Name: ..... Maths Class: .....

SYDNEY TECHNICAL HIGH SCHOOL

Year 10 Common

June 2014

Mathematics

Examination Booklet

Time allowed: 70 mins

Instructions:

- Write your name and class at the top of this page.
- These questions must be answered in the booklet provided
- Start every question on a new page
- Attempt all questions.
- Calculators may be used

Question 1: Simultaneous Equations and Trigonometry (15 marks)

(a) Solve simultaneously:

(i) $x + y = 8$ *(2 marks)*
$2x - y = 16$

(ii) $3x + 2y = 10$ *(3 marks)*
$4x + 3y = 13$

(b) Use the grid provided to draw and give the coordinates of the point of intersection of $y = x + 2$ and $y = -x + 4$. Clearly mark the point of intersection. *(4 marks)*

(c) A plane leaves Adelaide (A) and flies 1 200 km due north to Tyler Downs (T) station. From there it flies on a bearing 248° until it is due west of Adelaide.

(i) Draw a sketch to show the information above *(2 marks)*

(ii) How far has it flown on the bearing 248° when it is due west of Adelaide to the nearest kilometre? *(2 marks)*

(iii) Altogether it flies 5 000 km on the bearing 248°. It then flies due east until it is south of Adelaide. How far due south of Adelaide is the plane to 2 decimal places? *(2 marks)*

Question 2: Quadratic Equations (15 marks)

(a) Solve the following quadratic equations:

(i) $2x(5 - x) = 0$ *(1 mark)*

(ii) $2x^2 - 8 = 0$ *(2 marks)*

(iii) $(3x - 1)(40 - 2x) = 0$ *(2 marks)*

(iv) $2x^2 - x - 3 = 0$ *(3 marks)*

(v) $\frac{x}{x+2} = \frac{x-2}{2x+5}$ *(3 marks)*

(b) Solve the following equation $x^2 - 8x - 4 = 0$ by the method of completing the square. Leave your answer in exact form and fully simplified. *(3 marks)*

(c) Use the general quadratic formula to solve $3x^2 - 7x - 2 = 0$. Leave your answer in exact form. *(2 marks)*

Question 3: Probability (15 marks)

(a) A hundred tickets are to be sold in a raffle, a man buys ten tickets. Two different tickets are to be drawn out for first and second prizes respectively. .

(i) Fill in the probability tree provided. ***(2 marks)***

Find the probabilities that:

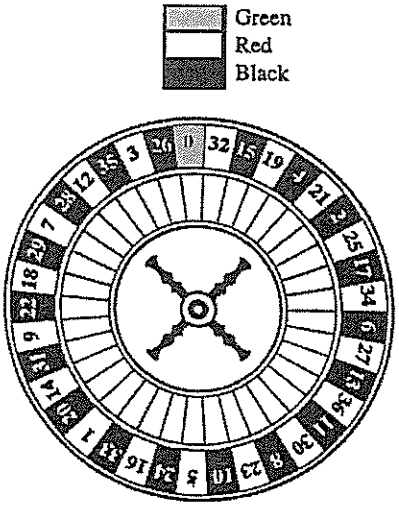
(ii) he wins the first prize. ***(1 mark)***

(iii) he wins both prizes. ***(1 mark)***

(iv) he loses both prizes. ***(1 mark)***

(v) he wins at least one prize. ***(1 mark)***

(b) A roulette wheel has 37 numbers from 0 to 36. Each number is coloured red or black, except for 0, which is coloured green. (the number 0 is not included in any column or row)



0		
1	2	3
4	5	6
7	8	9
10	11	12
13	14	15
16	17	18
19	20	21
22	23	24
25	26	27
28	29	30
31	32	33
34	35	36

For a roulette wheel, what is the probability of spinning:

(i) a 7? ***(2 marks)***

(ii) an even number? ***(1 mark)***

(iii) a number from 1 to 18? ***(1 mark)***

What is the probability of winning when betting on:

(iv) a column of numbers? ***(1 mark)***

(v) two rows of numbers? ***(1 mark)***

(vi) 4 numbers? ***(1 mark)***

(c) Consider the numbers 1 to 9.

(i) Find the number of 4 digit numbers that can be made using the digits 1 to 9 if each number can only be used once. ***(1 mark)***

(ii) How many 4 digit numbers starting with 6 can be formed? ***(1 mark)***

Question 4: Consumer Arithmetic (15 marks)

(a) Neil borrowed \$40 000 to buy a car. He paid the money back, plus interest, in equal monthly payments of \$720 for the next 5 years.

