Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics

Preliminary Course

Assessment 1

April, 2015

Time allowed: 70 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- *Begin each question on a new page*
- Write using black or blue pen
- All answers are to be in the writing booklet provided

| Section 1 | Multiple Choice Questions 1-5 5 Marks |
| Section II | Questions 6-13 56 Marks |

5 Marks

Attempt questions 1-5

Use multiple choice answer sheet

1. What are the solutions of $2x^2 - 5x - 1 = 0$

(A) $x = \dfrac{-5 \pm \sqrt{17}}{4}$

(C) $x = \dfrac{-5 \pm \sqrt{33}}{4}$

(B) $x = \dfrac{5 \pm \sqrt{17}}{4}$

(D) $x = \dfrac{5 \pm \sqrt{33}}{4}$

2. Which inequality defines the domain of the function $f(x) = \dfrac{1}{\sqrt{x + 3}}$?

(A) $x > -3$

(C) $x < -3$

(B) $x \geq -3$

(D) $x \leq -3$

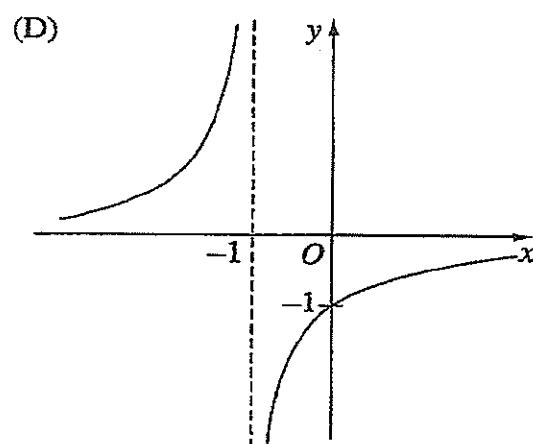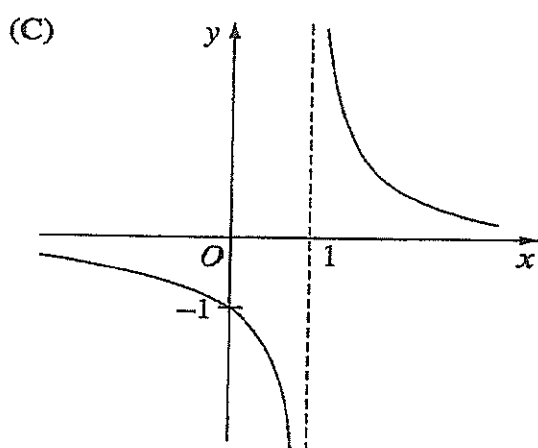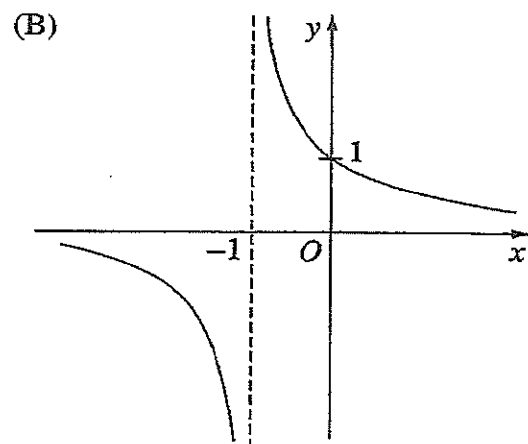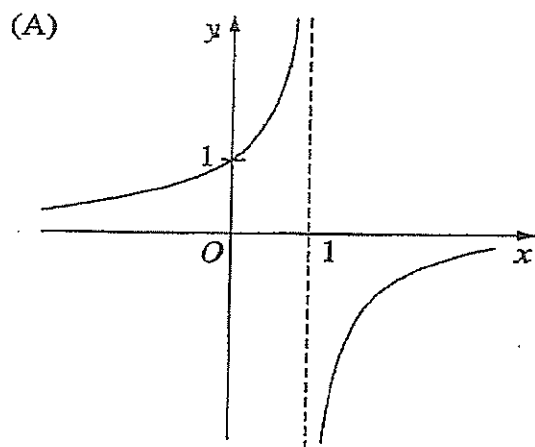3. What is 4.09784 correct to three significant figures?
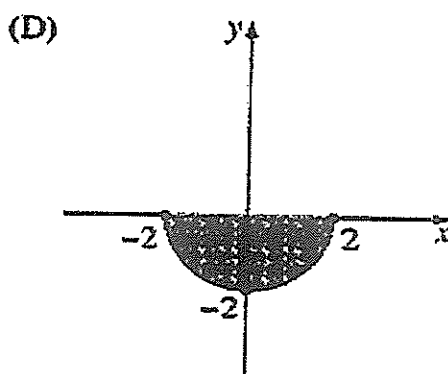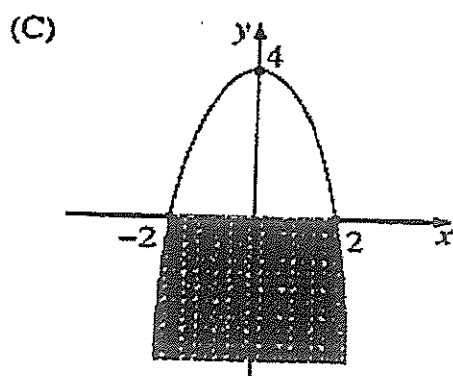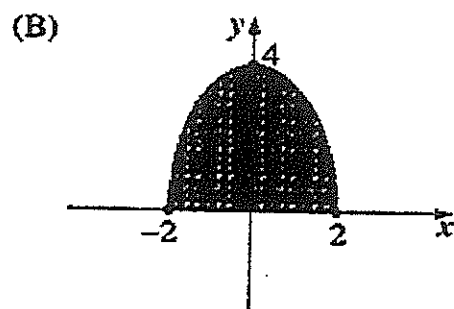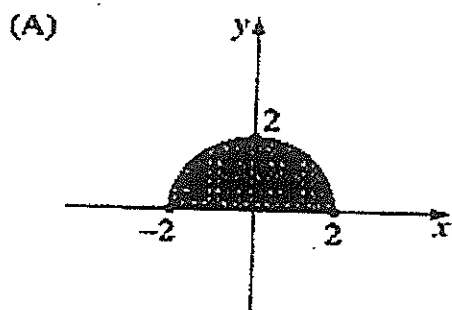
(A) 4.09   (B) 4.10   (C) 4.097   (D) 4.098

