Class Teacher________________________ Name__________________________

SYDNEY TECHNICAL HIGH SCHOOL

Mathematics Extension 1

Year 11 Preliminary Course

Assessment Task 3

September 2014

Time Allowed: **90 minutes**

General Instructions:

- Write using black or blue pen.
- Approved calculators may be used.
- Attempt all questions.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start each question on a new side of a page.

Total Marks 71

Section 1 – Multiple Choice **5 Marks** **Answer on sheet after question 5. Do not tear this sheet out.** **Allow 8 minutes for this section**	**Section 2** **66 Marks** **Allow 82 minutes for this section**

SECTION 1 – MULTIPLE CHOICE (FILL IN YOUR ANSWERS ON THE ANSWER SHEET PROVIDED-DO NOT TEAR THE SHEET OUT)

1. A parabola has its focus at (0, 4). The equation of its directrix is $x = -4$.

   Which of the following is the equation of the parabola?

   A. $x^2 = 16y$

   B. $(x + 2)^2 = 8(y-4)$

   C. $(y + 2)^2 = 8(x - 4)$

   D. $(y - 4)^2 = 8(x + 2)$

2. Which one of the following expressions represents the factored form of $8x^3 + 27$?

   A. $8x^3 + 27 = (2x + 3)(4x^2 + 6x + 9)$

   B. $8x^3 + 27 = (2x + 3)(4x^2 - 6x + 9)$

   C. $8x^3 + 27 = (2x - 3)(4x^2 - 6x - 9)$

   D. $8x^3 + 27 = (2x - 3)(4x^2 + 6x - 9)$

3. Consider the function $f(x) = \frac{x^4+3x^2}{x^4+3}$

   Which one of the following statements is correct?

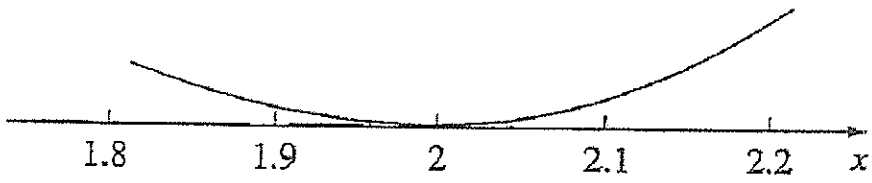
   A. $f(x)$ is odd and $\lim_{x\to\infty} f(x) = 1$

   B. $f(x)$ is even and $\lim_{x\to\infty} f(x) = 3$

   C. $f(x)$ is even and $\lim_{x\to\infty} f(x) = 1$

   D. $f(x)$ is odd and $\lim_{x\to\infty} f(x) = 3$

4. Part of the graph of $y = P(x)$, where $P(x)$ is a polynomial of degree four, is shown below.



Which of the following could be the polynomial $P(x)$?

A. $P(x) = x^2(x+2)^2$

B. $P(x) = (x+2)^4$

C. $P(x) = x(x-2)^3$

D. $P(x) = (x-1)^2(x-2)^2$

5. The normal to the graph of $y = \sqrt{b - x^2}$ has a gradient of 3 when $x = 1$.

The value of b is

A. $-\frac{10}{9}$

B. $\frac{10}{9}$

C. 4

D. 10

Name: ______________________________

Teacher: ____________________________

SECTION A: MULTIPLE CHOICE

Instructions:

- Circle the letter that best answers the question
- One mark each

1.	A	B	C	D
2.	A	B	C	D
3.	A	B	C	D
4.	A	B	C	D
5.	A	B	C	D

SECTION 2

QUESTION 6 (start a new page) **Marks**

(a) Solve $\frac{2x+1}{x-1} > 3$ 2

(b) (i) Sketch $y = x^2 - 1$ 1

(ii) Hence, on a separate diagram sketch $y = |x^2 - 1|$ 1

(c) $P(x)$ is an odd monic polynomial of degree 3. 2

If P(3)=0, sketch the polynomial.

(d) Differentiate $y = \frac{x+1}{\sqrt{x}}$ and express the derivative as a simplified fraction. 2

(e) Use the substitution $t = tan\frac{x}{2}$ to show that 3

$\frac{1+sinx}{1-cosx} = cot\frac{x}{2} + \frac{1}{2}cosec^2\frac{x}{2}$

QUESTION 7 (Start a new page) **Marks**

(a) The equation $2x^2 + px + q = 0$ has one root three times the other. 2

Show that $3p^2 = 32q$.

