Sydney Technical High School

Higher School Certificate

Assessment Task One
Term 4 2010

MATHEMATICS

Time allowed- seventy minutes

General Instructions

- Working Time – 70 minutes.
- Write using a black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown for every question.
- Begin each question on a new page

Total marks (**64**)

- Attempt Questions 1 - 8.
- All questions are of equal value.

NAME ______________________________ **TEACHER** ____________________

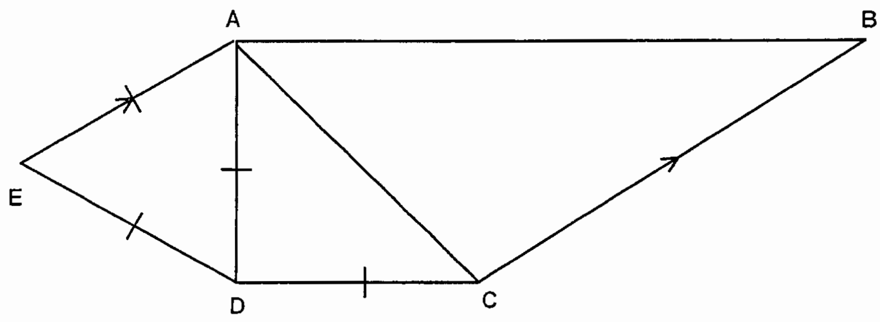
Qu 1	Qu 2	Qu 3	Qu 4	Qu 5	Qu 6	Qu 7	Qu 8	TOTAL
								/64

Question 1 **8 marks**

a) Write the equation of the circle with centre $(4,-1)$ and radius 7 units **2**

b) Solve the quadratic equation $x(2x-3)=5$ **2**

c) In the diagram below AE = ED = AD = DC, $\angle ADC = 90^\circ$ and AE || BC. $\angle BAC = 51^\circ$



i) Find the size of $\angle EAB$. Give reasons for your answer. **2**

ii) Find the size of $\angle ABC$. Give reasons for your answer. **2**

Question 2 (start a new page) **8 marks**

a) Derive the equation of the locus of a point P(x,y) which moves so as to be equidistant from the two points $A(-2,4)$ and $B(8,-3)$ **3**

b) For the parabola $(x-3)^2 = 20(y+5)$, find:

i) The coordinates of the vertex **1**

ii) The focal length **1**

iii) The coordinates of the focus **1**

iv) The equation of the directrix **1**

v) The equation of the axis of symmetry **1**

Question 3 (start a new page) **8 marks**

a) If α and β are the roots of $3x^2 + 4x - 12 = 0$, find without solving the equation:

i) $\alpha + \beta$ **1**

ii) $\alpha\beta$ **1**

iii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ **1**

iv) $\alpha^2 + \beta^2$ **2**

b) Find the values of the constants A, B and C such that **3**

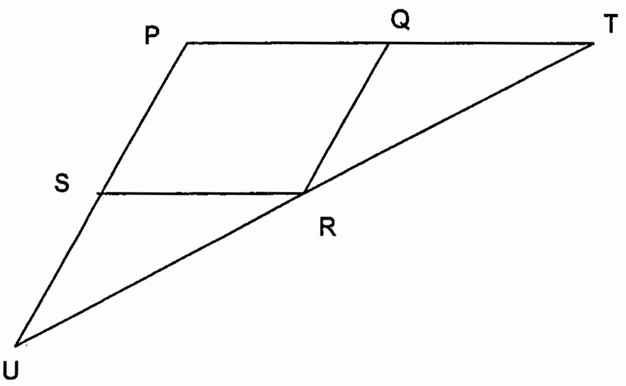
$3x^2 - 8x + 6 \equiv A(x-1)^2 + B(x-1) + C$

Question 4 (start a new page) **8 marks**

a) *PQRS* is a parallelogram. *PQ* is produced to *T* so that *QT*=*QR* and *PS* is produced to *U* so that *SU*=*PS*. It is now discovered that *T*, *R* and *U* are collinear.