(i) How much interest did he pay? *(2 marks)*

(ii) Calculate the annual rate of simple interest. *(2 marks)*

(b) Find the final amount when \$19500 is invested for 3 years at 4% pa:

(i) compounded yearly ***(2 marks)***

(ii) compounded monthly ***(1 mark)***

(c) Frank borrows \$400 000 to buy a house. The interest rate is 6% p.a. (compounded monthly). He makes repayments of \$2 400 each month.

(i) How much will be owing at the end of the first month after the interest has been added to the principal but no repayment has been made? ***(1 mark)***

(ii) How much interest will be charged in the second month? ***(3 marks)***

(d) Ray's X Box depreciates by 13% each year. If the cost at purchase was \$547, how much will it be worth after 4 years? ***(1 mark)***

(e) How many years would it take to cause a machine valued at \$53 000 to drop below \$21000 in value if the rate of depreciation is 12.5% pa.? (HINT: use trial and error) ***(3 marks)***

Question 5: Co-ordinate Geometry (15 marks)

(a) Find the equation of the line which:

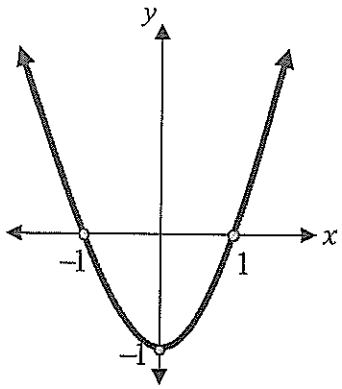
(i) is perpendicular to the y-axis and passes through the point $(-3, -2)$. ***(1 mark)***

(ii) passes through the point $(-2, 5)$ and parallel to $y = 2x + 4$. ***(2 marks)***

(iii) is perpendicular to $4x + 2y - 10 = 0$ and has a y-intercept of -2. ***(2 marks)***

(b) Use the space provided to show the region represented by the following inequations: $y < x + 3$ and $y \leq -x + 1$. ***(3marks)***

(c) Write the equation of the following parabola. ***(1 mark)***



(d) For the parabola $y = x^2 - 3x - 4$:

(i) Find the equation of the axis of symmetry. ***(1 mark)***

(ii) Find the minimum value of the parabola. ***(1 mark)***

(iii) Find the x and y – intercepts. ***(2 marks)***

(iv) Sketch the graph of the parabola clearly marking the x and y – intercepts and vertex. ***(2 marks)***

END OF EXAMINATION

Name: Solutions Maths Class: .....

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Topic	Question	Topic Total
Simultaneous. Eqns and Trigonometry	1	/15
Quadratic Equations	2	/15
Probability	3	/15
Consumer Arithmetic	4	/15
Co-ordinate Geometry	5	/15
	TOTAL	/75

Question 1: Simultaneous Equations and Trigonometry (15 marks)

(a) (i) $x+y=6$ ①

$2x-y=16$ ②

① + ②

2 marks

$3x=24$

$x=8$

$8+y=8$

$y=0$ ∴ POI $(8,0)$

(ii) $3x+2y=10$ ①

$4x+3y=13$ ②

$y=-1$

$3\times$①, $2\times$②

∴ POI $(4,-2)$

$9x+6y=30$ ③

$8x+6y=26$ ④

③ − ④

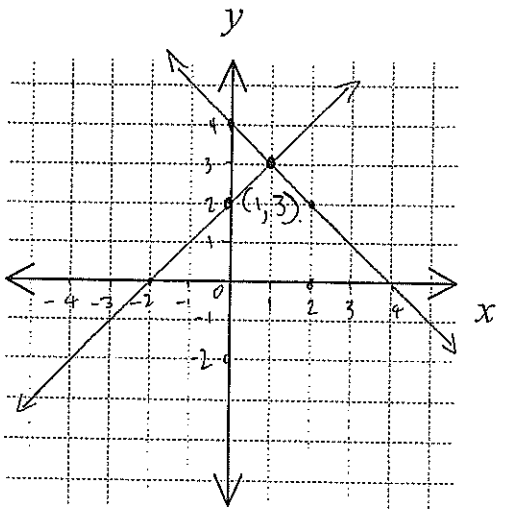
3marks

$x=4$

$3\times4+2y=10$

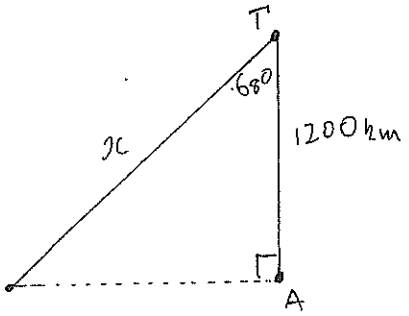
$2y=-2$

(b)