(b) For what values of k is $2x^2 - 5x + 4k$ positive definite? 2

(c) A parabola has equation $y^2 + 8y = -12x + 8$

(i) Find the coordinates of its vertex. 1

(ii) Sketch the parabola showing its x intercept. 1

(iii) On your sketch, display the focus and directrix. 2

(d) Find the acute angle between the lines $x - \sqrt{3}y - 2 = 0$ and $\sqrt{3}x - y + 3 = 0$ 3

QUESTION 8 (Start a new page) **Marks**

(a) Show that the equation $x^2 + (k+2)x + k = 0$ has two real roots for all real values of k. 2

(b) Solve the equation 3

$cos2x + 3cosx + 2 = 0$ for $0° \leq x \leq 360°$

(c) (i) Show that $(x+1)$ is a factor of $P(x) = x^3 - x^2 - 10x - 8$. 1

(ii) Hence express $P(x) = x^3 - x^2 - 10x - 8$ as a product of three linear factors 1

(iii) By sketching $P(x)$ or otherwise, solve the inequality 2

$$\frac{x^3 - 10x}{x^2 + 8} \geq 1$$

(d) Find the domain and range of the function $f(x) = 3\sqrt{4 - x^2}$ 2

QUESTION 9 (Start a new page) **Marks**

(a) Simplify $\frac{1}{p^2-pq}$ - $\frac{1}{pq-q^2}$ 2

(b) The polynomial $P(x) = x^3 + a^2x^2 + ax + b$ leaves a remainder of 2 when divided by x and a remainder of 13 when divided by $x + 1$.

(i) Show that $b = 2$ 1

(ii) Find the value of a 2

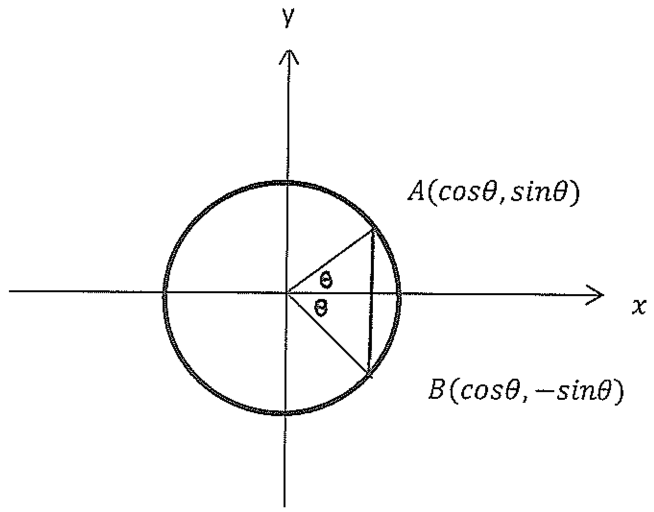
(c) (i) Express $sinx + 3cosx$ in the form $R\sin(x+\alpha)$ where $R > 0$ and $0° < \alpha < 90°$, giving the value of R in simplest exact form, and the value of α correct to the nearest degree. 2

(ii) Solve the equation $3cosx + sinx + 2 = 0$ for $0° \leq x \leq 360°$, giving the solutions correct to the nearest degree. 2

(d) Find the coordinates of point P on the curve $y = x\sqrt{x+3}$ where the tangent is parallel to the $x - axis$. 2

QUESTION 10 (Start a new page) **Marks**

(a) 2

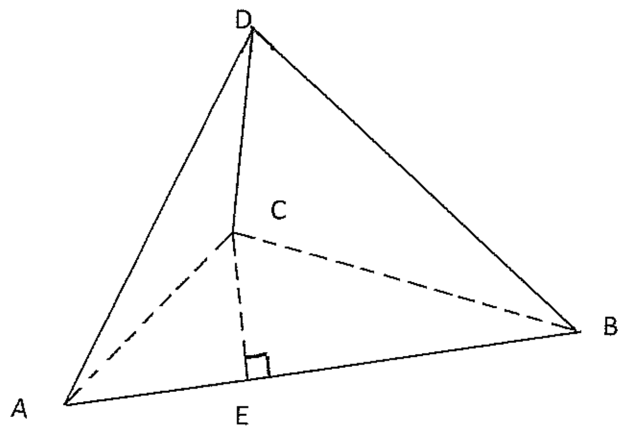


A $(cos\theta, sin\theta)$ and B $(cos\theta, -sin\theta)$, $0° < \theta < 90°$, are 2 points on the circle with centre at the origin and radius 1. Use the cosine rule in ΔAOB to show that $\cos 2\theta = 1 - 2sin^2\theta$.

