Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics Extension 1

HSC Course

Assessment 1

December, 2016

Time allowed: 90 minutes

General Instruction

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- BOSTES reference sheet is located at the end of the exam.

Section 1 Multiple Choice
Questions 1-7
7 Marks

Allow approximately 10 minutes for this section

Section II Questions 8-13
48 Marks

Allow approximately 80 minutes for this section

Section 1

7 marks
Attempt Questions 1 – 7
Allow about 10 minutes for this section
Use the Multiple Choice answer sheet for questions 1 – 7

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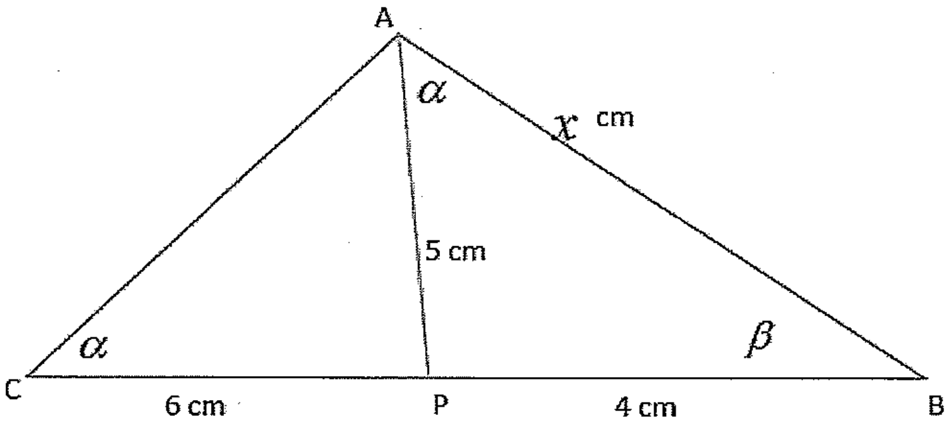
1. By considering ΔABC and Δ PBA the value of x is:

NOT TO SCALE



(A) 14

(B) $\frac{8}{3}$

(C) $2\sqrt{10}$

(D) $5\sqrt{2}$

---

2. For $x>1$, which one of the following expressions represents the limiting sum of this series?

$$1-\frac{1}{x}+\frac{1}{x^2}-\frac{1}{x^3}+\frac{1}{x^4}-\dots\dots\dots\dots$$

(A) $\frac{x}{1+x}$

(B) $\frac{x}{1-x}$

(C) $\frac{1+x}{x}$

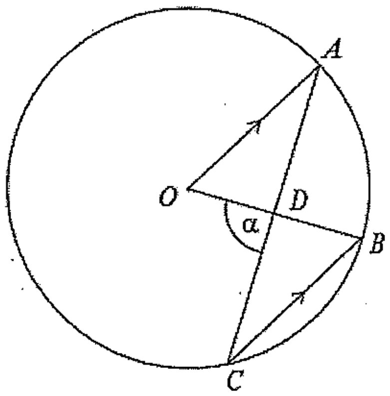
(D) $\frac{1-x}{x}$

---

3. The points A, B and C lie on the circle with centre O. OA is parallel to CB.

AC intersects OB at D and $\angle$ODC=α.

What is the size of $\angle$OAD in terms of α?



(A) $\frac{\alpha}{2}$

(B) $\frac{\alpha}{3}$

(C) $\frac{2\alpha}{3}$

(D) 3α

---

4. If $x=t^2$ and $y=\sqrt{t}$ which of the following is an expression for, $\frac{dy}{dx}$?

(A) $t^{0.5}$

(B) $x^{-0.25}$

(C) $\frac{3}{2}x^{-0.5}$

(D) $\frac{1}{4}t^{-1.5}$

---

5. When $g(x)$ is divided by x^2+x-12 the remainder is $(5x+9)$.

What is the remainder when $g(x)$ is divided by $(x+4)$?

