Name: ..... Maths Class: .....

SYDNEY TECHNICAL HIGH SCHOOL

(Est. 1911)

Year 11

Mathematics

Final Examination
Preliminary Course

September, 2014

Time allowed: 2 hours

General Instructions:

- Questions are not of equal value
- Approved calculators may be used
- All necessary working should be shown
- Begin each question on a new page
- Write using black or blue pen
- Full marks may not be awarded for careless work or illegible writing

Section I	Multiple Choice Questions 1-10 10 Marks
Section II	Questions 11-18 86 Marks

Section I **10 marks**

- Colour the correct response on the answer sheet provided
- Each question is worth 1 mark

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1 Use your calculator to evaluate $\dfrac{112.8}{16.1\times 2.93}$ correct to 3 significant figures

(A) 2.39 (B) 2.391 (C) 20.5 (D) 20.528

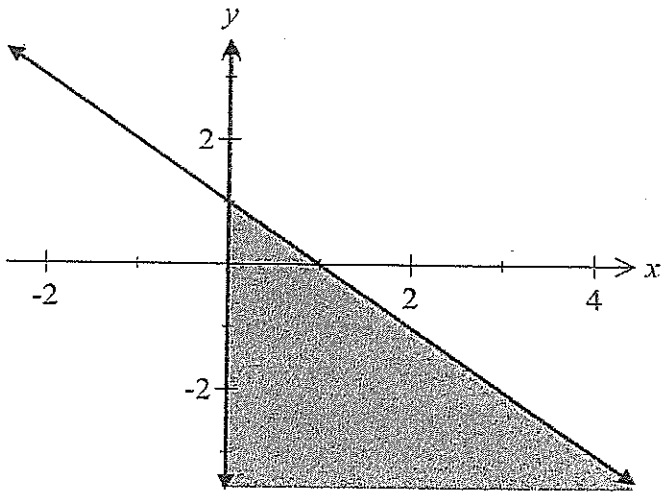
2 Simplify $\dfrac{a^6\times(a^8)^2}{a^2}$

(A) a^8 (B) a^{11} (C) a^{14} (D) a^{20}

3 Make y the subject of the equation $2y=1-xy$

(A) $y=\dfrac{1-x}{3}$ (B) $y=\dfrac{1}{2+x}$ (C) $y=\dfrac{1-xy}{2}$ (D) $y=\dfrac{1-2y}{x}$

4

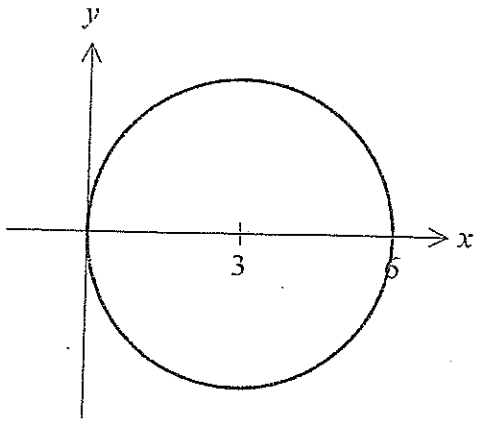


Which pair of inequalities best describes the shaded region in the above diagram

(A) $x+y\le 1$ and $x\ge 0$ (B) $x+y\le 1$ and $y\ge 0$

(C) $x+y\ge 1$ and $x\ge 0$ (D) $x+y\ge 1$ and $y\ge 0$

5



The equation of the circle drawn above is

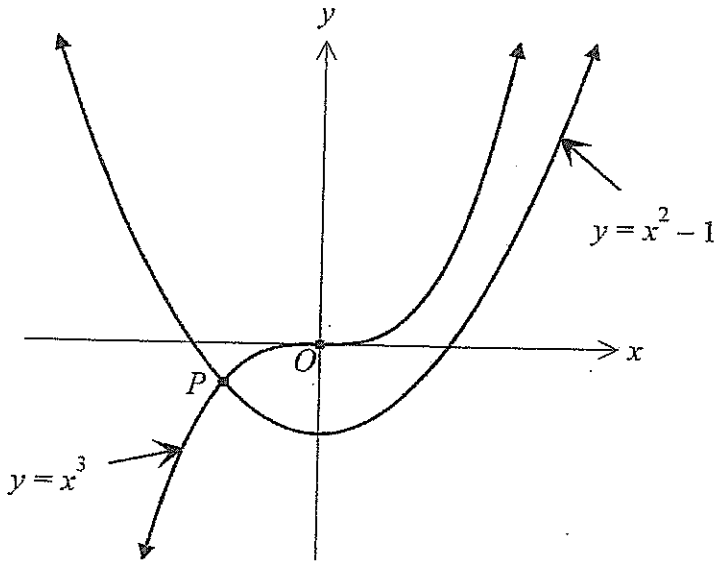
(A) $(x+3)^2+y^2=9$

(B) $(x+3)^2+y^2=36$

(C) $(x-3)^2+y^2=9$

(D) $(x-3)^2+y^2=36$

6



The graphs of $y=x^3$ and $y=x^2-1$ intersect at P.
Which equation gives the x co-ordinate of P?