(b) A $(8, \sqrt{50})$ and B $(1, \sqrt{18})$ are divided externally by a point P in the ratio of 3:1. Find the simplest exact form of this point. 3

(c) Show that $\tan 75° = 2 + \sqrt{3}$ 2

(d)



CD is a vertical flagpole of height 10 metres. It stands with its base on horizontal ground. A and B are points on the ground due South and due East of C respectively. The angle of elevation of D is 45° from A and 30° from B. E is the foot of the perpendicular from C to AB.

(i) Show that $\angle$ABC=30° 2

(ii) Find the angle of elevation of D from E correct to the nearest minute. 2

QUESTION 11 (Start a new page) **Marks**

(a) $P(x, y)$ is a variable point which moves in the number plane so that its distance from the point $A(3,3)$ is twice its distance from the origin. Find the equation of the locus of P. 2

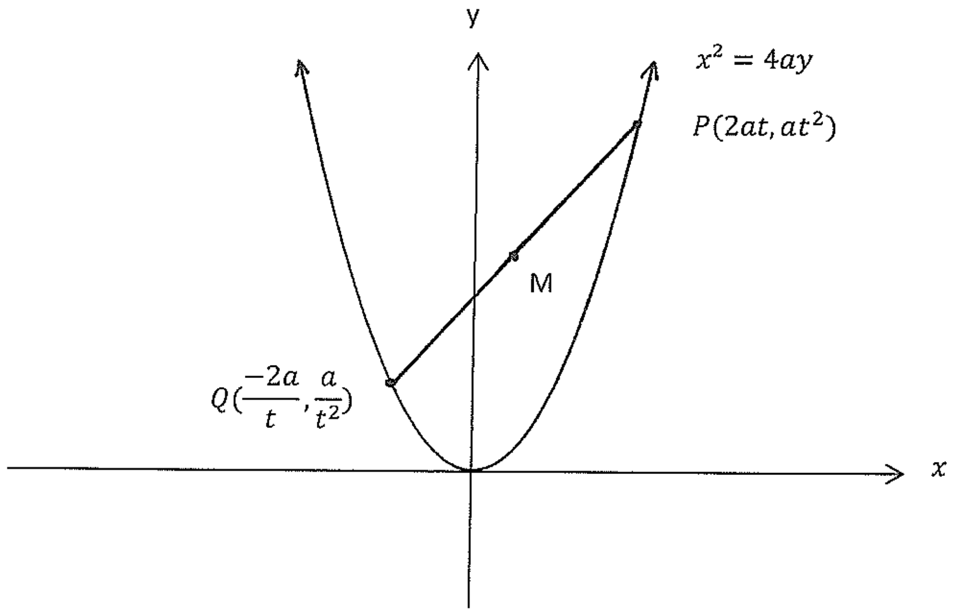
(b) The polynomial $P(x) = x^3 + 2x^2 - 4x - 1$ has zeros α, β and γ so that $P(x) = (x - \alpha)(x - \beta)(x - \gamma)$.

(i) Find the value of $(1 - \alpha)(1 - \beta)(1 - \gamma)$ 2

(ii) Find the value of $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$ 2

(c) Show that the equation of the normal at the point $P(2ap, ap^2)$ on the parabola $x^2 = 4ay$ is $x + py = 2ap + ap^3$. 2

(d)



$P(2at, at^2)$ and $Q(\frac{-2a}{t}, \frac{a}{t^2})$ are two points on the parabola $x^2 = 4ay$. M is the midpoint of the chord PQ. 3

As P and Q move on the parabola, find the locus of M.