4 marks

(c) (i)



2 marks

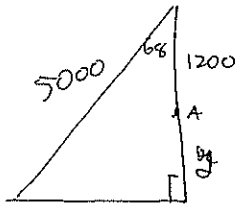
(ii) $\frac{1200}{x} = \cos 68$

$x = \frac{1200}{\cos 68}$

$= 3203.36$

2 marks

(iii) $\frac{y+1200}{5000} = \cos 68$



$y + 1200 = 5000 \cos 68$

$y = 5000 \cos 68 - 1200$

$= 673.03$ km

2 marks

Question 2: Quadratic Equations (15 marks)

(a) (i) $2x = 0 \quad 5 - x = 0$

$x = 0$ and 5

1 mark

(ii) $2x^2 = 8$

$x^2 = 4$

$x = \pm 2$

2 marks

(iii) $3x - 1 = 0$ & $40 - 2x = 0$

$x = \frac{1}{3}$ and 20

2 marks

(iv) $(2x - 3)(x + 1) = 0$

$2x - 3 = 0 \quad x + 1 = 0$

$x = \frac{3}{2}$ and -1

3 marks

(v) $x(2x + 5) = (x - 2)(x + 2)$

$2x^2 + 5x = x^2 - 4$

$x^2 + 5x + 4 = 0$

$(x + 4)(x + 1) = 0$

$x = -4$ and -1

3 marks

(b) $x^2 - 8x + 16 = 4 + 16$

$(x - 4)^2 = 20$

$x - 4 = \pm\sqrt{20}$

$x = 4 \pm 2\sqrt{5}$

3 marks

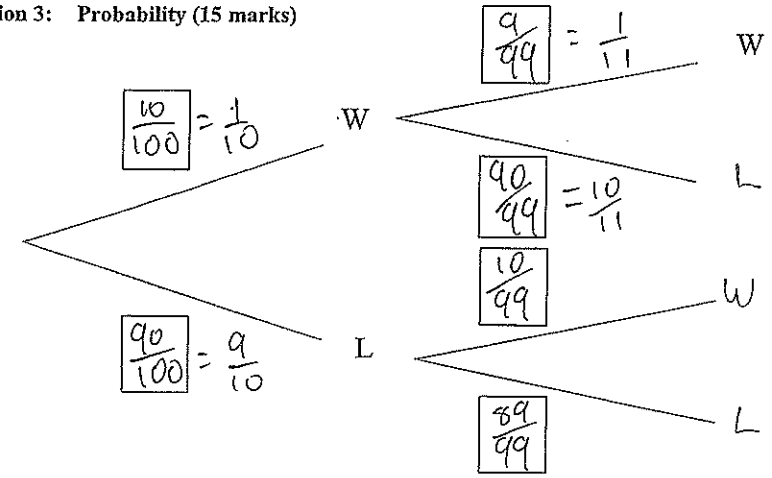
(c) $x = \frac{7 \pm \sqrt{(-7)^2 - 4 \times (3) \times (-2)}}{2 \times 3}$

$= \frac{7 \pm \sqrt{73}}{6}$

2 marks

Question 3: Probability (15 marks)

(a) (i)



2 marks

(ii) $P(W) = \frac{10}{100}$.

$= \frac{1}{10}$ or 0.1

1 mark.

(iii) $P(WW) = \frac{1}{10} \times \frac{1}{11}$

$= \frac{1}{110}$

1 mark

(iv) $P(LL) = \frac{9}{10} \times \frac{89}{99}$

$= \frac{801}{990} = \frac{89}{110}$

1 mark

(v) $P(W \geq 1) = 1 - \frac{89}{110}$

$= \frac{21}{110}$

1 mark

(b) (i) $\frac{1}{37}$

2 marks

(ii) $\frac{18}{37}$

1 mark

(iii) $\frac{18}{37}$

1 mark

(iv) $\frac{12}{37}$.

1 mark

(v) $\frac{6}{37}$

1 mark

(vi) $\frac{4}{37}$.

