

Name: Maths Class:

SYDNEY TECHNICAL HIGH SCHOOL

Year 10
MathematicsTERM 3
2016*Time allowed: 90 minutes****General Instructions:***

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided

Section 1 Multiple Choice
Questions 1-10
10 Marks

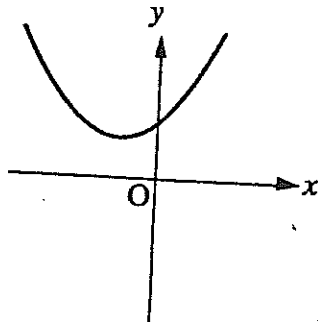
Section II Questions 11- 15
60 Marks

Section I

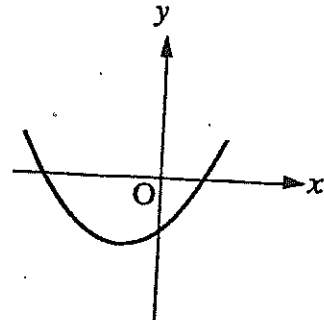
1.

Which of the following could be the graph of $y = x^2 - 2x + 9$?

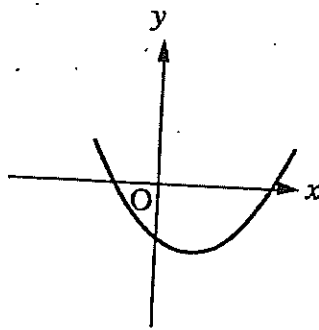
(A)



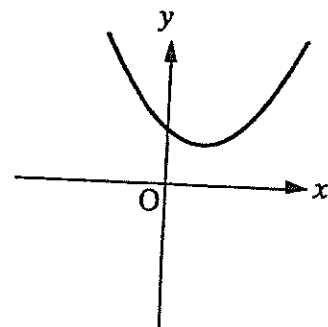
(B)



(C)



(D)



2.

In five spelling tests, Jenny made the following numbers of mistakes:

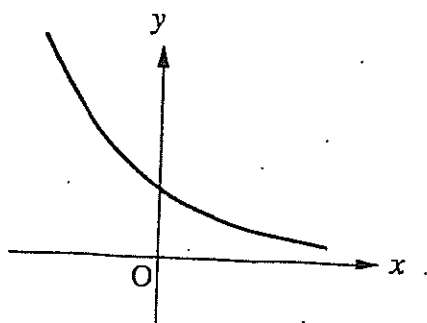
7 1 5 4 3

The mean number of mistakes is 4 and the standard deviation is 2.

If she makes no mistakes in either of the next two tests, then

- (A) the mean increases and the standard deviation increases.
- (B) the mean increases and the standard deviation decreases.
- (C) the mean decreases and the standard deviation increases.
- (D) the mean decreases and the standard deviation decreases.

3.



The graph shown could be part of the graph with equation

- (A) $y = 2^{-x}$
- (B) $y = 2^x$
- (C) $y = -\frac{1}{x}$
- (D) $y = \frac{1}{x}$

4

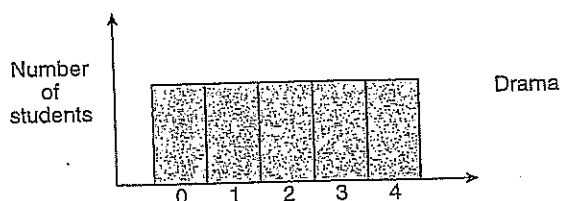
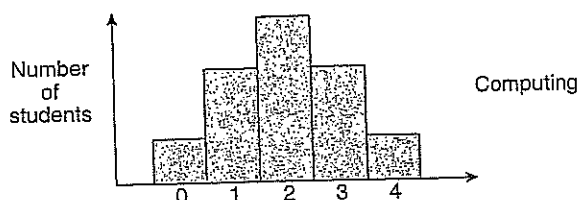
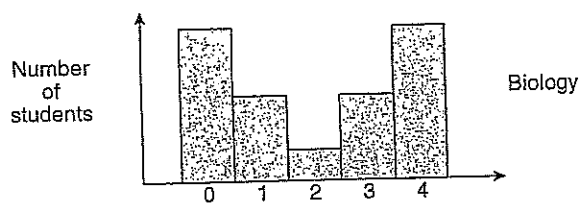
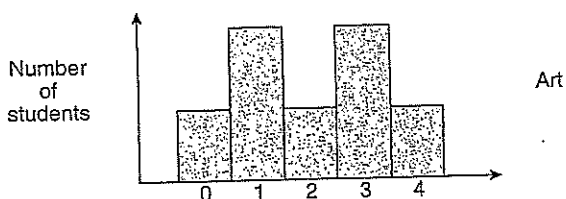
The mean of a set of ten scores is 14. Another two scores are included and the new mean is 16.