END OF PAPER

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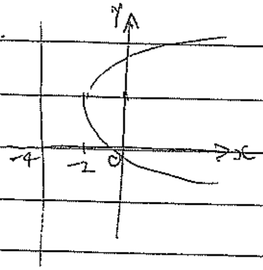
2014 Ext 1 Final Prelim. Solutions

1. $(y\;.)^2 = 8(x+2)$ 2. B



D

3. C   4. D   5. $\frac{dy}{dx} = \frac{1}{2}(b-x^2)^{-\frac{1}{2}} \times -2x$

$= \frac{-x}{\sqrt{b-x^2}}$

$-\frac{1}{3} = \frac{-1}{\sqrt{b-1}}$

$\frac{1}{9} = \frac{1}{b-1}$ D

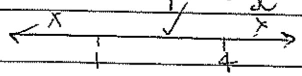
6. a) $\frac{2x+1}{x-1} > 3$

$x \neq 1$

$\frac{2x+1}{x-1} = 3$

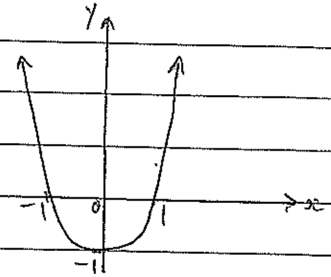
$2x+1 = 3x-3$

$4 = x$



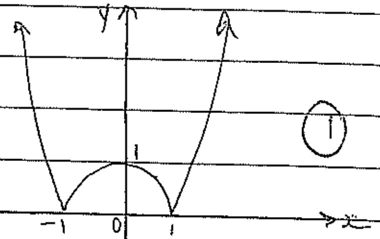
$1 < x < 4$ (2)

b) (i)



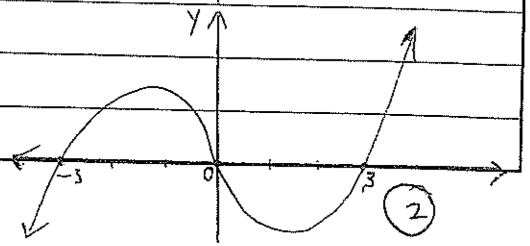
(1)

(ii)



(1)

c) $P(x) = x(x+3)(x-3)$



(2)

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d) $y = \frac{x+1}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{\sqrt{x} \times 1 - (x+1) \times \frac{1}{2}x^{-\frac{1}{2}}}{x}$

$= \frac{\sqrt{x} - \frac{x+1}{2\sqrt{x}}}{x} \times \frac{2\sqrt{x}}{2\sqrt{x}}$

$= \frac{2x - (x+1)}{2x^{\frac{3}{2}}}$

$= \frac{x-1}{2x^{\frac{3}{2}}}$ (2)

e) $\frac{1+\sin x}{1-\cos x} = \cot\frac{x}{2} + \frac{1}{2}\operatorname{cosec}^2\frac{x}{2}$

$LHS = \frac{1 + \frac{2t}{1+t^2}}{1 - \left(\frac{1-t^2}{1+t^2}\right)}$ (1)   where $t = \tan\frac{x}{2}$

$= \frac{1+t^2+2t}{1+t^2-1+t^2}$

$= \frac{1+t^2+2t}{2t^2}$

$= \frac{1}{2t^2} + \frac{1}{2} + \frac{1}{t}$

$= \frac{1}{2}\left(\cot^2\frac{x}{2} + 1\right) + \frac{1}{\tan\frac{x}{2}}$ (1)

$= \frac{1}{2}\operatorname{cosec}^2\frac{x}{2} + \cot\frac{x}{2}$

$= RHS$ (1)

7a) $2x^2 + px + q = 0$

Roots $\alpha, 3\alpha$

$\therefore 4\alpha = -\frac{p}{2}$ $3\alpha^2 = \frac{q}{2}$

$\alpha = -\frac{p}{8}$

$\alpha^2 = \frac{p^2}{64}$

$\therefore 3 \times \frac{p^2}{64} = \frac{q}{2}$

$3p^2 = 32q$

b) $2x^2 - 5x + 4k$

$\Delta < 0$

$25 - 4 \times 2 \times 4k < 0$

$25 - 32k < 0$

$32k > 25$

$k > \frac{25}{32}$

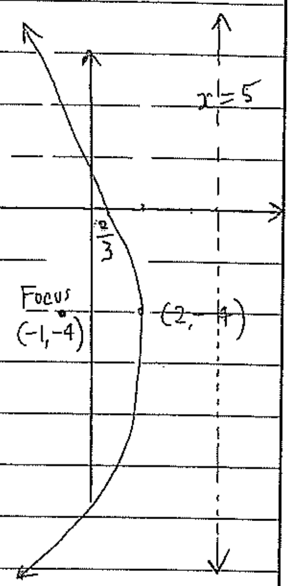
c) (i) $y^2 + 8y = -12x + 8$

$y^2 + 8y + 16 = -12x + 24$

$(y+4)^2 = -12(x-2)$

Vertex $(2, -4)$

(ii)



