SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

Year 11

2 Unit

April 2007

Common Test

Time Allowed: 70 mins

Name   :

Teacher :

Instructions:

- Begin each question on a new page
- Marks shown are approximate and may be varied
- Show necessary working
- Full marks may not be awarded if working is poorly set out or difficult to read
- Write all answers in simplest form

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
/8	/8	/8	/9	/8	/8	/8	/8	/65

Question 1

a)   Evaluate $\dfrac{2}{(1.11)^5 - 1}$ correct to 3 decimal places   2

b)   Express 68272 correct to 2 significant figures   1

c)   Write $\dfrac{x}{y} - 2$ as a single fraction   1

d)   Given that the surface area of an open cylinder is given by

$A = 2\pi r h + \pi r^2$

find the value of h when $A = 250$ and $r = 6$ correct to 1 decimal place   2

e)   Subtract $3x^2 - 1$ from $x^3 - 2x^2 + 3$ 2

Question 2

a)   Find the exact value of $x^2 - 4x$ if $x = 4\sqrt{3}$ 2

b)   Rationalise the denominator of $\dfrac{\sqrt{3}}{6 - \sqrt{3}}$ 2

c)   Simplify $\sqrt{\dfrac{a^3 b^7}{ab^3}}$ 2

d)   State the range of the function $y = x^2 - 1$ 1

e)   The diagram shows part of a function $y = f(x)$ 1

Copy this diagram onto your answer sheet.

Complete the graph of $y = f(x)$ given that it is an odd function.
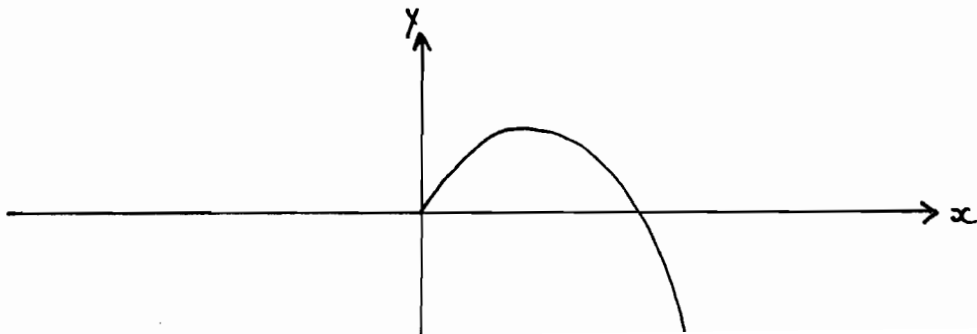
Question 3

a) Factorise fully

   (i) $x^4 - x^2$ 2

   (ii) $3a^2 + 2a - 8$ 2

   (iii) $xy + 6x - y - 6$ 2

b) Simplify $\dfrac{a^3 + 1}{a^2 - a + 1}$ 2

Question 4

a) Solve the following

   (i) $\dfrac{x-5}{4} + 3 = \dfrac{5x}{3}$ 2

   (ii) $4x^2 - 4x + 1 = 0$ 2

   (iii) $|x + 2| \le 2$ 2

   (iv) $|x - 1| = 3x - 1$ 3

Question 5

a) Sketch the following functions on separate number planes. Use a ruler to draw the axes. Label any important points.