What is the mean of the two additional scores?

- (A) 4
- (B) 16
- (C) 18
- (D) 26

5

Thirty students sat for tests in four different subjects. Each test was marked out of four. A histogram of the results for each subject is shown below.



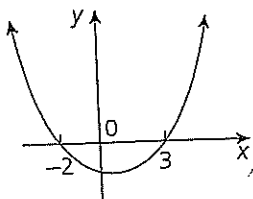
Which subject had marks with the highest standard deviation?

- (A) Art
- (B) Biology
- (C) Computing
- (D) Drama

6

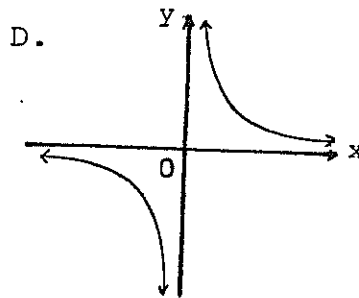
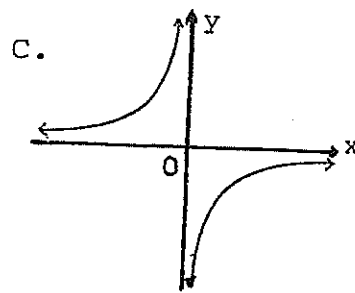
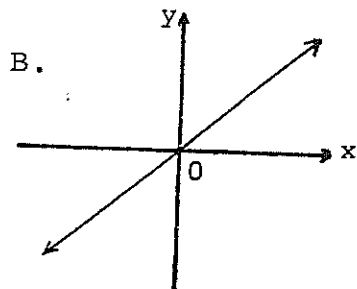
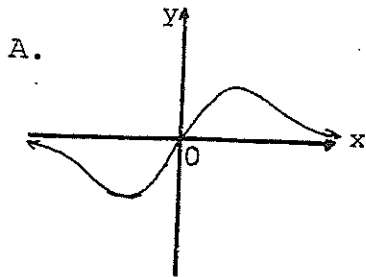
The equation of the curve could be:

- A $y = (x - 2)(x + 3)$
- B $y = (x + 2)(x - 3)$
- C $y = (x - 2)(x - 3)$
- D $y = (x + 2)(x + 3)$



7

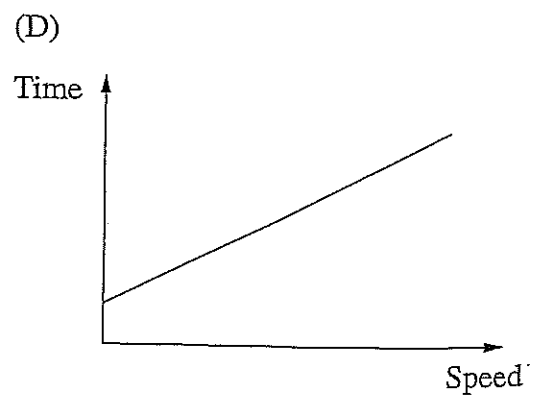
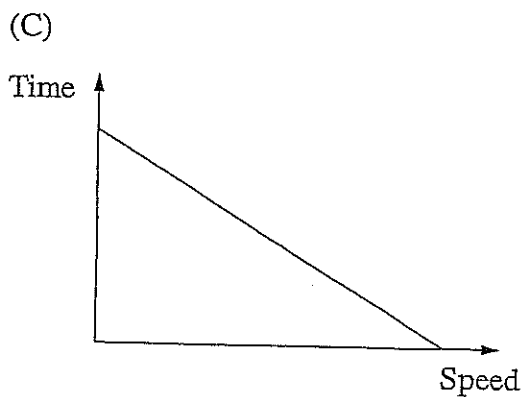
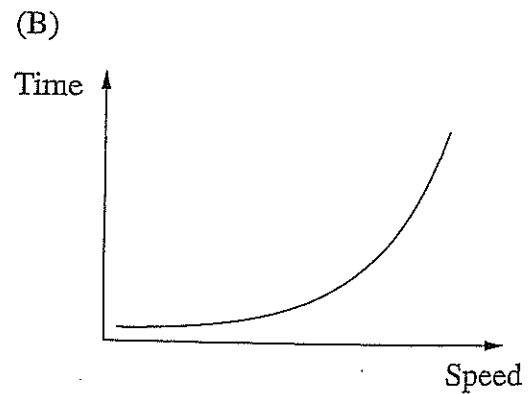
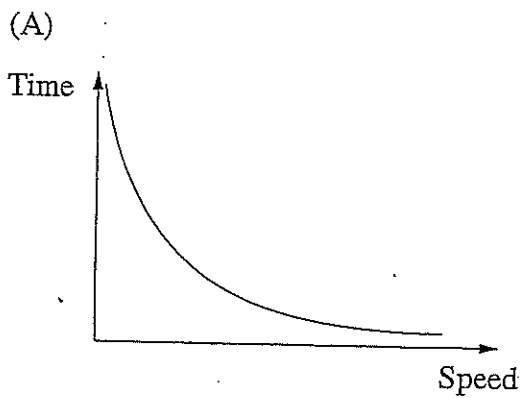
Which graph best represents $xy = 1$?



