = -12y$ 2
 iii) If this tangent meets the directrix at T, find the co-ordinates of T. 1

QUESTION 7

(Start a new page)

(6 marks)

MARKS

- a) For what values of k will the quadratic equation $(k - 2)x^2 - 2kx - 1 = 0$ have real and different roots. 3
- b) Find the sum of the powers of 2 from 1 to 1024 inclusive.
(ie $2^0 + 2^1 + 2^2 + \dots$) 3

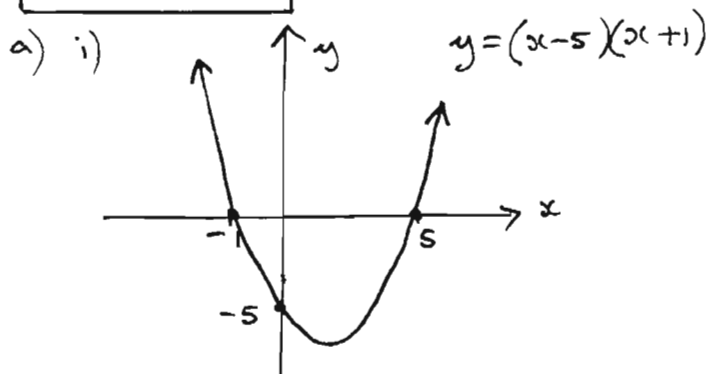
QUESTION 8

(Start a new page)

(7 marks)

- a) i) Express $0.\overline{34}$ as an infinite series by writing the first 4 terms. 1
- ii) By using the sum to infinity formula of the above series write $0.\overline{34}$ as a fraction. 2
- b) i) Let A and B be the fixed points $(-1, 0)$ and $(2, 0)$ and P be the variable point (x, y) .
If $PA = 2PB$ find the locus of $P(x, y)$. 3
- ii) Name the geometric shape traced out by the locus of $P(x, y)$. 1

Question 1



ii) $x > 5, x < -1$

b) $\frac{x+3}{x+5} = \frac{x+6}{12}$

$$12x + 36 = x^2 + 11x + 30$$

$$0 = x^2 - x - 6$$

$$(x-3)(x+2) = 0$$

$x = 3, -2$

c) $4 - (k-5)x - 2 + (k-7) = 0$

$$4 + 2k - 10 + k - 7 = 0$$

$$3k - 13 = 0$$

$k = 13/3$

Question 2

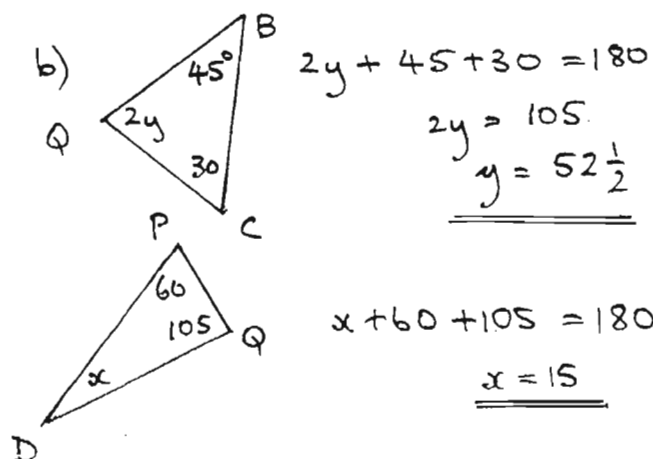
i) Let $u = x^2 \therefore u^2 - 5u + 4 = 0$

$$(u-4)(u-1) = 0$$

$$u = 4, u = 1$$

$$\therefore u^2 = 4 \quad u^2 = 1$$

$u = \pm 2 \quad u = \pm 1$



c) $x^2 + 8x + 16 + y^2 - 2y + 1 = 8 + 1$
 $(x+4)^2 + (y-1)^2 = 25$
Centre $(-4, 1)$ Radius 5

Question 3

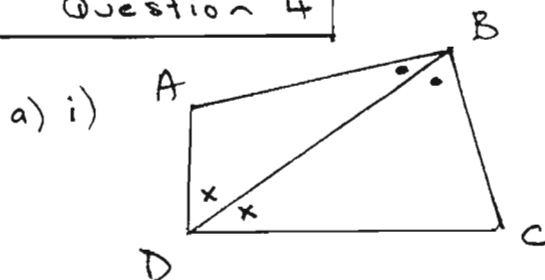
a) 5, 9, 13, ...
 $a = 5 \quad d = 4 \quad \text{A.P.}$
 i) $T_{46} = 5 + (46-1) \times 4$
 $T_{46} = 185$

ii) $147 = 5 + (n-1) \times 4$
 $142 = 4n - 4$
 $146 = 4n$
 $36\frac{1}{2} = n$
 n not a +ve integer \therefore
 147 not a term

b) $T_1 \quad T_2 \quad T_3 \quad T_{10}$
 8, 11, 14, ... 35
 $k=3 \quad k=4 \quad k=5 \quad k=12$
 $S_{10} = \frac{10}{2} (8 + 35)$
 $S_{10} = 215$