   (i) $y = \dfrac{2}{x}$ 1

   (ii) $y = 3 - x$ 2

   (iii) $y = 9 - x^2$ 2

b) Consider the function $y = \sqrt{16 - x^2}$

   (i) Explain why the domain of this function is restricted   1

   (ii) Find this domain   1

   (iii) Sketch the function on a number plane   1

Question 6

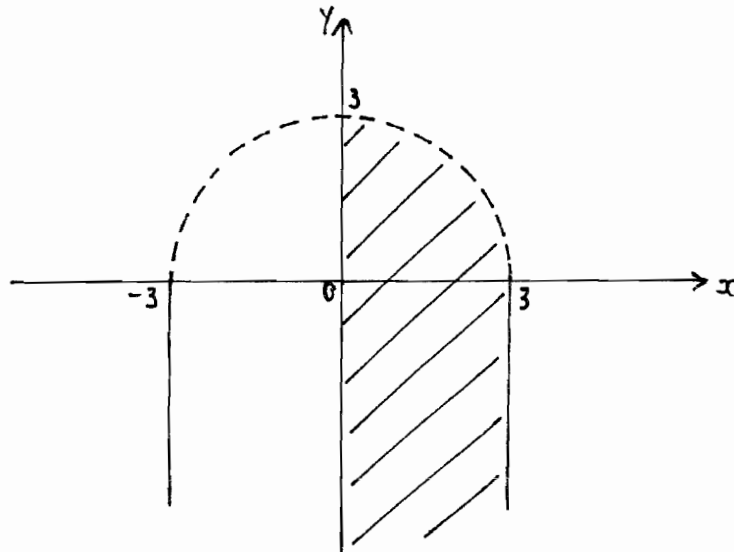   a) (i) Solve simultaneously: 2

$$2x + 3y = 4$$

$$3x + 6y = 2$$

       (ii) What does this solution represent in relation to the graphs of

        $2x + 3y = 4$ and $3x + 6y = 2$?   1

    b)   State the two inequalities which represent the shaded region below: 2

    c)   Express $0.3\dot{6}$ as a simple fraction.   3

Question 7

    a)   Find the values of a and b if $a + \sqrt{b} = 5 + 3\sqrt{6}$ 2

    b)   Simplify fully $\sqrt{98} \times \sqrt{48}$ 2

    c)   Solve $(x - 2)^2 = 3$ leaving your answer as a surd.   2

    d)   The function $f(x)$ is defined as   2

$$f(x) = \begin{cases} x^2 + 1 & if \ x > 3 \\ 3x & if \ -2 \le x \le 3 \\ 2 & if \ x < -2 \end{cases}$$

       Find $f(-3) + f(4) - f(3)$

Question 8

a) Simplify $\dfrac{2}{x-y} + \dfrac{y}{x^2 - y^2}$ 2

b) Simplify $f(x) + f(-x)$ if $f(x)$ is an odd function   1

c) Determine if the following function is odd, even or neither.   2

     Show all working. $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$.

d) (i) Expand $(x - \dfrac{1}{x})^2$ 1

     (ii) If $x - \dfrac{1}{x} = 3$, find the value of $x^2 + \dfrac{1}{x^2}$ without solving for x 2

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a) $\dfrac{2(x+y)}{x^2 - y^2} \quad i \quad y \qquad = \dfrac{2x + 3y}{}$

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b) $f(-x) = \dfrac{(-x)^2 + 1}{(-x)^2 - i} = \dfrac{x^2 + 1}{x^2 - 1} = f(x) \qquad \therefore \text{ even}$

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c)

Year 11 2 Unit Ass. Task   April 2007

Question 1

a) 2.919 ②

b) 68000 or 6.8×10^4 ①

c) $\dfrac{x}{y} - 2$

$\dfrac{x}{y} - \dfrac{2y}{y}$

$= \dfrac{x - 2y}{y}$ ①

d) $A = 2\pi rh + \pi r^2$

$250 = 2 \times \pi \times 6 \times h + \pi \times 6^2$ ①

$h = 3.6$ ①

e) $x^3 - 2x^2 + 3 - (3x^2 - 1)$ ①

$x^3 - 5x^2 + 4$ ①

Question 2

a) $x^2 - 4x$

$(4\sqrt{3})^2 - 4(4\sqrt{3})$ ①

$16 \times 3 - 16\sqrt{3}$

$48 - 16\sqrt{3}$ ①

b) $\dfrac{\sqrt{3}}{6 - \sqrt{3}} \times \dfrac{6 + \sqrt{3}}{6 + \sqrt{3}}$
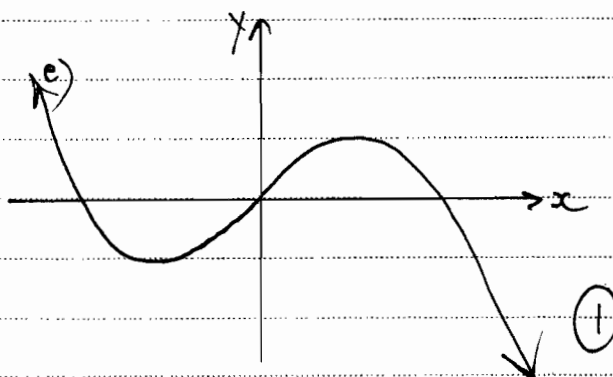
$\dfrac{6\sqrt{3} + 3}{33}$ ①