8

The time for a car to travel a certain distance varies inversely with its speed.

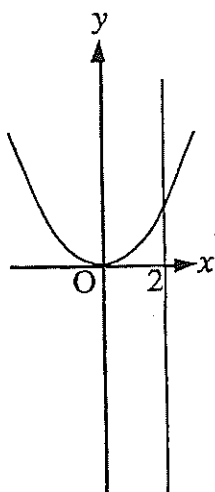
Which of the following graphs shows this relationship?



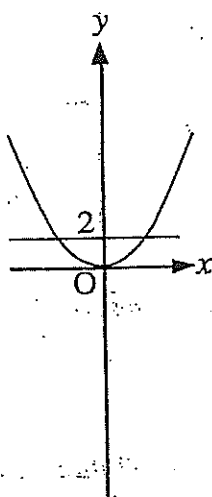
9

Which diagram shows sketches of $y = 2$ and $y = x^3$?

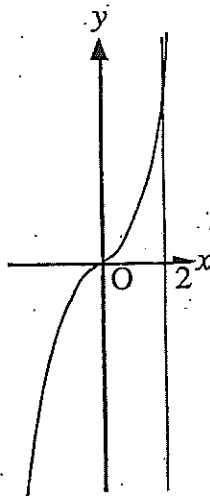
(A)



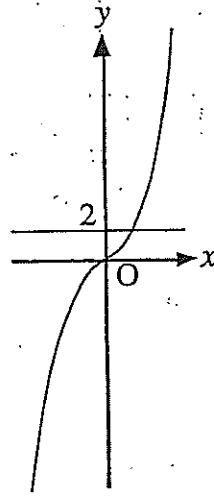
(B)



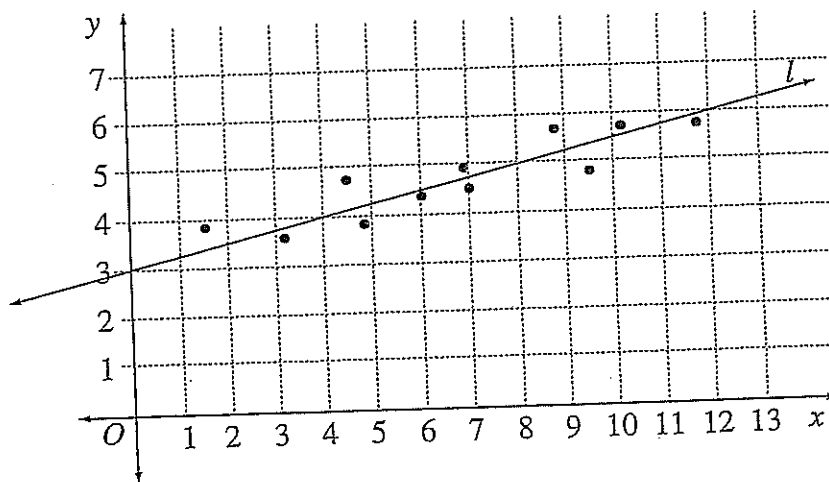
(C)



(D)



10.



A line of fit, l , is drawn through the points as shown.

What is the correct equation for line l ?

