Name: ..... Maths Class: .....

# SYDNEY TECHNICAL HIGH SCHOOL

## Year 11
# Mathematics Extension 1

## Preliminary Course
## Assessment 1

## Term 2, 2016

*Time allowed: 90 minutes*

***General Instructions:***

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A BOSTES Formulae booklet is provided at the rear of this Question Booklet, and may be removed at any time.

| | |
|---|---|
| Section 1 | Multiple Choice<br>Questions 1-6<br>6 Marks |
| Section II | Questions 7- 12<br>66 Marks |

## Section1

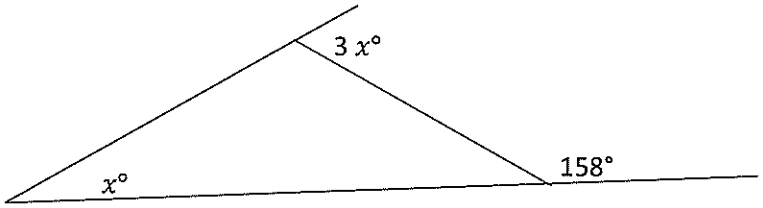
**Multiple choice: Answer on the sheet provided in your answer booklet**

1. $L + 2m - (L - 2n) - [2m + L - (2n - L)]$ simplifies to,

(A) $4n + 2L$
(B) $4n - 2L$
(C) $4m + 4n - 2L$
(D) $4m - 4n - 2L$

---

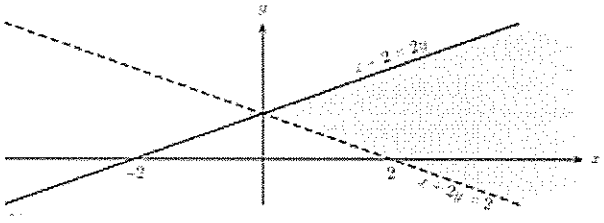
2. The value of $x$ is:




(A) 22
(B) 11
(C) 5.5
(D) 44

---

3. The shaded region in the diagram below is satisfied by:



(A) $x + 2 \geq 2y$ and $x + 2y > 2$
(B) $x + 2 \geq 2y$ and $x + 2y < 2$
(C) $x + 2 \leq 2y$ and $x + 2y > 2$
(D) $x + 2 \leq 2y$ and $x + 2y < 2$

---

4. Which of the following is an asymptote of the curve $y = \frac{x^2 - 4}{x}$ ?

(A) $y = x$
(B) $x = 2$
(C) $x = 1$
(D) $y = 0$

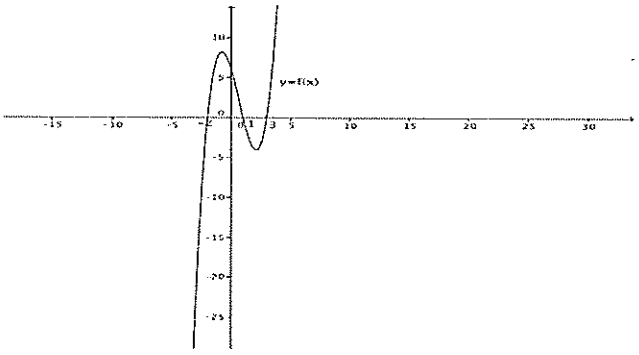
---

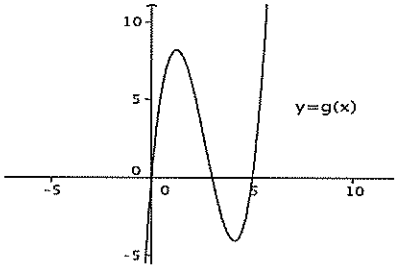
5. How many solutions does the equation $\boldsymbol{sin2\theta(tan\theta - 3) = 0}$ have in the domain $0° \leq \theta \leq 180°$ ?

(A) 2
(B) 3
(C) 4
(D) 7

---

6. Consider the graphs of $y = f(x)$ and $y = g(x)$,




Which of the following is true?