$\dfrac{2\sqrt{3} + 1}{11}$ ①

c) $\dfrac{\sqrt{a^3 b^7}}{\sqrt{a b^3}}$

$\sqrt{a^2 b^4}$ ①

$= ab^2$ ①

d) $y = x^2 - 1$

Range: All real $y \geqslant -1$ ①

e)

Question 3

a)(i) $x^4 - x^2$

$x^2(x^2 - 1)$ ①

$x^2(x-1)(x+1)$ ①

(ii) $3a^2 + 2a - 8$

$(3a - 4)(a + 2)$ ②

(iii) $xy + 6x - y - 6$

$x(y + 6) - 1(y + 6)$ ①

$(x - 1)(y + 6)$ ①
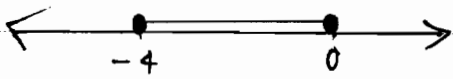
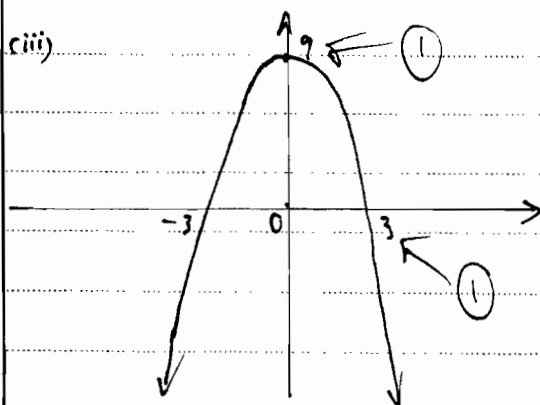b) $a^3 + 1$ $= (a + 1)(a^2 - a + 1)$ ① $= (a + 1)$ ①

Question 4

a)

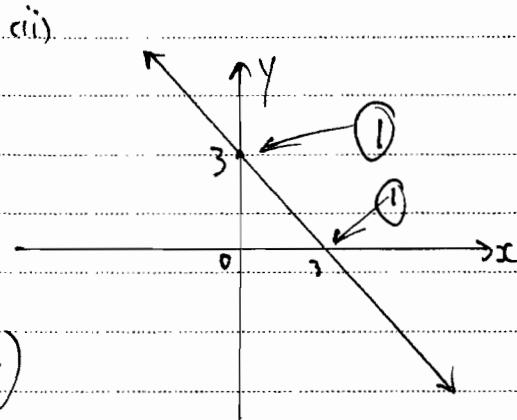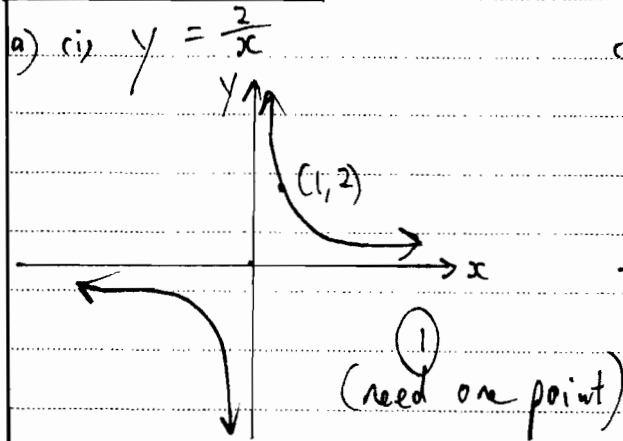(i) $\dfrac{x-5}{4} + 3 = \dfrac{5x}{3}$

$3(x-5) + 36 = 20x$

$3x - 15 + 36 = 20x$ ①

$21 = 17x$

$\underline{x = \frac{21}{17}}$ ①

(ii) $4x^2 - 4x + 1 = 0$

$(2x - 1)^2 = 0$ ①

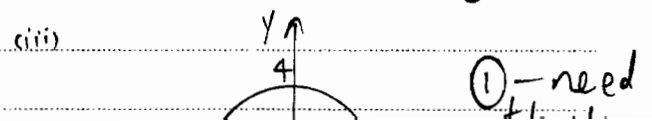$\underline{x = \frac{1}{2}}$ ①

(iii) $|x + 2| \leq 2$

$x + 2 \leq 2$ or $x + 2 \geq -2$ ①

$x \leq 0$ or $x \geq -4$

$\underline{-4 \leq x \leq 0}$ ①

(iv) $|x - 1| = 3x - 1$

$x - 1 = 3x - 1$ or $x - 1 = -(3x - 1)$

$0 = 2x$ $x - 1 = -3 \sim +$

$\underline{x = 0}$ ① or $4x = 2$

$\underline{x = \frac{1}{2}}$ ①

$\underline{\text{Check!} \quad x = \frac{1}{2} \text{ only}}$ ①

Question 5

a) (i) $y = \dfrac{2}{x}$

$(1, 2)$

①

(need one point)

