



HSC Assessment Task 2

March 2011

Mathematics

Time allowed – 70 minutes

Instructions

- Approved calculators may be used.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks awarded are shown on each question.
- Total marks – 52
- Attempt all questions.
- Start each question on a new page.

Q1 /10	Q2 /10	Q3 /10	Q4 /12	Q5 10	Total /52

Question 1

10 Marks

- a) Find the 16th term of the series $6 + 10 + 14 + \dots$

(2)

- b) Evaluate $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$

(2)

- c) For all x in the domain $0 < x < 4$, a function $g(x)$ satisfies $g'(x) < 0$ and $g''(x) < 0$. Sketch a possible graph of $y = g(x)$ in this domain.

(2)

- d) A closed box with a square base is to be made so that its volume is 100 cm^3 .

- i) Show that the surface area of the box is given by $A = 2x^2 + \frac{400}{x}$. (1)

- ii) Find the minimum possible surface area of the box, to one decimal place. (3)

Question 2**10 marks**

- a) The population P , of birds in a certain area is increasing over time t , but the rate of population growth is slowing.

Describe the sign of $\frac{dP}{dt}$ and $\frac{d^2P}{dt^2}$. (2)

- b) The tangent to a curve at point N has equation $5x - y - 1 = 0$.

i) If $\frac{dy}{dx} = 4x - 3$, for the curve, find the coordinates of N . (2)

ii) Find the equation of the curve (2)

- c) Which term of the series $\frac{7}{8} + \frac{1}{4} + \frac{1}{14} + \dots$ is equal to $\frac{16}{117649}$? (2)

- d) Find $\int \frac{3x^5 + 2x^2 - x}{x} dx$ (1)

- e) Find $\int x(\sqrt{x} + 1) dx$ (1)

Question 3**10 Marks**

- a) For the function $y = x^6 - 6x^4$
- i) Find the x coordinates of the points where the curve crosses the axes. (1)
 - ii) Determine the existence of any points of inflexion and state the coordinates. (2)
 - iii) Sketch the graph of $y = x^6 - 6x^4$ indicating clearly the intercepts, stationary points and points of inflexion. (3)
- b) For the curve $y = 2x^3 - 12x^2 - 5x - 3$ find the equation of the normal at the point of inflexion. (2)
- c) The sum to infinity of the series $x + \frac{2x}{3} + \frac{4x}{9} + \dots$ is 15. Evaluate x . (2)

Question 4

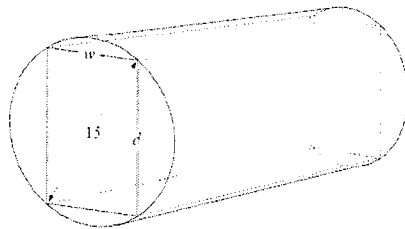
12 Marks

- a) Michael is in training for a local marathon. He is training by completing practice runs over the marathon course. So far he has completed three practice runs with times shown below.

Week 1	Week 2	Week 3
3 hours	2 hours 51 minutes	2 hours 42 minutes 27 seconds

- Show that these times form a geometric series with a common ratio $r = 0.95$.
(1)
- If this series continues, what would be his expected time in *Week 5*, to the nearest second?
(1)
- How many hours, minutes and seconds (to the nearest second) will he have run in total in his practice runs in these 5 weeks?
(1)
- If the previous winning time for the marathon was *2 hours and 6 minutes*, how many weeks must he keep practising to be able to run the marathon in less than the previous winning time?
(2)

b)



A rectangular beam of width w cm and depth, d cm is cut from a cylindrical pine log as shown.

The diameter of the cross section of the log (and hence the diagonal of the cross section of the beam) is 15 cm.

