Sydney Technical High School

Name: ....

Teacher: ....

Year 12

Mathematics

Trial HSC

August, 2017

Time allowed: 3 hours plus 5 minutes reading time

General Instructions:

- Reading time – 5 minutes
- Working time – 3 hours
- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A reference sheet is provided

Total marks – 100

Section I – 10 Marks
- Attempt Question 1-10 on the sheet provided
- Allow about 15 minutes for this section

Section II – 90 Marks
- Attempt Questions 11-16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1- 10.

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1. The solution to $x^2 - 4 < -3$ is:

(A) $-2 < x < 2$

(B) $x < -2,\ x > 2$

(C) $-1 < x < 1$

(D) $x < -1,\ x > 1$

2. An infinite geometric series has first term 4 and a limiting sum of 6.
What is the common ratio?

(A) $\frac{1}{6}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4}$

(D) $\frac{1}{3}$

3. What is a possible primitive function for $2x^{-4} + 5x$?

(A) $-\frac{2}{3x^3} + \frac{5x^2}{2} + 12$

(B) $-\frac{1}{6x^3} + \frac{5x^2}{2}$

(C) $\frac{2}{3x^3} + \frac{5x^2}{2} + 12$

(D) $\frac{1}{6x^3} + \frac{5x^2}{2}$

4. The quadratic equation $x^2 + 5x - 4 = 0$ has roots α and β. What is the value of $2\alpha^2\beta + 2\alpha\beta^2$?

(A) -20

(B) 40

(C) -40

(D) 20

5. What are the solutions of $tan2\theta = 1$ for $0 \leq \theta \leq 360°$?

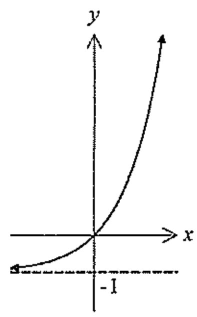
(A) $\theta = 45°, 225°$

(B) $\theta = 45°, 225°, 405°, 585°$

(C) $\theta = 22\frac{1}{2}°, 112\frac{1}{2}°$

(D) $\theta = 22\frac{1}{2}°, 112\frac{1}{2}°, 202\frac{1}{2}°, 292\frac{1}{2}°$

6. What is a possible equation for the following graph?



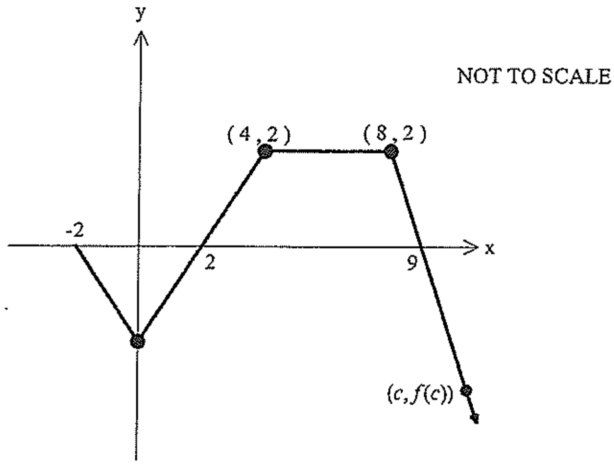
(A) $y = e^{x-1}$

(B) $y = e^x + 1$

(C) $y = e^x - 1$

(D) $y = e^{x+1}$

7. Consider the graph below:



For what value of C would $\int_{-2}^{C} f(x)dx = -2$ be true?

(A) 10

(B) 11

(C) 12

(D) 13

8. What is the value of $\sum_{n=1}^{5} n(n-1)$?

(A) 50

(B) 40

(C) 30

(D) 20

9. For what values of x is the curve $f(x) = 2x^3 + x^2$ both concave down and decreasing?

(A) $-\frac{1}{6} < x < 0$

(B) $-3 < x < 0$

(C) $-3 < x < -\frac{2}{12}$

(D) $-\frac{1}{3} < x < -\frac{1}{6}$

10. A parabola has a focus $(0,6)$ and directrix of $y = 2$.
What is the equation of the parabola?

(A) $x^2 = -8(y-4)$
(B) $x^2 = -16(y-5)$
(C) $x^2 = 8(y-4)$
(D) $x^2 = 16(y-5)$

Section II

Total marks – 90
Attempt Questions 11–16
Allow about 2 hours 45 minutes for this section.