(ii)

① ①

(iii)

① ①

b) (i) You can't $\sqrt{}$ a negative ①

(ii) $16 - x^2 \geq 0$

$(4 - x)(4 + x) \geq 0$

$\underline{-4 \leq x \leq 4}$ ①

(iii)

① — need

Question 6

a) ci) $2x + 3y = 4$ ①
$3x + 6y = 2$ ②

$2 \times ① - ②$
$4x + 6y = 8 \ -$
$3x + 6y = 2$
$x \qquad = 6$

$\therefore \ 2 \times 6 + 3y = 4$
$\qquad 3y = -8$

$\underline{x = 6, \qquad y = \frac{-8}{3}}$
    ①   ①

(ii) <u>Point of intersection</u>
                             ①

b) $x \geq 0$ <u>and</u> ①
$\underline{y < \sqrt{9 - x^2}}$
        ①

c) Let $x = 0.3666...$
$\therefore \ 10x = 3.6666... \ - \ ①$
$\therefore \ 9x = 3.3$
$\qquad x = \frac{3.3}{9}$
$\qquad = \frac{33}{90}$ ①
$\qquad \underline{x = \frac{11}{30}}$ ①

Question 7

a) $a + \sqrt{b} = 5 + 3\sqrt{6}$
$a + \sqrt{b} = 5 + \sqrt{54}$
$\underline{\therefore a = 5, \quad b = 54}$
    ①   ①

b) $\sqrt{28} \times \sqrt{48}$
$= 7\sqrt{2} \times 4\sqrt{3}$ ①
$\underline{= 28\sqrt{6}}$
     ①

c) $(x-2)^2 = 3$
$x - 2 = \pm\sqrt{3}$ (
$\underline{x = 2 \pm \sqrt{3}}$
        ①

d) $f(-3) + f(4) - f(3)$
$\quad 2 \ + 4^2 + 1 \ - 3 \times 3$ ①
$\quad 2 \ + 17 \ - 9$
$\underline{= 10}$
  ①

Question 8

a) $\dfrac{2}{x-y} + \dfrac{y}{x^2-y^2}$

$\dfrac{2}{x-y} + \dfrac{y}{(x-y)(x+y)}$

$\dfrac{2(x+y)}{(x-y)(x+y)} + \dfrac{y}{(x-y)(x+y)}$ ①

$= \dfrac{2x+3y}{(x-y)(x+y)}$ or $\dfrac{2x+3y}{x^2-y^2}$ ①

b) $f(x) + f(-x)$

$= 0$ ①

c) $f(x) = \dfrac{x^2+1}{x^2-1}$

odd if $f(-x) = -f(x)$ even if $f(x) = f(-x)$ ②

$\dfrac{(-x)^2+1}{(-x)^2-1} = -\left(\dfrac{x^2+1}{x^2-1}\right)$ $\dfrac{x^2+1}{x^2-1} = \dfrac{(-x)^2+1}{(-x)^2-1}$

$\dfrac{x^2+1}{x^2-1} \neq -\dfrac{x^2+1}{x^2-1}$ $\dfrac{x^2+1}{x^2-1} = \dfrac{x^2+1}{x^2-1}$ ① ✓

$\therefore$ Not odd $\therefore$ Even

d) (i) $\left(x - \dfrac{1}{x}\right)^2$

$= x^2 - 1 - 1 + \dfrac{1}{x^2}$

$= x^2 + \dfrac{1}{x^2} - 2$ ①

(ii) $\left(x - \dfrac{1}{x}\right)^2 = x^2 + \dfrac{1}{x^2} - 2$

$\therefore x^2 + \dfrac{1}{x^2} = \left(x - \dfrac{1}{x}\right)^2 + 2$ ①

$= 3^2 + 2$

$x^2 + \dfrac{1}{x^2} = 11$ ①