(A) $x^2-x^3+1=0$ (B) $x^3-x^2+1=0$

(C) $x^3+x^2-1=0$ (D) $x^3+x^2+1=0$

7 Factorise $y^2-(x+3)^2$

(A) $(y-x+3)(x+y+3)$

(B) $(y-x-3)(x+y+3)$

(C) $(y-x+3)^2$

(D) $(y-x-3)^2$

8 If $y=2\sqrt{x}$, then which expression for $\frac{dy}{dx}$ is correct?

(A) $\frac{dy}{dx}=\frac{1}{\sqrt{x}}$

(B) $\frac{dy}{dx}=\frac{2}{\sqrt{x}}$

(C) $\frac{dy}{dx}=\frac{\sqrt{x}}{2}$

(D) $\frac{dy}{dx}=2$

9 Given $180° < \theta < 270°$, which of the following is correct?

(A) $\sin\theta>0$ and $\tan\theta>0$

(B) $\sin\theta>0$ and $\tan\theta<0$

(C) $\sin\theta<0$ and $\tan\theta>0$

(D) $\sin\theta<0$ and $\tan\theta<0$

10 The expression $\frac{b+\sqrt{2b}}{\sqrt{b}}=$

(A) $\sqrt{2}+b$ (B) $\sqrt{3b}$ (C) $3\sqrt{b}$ (D) $\sqrt{2}+\sqrt{b}$

End of Section I

SECTION II

QUESTION 11 (12 Marks)

(a) Fully factorise $x^4 - 27x$ (2)

(b) Solve $|3x - 4| < 8$ (2)

(c) Fully simplify $\frac{4}{m^2-4} - \frac{1}{m-2}$ (3)

(d) Solve simultaneously $\left.\begin{aligned} 3a - 4b &= 11 \\ 2a + 3b &= -4 \end{aligned}\right\}$ (2)

(e) Express $0.21\dot{3}$ as a fraction in its simplest form (show all working steps) (1)

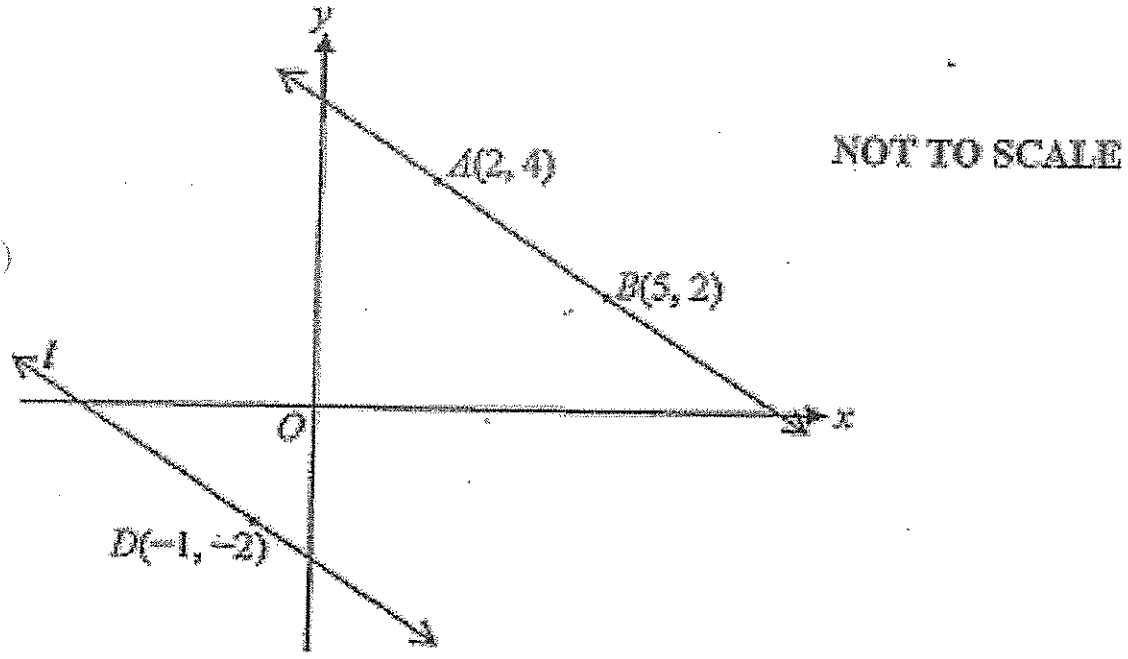
(f) Fully factorise $a^2 - b^2 - a + b$ (2)