Begin each question on a NEW page.

In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Begin a NEW page.

a) Evaluate $\dfrac{7.4^2 - e^2}{\sqrt{12 - \sqrt{2}}}$ to 4 significant figures. **2**

b) Rationalise the denominator of $\dfrac{5\sqrt{2}}{2\sqrt{2} - 3}$ **2**

c) Fully factorise $x^6 - 27$. **2**

d) Solve the equation $|5 - x| = 3x$. **2**

e) If $\sin\theta = \dfrac{7}{10}$ and $\tan\theta < 0$, find the exact value of $\sec\theta$. **2**

f) Simplify $\lim\limits_{x \to \infty} \dfrac{2x^3 - 4x^2 + 7}{x^3 + 3x + 1}$ **2**

g) Find the equation of the normal to the curve $y = 4e^{2(x-1)}$ at $x = 1$. **3**

End of Question 11

Question 12 (15 marks) Begin a NEW page.

a) Differentiate the following with respect to x

i. $(3x^2 + 4)^5$ **2**

ii. $x^2 \tan x$ **2**

iii. $\dfrac{\sin x}{e^{-x}}$ **2**

b) Find the area under the curve $y = |2x - 1|$ bounded by $x = -4$ and $x = 2$ **2**

c) Sketch the curve $y = 4\sin(2x) + 1$ between $-\pi \leq x \leq \pi$ showing all important features (You DO NOT need to find x-intercepts). (Make your graph **at least a third** of a page) **3**

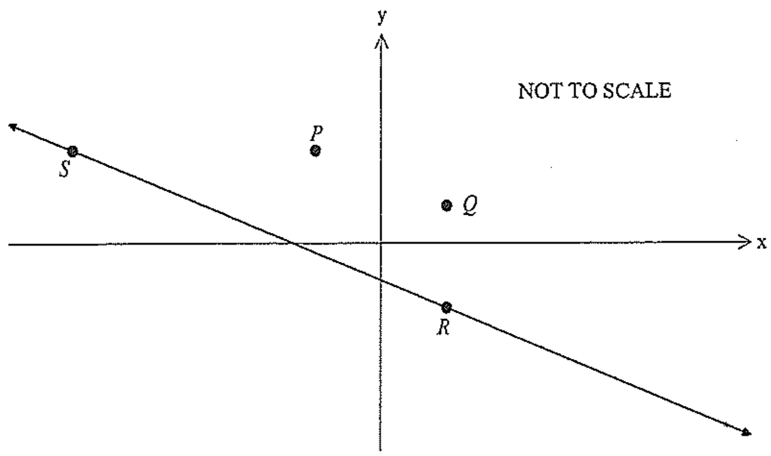
d) A function $y = f(x)$ has $\dfrac{dy}{dx} = 3x - 4$ and passes through (1,4). Find $f(x)$. **2**

e) Shade the region represented by the intersection of $x^2 + (y - 3)^2 \leq 4$ and $x + y > 3$. **2**

End of Question 12

Question 13 (15 marks) Begin a NEW page.

a) The points $P(-3,5)$ and $Q(3,2)$ are shown on the number plane below.