4. Which of the following graphs represents $y = \dfrac{1}{1-x}$?

(A)

(B)

(C)

(D)

5. Which diagram shows the region in the plane defined by $y \geq 0$ and $y \leq 4 - x^2$?

(A)

(B)

(C)

(D)

SECTION 2

56 Marks

Attempt questions 6-13

Marks are indicated next to question.

Start each question on a new page.

QUESTION 6 - (7 Marks)

(a)   Evaluate $\dfrac{1}{(1.04)^9-1}$ correct to 3 decimal places   2

(b)   Find $(2\sqrt{5})^3$ as an exact value   1

(c)   Simplify $\dfrac{b-a}{a-b}$ 1

(d)   Find the exact value of $8^{-2/3}$ 1

(e)   Solve $3 - x \leq 4$ and sketch the solution on a number line   2

QUESTION 7 – (7 Marks)   (Start a new page)   Marks

(a)   Draw a neat sketch (with a ruler) of $(x-1)^2 + (y+2)^2 = 9$ and state the   3
domain and range.

(b)   Solve $|2x - 1| > 9$ 2

(c)   Determine whether the function $f(x) = \dfrac{2x}{x^2+1}$ is odd, even or neither   2

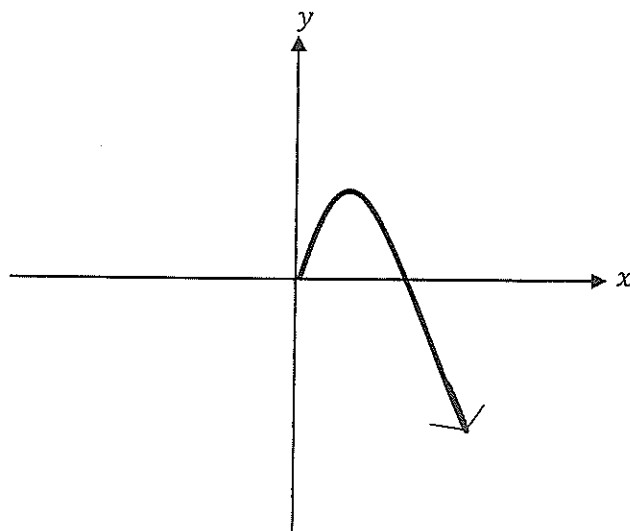QUESTION 8 – (7 marks)   (Start a new page)   **Marks**

(a) Express $\dfrac{2\sqrt{3}}{2-\sqrt{3}}$ in the form $a + \sqrt{b}$ 2

(b) If $f(x) = 8 - x^3$, find the value of

    (i) $f(2)$ 1

    (ii) x if $f(x) = 35$ 2

(c) Simplify $\sqrt{\dfrac{a^3 b^7}{a^5 b^3}}$ 2

QUESTION 9 – (7 marks)   (Start a new page)   **Marks**

(a)   Factorise fully:

    (i) $x^3 - 4x$ 2

    (ii) $ay - 3a + y^2 - 3y$ 2

    (iii) $8x^3 - 27$ 2

(b)   The diagram shows part of a function $y = f(x)$ 1

Copy or trace this diagram onto your answer sheet.

Complete the graph of $y = f(x)$ given that it is an odd function.