(A) -11

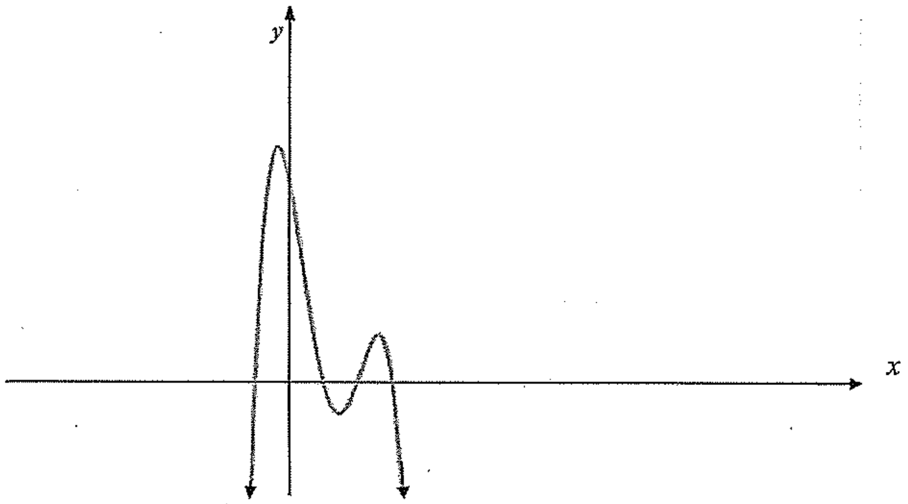
(B) -8

(C) 0

(D) 29

---

6. The graph below shows a polynomial function $y = P(x)$.



C

Which of the following could represent the equation of $y = P(x)$?

(A) $P(x) = (x+1)(x+2)(x+3)(x-1)$

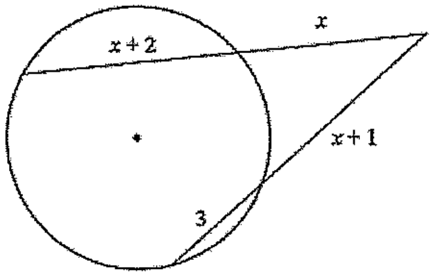
(B) $P(x) = (x+1)(x+2)(x+3)(1-x)$

(C) $P(x) = (x+1)(x-2)(x-3)(x-1)$

(D) $P(x) = (x+1)(x-2)(x-3)(1-x)$

---

7. Two secants from an external point cut off intervals on a circle as shown below.



C

What is the value of x?

(A) $\dfrac{1+\sqrt{14}}{2}$

(B) 4

(C) $\dfrac{-3+\sqrt{73}}{4}$

(D) 5

End of Section 1

Section II
Attempt Questions 8 – 13
Allow about 1 hour and 20 minutes for this section.
Answer each question in your answer booklet STARTING EACH QUESTION ON A NEW PAGE.

In Questions 8 – 13 your responses should include all relevant mathematical reasoning and / or calculations.

Question 8 - 8 marks

a. A series is given as;

$24 + 16 + 8 + 0 + \ldots\ldots\ldots$

i. Find an expression for the nth term of this series **(2)**

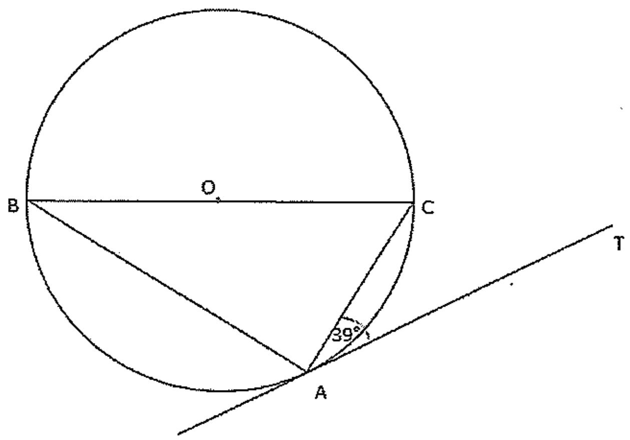
ii. Which term in this series has the value of -192? **(1)**

b. Show that the Cartesian equation of the curve defined by the parametric equation

$$x = t - 1, \qquad y = t^2 + t - 1$$

is a parabola and state the co-ordinates of its vertex. **(2)**

c. In the circle, centre O, BC is a diameter. AT is a tangent to the circle at the point A and $\angle$ TAC = 39°.