The equation of the line passing points S and R is $y = -\frac{1}{2}x - 2$

i. Find the gradient of PQ. Explain why PQRS is a trapezium. **2**

ii. Find the length of PQ in exact form. **2**

iii. Given that line QR is parallel to the y-axis, state the coordinates of R. **1**

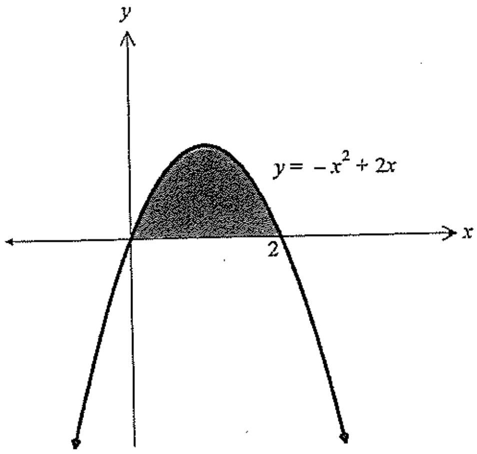
iv. Find the perpendicular distance from P to the line RS. **2**

v. If the length of RS is $\sqrt{95}$ units find the area of PQRS correct to 2 decimal places **2**

Question 13 continues on page 9

Question 13 (continued)

b) The graph of $y = -x^2 + 2x$ is shown below. **3**



Find the volume of the solid of revolution formed when the shaded region is rotated about the x-axis.

c) Given the function $f(x) = 3^{\cos x}$

i. Copy and complete the table for $y = f(x)$ in your exam booklet. (Round your answers to 3 decimals places) **1**

x	0	1	2	3	4
y	3.000				

ii. Apply the Trapezoidal rule with 4 subintervals to find an approximation of **2**

$$\int_0^4 3^{\cos x}\, dx$$

correct to 2 decimals places.

End of Question 13

Question 14 (15 marks) Begin a NEW page.

a) Consider the function

$$f(x) = \frac{x^2 + 15}{5x}$$

i. Show that the function is odd. **1**

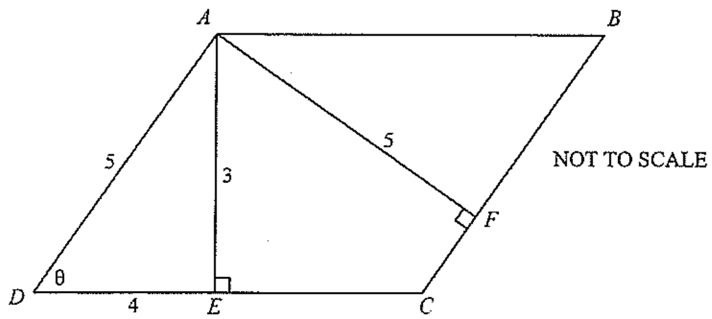
ii. Show that there is no value of x for which $f(x) = 0$. **1**

iii. State the vertical asymptote of $y = f(x)$. **1**

iv. Find the stationary points and determine their nature. **3**

v. Sketch the graph of $y = f(x)$ showing all important features. **2**

b) In the diagram below, $ABCD$ is a parallelogram.



Copy the diagram into your booklet

i. Prove that if $\angle ADE = \theta$, then $\angle EAF = \theta$ (give reasons). **2**

ii. Hence, using the cosine rule, find the exact length of EF **2**

c) Find the value of m for which the equation $(m-4)x^2 - 6x + 7 = 0$ has one root twice the other. **3**

End of Question 14

Question 15 (15 marks) Begin a NEW page.

a) Given the equation of the parabola $4y - 20 = x^2 + 12x + 36$:

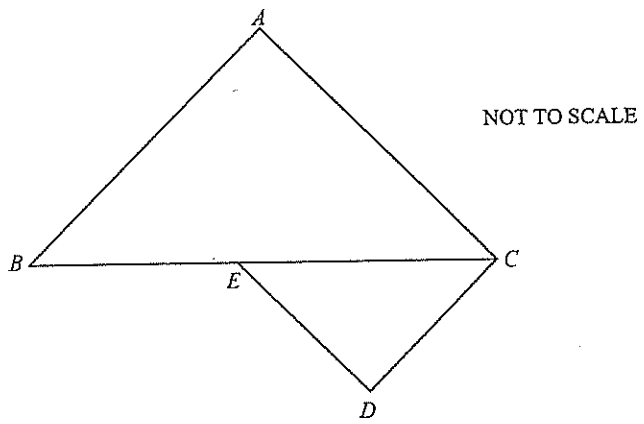
i. Find the coordinates of the vertex. **2**

ii. Find coordinates of the focus. **1**

iii. Find the equation of the directrix. **1**

b) Find $\int (10x-4)^5 \, dx$. **1**

c) In the diagram $CD \parallel AB$ and $DE \parallel CA$. $AC = 15cm, AB = 18cm, CD = 8cm$ and $BE = 12cm$.