i) Show that *SU*=*RQ* **2**

ii) Prove *PQRS* is a rhombus **3**



b) For the function $y = kx^2 - 4\sqrt{3}x + k - 1$,

i) find an expression for the discriminant. **2**

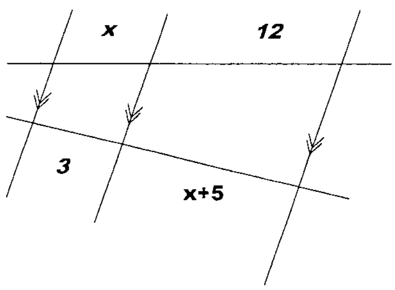
ii) for what values of k is the function positive definite. **2**

Question 5 (start a new page) **8 marks**

a) Find the axis of symmetry and the vertex of the parabola $y = 5x^2 + 10x + 2$. **2**

b) The roots of the quadratic equation $px^2 - x + q = 0$ are -1, 3. Find p and q. **3**

c) Solve for x **3**

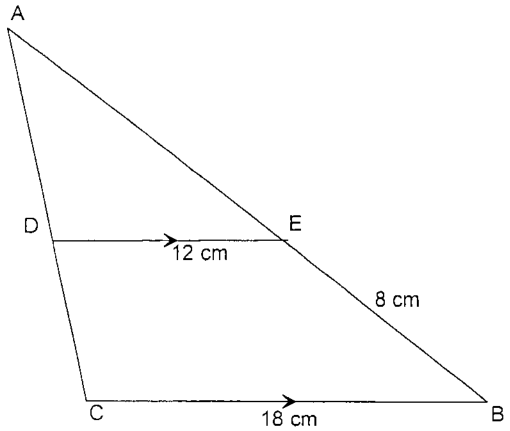


Question 6 (start a new page) **8 marks**

a) Find the discriminant of the following equation and state the nature of the roots $2x^2 + 3x + 5 = 0$ **2**

b) Solve $2(x^2 + 1)^2 - 19(x^2 + 1) - 10 = 0$ **3**

c) In the diagram below DE || CB, DE = 12 cm , CB = 18 cm and EB = 8 cm.



(i) Prove that Δ ADE ||| Δ ACB **2**

(ii) Find the length of AE. **1**

Question 7 (start a new page) **8 marks**

a) A parabola whose equation is $y = ax^2$, where a is a constant, has the line $y = 12x + 3$ as a tangent.

i) By equating the two given equations, find a quadratic equation in terms of x and a. **1**

ii) By using the discriminant of the quadratic equation found, find the value of a. **2**

iii) Find the coordinates of the point of contact between the tangent and the parabola. **2**

iv) Sketch the parabola and the tangent line, showing the co-ordinates of intercepts and the point of contact **2**

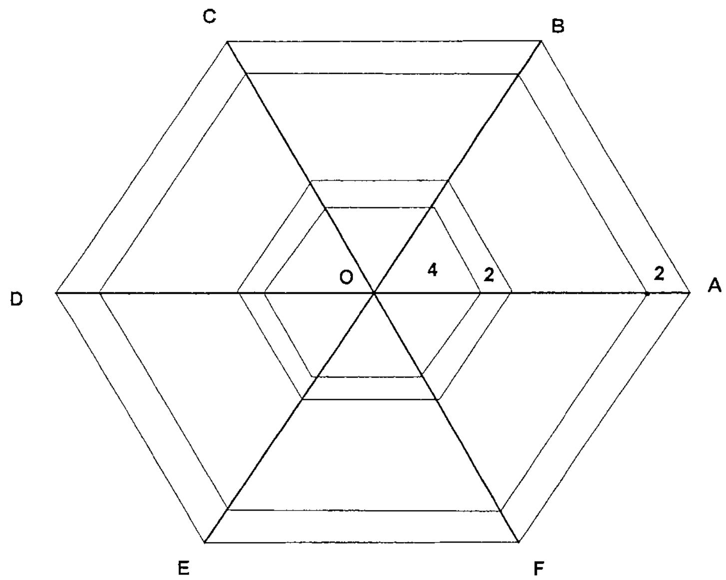
Question 8 (start a new page) **8 marks**