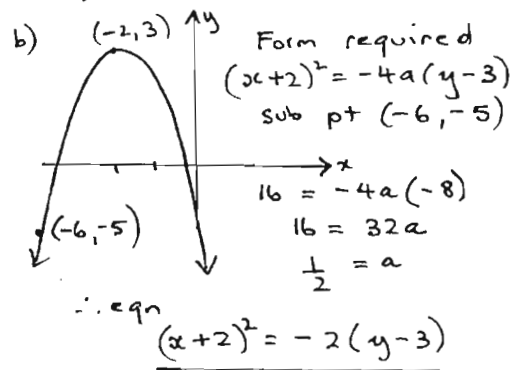
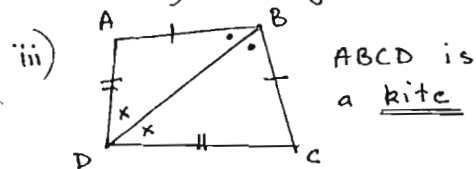
c) $\frac{c}{a} = 18 \therefore \frac{3k}{1} = 18$
 $k = 6$

Question 4



ii) In Δ 's ABD, CBD
 $\hat{A}BD = \hat{C}BD$ (DB bisects $\hat{A}BC$)
 $\hat{A}DB = \hat{C}DB$ (" "
 DB is common
 $\therefore \Delta ABD \equiv \Delta CBD$ (AAS)

$AB = BC$
 $AD = DC$ } corresponding sides
in congruent triangles



Question 5

a) $T_3 = \frac{3}{4}$ $T_7 = 12$

$$ar^2 = \frac{3}{4} - \textcircled{1} \quad ar^6 = 12 - \textcircled{2}$$

$$\textcircled{2} \div \textcircled{1} \quad \frac{ar^6}{ar^2} = \frac{12}{3/4}$$

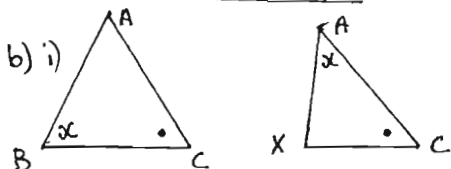
$$r^4 = 16$$

$$r = \pm 2$$

$$r = 2 \quad a = 3/16$$

GP: $\frac{3}{16}, \frac{3}{8}, \frac{3}{4}, \dots$

or $\frac{3}{16}, -\frac{3}{8}, \frac{3}{4}, \dots$



In Δ 's CAX, CBA
 \hat{C} is common
 $\hat{CAX} = \hat{ABC} = x$ (given)
 $\therefore \Delta CAX \parallel \Delta CBA$ (equiangular)

ii) $\frac{CA}{CX} = \frac{CB}{CA}$ (corresp sides
similar Δ 's)

$(CA)^2 = CX \cdot CB$

Question 6

a)

i) 406, \dots , 595

ii) $T_1 = 406$ $d = 7$

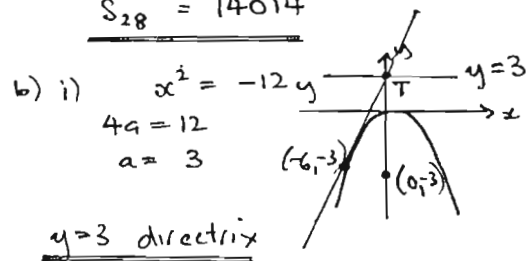
$$595 = 406 + (n-1) \times 7$$

$$189 = 7n - 7$$

$$n = 28 \quad \therefore T_{28} = 595 = l$$

$$S_{28} = \frac{28}{2} (406 + 595)$$

$S_{28} = 14014$



ii) $y = \frac{x^2}{-12}$

$\frac{dy}{dx} = \frac{2x}{-12} = \frac{x}{-6}$

$m = 1$ at $(-6, -3)$

eqn tang: $y + 3 = 1(x + 6)$
 $0 = x - y + 3$

iii) $T(0, 3)$

Question 7

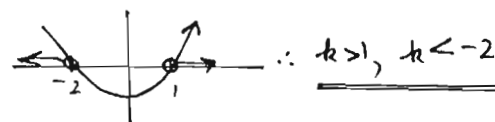
a) Real diff roots if $\Delta > 0$
 $\Delta = (-2k)^2 - 4(k-2)x - 1$

$$\Delta = 4k^2 + 4k - 8$$

$$\therefore 4k^2 + 4k - 8 > 0$$

$$k^2 + k - 2 > 0$$

$$(k+2)(k-1) > 0$$



b) $2^0 + 2^1 + 2^2 + \dots + 2^{10}$
GP $a = 1$ $r = 2$ $n = 11$

$S_{11} = \frac{1(2^{11} - 1)}{2 - 1}$

$S_{11} = 2047$

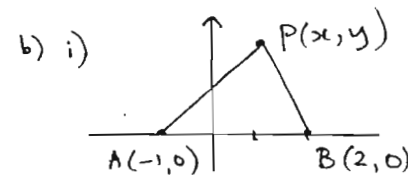
Question 8

a) i) $\frac{3}{10} + \left[\frac{4}{100} + \frac{4}{1000} + \frac{4}{10000} + \dots \right]$
G.P $a = \frac{4}{100}$ $r = \frac{1}{10}$

ii) $0.3\dot{4} = \frac{3}{10} + \frac{4/100}{1 - 1/10}$

$$= \frac{3}{10} + \frac{2}{45}$$

$$= \frac{31}{90}$$



$PA = 2PB$

$\sqrt{(x+1)^2 + (y-0)^2} = 2\sqrt{(x-2)^2 + (y-0)^2}$

$x^2 + 2x + 1 + y^2 = 4(x^2 - 4x + 4 + y^2)$

$x^2 + 2x + 1 + y^2 = 4x^2 - 16x + 16 + 4y^2$

$0 = 3x^2 - 18x + 3y^2 + 15$

$0 = x^2 - 6x + y^2 + 5$

ii) circle