QUESTION 12 **(12 Marks) (Start a New Page)**

(a) (i) Sketch the graph of $y = \frac{1}{x-3}$,showing its main features (2)

(ii) State the domain for the function $y = \frac{1}{x-3}$ (1)

(b) Consider the diagram below with points *A* (2, 4) and *B* (5,2).



(i) Find the gradient of the line through *AB*. (1)

(ii) Show that the equation of the line l parallel to the line *AB* and passing through *D* $(-1,-2)$ is $2x + 3y + 8 = 0$. (2)

(iii) Show the distance *OA* is $2\sqrt{5}$ units. (1)

(iv) Find the equation of the circle with centre at the origin and passing through *A*. (1)

(v) The radius *AO* of the circle in part (iv) is produced to *C* on the circumference. Find the co-ordinates of the point *C*. (2)

(vi) State the perpendicular distance from the point *B* to the line l. (2)

QUESTION 13 (12 Marks) (Start a new page)

(a) Fully simplify $\frac{sin^2\theta}{1-cos\theta} - \frac{sin^2\theta}{1+cos\theta}$ (3)

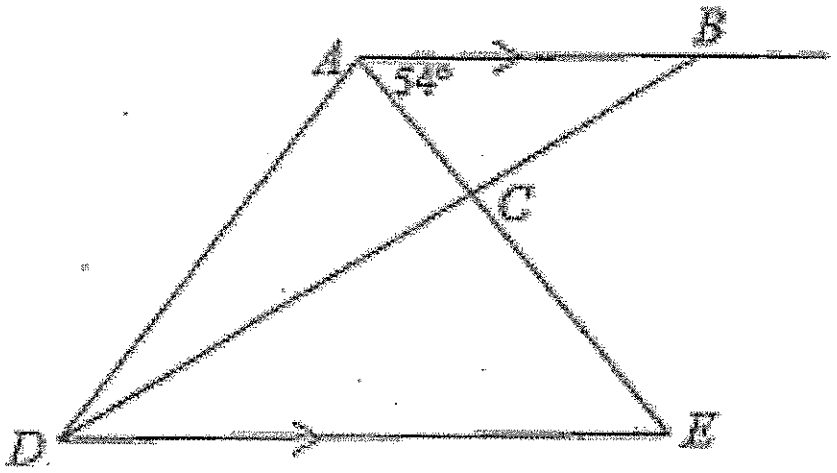
(b) (i) Show that f(x) $= x\sqrt{4-x^2}$ is an odd function. (1)

(ii) If $f(x-1) = \sqrt{3}$, state the value of f($1-x$), where f(x) is the function in part (i). (1)

(c) Solve $2\sin x + 1 = 0$ for $0° \leq x \leq 360°$ (2)

(d) Solve $x^2 - 11x + 24 > 0$ (1)

(e) In the diagram *AB//DE, AE=DE, AE⊥BD and ∠BAC=54°*



Copy the diagram into your answer booklet showing all given information.

Calculate *∠ADB* giving reasons. (3)

(f) Find the value of $\lim_{x\to 3}\left(\frac{x^2-9}{x-3}\right)$ (1)