a) Write down the formula for:

i) the nth term of an arithmetic series with first term a and the common difference d **1**

ii) the sum of the first n terms of this series **1**

A particular spider's web consists of a series of regular hexagons with a common centre O, held together by rays through O, as in the figure, where only some of the hexagons are shown.



NOT TO SCALE LENGTHS IN CM

The vertices of the smallest hexagons are 4cm from O. The vertices of the next hexagons are 2cm further away and they continue at 2cm intervals along the rays until the vertices of the last hexagon ABCDEF are 60cm from O.

iii) How many hexagons are there? **1**

iv) What is the length, in cm, of the perimeter of the smallest hexagon? **1**

v) What is the total length of thread used by the spider in making this web (including the six rays from O)? **1**

b) i) If $4^{x+1} = 2^a$ find a **1**

ii) Hence solve $4^{x+1} - 12(2^x) + 8 = 0$ **2**

END OF EXAM

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Question 1

a) $(x-4)^2+(y+1)^2=49$
$(x-4)^2+(y+1)=7$

b) $x(2x-3)=5$
$2x^2-3x-5=0$
$(x+1)(2x-5)=0$
$x=-1 \quad x=\frac{5}{2}$

c) i) $\angle EAD=60°$ (equilateral triangle)
$\angle DAC=45°$ (right-angled isosceles $\triangle$)
$\angle CAB=51°$ (given)
$\therefore \angle EAB=156°$

ii) $\angle EAB+\angle ABC=180°$
$156°+\angle ABC=180°$
$\angle ABC=24°$
(interior angles in $\parallel$ lines $AE \parallel BC$)

Question 2

a) $P(x,y)$
$A(-2,4)$
$B(8,-3)$
$PA^2=PB^2$
$(x+2)^2+(y-4)^2=(x-8)^2+(y+3)^2$
$x^2+4x+4+y^2-8y+16=x^2-16x+64+y^2+6y+9$

b) $(x-3)^2=20(y+5)$
i) vertex $(3,-5)$
ii) focal length $=5$
iii) focus $(3,0)$
iv) directrix $y=-10$
v) $x=3$

Question 3

a) $3x^2+4x-12=0$
i) $\alpha+\beta=-4/3$

ii) $\alpha\beta=-12/3=-4$

iii) $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}=-\frac{4}{3}\div-4=\frac{1}{3}$

iv) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$
$=(-4/3)^2-2\times-4 \quad = 9\frac{7}{9}$

B) $3x^2-8x+6\equiv A(x-1)^2+B(x-1)+C$

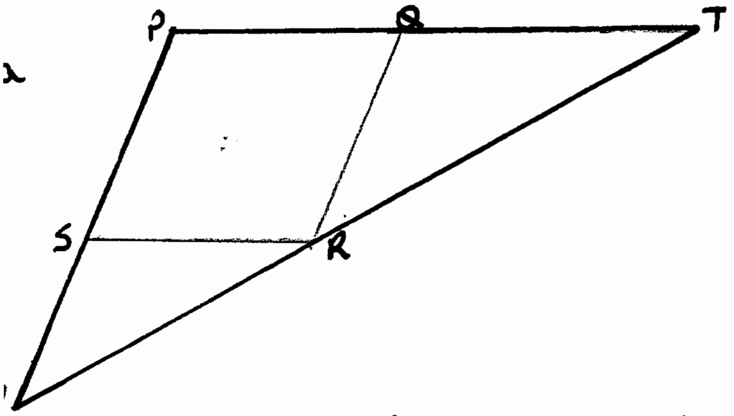
R.H.S $=A(x-1)^2+B(x-1)+C$
$=A(x^2-2x+1)+Bx-B+C$
$=Ax^2-2Ax+Bx+A-B+C$
$A=3$