Copy the diagram into your booklet adding in all given information.

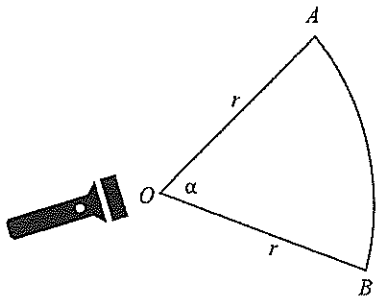
i. Prove $\Delta ABC \,|||\, \Delta DCE$ **2**

ii. Hence find the length of BC. **2**

Question 15 continues on page 12

Question 15 (continued)

d) The diagram below shows a sector OAB of a circle with centre O and a radius $r$cm created by the light of a torch.



i. Show that the perimeter of the light sector OAB is $r(2+\alpha)$ **1**

ii. Given that the perimeter of the light sector OAB is 6m, show that the area illuminated is given by: **2**

$$A = \frac{18\alpha}{(\alpha+2)^2}$$

iii. Hence show that the maximum illuminated area is 2.25m^2. **3**

End of Question 15

Question 16 (15 marks) Begin a NEW page.

a) Show that 3

$$\frac{\cos A}{1+\sin A}+\frac{1+\sin A}{\cos A}=\frac{2}{\cos A}$$

b) The 4th term of an arithmetic sequence is 18 and the sum of the first 10 terms is 195. Find the first term. 3

c) Mr Steve has a travel fund of $55 000. The account accrues interest at 5.4% p.a. compounded monthly. He withdraws $1 500 per month, after interest is paid, to pay for his travel adventures.

i. Show that the amount left at the end of the 2nd month is given by $A_2 = 55000 \times 1.0045^2 - 1500(1.0045+1)$ 2

ii. If A_n is the amount left after n months, show that: 2

$$A_n = 55000(1.0045)^n - 1500\left[\frac{1.0045^n - 1}{0.0045}\right]$$

iii. Hence find the number of months Mr Steve can travel before his funds run out. 2

iv. If after 12 months Mr Steve decides to travel overseas, and increases his withdrawals to $3 000 per month, how many more months can he now afford to travel. 3

End of Paper

Solutions

MULTIPLE CHOICE ANSWER SHEET

Sydney Technical High School

Mathematics 2 unit Trial HSC
August 2017

Completely fill the response oval representing the most correct answer.

Do not remove this sheet from the answer booklet.

1. A ○ B ○ C ● D ○
2. A ○ B ○ C ○ D ●
3. A ● B ○ C ○ D ○
4. A ○ B ● C ○ D ○
5. A ○ B ○ C ○ D ●
6. A ○ B ○ C ● D ○
7. A ○ B ○ C ● D ○
8. A ○ B ● C ○ D ○
9. A ○ B ○ C ○ D ●
10. A ○ B ○ C ● D ○

Q11

a) $14.5596....$

$\doteqdot 14.56$ (4 sig figs)

b) $\dfrac{5\sqrt{2}}{2\sqrt{2}-3} \times \dfrac{2\sqrt{2}+3}{2\sqrt{2}+3}$

$= \dfrac{10\times 2 + 15\sqrt{2}}{8-9}$

$= \dfrac{20+15\sqrt{2}}{-1}$

$= -20-15\sqrt{2}$

c) $x^6 - 27$

$=(x^2)^3 - 3^3$

$= (x^2-3)(x^4+3x^2+9)$

d) $|5-x| = 3x$

$5-x = -3x$

$2x = -5$

$x = -5/2$

test: $|5+\frac{5}{2}| = \frac{15}{2}$

$3(-\frac{5}{2}) = -\frac{15}{2}$

$\therefore$ LHS $\neq$ RHS.