1 mark

(c) (i) $9 \times 8 \times 7 \times 6$

$= 3024$

1 mark

(ii) $1 \times 8 \times 7 \times 6$ or $1 \times 9 \times 9 \times 9$

$= 336$ $= 729$

1 mark

Question 4: Consumer Arithmetic (15 marks)

(a) (i) $I = 720 \times 5 \times 12 - 40000$

$= \$3200$ — 2 marks

(ii) $40000 \times r \times 5 = 3200$

$200000r = 3200$

$r = \frac{3200}{200000} \times 100 = 1.6\%$ — 2 marks

(b) (i) $A = 19500(1+0.04)^3$

$= \$21934.84$ — 2 marks

(ii) $A = 19500\left(1+\frac{0.04}{12}\right)^{3\times12}$

$= \$21981.80$ — 1 mark

(c) (i) $400000\left(1+\frac{0.06}{12}\right)$

$= \$402000$ — 1 mark

(ii) $402000 - 2400$

$= 399600 \times \frac{0.06}{12}$

$= \$1998$ — 3 marks

(d) $D = 547(1-0.13)^4$

$= \$519.11$ — 1 mark

(e) $53000(1-0.125)^n < 2100$ — 3 marks

$n = 7$, after 7 years

Question 5: Co-ordinate Geometry (15 marks)

(a) (i) $y = -2$ — 1 mark

(ii) $m = 2$ $\quad y - 5 = 2(x+2)$

$y - 5 = 2x + 4$

$y = 2x + 9$

or $2x - y + 9 = 0$ — 2 marks

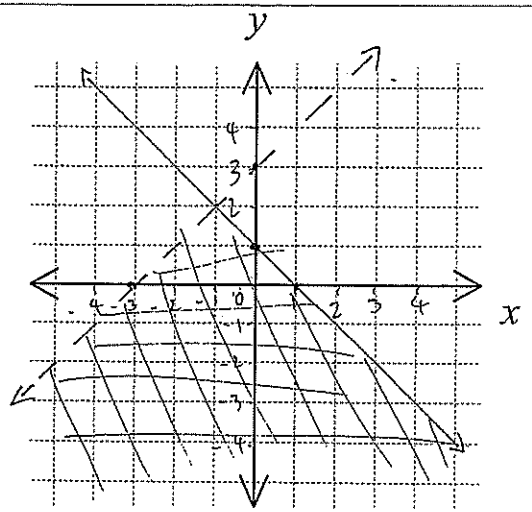
(iii) $2y = -4x + 10$ $\quad \therefore \quad y = \frac{1}{2}x - 2$

$y = -2x + 5$

$m = -2$

$\therefore m_p = \frac{1}{2}$ — 2 marks

(b)



3 marks

(c) $y = x^2 - 1$ or $y = (x-1)(x+1)$ — 1 mark

(d) (i) $x = \frac{-(-3)}{2 \times 1}$

$= \frac{3}{2}$ or $1\frac{1}{2}$

1 mark

(ii) $y = \left(\frac{3}{2}\right)^2 - 3\left(\frac{3}{2}\right) - 4$

$= \frac{-25}{4}$ or $-6\frac{1}{4}$

1 mark

(iii) $y = 0$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = 4, -1$

x-intercepts

$x = 0$

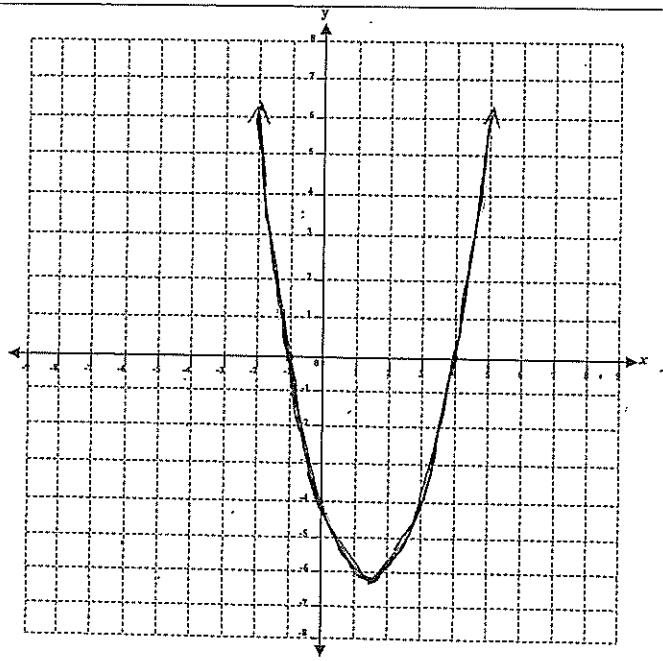
$y = 0^2 - 3 \times 0 - 4$

$= -4$

y-intercept

2 marks

(iv)



2 marks

END OF EXAMINATION