- (A) $y = \frac{x}{4} + 3$
- (B) $y = \frac{x}{4} - 3$
- (C) $y = 4x - 12$
- (D) $y = 4x + 3$

Section 2

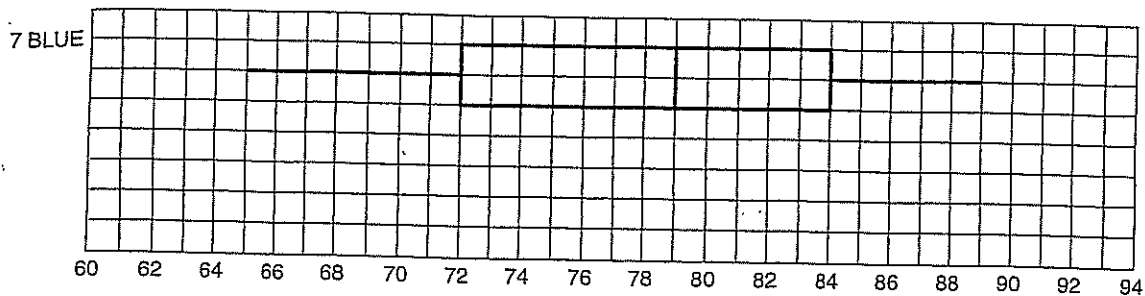
QUESTION 11 (14 marks)

a)

The table below shows test results for students in two ungraded classes.

7 Blue	68	78	85	89	80	76	65	83
7 Green	69	76	84	82	82	72	76	83

The box-and-whisker plot drawn is for 7 Blue.



- State the 5 summary statistics required to draw the box-and-whisker plot for 7 Green.
- On the same grid as 7 Blue, draw a box-and-whisker plot for 7 Green.
- Based on these test results, are the two classes of equal ability? Justify your answer.

b)

Jillian received a threatening letter. It is being analysed by the police in an attempt to identify its writer.

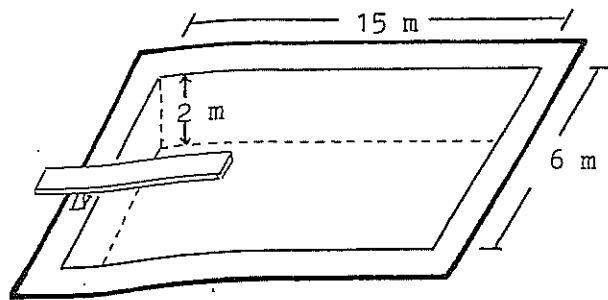
The number of words per sentence has been counted. These data are summarised in the following table.

Number of words per sentence	Frequency
2	2
3	3
4	6
5	6
6	7
7	5
8	4
9	4
10	3
Total	40

- How many sentences were in this letter?
- How many sentences contained more than five words?
- Calculate the mean number of words per sentence, and the standard deviation.
- Jillian received a second letter. The police suspect that this second letter was written by a different person, because the sentences were longer on average and more consistent in length.

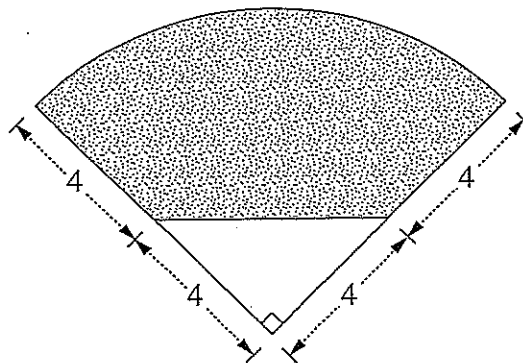
Write a statement comparing the mean number of words per sentence and the standard deviation for the second letter to your results in part (iii).

c)



An in-ground pool is 6 m wide, 15 m long and 2 m deep. The walls and floor of the pool are to be repainted. What is the total surface area to be repainted?

d)



NOT TO SCALE

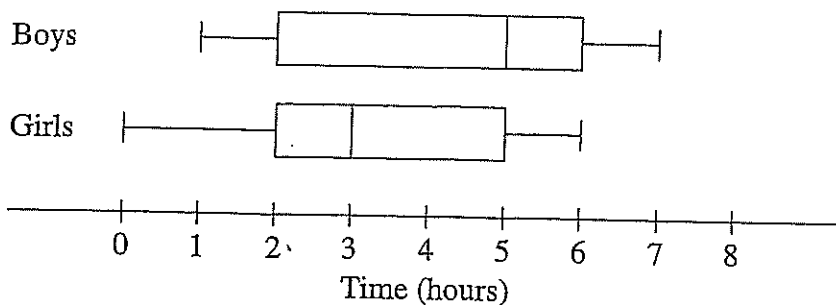
All measurements are in centimetres.