The strength S of the beam is proportional to the product of its width and the square of its depth, so that

$$S = kd^2 w$$

Question 4 (continued)

i) Show that $S = k(225w - w^3)$ (1)

ii) What numerical dimensions will give a beam of maximum strength? (2)

iii) A square beam with diagonal 15cm could have been cut from the log. Show that the rectangular beam of maximum strength is more than 8% stronger than this square beam. (3)

b) Differentiate $3x^2(7x - 1)^5$. (1)

Question 5**10 Marks**

- a) (i) Grandma wants to invest a certain amount of money for her grandchild so that she will have \$5 000 in ten years time. If the bank account pays 5% *p.a.* interest, compounding quarterly, how much will she need to invest now? (2)

- b) Tiarn borrows \$500 000 to buy a house. An interest rate of 9% *p.a.* compounded monthly is charged on the outstanding balance. The loan is to be repaid in equal monthly installments (R) over a 25 year period.

Show the amount owing after 3 months is:

i)
$$A_3 = 500000(1.0075)^3 - R[1 + 1.0075 + 1.0075^2]$$
 (1)

- ii) Assuming this pattern continues the monthly repayment can be calculated using:

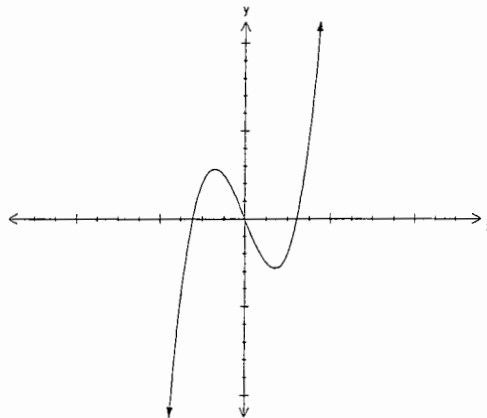
$$A_n = 500000(1.0075)^n - R[1 + 1.0075 + 1.0075^2 + \dots + 1.0075^{n-1}]$$

How much should Tiarn be paying each month? (2)

- iii) How much interest does Tiarn pay over the 25 years? (1)

- iv) What is the equivalent simple interest rate of this loan? (1)

- c) Below is a graph of $f'(x)$. Carefully copy the graph into your exam booklet and draw a neat sketch of the graph of $f(x)$ showing any significant points. (3)



1)

2) $6+10+14+\dots$
AP $a=6$ $d=4$

$$T_{16} = a + (n-1)d$$

$$= 6 + (15)4$$

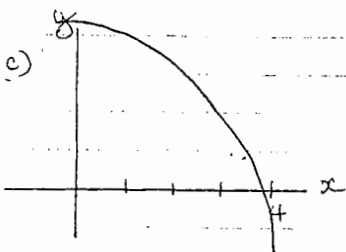
$$= 66$$

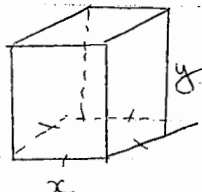
3) $\sum_{n=1}^{\infty} \left(\frac{1}{5}\right)^n$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{\frac{1}{5}}{1-\frac{1}{5}}$$

$$= \frac{1}{4}$$



1) 

$$V = x^2 y$$

$$100 = x^2 y$$

$$y = \frac{100}{x^2}$$

i) $S.A = 2x^2 + 4xy$

$$= 2x^2 + 4x \frac{100}{x^2}$$

$$= 2x^2 + \frac{400}{x}$$

ii) $A = 2x^2 + 400x^{-1}$

$$\frac{dA}{dx} = 4x - 400x^{-2}$$

$$= 4x^3 - 400$$

stat pt $\frac{dA}{dx} = 0$

$$4x^3 - 400 = 0$$

$$x^3 = 100$$

$$x \approx 4.64$$

check:

x	0	4.64	10
$f'(x)$	-	0	+

$\therefore x \approx 4.64$ min

min SA = 129.295

$$= 129.3$$

02

(a) $\frac{dp}{dt} > 0$ $\frac{d^2p}{dt^2} < 0$

(b) $5x - y = 0$

(i) $y = 5x - 1$

$$n = 5$$

$$4x - 3 = 5$$

$$x = 2 \quad y = 9 \quad N \Rightarrow (2, 9)$$

(ii) $\int 4x - 3 dx = 2x^2 - 3x + C$

at (2, 9) $9 = 8 - 6 + C$

$$C = 7$$

$$y = 2x^2 - 3x + 7$$

02

(c) $\frac{7}{8} + \frac{1}{4} + \frac{1}{16} + \dots$ (cont'd)