d) $x - \sqrt{3}y - 2 = 0$ $\sqrt{3}x - y + 3 = 0$

$y = \frac{1}{\sqrt{3}}x - \frac{2}{\sqrt{3}}$ $y = \sqrt{3}x + 3$

$m_1 = \frac{1}{\sqrt{3}}$, $m_2 = \sqrt{3}$

$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$

$= \left|\frac{\frac{1}{\sqrt{3}} - \sqrt{3}}{1 + \frac{1}{\sqrt{3}} \times \sqrt{3}}\right| = \frac{\sqrt{3} - \frac{1}{\sqrt{3}}}{2}$

$\theta = 30^\circ$

Teacher Name: Student Name:

8a) $x^2 + (k+2)x + k = 0$

2 Real roots if

$\Delta \geq 0 \Rightarrow b^2 - 4ac \geq 0$

$(k+2)^2 - 4 \times 1 \times k \geq 0$

$k^2 + \cancel{4k} + 4 - \cancel{4k} \geq 0$

$k^2 + 4 \geq 0$

True for all k

$\therefore$ Two real roots.

b) $\cos 2x + 3\cos x + 2 = 0$

$2\cos^2 x - 1 + 3\cos x + 2 = 0$

$2\cos^2 x + 3\cos x + 1 = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = -\frac{1}{2}, \quad -1$

$x = 120^\circ, 240^\circ, 180^\circ$

c)(i) $P(-1) = 0$ need to show

$P(-1) = (-1)^3 - (-1)^2 + 10 - 8$

$= -1 - 1 + 10 - 8$

$= 0$

$\therefore (x+1)$ is a factor.

(ii) $x^3 - x^2 - 10x - 8$

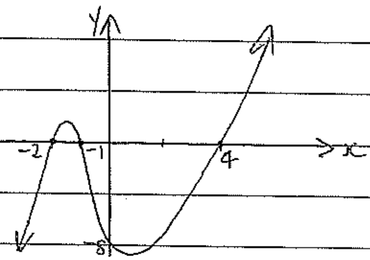
$= (x+1)(x^2 - 2x - 8)$ by inspection

$= (x+1)(x+2)(x-4)$

c(iii) $\frac{x^3 - 10x}{x^2 + 8} \geq 1$

$x^3 - 10x \geq x^2 + 8$

$x^3 - x^2 - 10x - 8 \geq 0$



$-2 \leq x \leq -1, \quad x \geq 4$

d) $f(x) = 3\sqrt{4 - x^2}$

D: $4 - x^2 \geq 0$

$(2-x)(2+x) \geq 0$

$-2 \leq x \leq 2$

R: $0 \leq y \leq 6$

Teacher Name: Student Name:

9a. $\frac{1}{p(p-q)} - \frac{1}{q(p-q)}$

$= \frac{pq - q^2 - p^2 + pq}{pq(p-q)^2}$

$= \frac{-(p^2 - 2pq + q^2)}{pq(p-q)^2}$

$= \frac{-\cancel{(p-q)^2}}{pq\cancel{(p-q)^2}}$

$= \frac{-1}{pq}$

b)(i) $P(0) = 2 = b$

$\therefore b = 2$

(ii) $13 = -1 + a^2 - a + 2$

$0 = a^2 - a - 12$

$0 = (a+3)(a-4)$

$a = -3$ or 4

c(i) $R = \sqrt{1^2 + 3^2} = \sqrt{10}$

$\tan\alpha = 3$

$\alpha = 71°34'$

$\sqrt{10}\sin(x + 72)$

(ii) $\sqrt{10}\sin(x+72) = -2$

$\sin(x+72) = \frac{-2}{\sqrt{10}}$

$x + 72 = 219°14', 320°46$

$x = 248°46', 147°14$

$x = 249°, 147°$

d) $y = x\sqrt{x+3}$

$y = x(x+3)^{\frac{1}{2}}$

$y' = (x+3)^{\frac{1}{2}} + x \times \frac{1}{2}(x+3)^{-\frac{1}{2}}$

$y' = \sqrt{x+3} + \frac{x}{2\sqrt{x+3}}$

$y' = 0 = \sqrt{x+3} + \frac{x}{2\sqrt{x+3}}$ (tangent parallel to x axis)