$5-x = 3x$

$5 = 4x$

$x = \frac{5}{4}$

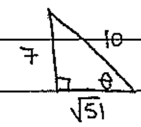
test: $|5-\frac{5}{4}| = \frac{15}{4}$

$3(\frac{5}{4}) = \frac{15}{4}$

$\therefore$ LHS = RHS.

$\therefore x = \frac{5}{4}$ only

e)



$\tan\theta < 0$, $\sin\theta > 0$

$\therefore$ quadrant 2.

$\sec\theta = -\frac{10}{\sqrt{51}}$ or $-\frac{10\sqrt{51}}{51}$

f. $\displaystyle\lim_{x\to\infty} \frac{2x^3-4x^2+7}{x^3+3x+1} = 2$

g. $\frac{dy}{dx} = 8e^{2(x-1)}$

at $x=1$

$m_T = 8$; $m_N = -\frac{1}{8}$

Using $m = -\frac{1}{8}$ and $(1,4)$

eqn of normal

$y - 4 = -\frac{1}{8}(x-1)$

$y - 4 = -\frac{x}{8} + \frac{1}{8}$

$y = -\frac{x}{8} + \frac{33}{8}$

$x + 8y - 33 = 0$

Q12.

a) i. $\frac{d}{dx}(3x^2+4)^5 = 5\times 6x(3x^2+4)^4$

$= 30x(3x^2+4)^4$

ii. $\frac{d}{dx} x^2\tan x \Rightarrow$

$u = x^2 \quad v = \tan x$

$u' = 2x \quad v' = \sec^2 x$

$\frac{d}{dx} x^2 \tan x = 2x\tan x + x^2\sec^2 x$

iii. $\frac{d}{dx} \frac{\sin x}{e^{-x}}$

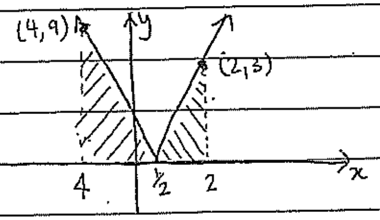
$u = \sin x \quad v = e^{-x}$

$u' = \cos x \quad v' = -e^{-x}$

$\frac{d}{dx}\frac{\sin x}{e^{-x}} = \frac{\cos x e^{-x} + e^{-x}\sin x}{(e^{-x})^2}$

$= \frac{\cos x + \sin x}{e^{-x}}$

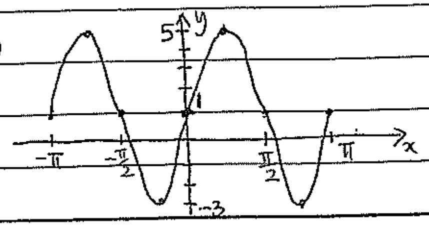
b)



$\int_{-4}^{2} |2x-1|\,dx = \frac{1}{2}\times\frac{9}{2}\times 9 + \frac{1}{2}\times\frac{3}{2}\times 3$

$= 22\frac{1}{2}\ u^2$

c)



d) $\frac{dy}{dx} = 3x-4$

$y = \int 3x-4\,dx$

$= \frac{3}{2}x^2 - 4x + C$

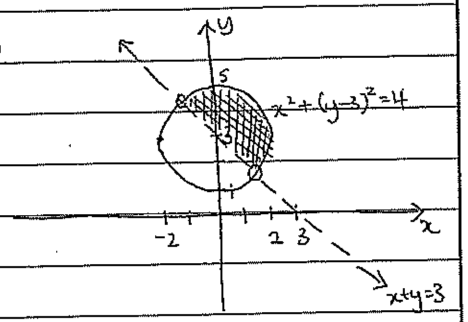
sub (1,4)

$4 = \frac{3}{2} - 4 + C$

$C = \frac{13}{2}$

$\therefore y = \frac{3}{2}x^2 - 4x + \frac{13}{2}$

e)