(A) $g(x) = f(x) + 2$
(B) $g(x) = f(x) - 2$
(C) $g(x) = f(x + 2)$
(D) $g(x) = f(x - 2)$

***End of Section 1***

**Section 2**

**Show full working out for each question in the answer booklet provided:**

**Remember to start EACH question on a NEW page**

---

**Question 7 ( 11 MARKS )**

a. Fully factorise,

i. $64x^4 - xy^3$ 2.

ii. $x^2 + 6xy + 9y^2 - 16$ 2.

b. Fully simplify,

i. $\frac{x^2+4x}{x^3-9x} \div \frac{x^2+2x-8}{x^2+x-6}$ 2.

ii. $\frac{3^n \times 9^{n+1}}{27^{2n}}$ 2.

c. Write down the exact value of,

i. $tan420°$ 1.

ii. $12sin^2 30°(1 + sec^2 30°)$ 2.

***START A NEW PAGE***

**Question 8 ( 11 marks )**

a. Solve $|2x - 1| > 8$ 2.

b. Solve simultaneously $y - 4 + x = 0$ and $xy = 4$ 2.

c. Solve;

i. $|2x + 1| = 3x - 2$ 3.

ii. $\frac{x^2+x-6}{x} \geq 0$ 2.

d. Show that $f(x) = \frac{x^3}{2x^2-1}$ is an odd function. 2.

*START A NEW PAGE*

**Question 9 ( 11 marks )**

a. Write down the natural domain for each of these relations. 3.

i. $y = \frac{x-1}{x^2+4}$

ii. $y = \frac{1}{1-x^2}$

iii. $y = \sqrt{x+2} + \sqrt{4-x}$

b. Write down the domain and range for the function represented by the equation

$y = x^2 - 4x - 28$ 2.

c. For the function $f(x) = x^2 - 1$, evaluate $\frac{f(m)-f(n)}{n-m}$ given $n \neq m$ 2.

d. Consider the function $f(x) = \frac{2x+1}{x-2}$ 4.

i. Write down the equation of the horizontal asymptote.

ii. Write down the coordinates of the $y$ – intercept.

iii. Sketch the curve $y = f(x)$, neatly on a Cartesian plane, showing all intercepts and asymptotes.

*START A NEW PAGE*

**Question 10 ( 11 marks )**

a. Find the exact value of $sec\theta$, given $sin\theta = \frac{12}{13}$ and $tan\theta < 0$. 2.

b. Solve each of the following for the domain $0° \leq \theta \leq 360°$,

i. $tan^2\theta = \frac{1}{3}$ 2.

ii. $sin^2\theta = sin\theta cos\theta$ 3.

iii. $cos2\theta = 1$ 2.

c. Show that

$$\frac{\sin\theta}{1-\cos\theta} + \frac{1-\cos\theta}{\sin\theta} = 2cosec\theta$$ 2.

*START A NEW PAGE*

**Question 11 ( 11 marks )**

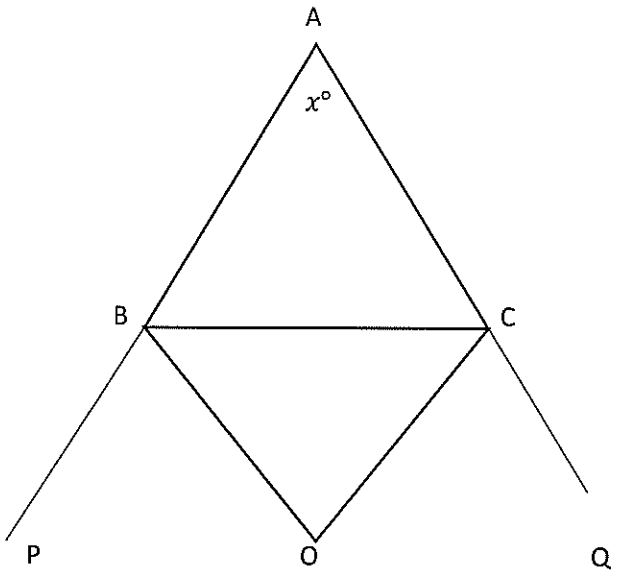
a. On separate diagrams show the regions defined by;

i. $y + 3 > |x|$ 2.

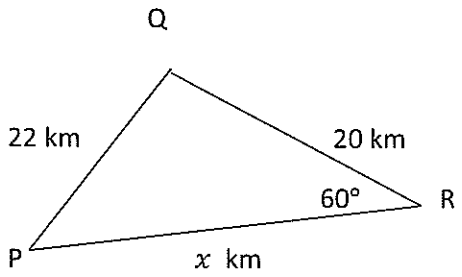
ii. $y + 3 \geq |x|$ and $y \leq \sqrt{9 - x^2}$ 2.

b. In the triangle ABC, the sides AB and AC are produced and the exterior angles are bisected by BO and CO respectively.