What is the area of the shaded part of this quadrant, to the nearest square centimetre

QUESTION 12 (11 marks) (Start a new page)

a)

In a school, boys and girls were surveyed about the time they usually spend on the internet over a weekend. These results were displayed in box-and-whisker plots, as shown below.



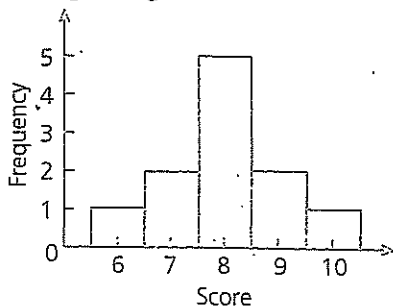
- (i) Find the interquartile range for boys. 1
- (ii) What percentage of girls usually spend 5 or less hours on the internet over a weekend? 1
- (iii) Jenny said that the graph shows that the same number of boys as girls usually spend between 5 and 6 hours on the internet over a weekend. 1

Under what circumstances would this statement be true?

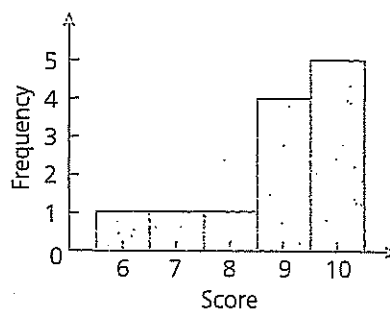
b)

Match the terms 'negatively skewed', 'positively skewed', 'bi-modal' and 'normally distributed' to each of the following histograms:

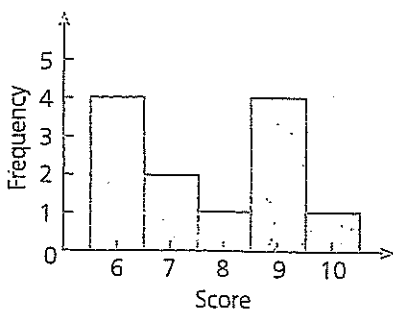
i)



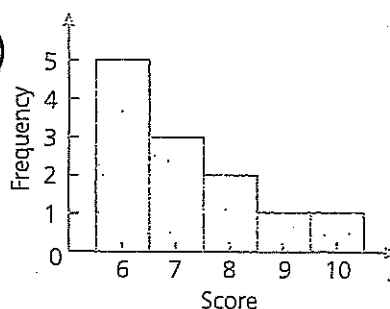
ii)



iii)



iv)



c)

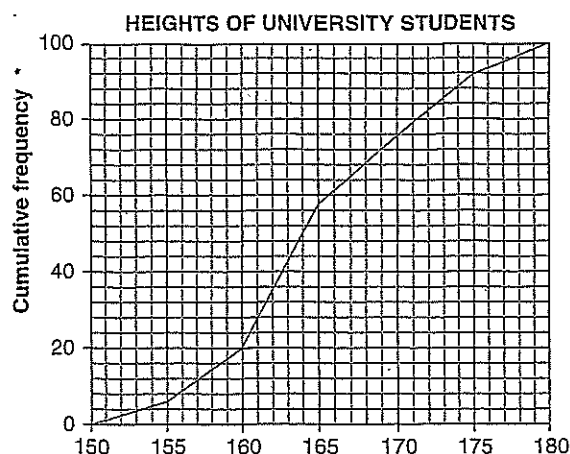
Data was collected from 30 students on the number of text messages they had sent in the previous 24 hours. The set of data collected is displayed.

Male		Female
9 9 8 7 6 5 5 4 2 1	0	8 9
1 1 0 0	1	1 1 2 5 6 8 8 8
0	2	0 1 7
	3	4
	4	
	5	
	6	
1	7	

- (i) What is the outlier for this set of data? 1
- (ii) What is the interquartile range of the data collected from the female students? 1

d)

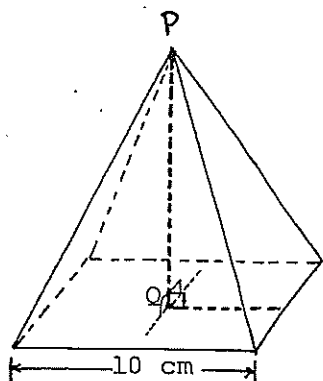
Gary surveyed 100 university students and recorded their heights. He drew the following cumulative frequency graph to represent the information.