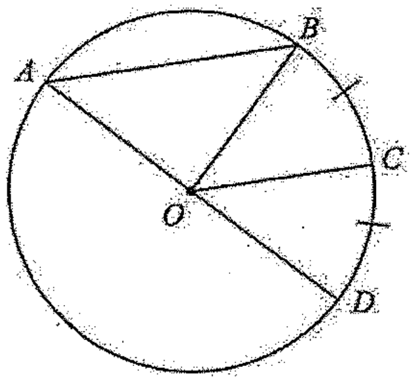
i. Explain the theorem that supports $\angle ABC = 39°$ **(1)**

ii. Hence, find, with reasons, the size of $\angle BCA$ **(2)**

Question 9 - 8 marks

Begin this question on a NEW PAGE in your answer booklet.

a. In the diagram below, the points A, B, C and D are concyclic. The point O is the centre of the circle, AD is a diameter of the circle and the arc lengths BC and CD are equal.



i. State why $\angle$ BOC= $\angle$ COD **(1)**

ii. Show that AB is parallel to OC, giving clear and full reasons. **(2)**

b. The roots of $x^3 - x^2 - 5x + 2 = 0$ are given as α, β and χ

i. Find the value of $\alpha\beta\chi$ **(1)**

ii. Find the value of $\alpha\beta + \alpha\chi + \beta\chi$ **(1)**

iii. Show that $\beta + \chi = 1 - \alpha$ and $\dfrac{\beta+\chi}{\alpha} = \dfrac{1}{\alpha} - 1$ **(2)**

iv. Hence, evaluate,

$$\frac{\beta+\chi}{\alpha} + \frac{\chi+\alpha}{\beta} + \frac{\alpha+\beta}{\chi}$$ **(1)**

Question 10 - 8 marks

Begin this question on a NEW PAGE in your answer booklet.

a. Consider an Arithmetic series where,

$$T_5 = 3 \times T_2 \text{ and } S_6 = 144$$

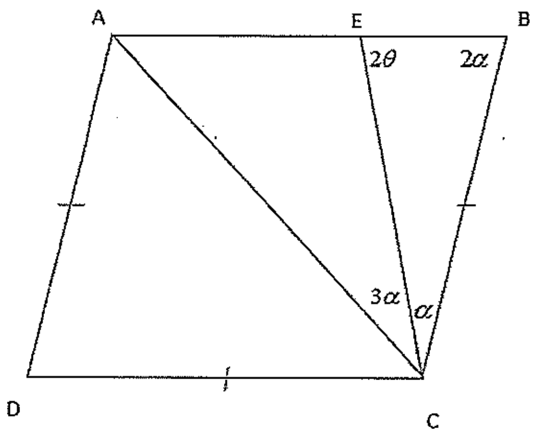
Find the value of the third term. **(3)**

b. Evaluate $\sum_{n=0}^{5} 3n^2 - 1$ **(1)**

c. ABCD is a rhombus. The point E lies on side AB as shown below.

$\angle$CBE$=2\alpha$ and $\angle$CEB$=2\theta$

$\angle$ACE$=3\alpha$ and $\angle$ECB$=\alpha$



i. Without finding the value of α or θ, Show that $\theta = \dfrac{7\alpha}{2}$

(include **clear** reasoning) **(2)**

ii. Find the values of α and θ **(2)**

Question 11 - 8 marks

Begin this question on a NEW PAGE in your answer booklet.

a. Show that;

$$(a+2)(a^2-3a+5)=a^3-a^2-a+10 \qquad \textbf{(1)}$$

b. Each term of the arithmetic sequence $a, a+d, a+2d, ...$ is added to the corresponding term of the geometric sequence b, ab, ba^2 ,...... to form a third sequence, S.

i. Show that the first three terms of the sequence S can be written as;

$$(a+b),(a+ab+d),(a+ba^2+2d) \qquad \textbf{(1)}$$

ii. Given that the first three terms of the sequence S have the values -1, -2 and 6 respectively. Show that,

$$a^3-a^2-a+10=0 \qquad \textbf{(2)}$$

iii. Given that a is real find the value of a and b. **(2)**

iv. Hence, show that the nth term of S is given by,

$$T_n=2(n-2)+(-2)^{n-1} \qquad \textbf{(2)}$$

Question 12 - 8 marks

Begin this question on a NEW PAGE in your answer booklet.

Hilton borrows \$400 000 from a bank. The loan is to be repaid in equal monthly repayments of $\$M$, at the end of each month, over 30 years. Reducible interest is charged at 3.6% p.a., calculated monthly.