Q13.

a) i. $m_{PQ} = \frac{5-2}{-3-3}$

$= \frac{3}{-6}$

$= -\frac{1}{2} = m_{RS}$

$\therefore$ PQRS is a trapezium as PQ ∥ RS.

ii. $d_{PQ} = \sqrt{(-3-3)^2 + (5-2)^2}$

$= \sqrt{36+9}$

$= \sqrt{45}$

$= 3\sqrt{5}$

iii. $R\left(3, -\frac{7}{2}\right)$

iv. $d_{PRS} = \frac{|1(-3)+2(5)+4|}{\sqrt{1^2+2^2}}$

$= \frac{|11|}{\sqrt{5}}$

$= \frac{11\sqrt{5}}{5}$

v. $A = \frac{11\sqrt{5}}{5}\times\frac{1}{2}\left(\sqrt{95}+3\sqrt{5}\right)$

$= \frac{11\sqrt{5}}{10}\left(\sqrt{95}+3\sqrt{5}\right)$

$= 40.473\ldots$

$\doteqdot 40.47\ u^2$ (2 d.p.)

b) $V = \pi\int_0^2 (-x^2+2x)^2\,dx$

$= \pi\int_0^2 x^4 - 4x^3 + 4x^2\,dx$

$= \pi\left[\frac{x^5}{5} - x^4 + \frac{4}{3}x^3\right]_0^2$

$= \pi\left(\frac{16}{15}\right)$

$= \frac{16\pi}{15}\ u^3$

$\doteqdot 3.3510\ u^3$ (4 d.p.)

c) i.

x	0	1	2	3	4
y	3.00	1.810	0.633	0.337	0.488

ii. $\int_0^4 3^{\cos x}\,dx \doteqdot$

$\doteqdot \frac{1}{2}(3+0.488+2(1.810+0.633+0.337))$

$\doteqdot 4.524$

$\doteqdot 4.52$ (2 d.p.)

Q14.

a) i. $f(-x) = \frac{(-x)^2+15}{5(-x)}$

$= \frac{x^2+15}{-5}$

$= -\frac{x^2+15}{5}$

$= -f(x)$

$\therefore$ odd function.

ii. let $f(x) = 0$

$0 = \frac{x^2+15}{5x}$

$0 = x^2+15$

$x^2 = -15$

no solution

$\therefore$ no x-value, $f(x) \neq 0$.

iii. $x = 0$

iv. $y = \frac{x^2+15}{5x}$

$= \frac{x}{5} + \frac{3}{x}$

$\frac{dy}{dx} = \frac{1}{5} - \frac{3}{x^2}$

at $\frac{dy}{dx} = 0$, stationary pt.

$0 = \frac{1}{5} - \frac{3}{x^2}$

$\frac{3}{x^2} = \frac{1}{5}$

$15 = x^2$

$x = \pm\sqrt{15}$.

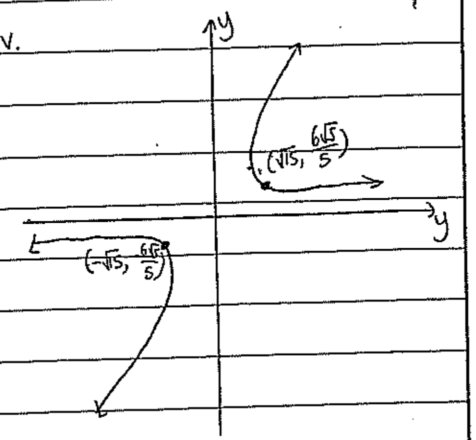
test:

x	-4	$-\sqrt{15}$	-3	3	$\sqrt{15}$	4
y'	$\frac{1}{80}$	0	$-\frac{2}{15}$	$-\frac{2}{15}$	0	$\frac{1}{80}$
	/	—	\	\	—	/

so: $(-\sqrt{15}, -\frac{6\sqrt{5}}{5})$ maximum

$(\sqrt{15}, \frac{6\sqrt{5}}{5})$ minimum

v.