- (i) Use the graph to estimate the median height.
- (ii) Use the graph to estimate the interquartile range.

QUESTION 13 (Start a new page) (11 marks)

a)



The figure shows a square based pyramid with identical triangular faces.

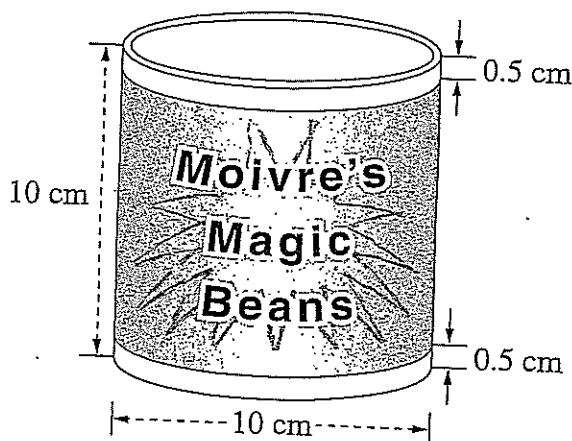
$PQ = 12$ cm

What is the total surface area (in cm^2) of this pyramid?

2

b)

Moivre's manufacturing company produces cans of Magic Beans. The can has a diameter of 10 cm and a height of 10 cm.



- (i) Cans are packed in boxes that are rectangular prisms with dimensions $30 \text{ cm} \times 40 \text{ cm} \times 60 \text{ cm}$.

1

What is the maximum number of cans that can be packed into one of these boxes?

- (ii) The shaded label on the can shown wraps all the way around the can with no overlap.

2

What area of paper is needed to make the labels for all the cans in this box when the box is full? (in m^2 correct to 2 decimal places)

- (iii) The company is considering producing larger cans. Monica says if you double the diameter of the can this will double the volume. Is Monica correct? Justify your answer with suitable calculations.

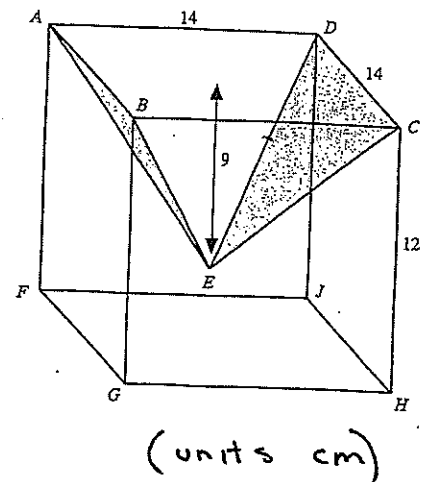
2

c)

$ABCDE$ is a square pyramid which fits exactly into a square prism as shown. The pyramid is used to strain cooking oil so the oil can be reused.

i) Find the volume of the pyramid.

ii) What fraction of the square prism is occupied by the pyramid?

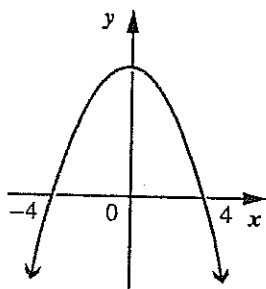


2

2

Question 14 (Start a new page) (12 marks)

a)

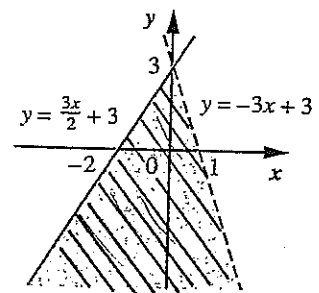


A sketch of $y = k - x^2$ is given. Find the value of k .

1

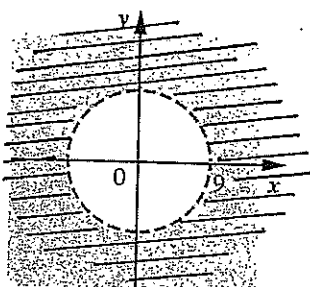
b)

Write a pair of inequalities that describe the shaded region.