QUESTION 10 – (7 marks)   **(Start a new page)**

(a)   Solve the following equations

    (i) $2x^2 - 5x + 3 = 0$

             2

    (ii) $\frac{x-4}{3} + 2 = \frac{3x}{5}$

             2

    (iii) $|x + 1| = 5 - 3x$

             3

QUESTION 11 – (7 marks)   **(Start a new page)   Marks**

(a)   For the parabola $y = x^2 + 4x + 3$, find

    (i)   the y intercept

             1

    (ii) the x intercepts
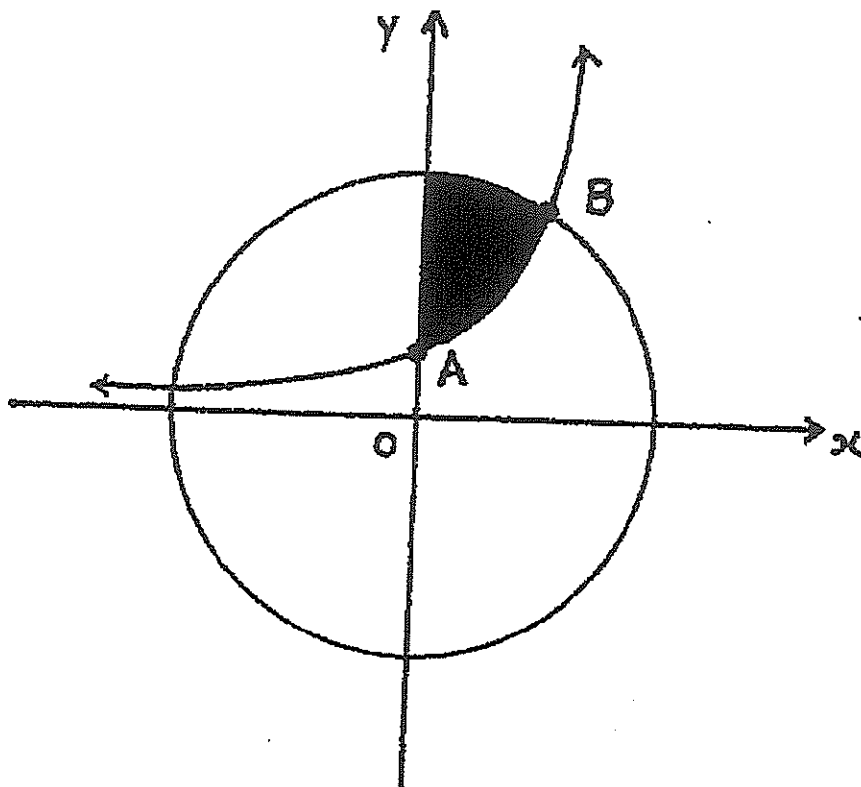
             2

    (iii) the coordinates of the vertex.

             2

    (iv) Sketch the parabola showing all important features

             2

(a)   A circle, centre the origin, and an exponential function of the form $y = a^x$ are shown on the diagram below. A and B lie on the curves as shown. B has coordinates (1,3)

Find

(i)   the coordinates of A.   1

(ii)   the equation of the circle.   2

(iii)   the equation of the exponential function.   1

(iv)   Give the three inequations which describe the shaded region above.   3

QUESTION 13 – (7 marks)   (Start a new page)

(a)   Sketch $y = 1 - \frac{1}{x-2}$ on a number plane showing all important features. Use a ruler.   2

(b)   Factorise $(a + b)^2 - a - b$ fully.   2

(c)   Simplify $\dfrac{a^{-1} + b^{-1}}{a^{-2} - b^{-2}}$ 3

END OF PAPER

Year 11 2015 2 Unit
Task 1 Solutions

1. D   2. A   3. B   4. A   5. B

6a. 2.3620 ①
① for 3 d.p.

b. $8 \times 5\sqrt{5}$
$40\sqrt{5}$

c. $\frac{b-a}{a-b}$
$= \frac{-(a-b)}{a-b}$
$= -1$

d. $\frac{1}{8^{2/3}}$
$\frac{1}{2^2}$
$= \frac{1}{4}$

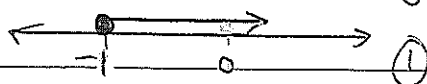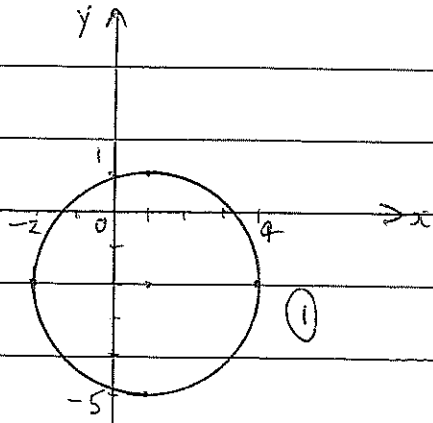e. $3 - x \leq 4$
$-x \leq 1$
$x \geq -1$ ①

①

7. a.

①

b. $|2x - 1| > 9$

$2x - 1 > 9$ and $2x - 1 < -9$
$2x > 10$ $2x < -8$
$x > 5$ ① $x < -4$
                              ①

D: $-2 \leq x \leq 4$ ①
R: $-5 \leq y \leq 1$ ①

c. $y = \frac{2x}{x^2+1}$

Odd if

$f(-x) = -f(x)$ ①

$f(-x) = \frac{2x - x}{(-x)^2 + 1}$
$= \frac{-2x}{x^2+1}$
$= -\frac{2x}{x^2+1}$
① $= -f(x)$

$\therefore f(x)$ is odd

8. a. $\dfrac{2\sqrt{3}}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \dfrac{4\sqrt{3}+6}{1}$