Let $\$A_n$ be the amount owing after the nth repayment.

i. Show that the amount Hilton owes immediately after his second repayment can be written as:

$$A_2 = 400000(1.003)^2 - M(1+1.003)$$ **(1)**

ii. Write an expression for A_n, in terms of M and n, and hence calculate how much Hilton would need to pay each month to fully repay the loan in exactly 30 years. **(2)**

iii. However, just after Hilton makes his 120th payment, Hilton decides to increase his repayments, from \$M a month, to \$2100 per month. In how many more months will he pay off the remainder of the loan?

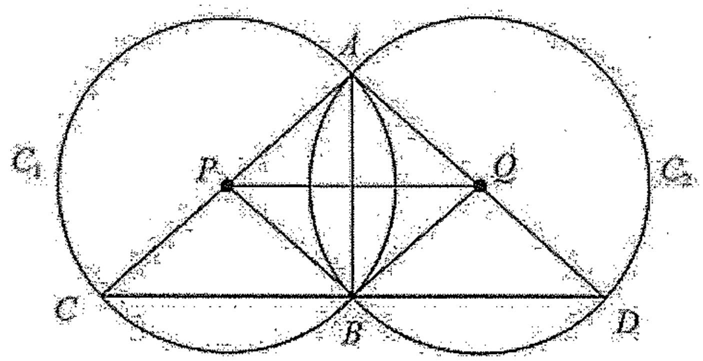
(3)

iv. How much money will Hilton save as a result of changing his repayment amount? **(2)**

Question 13 - 8 marks

Begin this question on a NEW PAGE in your answer booklet.

a. Two circles C_1 and C_2 centres at P and Q with equal radii intersect at A and B respectively. AC is a diameter in circle C_1 and AD is a diameter in C_2.



Redraw the diagram in your answer booklet.

i. Show that ΔABC is congruent to, ΔABD. **(2)**

ii. Show that $PB \parallel AD$. **(2)**

iii. Show that $PQDB$ is a parallelogram **(1)**

b. The variable point P has coordinates, $P(a\cos 2\theta, a\sin\theta)$.

i. Show that P lies on the curve $y^2 = -\frac{a}{2}(x-a)$ **(1)**

ii. Sketch the locus of P as θ varies, taking into account any restrictions on x and y. **(2)**