AP $r = \frac{1}{2}$ $a = \frac{7}{8}$

$$T_n = ar^{n-1}$$

$$\frac{16}{117649} = \frac{7}{8} \times \left(\frac{2}{7}\right)^{n-1}$$

$$0.00155\dots = \left(\frac{2}{7}\right)^{n-1}$$

$$\log \dots = (n-1) \log \frac{2}{7}$$

$$n = 8$$

(d) $\int \frac{3x^5 + 2x^2 - x}{x} dx$

$$= \int 3x^4 + 2x - 1 dx$$

$$= \frac{3x^5}{5} + x^2 - x + C$$

(e) $\int x(x^{1/2} + 1) dx$

$$= \int x^{3/2} + x dx$$

$$= \frac{2x^{5/2}}{5} + \frac{x^2}{2} + C$$

$$= \frac{2\sqrt{x^5}}{5} + \frac{x^2}{2} + C$$

03

(a) $y = x^6 - 6x^4$

at $y = 0$ $x^4(x^2 - 6) = 0$

$$x = 0, x = \pm\sqrt{6}$$

(ii) $y = x^6 - 6x^4$

$$y' = 6x^5 - 24x^3$$

stat pts at $y' = 0$

$$6x^5 - 24x^3 = 0$$

$$x^3(6x^2 - 24) = 0$$

$$6x^2 = 24$$

$$x^2 = 4$$

$$x = \pm 2$$

x	-10	-2	-1	0	1	2	10
$f'(x)$	-	0	+	0	-	0	-

$$y'' = 30x^4 - 72x^2$$

$$30x^4 - 72x^2 = 0$$

$$x^2(30x^2 - 72) = 0$$

$$30x^2 = 72$$

$$x^2 = \frac{12}{5}$$

$$x = \pm\sqrt{\frac{12}{5}}$$

$$\approx \pm 1.55$$

poss. horiz. pt inflex at $x = 0$.

x	-1	0	1
$f''(x)$	-	0	-

$x = 0$ not hor pt. inflex

3rd (cont'd)

pts

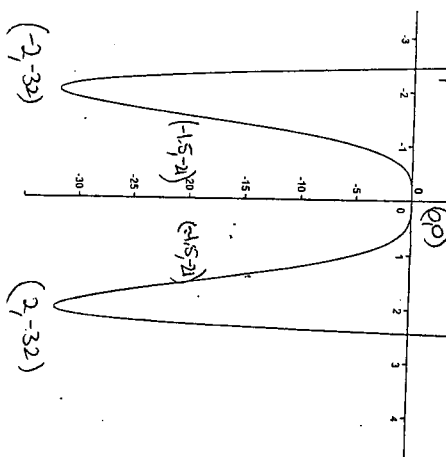
(-2, -32) min

(0, 0) max

(2, -32) min

($\sqrt{2/5}$, $-20\sqrt{2/5}$) pt inflex

($-\sqrt{2/5}$, $-20\sqrt{2/5}$)