$-8=-2A+B$
$-8=-2\times3+B$
$B=-2$

$A-B+C=6$
$-3--2+C=6$
$C=1$

$A=3$
$B=-2$
$C=1$

Question 4



i) $\angle RSU = \angle QPS$ (corresponding $\angle$'s $SR \parallel PT$)

$\angle TQR = \angle QPS$ (corresponding $\angle$'s $QR \parallel PU$)

$\therefore \angle TQR = \angle RSU$

$QR = PS$ (opp sides of parallelogram)

$SU = PS$ (given).

$QR = SU$

ii) In ΔTQR & RSU

* $\angle TQR = \angle RSU$ (proven above)
* $QR = SU$ (proven above)
* $\angle RTQ = \angle URS$ (corresponding $\angle$'s $PT \parallel SR$)

$\therefore \Delta TQR \equiv \Delta RSU$ (AAS)

ie $QT = SR$

$QT = QR$

$QR = SR$

(adjacent sides of the parallelogram PQRS)

$\therefore$ PQRS is a rhombus

b i) $y = kx^2 - 4\sqrt{3}x + k - 1$

$\Delta = b^2 - 4ac$

$\Delta = (-4\sqrt{3})^2 - 4 \times k\,(k-1)$

$\Delta = 48 - 4k^2 + 4k$

ii) positive definite

$a > 0$ $\quad 48 + 4k - 4k^2 < 0$

$\Delta < 0$ $\quad 4k^2 - 4k - 48 > 0$

$k^2 - k - 12 > 0$

$(k+3)(k-4) > 0$

$k < -3 \qquad k > 4$

only solution $k > 4$

Question 5

a) $y = 5x^2 + 10x + 2$

$x = -10/2\times5 = -1$

vertex $(-1, -3)$

b) $px^2 - x + q = 0$

$x^2 - \frac{1}{p}x + \frac{q}{p} = 0$

$\alpha + \beta = \frac{1}{p}$

$\alpha + \beta = -1 + 3 = \frac{1}{p}$

$p = \frac{1}{2}$

$\alpha\beta = q/p = -1 \times 3 = \frac{q}{p}$

$-3 = q \div \frac{1}{2}$

$q = -3/2$

$p = \frac{1}{2}$

$q = \frac{-3}{2}$

c) $\frac{x}{12} = \frac{3}{x+5}$

$x^2+5x-36=0$
$(x+9)(x-4)=0$
$x=-9, 4$ but $x>0$
$\therefore x=4$

Question 6

a) $2x^2+3x+5=0$
$\Delta = b^2-4ac$
$\Delta = 3^2-4\times 2\times 5$
$\Delta = -31<0$
$\therefore \Delta<0 \quad a>0$
No real roots
Positive definite.

b) $2(x^2+1)^2-19(x^2+1)-10=0$
let $m=x^2+1$
$2m^2-19m-10=0$
$(2m+1)(m-10)=0$
$m=-\frac{1}{2} \qquad m=10$

$x^2+1=-\frac{1}{2} \qquad x^2+1=10$
$x^2=-\frac{3}{2} \qquad x^2=9$
No Solution $\qquad x=\pm 3$

ci) In ΔADE & ΔABC
$\angle A$ is common
$\angle AED = \angle ABC$ (corresponding $\angle$'s
$\angle EDA = \angle ACB$ (in parallel lines)
$\therefore \Delta ADE \parallel \Delta ABC$ (equiangular)

cii) let $AE=x$
$\frac{AB}{AE} = \frac{CB}{DE}$
$\frac{x+8}{x} = \frac{18}{12}$
$12x+96=18x$
$6x=96$
$x=16$
$AE = 16\text{ cm}$