END OF TASK

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REFERENCE SHEET

– Mathematics –

– Mathematics Extension 1 –

– Mathematics Extension 2 –

Factorisation

$a^2-b^2=(a+b)(a-b)$

$a^3+b^3=(a+b)(a^2-ab+b^2)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

Angle sum of a polygon

$S=(n-2)\times180°$

Equation of a circle

$(x-h)^2+(y-k)^2=r^2$

Trigonometric ratios and identities

$\sin\theta=\frac{\text{opposite side}}{\text{hypotenuse}}$

$\cos\theta=\frac{\text{adjacent side}}{\text{hypotenuse}}$

$\tan\theta=\frac{\text{opposite side}}{\text{adjacent side}}$

$\operatorname{cosec}\theta=\frac{1}{\sin\theta}$

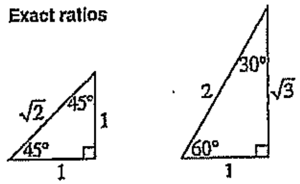
$\sec\theta=\frac{1}{\cos\theta}$

$\tan\theta=\frac{\sin\theta}{\cos\theta}$

$\cot\theta=\frac{\cos\theta}{\sin\theta}$

$\sin^2\theta+\cos^2\theta=1$

Exact ratios



Sine rule

$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

Cosine rule

$c^2=a^2+b^2-2ab\cos C$

Area of a triangle

$\text{Area}=\frac{1}{2}ab\sin C$

Distance between two points

$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

Perpendicular distance of a point from a line

$d=\frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

Slope (gradient) of a line

$m=\frac{y_2-y_1}{x_2-x_1}$

Point-gradient form of the equation of a line

$y-y_1=m(x-x_1)$

***n*th term of an arithmetic series**

$T_n=a+(n-1)d$

Sum to *n* terms of an arithmetic series

$S_n=\frac{n}{2}[2a+(n-1)d]$ or $S_n=\frac{n}{2}(a+l)$

***n*th term of a geometric series**

$T_n=ar^{n-1}$

Sum to *n* terms of a geometric series

$S_n=\frac{a(r^n-1)}{r-1}$ or $S_n=\frac{a(1-r^n)}{1-r}$

Limiting sum of a geometric series

$S=\frac{a}{1-r}$

Compound interest

$A_n=P\left(1+\frac{r}{100}\right)^n$

Differentiation from first principles

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Derivatives

If $y=x^n$, then $\frac{dy}{dx}=nx^{n-1}$

If $y=uv$, then $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$

If $y=\frac{u}{v}$, then $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

If $y=F(u)$, then $\frac{dy}{dx}=F'(u)\frac{du}{dx}$

If $y=e^{f(x)}$, then $\frac{dy}{dx}=f'(x)e^{f(x)}$

If $y=\log_e f(x)=\ln f(x)$, then $\frac{dy}{dx}=\frac{f'(x)}{f(x)}$

If $y=\sin f(x)$, then $\frac{dy}{dx}=f'(x)\cos f(x)$

If $y=\cos f(x)$, then $\frac{dy}{dx}=-f'(x)\sin f(x)$

If $y=\tan f(x)$, then $\frac{dy}{dx}=f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha+\beta=-\frac{b}{a}$ $\qquad$ $\alpha\beta=\frac{c}{a}$

Equation of a parabola

$(x-h)^2=\pm 4a(y-k)$

Integrals

$$\int(ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+C$$

$$\int e^{ax+b}\,dx=\frac{1}{a}e^{ax+b}+C$$

$$\int\frac{f'(x)}{f(x)}dx=\ln|f(x)|+C$$

$$\int\sin(ax+b)dx=-\frac{1}{a}\cos(ax+b)+C$$

$$\int\cos(ax+b)dx=\frac{1}{a}\sin(ax+b)+C$$

$$\int\sec^2(ax+b)dx=\frac{1}{a}\tan(ax+b)+C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)dx\approx\frac{b-a}{2}\left[f(a)+f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)dx\approx\frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

Logarithms – change of base

$$\log_a x=\frac{\log_b x}{\log_b a}$$

Angle measure

$180°=\pi$ radians

Length of an arc

$l=r\theta$

Area of a sector

$\text{Area}=\frac{1}{2}r^2\theta$

Angle sum identities

$\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$

$\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$

$\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

***t* formulae**

If $t=\tan\frac{\theta}{2}$, then

$$\sin\theta=\frac{2t}{1+t^2}$$

$$\cos\theta=\frac{1-t^2}{1+t^2}$$

$$\tan\theta=\frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta=a,\quad \theta=n\pi+(-1)^n\sin^{-1}a$

$\cos\theta=a,\quad \theta=2n\pi\pm\cos^{-1}a$

$\tan\theta=a,\quad \theta=n\pi+\tan^{-1}a$

Division of an interval in a given ratio

$$\left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$

Parametric representation of a parabola

For $x^2=4ay$,

$x=2at,\quad y=at^2$

At $(2at,\ at^2)$,

tangent: $y=tx-at^2$

normal: $x+ty=at^3+2at$

At $(x_1,\ y_1)$,

tangent: $xx_1=2a(y+y_1)$

normal: $y-y_1=-\frac{2a}{x_1}(x-x_1)$

Chord of contact from $(x_0,\ y_0)$: $xx_0=2a(y+y_0)$

Acceleration

$$\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Simple harmonic motion

$x=b+a\cos(nt+\alpha)$

$\ddot{x}=-n^2(x-b)$

Further integrals

$$\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\frac{x}{a}+C$$

$$\int\frac{1}{a^2+x^2}dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

Sum and product of roots of a cubic equation

$\alpha+\beta+\gamma=-\frac{b}{a}$

$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}$

$\alpha\beta\gamma=-\frac{d}{a}$

Estimation of roots of a polynomial equation

Newton's method

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

Binomial theorem

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

2016 HSC EXTENSION 1 TASK 1 SAMPLE SOLUTIONS

1. C 2. A 3. B 4. D 5. A 6. D 7. B

Section 2

Question 8

a) $24 = a$

$d = -8$

i. $T_n = a + (n-1)d$

$= 24 - 8(n-1)$

$T_n = 32 - 8n$

ii. $-192 = 32 - 8n$

$8n = 224$

$n = 28$

$\therefore$ $\underline{28\text{th term}}$

b)

$x = t - 1 \quad y = t^2 + t - 1$

$x + 1 = t$ into y

$\therefore y = (x+1)^2 + (x+1) - 1$

$y = x^2 + 2x + 1 + x$

$y = x^2 + 3x + 1$

Vertex $\left(-3/2, -1\tfrac{1}{4}\right)$

c) i. $\angle ABC = \angle CAT$

angle between the tangent and the chord equals the angle in the alternate segment.

ii.