Given angle ABC = $x°$, write down, with clear reasoning, an expression for angle BOC, in terms of $x$. 3.




c. Consider triangle PQR. 4.




i. Show that $x^2 - 20x - 84 = 0$

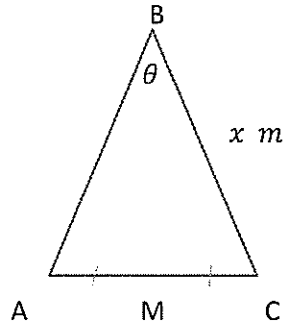
ii. Hence, calculate the area of triangle PQR, correct to 3 significant figures.

*START A NEW PAGE*

**Question 12 ( 11 marks )**

a. Simplify the expression $\frac{\sqrt{x^2}}{x}$ for all values of $x, x \neq 0$. 2.

b. Triangle ABC, of perimeter 4 metres, is isosceles with AB = BC = $x$ metres, and angle BAC = $\theta$. M is the midpoint of the base AC, with BM being the altitude of the triangle.



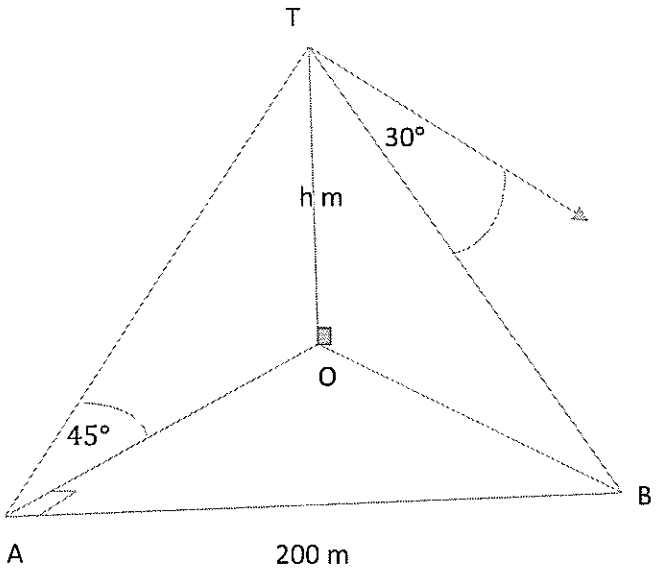

Find simplified expressions, in terms of $x$, for

i. $\cos\frac{\theta}{2}$ 2.

ii. The area of triangle ABC. 2.

***Question 12 continues on the NEXT page .....***

c. A man stands at a point A due south of a tower OT of height $h$ metres. From this point the angle of elevation to the top of the tower, T, is 45°. A woman stands 200 metres due ~~west~~ east of the man, at the point B. The angle of depression form the top of the tower to the woman is 30°.




i. Find the distance OB, in terms of $h$. 1.

ii. Show that $h = 100\sqrt{2}$ 2.

iii. Calculate the bearing of B from the base of the tower O, answer to the nearest degree. 2.

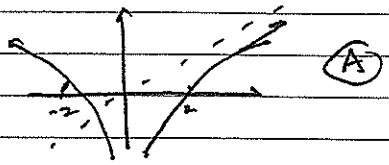
***END OF TASK***

# SOLUTIONS

## SECTION I

1. $L + 2m - L + 2n - 2m - L + 2n - L$
$= -2L + 4n$. (B)

2. (B) 3. (A) 4.




5. ~~(B)~~ (C)

6. D. #.

## SECTION II

QUESTION 7:

(a) (i) $x(4x-y)(16x^2+4xy+y^2)$

(ii) $(x+3y)^2-16 = (x+3y+4)(x+3y-4)$

(b) (i) $\frac{x(x+4)}{x(x+3)(x-3)} \times \frac{(x+3)(x-2)}{(x+4)(x-2)} = \frac{1}{x-3}$

(ii) $\frac{3^n \times 3^{2n+2}}{3^{6n}} = 3^{2-3n}$

(c) (i) $\sqrt{3}$ (ii) $12\left(\frac{1}{4}\right)\left(1+\frac{4}{3}\right) = 7$

QUESTION 8:

(a) $2x-1 \geq 8$ OR $2x-1 < -8$

$\therefore x \geq 9/2$ OR $x < -7/2$

(b) $x(x+4) = 4 \rightarrow x^2+4x+4=0$

$\therefore \begin{cases} x = 2 \\ y = 2 \end{cases}$

(c) $2x+1 = 3x-2$ OR $2x+1 = -3x+2$

$3 = x$ $5x = 1$

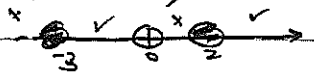
$x = 1/5$

TESTING ✓ ×

$\therefore x = 3$

(d) (i) C.V $x \neq 0$

$x = -3$ OR $x = 2$




$-3 \leq x < 0$ OR $x \geq 2$

(ii) $f(-a) = \frac{-a^3}{2a^2-1}$

$= -\left(\frac{a^3}{2a^2-1}\right)$

$= -f(a)$

## QUESTION 9:

(a) (i) all $x \in \mathbb{R}$ (ii) all $x$, $x \neq \pm 1$

(iii) $x \geq -2$ and $x \leq 4$ i.e. $-2 \leq x \leq 4$

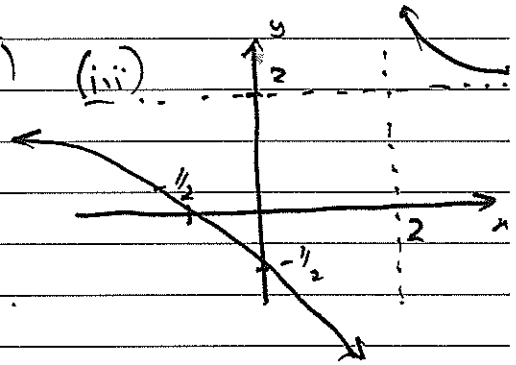
(b) $y = (x-2)^2 - 32$ $\therefore$ D: all $x$

R: $y \geq -32$

(c) $\dfrac{f(m)-f(n)}{n-m} = \dfrac{m^2-n^2}{n-m}$

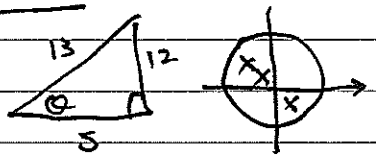
$= -(m+n)$

(d) (i) $y = 2$ (ii) $(0, -\frac{1}{2})$ (iii)




## QUESTION 10:

(a)




$\operatorname{cosec}\theta = \dfrac{1}{\cos\theta}$

$= -\ 13/5$

(b)(i) $30°, 150°, 210°, 330°$ (ii) $\sin\theta(\sin\theta - \cos\theta) = 0$

$\therefore \sin\theta = 0$ or $\tan\theta = 1$

(iii) $2\theta = 0°, 180°, 360°, 540°, 720°$ $\therefore$ $0°, 180°, 360°$ OR $45°, 225°$

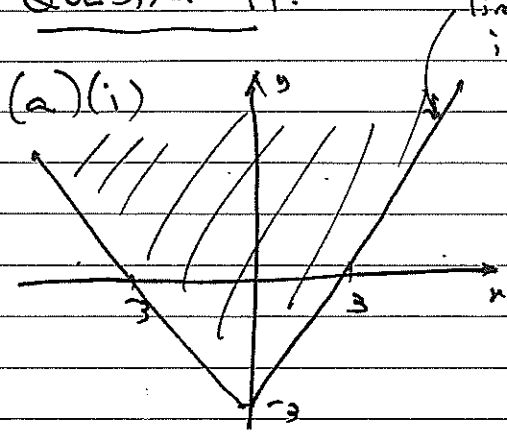
$\therefore \theta = 0°$ or $90°$ or $180°$, or $270°$ or $360°$

(c) $\dfrac{\sin\theta(1+\cos\theta)}{\sin^2\theta} + \dfrac{1-\cos\theta}{\sin\theta} = 2/\sin\theta$