$= 6 + \sqrt{48} = a + \sqrt{b}$

$a = 6$ ①, $b = 48$ ①

b. i) $f(x) = 8 - x^3$

$f(2) = 8 - 2^3$

$= 0$

ii) $35 = 8 - x^3$

$x^3 = -27$ ①

$x = -3$ ①

c. $\sqrt{\dfrac{a^3 b^7}{a^5 b^3}}$

$= \sqrt{\dfrac{b^4}{a^2}}$ ①

$= \dfrac{b^2}{a}$ ①

9. a. i) $x^3 - 4x$

$x(x^2 - 4)$ ①

$x(x-2)(x+2)$ ①

ii) $ay - 3a + y^2 - 3y$

$a(y-3) + y(y-3)$ ①

$(y-3)(a+y)$ ①

iii) $8x^3 - 27$

$(2x)^3 - 3^3$

$(2x-3)(4x^2 + 6x + 9)$ ②

give one for

$(2x+3)(4x^2 - 6x + 9)$

b.
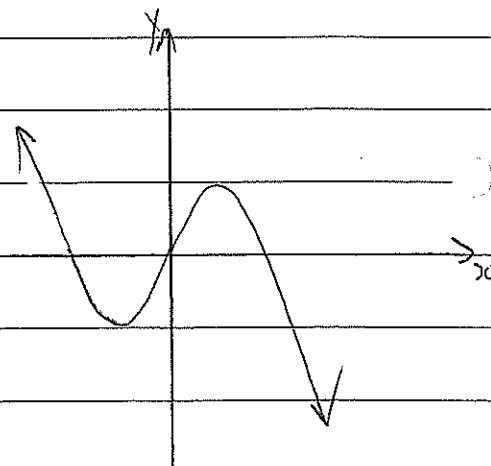
10. a-ci) $2x^2 - 5x + 3 = 0$ (ii) $\dfrac{x-4}{3} + 2 = \dfrac{3x}{5}$

$\qquad (2x-3)(x-1) = 0$ ① $5(x-4) + 30 = 3 \times 3x$

$\qquad x = \dfrac{3}{2}, 1$ $5x - 20 + 30 = 9x$ ①

$\qquad\qquad\qquad\qquad\qquad\qquad 10 = 4x$

$\qquad\qquad\qquad\qquad\qquad\qquad x = 2\frac{1}{2}$ ①

(iii) $|x+1| = 5 - 3x$

$\quad x+1 = 5-3x, \quad x+1 = -(5-3x)$ ①

$\quad 4x = 4 \qquad\qquad x+1 = -5 + 3x$

$\qquad x = 1 \qquad\qquad\quad 6 = 2x$

$\qquad\qquad\qquad\qquad\qquad x = 3$ ①

Need to check solutions.