b) i. $\angle ECF = 180° - \theta°$

(cointerior angles, $AD \parallel BC$)

$180° - \theta° + 90° + 90° + \angle EAF = 360°$

(angle sum of quadrilateral)

$\angle EAF = \theta°$

ii. $EF^2 = 3^2 + 5^2 - 2(3)(5)\cos\theta$

$\cos\theta = \frac{4}{5}$ (from $\triangle ADE$)

$EF^2 = 9 + 25 - 30 \times (\frac{4}{5})$

$= 10$

$EF = \sqrt{10}$

c) Roots are α and 2α

so $\alpha + 2\alpha = -\frac{b}{a}$

$3\alpha = \frac{6}{m-4}$

$\alpha = \frac{2}{m-4}$

$\alpha \times 2\alpha = \frac{c}{a}$

$2\alpha^2 = \frac{7}{m-4}$

sub $\alpha = \frac{2}{m-4}$

$2\left(\frac{2}{m-4}\right)^2 = \frac{7}{m-4}$

$\frac{8}{(m-4)^2} = \frac{7}{m-4}$

for quadratic, $m-4 \neq 0$ so divide off.

$\frac{8}{m-4} = 7$

$8 = 7m - 28$

$7m = 36$

$m = \frac{36}{7}$ or $5\frac{1}{7}$

Q15

a) $(x+6)^2 = 4(y-5)$

i. $V(-6,-5)$

ii. $S(-6,-4)$

iii. $y=-6$

b) $\frac{(10x-4)^6}{60} + C$

c) i. In $\triangle ABC$ and $\triangle DCE$

1. $\angle ACB = \angle DEC$

(alternate angles, $AC \parallel ED$)

2. $\angle ABC = \angle ECD$

(alternate angles, $AB \parallel CD$)

$\therefore \triangle ABC \parallel\!\mid \triangle DCE$ (equiangular)

ii. $\frac{AB}{DC} = \frac{BC}{CE} = \frac{AC}{DE}$

(matching sides in ratio, similiar triangles)

$\frac{18}{8} = \frac{12+CE}{CE}$

$18CE = 96 + 8CE$

$10CE = 96$

$CE = 9.6$

$BE = 9.6 + 12$

$= 21.6\text{ cm}$

d) i. $p = r + r + r\theta \quad (l = r\theta)$

$= r(2+\theta)$

ii. $6 = r(2+\theta)$

$r = \frac{6}{2+\theta}$

Area of sector $= \frac{1}{2}r^2\theta$

$= \frac{1}{2}\times\left(\frac{6}{2+\theta}\right)^2 \times \theta$

$= \frac{18\theta}{(2+\theta)^2}$

iii. $A = \frac{18\theta}{(2+\theta)^2}, \quad \theta \neq -2$

$u = 18\theta \qquad v = (2+\theta)^2$

$u' = 18 \qquad v' = 2(2+\theta)$

$\frac{dA}{d\theta} = \frac{18(2+\theta)^2 - 36\theta(2+\theta)}{(2+\theta)^4}$

$= \frac{18(2+\theta) - 36\theta}{(2+\theta)^3}$

For max, $\frac{dA}{d\theta} = 0$

$0 = \frac{18(2+\theta) - 36\theta}{(2+\theta)^3}$

$0 = 36 + 18\theta - 36\theta$

$0 = 36 - 18\theta$

$18\theta = 36$

$\theta = 2$

test

θ	1	2	3
A'	$\frac{2}{3}$	0	$-\frac{18}{125}$
	/	—	\

$\therefore$ max

Max Area $= \frac{18\times 2}{(2+2)^2}$

$= 2.25\text{ m}^2$

Q16.