QUESTION 16 **(10 Marks) (Start a new page)**

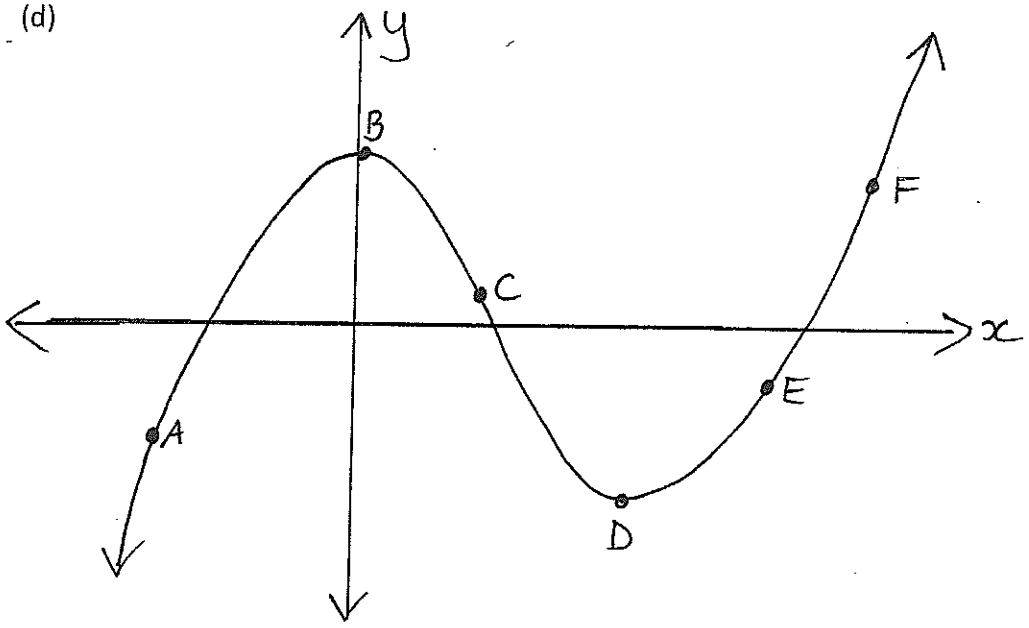
(a) Find the values of *k* for which the quadratic equation $x^2 - (k+2)x + 4 = 0$ has no real roots. (2)

(b) (i) Sketch the graphs of $y = 3 - x$ and $y = |x| - 1$ (2)

(ii) Write down the co-ordinates of all, if any, points of intersection of $y = 3 - x$ and $y = |x| - 1$ (1)

(c) Solve $x^6 - 28x^3 + 27 = 0$ (3)

(d)



Where on the sketch is $(i)\ \frac{dy}{dx} = 0$ (1)

$(ii)\ \frac{dy}{dx} < 0$ (1)

QUESTION 17 (10 Marks) (Start a new page)

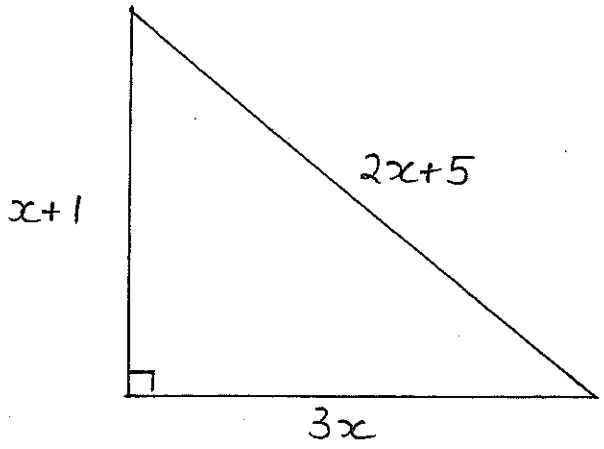
(a) Find the exact value of $\cot(-60°)\ cosec240°$ (1)

(b) Simplify $\frac{3^n \times 9^{n+1}}{27^n}$ (2)

(c) Find the centre and radius of the circle with equation

$$x^2 - 2x + y^2 + 6y - 6 = 0$$ (2)

(d) Form an equation to find the value of x in the following and then solve it to find x (3)



(e) If $cos\theta = \frac{-5}{13}$ and $180° < \theta < 360°$, find $\tan\theta$ as a fraction. (2)

QUESTION 14 **(11 Marks) (Start a new page)**

(a) A function is defined as

$$f(x) = \begin{cases} 1-x & x < 0 \\ x^2+1 & x \geq 0 \end{cases}$$

(i) Find the value of $f(-2) + f(3)$ (1)

(ii) Sketch the graph of $y = f(x)$ (2)

(b) Differentiate $f(x) = x^2 + x$ from first principles. (2)

(c) Differentiate with respect to x

(i) $y = x^2(5x+3)^5$ (2)

(ii) $y = \frac{x^2+1}{3-x}$ (2)

(d) Find the equation of the tangent to the curve $y = x^3 + 5x - 4$ at the point where $x = 1$ (2)

QUESTION 15 **(10 Marks) (Start a new page)**

(a) The roots of the equation $2x^2 - 7x + 12 = 0$ are $\propto$ and β. Find the value of