$\angle BAC = 90^\circ$ (angle in the semi-circle)

Now, $90^\circ + 39^\circ + \angle BCA = 180^\circ$

(angle sum ΔBCA)

$\therefore \angle BCA = 51^\circ$

Question 9

a)

i. Equal arcs subtend equal angles at the centre.

ii.

Let $\angle COD = x$

$\therefore \angle BOC = x$

Now $\angle AOB + 2x = 180$

(straight line)

$\therefore \angle AOB = 180 - 2x$

as $AO = BO$ (radii)

$\angle OAB = \angle OBA$

(equal angles opposite equal sides ΔAOB)

Now

$\angle AOB + 2\angle ABO = 180$

(angle sum ΔAOB)

$\therefore \angle ABO = x$

$\therefore$ as $\angle ABO = \angle BOC$

$(= x)$

then as they are equal alternate angles AB is parallel to OC.

con't

i. $\alpha\beta\gamma = -d/a$

$= -2$

ii. $\alpha\beta + \alpha\gamma + \beta\gamma = \frac{c}{a}$

$= -5$

iii.

$\alpha + \beta + \gamma = \frac{-b}{a}$

$= 1$

$\therefore \beta + \gamma = 1 - \alpha$

Now $\frac{\beta+\gamma}{\alpha} = \frac{1-\alpha}{\alpha}$

$= \frac{1}{\alpha} - 1$

iv.

$\frac{\beta+\gamma}{\alpha} + \frac{\gamma+\alpha}{\beta} + \frac{\alpha+\beta}{\gamma}$

$= \frac{1-\alpha}{\alpha} + \frac{1-\beta}{\beta} + \frac{1-\gamma}{\gamma}$

$= \frac{1}{\alpha} - 1 + \frac{1}{\beta} - 1 + \frac{1}{\gamma} - 1$

$= \left(\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}\right) - 3$

$= \frac{\beta\gamma + \alpha\gamma + \alpha\beta}{\alpha\beta\gamma} - 3$

$= \frac{-5}{-2} - 3$

$= \frac{-1}{2}$

a.

$T_5 = 3T_2$

$a + 4d = 3(a+d)$

$d = 2a$

$S_6 = \frac{6}{2}(2a + 5d) = 144$

$3(d + 5d) = 144$

$3(6d) = 144$

$6d = 48$

$d = 8$

$\therefore a = 4$

$T_3 = a + 2d$

$= 4 + 16$

$= 20$

b. $\sum_{n=0}^{5} 3n^2 - 1$

$(-1) + (2) + (11) + (26) + (47) + 74$

sum $= 159$

c.

i. $\angle ECD = \angle BEC$

$\angle ECD = 2\theta$ (alternate angles $AB \parallel CD$ opposite sides rhombus parallel)

$\angle ACD = \angle ACB$ (~~diagonals of rhombus~~ bisect interior angles)

$= 4\alpha$

$\therefore \angle ECD = \angle ECA + \angle ACD$ (adjacent angle

$= 7\alpha$

ie $7\alpha = 2\theta$ ($\angle ECD$)

and $\theta = 7\alpha/2$.

ii. $2\alpha + 8\alpha = 180^\circ$ (co-interior angles $AB \parallel CD$

$\alpha = 18^\circ$

$\therefore \theta = 63^\circ$

Question 4

a. $(a+2)(a^2-3a+5)$
$= a(a^2-3a+5) + 2(a^2-3a+5)$
$= a^3 - a^2 - a + 10$

b.

	T_1	T_2	T_3
AP	a	$a+d$	$a+2d$
GP	b	ab	$\cancel{ab^2}$ ba^2

$\therefore S = (a+b) + (a+ab+d) + (a + \cancel{ab^2}\ ba^2 + 2d)$

ii. $\therefore a+b=-1$ ①
$a + d + ab = -2 \longrightarrow a + d + a(-1-b) \overset{②}{=} -2$
$d = a^2 - 2$

and $a + 2d + ba^2 = 6$ ③
$\therefore$ sub into ① & ② into ③
$a + 2(a^2-2) + a^2(-1-a) = 6$
$a + 2a^2 - 4 - a^2 - a^3 - 6 = 0$
$-a^3 + a^2 + a - 10 = 0$
$\therefore a^3 - a^2 - a + 10 = 0$

ii) from part (a)
$(a+2)(a^2-3a+5) = 0$
as a is real
$a = -2$
$\therefore a = -2$
$b = 1$
$d = 2$