a) LHS $= \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$

$= \frac{\cos^2 A + (1+\sin A)^2}{\cos A(1+\sin A)}$

$= \frac{\cos^2 A + 1 + 2\sin A + \sin^2 A}{\cos A(1+\sin A)}$

$= \frac{\cos^2 A + \sin^2 A + 1 + 2\sin A}{\cos A(1+\sin A)}$

$= \frac{2 + 2\sin A}{\cos A(1+\sin A)}$

$= \frac{2(1+\sin A)}{\cos A(1+\sin A)}$

$= \frac{2}{\cos A}$

$=$ RHS.

b) $T_4 = 18$

$18 = a + 3d$ ①

$S_{10} = 195$

$195 = 5(2a+9d)$ ②

$3\times$① : $54 = 3a + 9d$

② : $39 = 2a + 9d$

$15 = a + 0$

$\therefore a = 15$

c) i. $A_1 = 55000 \times 1.0045 - 1500$

$A_2 = A_1 \times 1.0045 - 1500$

$= (55000\times 1.0045 - 1500)\times 1.0045 - 1500$

$= 55000\times 1.0045^2 - 1500\times 1.0045 - 1500$

$= 55000\times 1.0045^2 - 1500(1.0045+1)$

ii. $A_3 = 55000\times 1.0045^3 - 1500(1.0045^2 + 1.0045 + 1)$

$A_n = 55000\times 1.0045^n - 1500(1 + 1.0045 + \dots + 1.0045^{n-1})$

$= 55000\times 1.0045^n - 1500\left[\frac{1(1.0045^n - 1)}{1.0045 - 1}\right]$

$= 55000\times 1.0045^n - 1500\left[\frac{1.0045^n - 1}{0.0045}\right]$

iii. $0 = 55000\times 1.0045^n - 1500\left(\frac{1.0045^n - 1}{0.0045}\right)$

$1500\left(\frac{1.0045^n - 1}{0.0045}\right) = 55000\times 1.0045^n$

$1.0045^n - 1 = \frac{55000\times 0.0045}{1500}\times 1.0045^n$

$1.0045^n - \frac{33}{200}(1.0045^n) = 1$

$\frac{167}{200}(1.0045^n) = 1$

$1.0045^n = \frac{200}{167}$

$\frac{\ln\frac{200}{167}}{\ln 1.0045} = n$

$n = 40.16\dots$

$\doteqdot 40$ months.

iv. $A_{12} = 55000\times 1.0045^{12} - 1500\left(\frac{1.0045^{12} - 1}{0.0045}\right)$

$\doteqdot 39592.37071\dots$

1st month after 12 (i.e. 13th)

$A_1 = A_{12}\times 1.0045 - 3000$

$A_2 = (A_{12}\times 1.0045 - 3000)1.0045 - 3000$

$= A_{12} \times 1.0045^2 - 3000(1+1.0045)$

$A_3 = A_{12} \times 1.0045^3 - 3000(1+1.0045+1.0045^2)$

$A_n = A_{12} \times 1.0045^n - 3000(1+1.0045+\dots+1.0045^{n-1})$

$= A_{12} \times 1.0045^n - 3000\left[\dfrac{1.0045^n - 1}{0.0045}\right]$

let $A_n = 0$

$0 = A_{12} \times 1.0045^n - 3000\left[\dfrac{1.0045^n - 1}{0.0045}\right]$

$3000\left[\dfrac{1.0045^n - 1}{0.0045}\right] = A_{12} \times 1.0045^n$

$1.0045^n - 1 = \dfrac{A_{12} \times 0.0045}{3000} \times 1.0045^n$

$1.0045^n - \left(\dfrac{A_{12} \times 0.0045}{3000}\right) \times 1.0045^n = 1$

$1.0045^n\left(1 - \dfrac{A_{12} \times 0.0045}{3000}\right) = 1$

$1.0045^n = 1 \div \left(1 - \dfrac{A_{12} \times 0.0045}{3000}\right)$

$= 1.063138245\dots$

$n = \dfrac{\ln 1.063138245\dots}{\ln 1.0045}$

$= 13.6361\dots$

$\doteqdot 13$

$\therefore$ He can travel 13 more months.

(can travel 25 months altogether)