2

c)

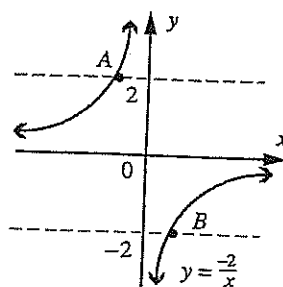


What inequality describes the shaded region?

2

d)

State the coordinates of A and B in this diagram.



2

e)

- i) Sketch the exponential graph $y = 3^x$
- ii) Is y ever negative?
- iii) At what point does the curve cut the y -axis?
- iv) Describe what happens as x gets smaller.

4

f)

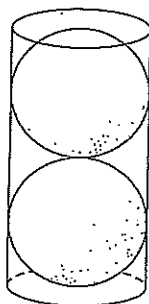
Given the parabola $y = 4 - 2x - x^2$
find i) the axis of symmetry
ii) the maximum value

2

QUESTION 15 (Start a new page) (12 marks)

a)

Two identical spheres fit exactly inside a cylindrical container, as shown.



2

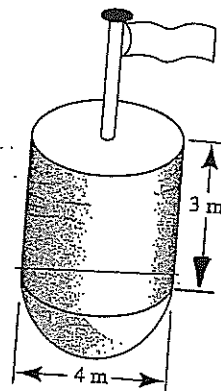
The diameter of each sphere is 12 cm.

What is the volume of the cylindrical container, to the nearest cubic centimetre?

b)

A buoy is constructed from a hemisphere and cylinder as shown.
Neglecting the flag, find:

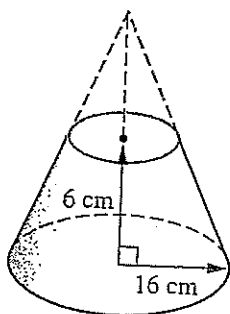
- i) the volume of the buoy (in terms of π)
- ii) its surface area. (in terms of π)



2

2

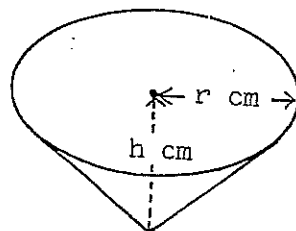
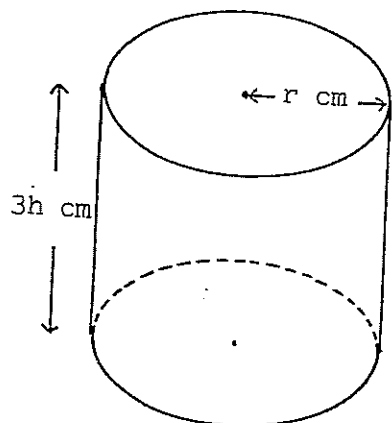
c)



This frustum is made by removing a cone 6 cm high from the top of a larger cone, 12 cm high. Given that the radius of the top circle is 8 cm, find the total surface area of this frustum (to the nearest cm^2).

3

d)



- i) Write a formula for the volume of both the cylinder and the cone in terms of r, h and π .

2

- ii) How many times must the cone be filled with water and emptied into the cylinder in order to fill the cylinder?

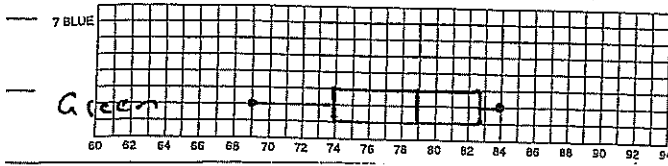
1

m/c

D 3 A 5 B 7 D 9 D

2 A 4 D 6 B 8 A 10 A

Question 11



69, 72, 76, 76, 82, 83, 84

i) Lowest score

69

Highest score

84

Median

79

Lower quartile

74

Upper quartile

82.5

do not need to write these values

iii) Green more consistent in ability

i) 40

ii) 23

iii) $\bar{x} = 6.05$ $G^* = 2.2$ iv) 2nd letter higher mean no. of words and smaller SD \therefore more consistent in length

$$SA = 2(2 \times 15) + 2(2 \times 6) + (6 \times 15) \\ = 174 \text{ m}^2$$

$$A = \frac{\pi \times 8^2}{4} - \frac{4 \times 4}{2}$$

$$= 42 \text{ cm}^2 \text{ (nearest cm}^2\text{)}$$

C

Question 12

a) i) IQR boys = 4

ii) 75%

iii) true if total no. of boys girls was equal

b) i) normal

ii) neg. skewed

iii) bimodal

iv) pos. skewed

c) i) 71 (male) outlier

ii) $20 - 11 = 9$ IQR female

d) i) median 164

ii) $169.5 - 160.5 = 9$ IQR (± 1)