$-\sqrt{x+3} = \frac{x}{2\sqrt{x+3}}$

$x+3 = \frac{x^2}{4(x+3)}$

$4(x+3)^2 = x^2$

$4x^2 + 24x + 36 = x^2$

$3x^2 + 24x + 36 = 0$

$x^2 + 8x + 12 = 0$

$(x+2)(x+6) = 0$

$x = -2, \cancel{-6}$ not in di

$(-2, -2)$

Student Name: Teacher Name:

10.a) $\cos 2\theta = \frac{1^2 + 1^2 - 4\sin^2\theta}{2}$ in ΔAOB

$\cos 2\theta = 1 - 2\sin^2\theta$

b) $x = \frac{-3 \times 1 + 1 \times 8}{-3+1}$, $y = \frac{-3 \times \sqrt{18} + \sqrt{50}}{-3+1}$

$x = \frac{5}{-2}$ $y = \frac{-9\sqrt{2} + 5\sqrt{2}}{-2}$

$= \frac{-4\sqrt{2}}{-2}$

$= 2\sqrt{2}$

$(-2\frac{1}{2}, 2\sqrt{2})$

c) $\tan 75 = \tan(45 + 30)$

$= \frac{\tan 45 + \tan 30}{1 - \tan 45 \tan 30}$

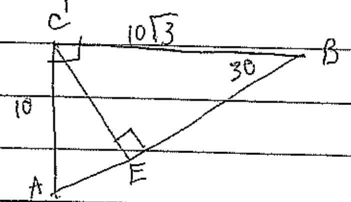
$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}+1}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{4 + 2\sqrt{3}}{2}$

$= 2 + \sqrt{3}$

d) Top View.



AC is 10 (ΔACD isosceles)

$\tan 30 = \frac{10}{BC}$

$\frac{1}{\sqrt{3}} = \frac{10}{BC}$ $\therefore BC = 10\sqrt{3}$

$\therefore \tan\angle ABC = \frac{10}{10\sqrt{3}}$

$\angle ABC = 30°$

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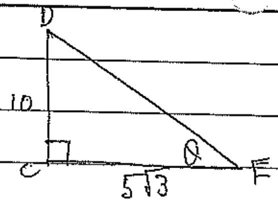
(ii) $\sin 30 = \frac{CE}{10\sqrt{3}}$

$\frac{1}{2} = \frac{CE}{10\sqrt{3}}$

$CE = 5\sqrt{3}$



$\tan\theta = \frac{10}{5\sqrt{3}}$

$= \frac{2}{\sqrt{3}}$

$\therefore \theta = 49°6'$

11. a) $\sqrt{(x-3)^2+(y-3)^2} = 2\sqrt{x^2+y^2}$

$x^2-6x+9+y^2-6y+9 = 4x^2+4y^2$

$0 = 3x^2+6x+3y^2+6y-18$

$0 = x^2+2x+y^2+2y-6$

b) (i) $(1-\alpha)(1-\beta)(1-\gamma) = P(1)$

$\therefore P(1) = 1+2-4-1$

$= -2$

(ii) $\alpha+\beta+\gamma = -2$

$\therefore (\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$

$= (-2-\alpha)(-2-\beta)(-2-\gamma)$

$= P(-2)$

$= (-2)^3+2(-2)^2-4(-2)-1$

$= -8+8+8-1$

$= 7$

c) $y = \frac{x^2}{4a}$

$\frac{dy}{dx} = \frac{2x}{4a}$

$\frac{dy}{dx} = \frac{x}{2a}$

At $x = 2ap$, m of tangent is p $\therefore$ gradient of normal is $\frac{-1}{p}$

$y - ap^2 = \frac{-1}{p}(x-2ap)$

$py - ap^3 = -x+2ap$

$x + py = 2ap + ap^3$

Student Name: Teacher Name:

d) M is $\left(\frac{2at - \frac{2a}{t}}{2}, \frac{at^2+\frac{a}{t^2}}{2}\right)$

$x = at - \frac{a}{t}$

$x = a(t-\frac{1}{t})$

$x^2 = a^2(t^2-2+\frac{1}{t^2})$

$x^2 = a^2(\frac{2y}{a}-2)$ from y equation

$x^2 = 2ay - 2a^2$ as req'd.

$y = \frac{a}{2}(t^2+\frac{1}{t^2})$

$\frac{2y}{a} = t^2+\frac{1}{t^2}$

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