(i) $\propto + \beta$ (1)

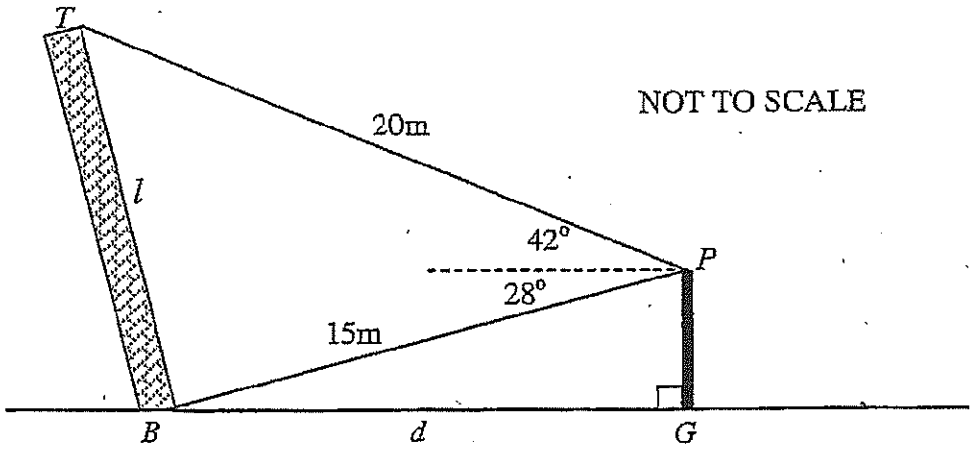
(ii) $\propto \beta$ (1)

(iii) $\frac{1}{\propto} + \frac{1}{\beta}$ (1)

(iv) $\propto^2 + \beta^2$ (1)

(b) From a point *P* above the ground, the angle of elevation to the top of a leaning wall *T* is 42° and the angle of depression to the base of the leaning wall B is 28°.

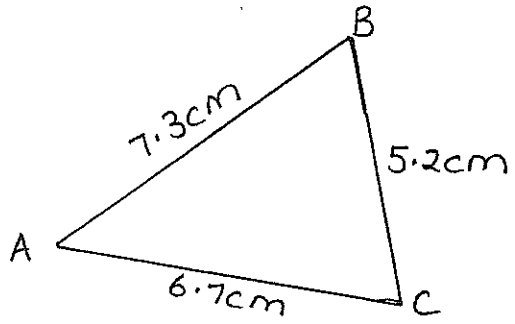
A wire 20m long is attached from this point *P* to the top of the leaning wall T and another 15m wire is attached to the base of the leaning wall B.



(i) Find the horizontal distance *(d)* in the diagram above (correct to 1 decimal place) (1)

(ii) Calculate the slant height (l) of the leaning wall (correct to 1 decimal place) (2)

(c)



(i) Calculate the size of the smallest angle in ΔABC. Give your answer correct to the nearest minute. (2)

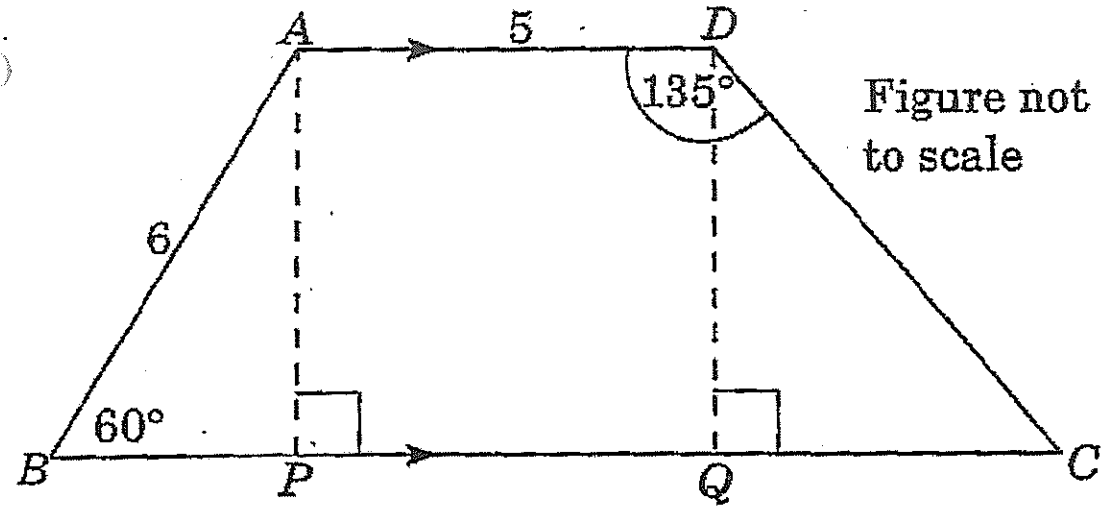
(ii) Hence find the area of the triangle correct to the nearest cm² (1)