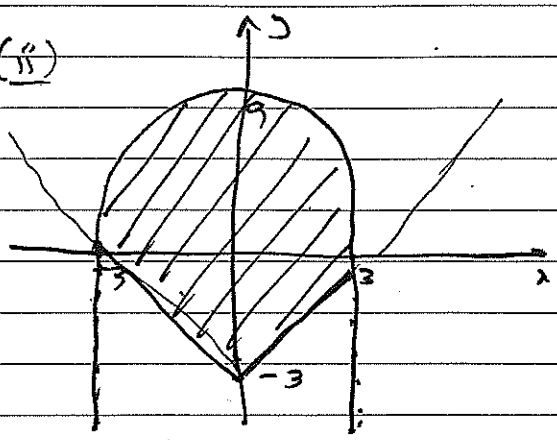
$= 2\operatorname{cosec}\theta$

## QUESTION 11:

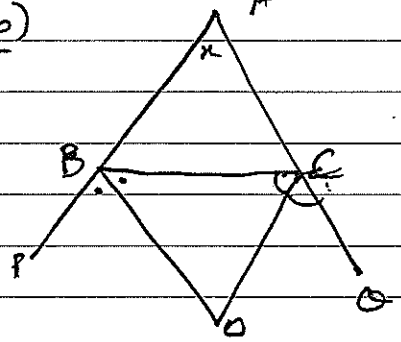
(a)(i)




(ii)




(b)




Let $\angle ACB = y$ $\therefore \angle ABC = 180-(x+y)$

and $\angle BCQ = 180-y$.

$\angle CBP = x+y$

(exterior angle of $\triangle ABC$)

$\therefore \angle OBC = \frac{1}{2}(x+y)$ and

$\angle OCB = 90-\frac{1}{2}y$

$\therefore \angle O = 180-\left(\frac{1}{2}(x+y)+90-\frac{1}{2}y\right)$

(angle sum of $\triangle BCO$)

$\therefore \angle BOC = 180-\frac{1}{2}x-\frac{1}{2}y-90+\frac{1}{2}y$

$= 90-\frac{1}{2}x$.

(c) (i) By cosine rule, $22^2 = 20^2+x^2-2\cdot 20\cdot x\cos 60^\circ$

$\therefore 484 = 400+x^2-20x$

$\therefore x^2-20x-84=0$.

(ii) Area $\triangle PQR = \frac{1}{2}\cdot 20\cdot x\sin 60^\circ$

$= 5\sqrt{3}x$

where $x = \dfrac{20\pm\sqrt{400+336}}{2}$

$= \dfrac{20\pm\sqrt{736}}{2}$

$\approx 23.56$

$\therefore$ Area $\doteq 204.08$

$\approx 204$ km$^2$

QUESTION 12:

(a) For $x > 0$; $\sqrt{x^2}/x = 1$

for $x < 0$, $\sqrt{x^2}/x = -1$

(b) (i) $AC = 4 - 2x$

$\therefore AM = 2 - x$

$\therefore BM = \sqrt{x^2 - (2-x)^2}$

$= \sqrt{4x - 4}$

(ii) Area $= \frac{1}{2} AC \times BM$

$\therefore A = \frac{1}{2}(4-2x)2\sqrt{x-1}$

$= (4-2x)\sqrt{x-1}$

$\therefore \cos \theta/2 = \sqrt{4x-4}/x$

$= \frac{2\sqrt{x-1}}{x}$

(c) (i) $h/OB = \tan 30°$

$\therefore OB = h/\frac{1}{\sqrt{3}}$

$= h\sqrt{3}$

(ii) In $\triangle AOT$, $\frac{h}{AO} = \tan 45$

$\therefore h = AO$

In $\triangle AOB$,

$AO^2 + 40000 = OB^2$

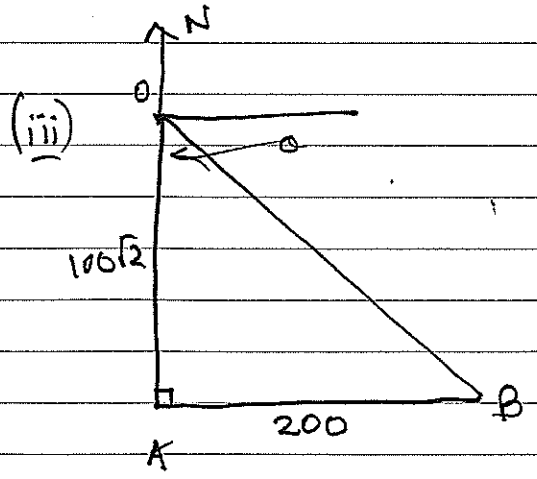
$\therefore AO^2 = 3h^2 - 40000$

$\therefore h^2 = 3h^2 - 40000$

$\therefore h^2 = 20000$

$\therefore h = 100\sqrt{2}$

(iii)




$\tan\theta = 200/100\sqrt{2}$

$= 2/\sqrt{2}$

$= \sqrt{2}$

$\therefore \theta = 54°44'$

$\therefore$ Bearing is $125°16'$