Only solution is $x = 1$ ①

11. a-ci) $y = x^2 + 4x + 3$ (ii) When $y = 0$ (iii) $x = \dfrac{-b}{2a}$

$\quad y$ intercept is 3 $0 = x^2 + 4x + 3$ $= \dfrac{-4}{2}$

$\qquad\qquad\qquad\qquad 0 = (x+1)(x+3)$ $= -2$ ①

$\qquad\qquad\qquad\qquad x = -1$ or -3 sub. in
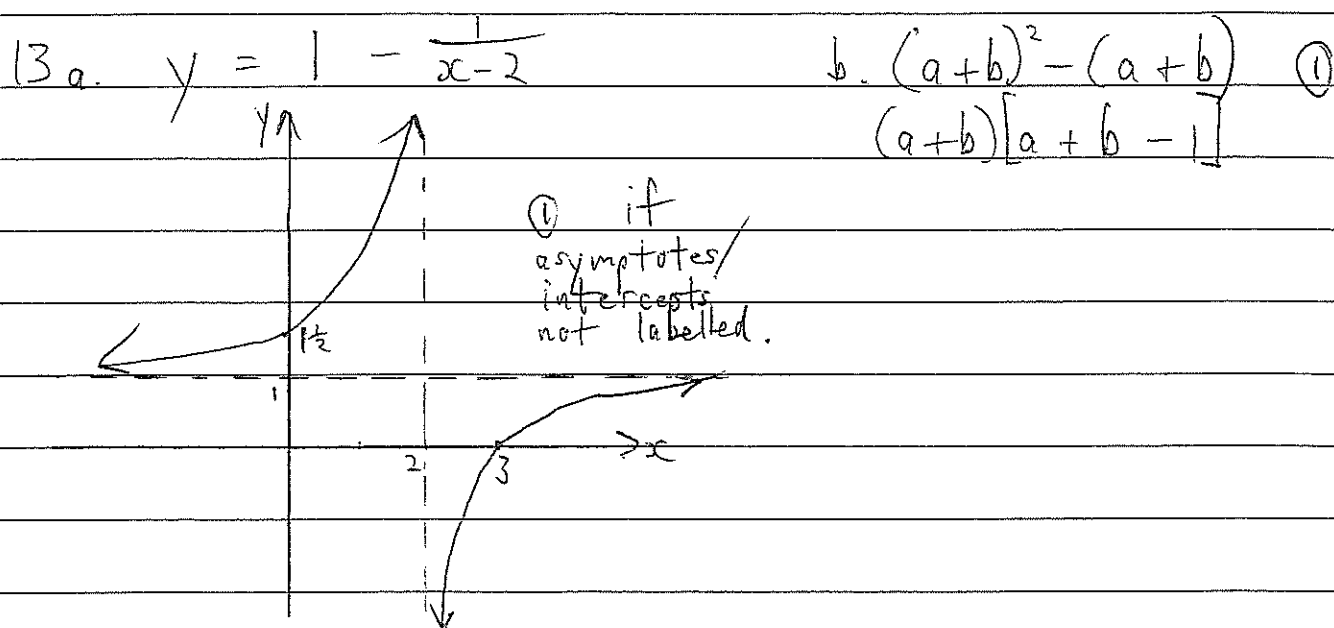
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad (-2, -1)$ ①

(iv)
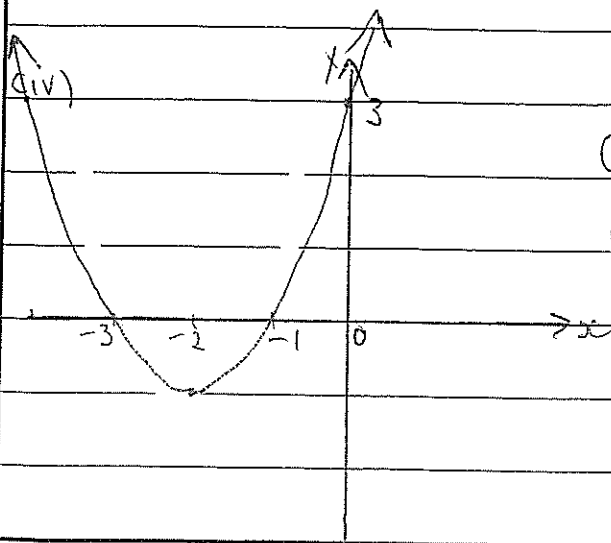
Only ① if intercepts not marked.

12. (a) ci) $(0,1)$ cii) radius of circle

is $\sqrt{1^2 + 3^2} = \sqrt{10}$ ①

∴ Equation is

$$x^2 + y^2 = 10 \qquad ①$$

ciii) $y = a^x$ passes

through $(1,3)$

$3 = a^1$

∴ $a = 3$

$y = 3^x$

civ) $x^2 + y^2 \leq 10$

$x \geq 0$

$y \geq 3^x$

13 a. $y = 1 - \frac{1}{x-2}$

① if asymptotes/ intercepts not labelled.

b. $(a+b)^2 - (a+b)$ ①

$(a+b)[a+b-1]$

c. $\dfrac{a^{-1} + b^{-1}}{a^{-2} - b^{-2}} = \dfrac{\frac{1}{a} + \frac{1}{b}}{\frac{1}{a^2} - \frac{1}{b^2}}$

$= \dfrac{a+b}{ab} \div \dfrac{b^2 - a^2}{a^2 b^2}$ ①

$= \dfrac{a+b}{ab} \times \dfrac{a^2 b^2}{(b-a)(b+a)}$ ①

$= \dfrac{ab}{b-a}$ ①