QUESTION 18 (9 Marks) (Start a new page)

(a) The line $5x + ky = 4$ passes through the point $(-2,1)$. Find the value of k. (1)

(b) If $f(x) = \sqrt[3]{x}$, find $f'(8)$ as a fraction. (2)

(c)



The diagram shows a trapezium *ABCD* in which *AD* is parallel to *BC*. *AB = 6, AD = 5,* $\angle ABC = 60°$ and $\angle ADC = 135°$.

Perpendiculars are drawn from *A* and *D* to meet *BC* at *P* and *Q*.

(i) Show that *BP = 3* (1)

(ii) Show that $AP = 3\sqrt{3}$ (1)

(iii) Find the exact value of *BC* (2)

(iv) Find the area of the trapezium *ABCD*. Leave your answer in surd form. (2)

END OF PAPER

2U Maths Yr 11 Final

Section I

1. A
2. D
3. B
4. A
5. C
6. B
7. B
8. A
9. C
10. D

QUESTION 11

(a) $= x(x^3-27)$

$= x(x-3)(x^2+3x+9)$

(b) $3x-4<8$ $\quad$ $3x-4>-8$

$3x<12$ $\quad$ $3x>-4$

$x<4$ $\quad$ $x>-\frac{4}{3}$

$-\frac{4}{3}<x<4$

(c) $\frac{4}{(m-2)(m+2)} - \frac{1}{m-2}$

$= \frac{4-(m+2)}{(m-2)(m+2)}$

$= \frac{4-m-2}{(m-2)(m+2)}$

$= \frac{2-m}{(m-2)(m+2)}$

$= \frac{-(m-2)}{(m-2)(m+2)}$

$= \frac{-1}{m+2}$

(d) $3a-4b=11$ $\times 2$

$2a+3b=-4$ $\times 3$

$6a-8b=22$

$6a+9b=-12$ $-$

$-17b=34$

$b=-2$

$3a+8=11$

$3a=3$

$a=1$

(e) Let $x=0.213333\ldots$

$100x=21.3333\ldots$

$1000x=213.3333\ldots$

$900x=192$

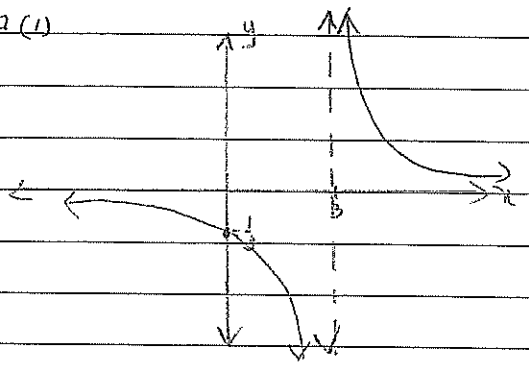
$x=\frac{192}{900}=\frac{16}{75}$

(f) $=(a-b)(a+b)-(a-b)$

$=(a-b)(a+b-1)$

QUESTION 12

a (i)



(ii) Domain: all real x, $x \neq 3$

(b) (i) $m = \frac{2-4}{5-2} = \frac{-2}{3}$

(ii) $(-1,-2)$ $m = -\frac{2}{3}$

$y+2 = -\frac{2}{3}(x+1)$

$3y+6 = -2x-2$

$2x+3y+8=0$

(iii) $(0,0)$ $(2,4)$

$d = \sqrt{(4-0)^2+(2-0)^2}$

$d = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

(iv) $x^2+y^2 = (2\sqrt{5})^2$

$x^2+y^2 = 20$

(v) $(0,0) = \left(\frac{x+2}{2}, \frac{y+4}{2}\right)$

$\frac{x+2}{2} = 0$ $\quad \frac{y+4}{2} = 0$

$x = -2$ $\quad y = -4$ $\quad C(-2,-4)$

(vi) $d = \frac{|5(2)+3(2)+8|}{\sqrt{5^2+3^2}}$ $\quad (\overset{x}{5},\overset{y}{2})$ $\quad \overset{a}{2}x+\overset{b}{3}y+\overset{c}{8}=0$

$d = \frac{|10+6+8|}{\sqrt{13}} = \frac{24}{\sqrt{13}} = \frac{24\sqrt{13}}{13}$.