Question 7

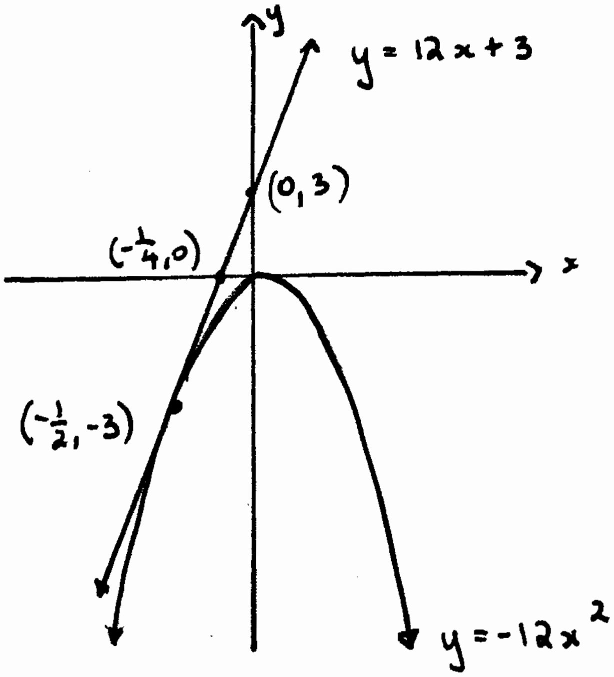
ai) $y=ax^2$
$y=12x+3$

$ax^2=12x+3$ or
$ax^2-12x-3=0$

ii) Since the line is a tangent to the parabola (one point of contact) the roots are equal
$\Delta=0 \quad (-12)^2-4\times a\times -3=0$
$144+12a=0$
$144=-12a$
$a=-12$

iii) Point of contact $a=-12$
$ax^2-12x-3=0$
$-12x^2-12x-3=0$
$4x^2+4x+1=0$
$(2x+1)^2=0$
$x=-\frac{1}{2}$
$y=-3$

iv)



Question 8

a i) $T_n = a + (n-1)d$

ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

iii) $a = 4$ $\quad 4, (4+2), (4+2+2), \ldots, 60$

$d = 2$ $\quad 4, 6, 8, \ldots, 60$

$T_n = 60$ $\quad 60 = 4 + (n-1)2$

$60 = 4 + 2n - 2$

$58 = 2n$

$n = 29$

$\therefore$ There are 29 hexagons

iv) Each regular hexagon can be divided into 6 equilateral triangles. The sides of the smallest regular hexagon are 4cm. Perimeter of smallest regular hexagon is $6 \times 4 = 24$cm

v) Sum of hexagon perimeters

$= (6\times4) + (6\times6) + (6\times8) + \ldots + (6\times60)$

$= 24 + 36 + 48 + \ldots + 360$

$a = 24$ $\quad S_n = \frac{29}{2}[2\times24 + (29-1)\times12]$

$d = 12$ $\quad = 14.5 \times 336$

$n = 29$ $\quad = 5568$ cm

length of 6 rays $\quad 6\times60 = 360$

$\therefore$ Total length $5568 + 360 = 5928$cm

b) i) $4^{x+1} = 2^a$

$2^{2(x+1)} = 2^a$

$a = 2x + 2$

$4^{x+1} - 12(2^x) + 8 = 0$

$(2^2)^{x+1} - 12(2^x) + 8 = 0$

$2^{2x+2} - 12\,2^x + 8 = 0$

$(2^x)^2 \cdot 2^2 - 12(2^x) + 8 = 0$

let $m = 2^x$

$4m^2 - 12m + 8 = 0$

$(2m-4)(2m-2) = 0$

$m = 2 \qquad m = 1$

$2^x = 2 \qquad 2^x = 1$

$x = 1 \qquad x = 0$