(b) $y = 2x^3 - 12x^2 - 5x - 3$

$y' = 6x^2 - 24x - 5$

$y'' = 12x - 24$

pt inflex $12x - 24 = 0$
 $x = 2$

x	1	2	3
y	-	0	+

at $x = 2$ $y = -45$

$M_L = \frac{1}{29}$

$(y + 45) = \frac{1}{29}(x - 2)$

$29y + 1305 = x - 2$

$29y = x - 1307$

$x - 29y - 1307 = 0$

(c) $x + \frac{2x}{3} + \frac{4x}{9} + \dots$

$a = x$ $r = \frac{2}{3}$

$S_{\infty} = \frac{a}{1-r}$

$15 = \frac{x}{1/3}$

$x = 5$

(i) $\frac{w_2}{w_1} = \frac{19}{20}$

$\frac{w_3}{w_2} = \frac{19}{20}$

\therefore AP $a = 3hrs$ $r = 0.95$

(ii) $w_4 - 2h$ $34m$ $19.65s$

$w_5 - 2h$ $36m$ $36.67s$

(iii) Tot - $13h$ $34m$ $23.32s$

(iv) $T_n = ar^{n-1}$

$2h$ $6m = 3 \times 0.95^{n-1}$

$0.95^{n-1} = \frac{42m}{3}$

$n-1 = \frac{\log 42m}{\log 0.95}$

$n-1 = 6.95$

$n = 7.95$

$n = 7$

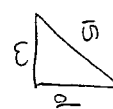
$3h \times 0.95^7 = 2h$ $5m$ $42.4s$

(b) $S = kd^2 w$

$S = k(225 - w^2)w$
 $= k(225w - w^3)$

$w^3 + d^2 = 225$

$d^2 = 225 - w^3$



(ii) $\frac{dS}{dw} = 225k - 3kw^2$

stat pt $225k - 3kw^2 = 0$

$w^2 = \frac{225}{3}$

$w = \sqrt{75}$

cannot be -ve

$w \approx 8.66$

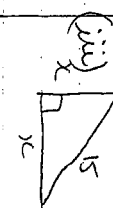
w		8.66	
f(w)	+	0	-

\therefore 8.66 max

$d = \sqrt{150}$

$= 12.247$

$= 12.25$



$2x^2 = 15^2$

$x^2 = 112.5$

$x = 10.60$

$S \propto d^2 w$

$\propto 150 \times 8.66$

$\propto 1299$

$S \propto d^2 w$

$\propto 112.5 \times 106$

$\propto 1192.5$

strength = $1299 - 1192.5$

$= 106.5$

strength = $\frac{106.5}{1192.5} \times 100$

0.0089

04

$$(b) \quad 3x^2(7x-1)^5$$

$$y' = 3x^2(5)(7x-1)^4(7) + (7x-1)^5 6x$$

$$= 105x^2(7x-1)^4 + (7x-1)^5 6x$$

$$= (7x-1)^4 [105x^2 + (7x-1)6x]$$

$$= (7x-1)^4 [105x^2 + 42x^2 - 6x]$$

$$= (7x-1)^4 (147x^2 - 6x)$$

05

$$(a) (i) \quad V = P(1+r)^n$$

$$5000 = P(1.0125)^{40}$$

$$P = \$3042.07$$

$$(b) \quad \$500,000 \quad i = \frac{9}{1200} = 0.0075$$

$$n = 25 \times 12 = 300$$

i)

$$A_1 = 500,000(1.0075) - M$$

$$A_2 = A_1(1.0075) - M$$

$$= [500,000(1.0075) - M](1.0075) - M$$

$$= 500,000(1.0075)^2 - M(1.0075 + 1)$$

$$A_3 = 500,000(1.0075)^3 - M(1.0075^2 + 1.0075 + 1)$$

$$(ii) \quad A_{300} = 500,000(1.0075^{300}) - M(1 + 1.0075 + \dots + 1.0075^{300})$$

a.p

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{1.0075^{300} - 1}{0.0075}$$

(CONT'D)

$$0 = 500,000(1.0075)^{300} - M \left(\frac{1.0075^{300} - 1}{0.0075} \right)$$

$$M = 4195.9818$$

$$M = \$4,195.98$$

$$(iii) \quad \text{Int} = 4195.98 \times 300 - 500,000$$

$$= \$758,794$$

$$(iv) \quad S.I = \frac{PRT}{100}$$

$$758,794 = \frac{500,000 \times R \times 25}{100}$$

$$R = 6.07\% \text{ p.a.}$$

(c)