QUESTION 13

(a) $\frac{\sin^2\theta}{1-\cos\theta} - \frac{\sin^2\theta}{1+\cos\theta}$

$= \frac{\sin^2\theta(1+\cos\theta) - \sin^2\theta(1-\cos\theta)}{(1-\cos^2\theta)(1+\cos\theta)}$

$= \frac{\sin^2\theta + \sin^2\theta\cos\theta - \sin^2\theta + \sin^2\theta\cos\theta}{1-\cos^2\theta}$

$= \frac{2\sin^2\theta\cos\theta}{\sin^2\theta} = 2\cos\theta$

(b) (i) $f(x) = x\sqrt{4-x^2}$

$f(-x) = -x\sqrt{4-(-x)^2} = -x\sqrt{4-x^2}$

$-f(x) = -x\sqrt{4-x^2}$

$\therefore$ as $f(-x) = -f(x)$ function is odd

(ii) $f(1-x) = -\sqrt{3}$

(c) $2\sin x + 1 = 0$ $\quad 0 \leq x \leq 360$

$\sin x = -\frac{1}{2}$

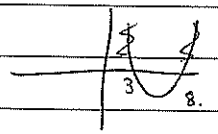
$x = 210°, 330°$

S	A
T	C

30°

(d) $(x-8)(x-3) > 0$

$x < 3$ $\quad x > 8$.



(e) $\angle AED = 54°$ (alt angles eq in $\parallel$ lines)

$\angle EAD = \angle ADE = 63°$ (equal angles of isosceles triangle)

$\angle ABC = 36$ (angle sum of $\triangle ABC$

$\therefore \angle ADB = 27°$ angle sum of $\triangle ADB$

(f) $= \lim_{x \to 3} \frac{(x-3)(x+3)}{(x-3)}$

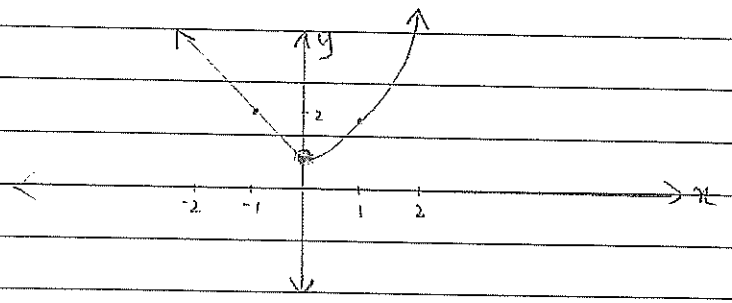
$= 6$.

QUESTION 14

(a) (i) $f(-2) + f(3)$

$= (1--2) + (9+1) = 13$

(ii)



(b) $f(x) = x^2 + x$

$f(x+h) = (x+h)^2 + x + h$

$= x^2 + 2xh + h^2 + x + h$

$f'(x) = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 + x + h - x^2 - x}{h}$

$= \lim_{h \to 0} \frac{h(2x + h + 1)}{h}$

$= 2x + 1$

(c) (i) $y = x^2(5x+3)^5$

$y' = (5x+3)^5 \cdot 2x + x^2 \cdot 5(5x+3)^4 \cdot 5$

$= 2x(5x+3)^5 + 25x^2(5x+3)^4$

$= x(5x+3)^4[2(5x+3) + 25x]$

$= x(5x+3)^4(10x + 6 + 25x$

$= x(5x+3)^4(35x+6)$

(ii) $y = \frac{x^2+1}{3-x}$ $\quad u, v$

$y' = \frac{(3-x)2x - (x^2+1)\cdot -1}{(3-x)^2}$

$y' = \frac{6x - 2x^2 + x^2 + 1}{(3-x)^2}$

$y' = \frac{6x - x^2 + 1}{(3-x)^2}$

(d) $y = x^3 + 5x - 4$

$y' = 3x^2 + 5$

At $x = 1$ $\quad y' = 8$

At $x = 1$ $\quad y = 2$

Eqn of tangent

$y - 2 = 8(x - 1)$

$y - 2 = 8x - 8$

$y = 8x - 6$

QUESTION 15

(a) (i) $\alpha + \beta = \frac{7}{2}$

(ii) $\alpha\beta = \frac{12}{2} = 6$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{7}{2}}{6} = \frac{7}{12}$

(iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= \left(\frac{7}{2}\right)^2 - 2.6$