$T_n = -2 + (n-1)\times 2 + 1(-2)^{n-1}$
$= 2n - 4 + (-2)^{n-1}$
as required

$\therefore$
$S = [-2+1] + [0+-2] + [2+4] + \ldots$

ie $(-2+0+2\ldots) + (1+-2+4+\ldots) \Rightarrow$ AP + GP.
$T_n = a + (n-1)d + ar^{n-1}$

Question 12:

$n = 360$

$3.6\% = 0.003$ p. month

i. $A_1 = 400\,000(1.003) - M$

$A_2 = [400\,000(1.003) - m](1.003) - m$

$= 400\,000(1.003)^2 - m[1.003 + 1]$

ii. $A_n = 400\,000(1.003)^n - M[1.003^{n-1} + \ldots + 1]$

$A_n = 400\,000(1.003)^n - M\left[\dfrac{1(1.003^n - 1)}{0.003}\right]$

Now $A_n = 0$ $n = 360$

$M\left[\dfrac{1.003^{360} - 1}{0.003}\right] = 400\,000(1.003)^{360}$

$m = \$1818.58$

iii. $A_{120} = 400\,000(1.003)^{120} - 1818.58\left[\dfrac{1.003^{120} - 1}{0.003}\right]$

$= \$310\,809.60$

Now

$A_n = 0 = 310809.60(1.003)^n - 2100\left[\dfrac{1.003^n - 1}{0.003}\right]$

$0 = 310809.60(1.003)^n - 700\,000(1.003^n - 1)$

$0 = -389190.4(1.003)^n + 700\,000$

$1.003^n = 1.7986\ldots$

$n = \dfrac{\log 1.7986\ldots}{\log 1.003}$

$= 195.963$

$= 196$ payments

iv. $360 \times 1818.58 = 654688.8$

$120 \times 1818.58 +$

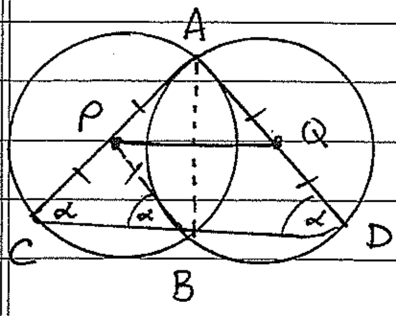
196×2100

$= 629829.60$

$\therefore$ Saved

$\$24859.20$

Question 13.



1) As C_1 and C_2 have equal radii, given

$AC = AD$

(equal diameters of equal circles)

$\angle ABC = \angle ABD = 90^\circ$

C

(angle in semi-circles C_1 and C_2 AB is a common chord)

$\therefore \Delta ABC \equiv \Delta ABD$ (RHS)

ii) Join PB

Let $\angle ADB = \alpha$

$\therefore \angle ACB = \angle ADB$

$= \alpha$

(corresponding angles in congruent triangles)

C

$PC = PB$ (radii of circle C_1)

$\therefore \angle PBC = \angle PCB$ (equal angles opposite equal sides)

$= \alpha$

Now

$\angle PBC = \angle ADB$

$= \alpha$

$\therefore$ The corresponding angles of $\angle PBC$ & $\angle ADB$ are equal making $PB \parallel AD$.

iii) As $PB = QD$ equal radii of equal circles

and $PB \parallel AD$ (part ii)

PQDB is a parallelogram as one pair of opposite sides are equal and parallel.

13 b)

$x = a\cos 2\theta$, — ① $y = a\sin\theta$ — ②

1. $\frac{x}{a} = 1 - 2\sin^2\theta$ — ③

(as $\cos 2\theta = 1 - 2\sin^2\theta$)

sub ② into ③

$\frac{x}{a} = 1 - 2\left(\frac{y}{a}\right)^2$

$\frac{x}{a} = 1 - \frac{2y^2}{a^2}$

$ax = a^2 - 2y^2$

$2y^2 = a^2 - ax$

$y^2 = -\frac{a}{2}(x - a)$

ii) as $-1 \le \sin\theta \le 1$

$-a \le y \le a$

$\therefore$ when $y = a$ $x = -a$

making $x \geq -a$

vertex @ $(a, 0)$