$= \frac{1}{4}$

(b) (i) $\cos 28 = \frac{d}{15}$

$d = 13.2$

(ii) $l^2 = 15^2 + 20^2 - 2 \times 15 \times 20 \times \cos 70$

$l^2 = 419.787914$

$l = 20.5$

(c) (i) $\cos A = \frac{7.3^2 + 6.7^2 - 5.2^2}{2 \times 7.3 \times 6.7} = \frac{71.14}{97.82}$

$A = 43°21'$

(ii) $A = \frac{1}{2} \times 6.7 \times 7.3 \times \sin 43°21'$

$A = 16.787 cm^2$

$A = 17 cm^2$

QUESTION 16

(a) $\Delta = (k+2)^2 - 4.1.4$

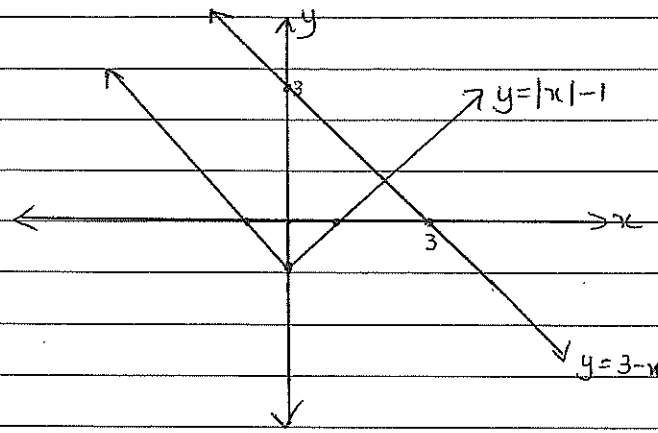
$\Delta = (k+2)^2 - 16$

For no real roots $(k+2-4)(k+2+4) < 0$

$(k-2)(k+6) < 0$

$-6 < k < 2$

b (i)



$3-x = x-1$

$-2x = -4$

$x = 2$

$y = 1$

(2,1) pt of intersection

(d) Solve $x^6 - 28x^3 + 27 = 0$

Let $y = x^3$

$y^2 - 28y + 27 = 0$

$(y-27)(y-1) = 0$

$y = 27 \quad y = 1$

$x^3 = 27 \quad x^3 = 1$

$x = 3, 1$

(e) (i) B + D

(ii) C

QUESTION 17

(a) $\frac{-1}{\sqrt{3}} \times \frac{-2}{\sqrt{3}} = \frac{2}{3}$

(b) $\frac{3^n \times (3^2)^{n+1}}{(3^3)^n}$

$= \frac{3^n \times 3^{2n+2}}{3^{3n}}$

$= \frac{3^{3n+2}}{3^{3n}} = 9$

(c) $x^2 - 2x + 1 + y^2 + 6y + 9 = 6 + 1 + 9$

$(x-1)^2 + (y+3)^2 = 16$

Centre $(1,-3)$ radius $= 4$

(d) $(x+1)^2 + (3x)^2 = (2x+5)^2$

$x^2 + 2x + 1 + 9x^2 = 4x^2 + 20x + 25$

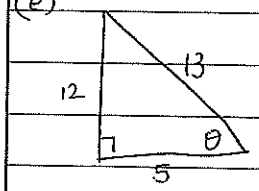
$6x^2 - 18x - 24 = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1 \quad x \neq -1 \quad \therefore x = 4$

(e)



$\tan\theta = \frac{12}{5}$

S | A

T | C

QUESTION 18

(a) $5(-2)+k=4$

$-10+k=4$

$k=14$

(b) $f(x)=x^{1/3}$

$f'(x)=\frac{1}{3}x^{-2/3}$

$f'(8)=\frac{1}{3}(8)^{-2/3}$

$=\frac{1}{12}$

(c) (i) $\cos 60=\frac{BP}{6}$

$BP=6\cos 60$

$BP=6\times\frac{1}{2}=3$

(ii) $AP^2=6^2-3^2$

$AP^2=36-9$

$AP^2=27$

$AP=\sqrt{27}=3\sqrt{3}$.

(iii) $\frac{QC}{3\sqrt{3}}=\tan 45$

$QC=3\sqrt{3}$.

$\therefore BC=8+3\sqrt{3}$.

(iv) $A=\frac{1}{2}.3\sqrt{3}\left(5+8+3\sqrt{3}\right)$

$A=\frac{3\sqrt{3}}{2}\left(13+3\sqrt{3}\right)$ U^2