Question 13

a) slant height 13

$$\therefore SA = 10^2 + 4\left(\frac{1}{2} \times 10 \times 13\right) \\ = 360 \text{ cm}^2$$

b) i) 72 cans

$$\text{ii) } SA = (2\pi \times 5 \times 9) \times 72 \\ = 20357.5204 \text{ cm}^2 \\ = 2.04 \text{ m}^2$$

iii) $V_{\text{small}} = \pi \times 5^2 \times 10 = 250\pi \text{ cm}^3$

$$V_{\text{large}} = \pi \times 10^2 \times 10 = 1000\pi \text{ cm}^3$$

Monica incorrect you increase volume by 4

$$c) i) V_{\text{pyramid}} = \frac{1}{3} \cdot 14^2 \cdot 9$$

$$= 588 \text{ cm}^3$$

$$ii) V_{\text{prism}} = 14 \times 14 \times 12$$

$$= 2352 \text{ cm}^3$$

$$\text{Fraction } \left(\frac{\text{pyramid}}{\text{prism}} \right) = \frac{588}{2352}$$

$$= \frac{1}{4}$$

Question 14

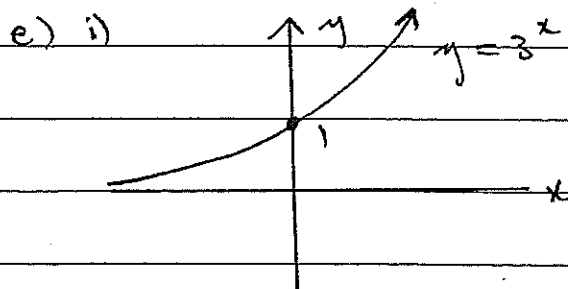
$$a) \underline{k = 16}$$

$$b) y \leq \frac{3x}{2} + 3$$

$$\underline{y < -3x + 3}$$

$$c) \underline{x^2 + y^2 > 81}$$

$$d) \underline{A(-1, 2) \quad B(1, -2)}$$



$$ii) \underline{y > 0 \quad \therefore \text{not negative}}$$

$$iii) \underline{y = 1}$$

$$iv) \underline{y \text{ approaches zero}}$$

$$f) y = 4 - 2x - x^2$$

$$\text{axis of sym} = \frac{-(-2)}{-2} = -1$$

$$\therefore \underline{x = -1}$$

$$\text{max when } x = -1 \text{ is } \underline{y = 5}$$

Question 15

a)

$$V = \pi \cdot 6^2 \cdot 24$$

$$\underline{V = 2714 \text{ cm}^3}$$

b) i) $V = \pi \cdot 2^2 \cdot 3 + \frac{1}{2} \left(\frac{4\pi \cdot 2^3}{3} \right)$

$$= 12\pi + \frac{16\pi}{3}$$

$$= \underline{\underline{\frac{52\pi}{3} \text{ m}^3}}$$

ii) $SA = (\pi \cdot 2^2) + (2 \cdot \pi \cdot 2 \cdot 3) + \frac{4\pi \cdot 2^2}{3}$

↑
circle
↑
curve area
↑
hemisphere

$$= \underline{\underline{24\pi \text{ m}^2}}$$

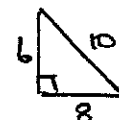
c)

Top circle = $\pi \cdot 8^2$

Btm circle = $\pi \cdot 16^2$

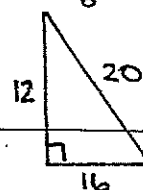
Curved SA

small cone = $\pi \cdot 8 \cdot 10$



curved SA

large cone = $\pi \cdot 16 \cdot 20$



$$\therefore \text{Total SA} = 64\pi + 256\pi + (320\pi - 80\pi)$$

$$= 560\pi$$

$$= \underline{\underline{1759 \text{ cm}^2}}$$

d) $V_{\text{cylinder}} = \underline{\underline{\pi r^2 \cdot 3h}}$

i)

$$V_{\text{cone}} = \underline{\underline{\frac{1}{3} \pi r^2 h}}$$

ii)

$$\frac{\text{cylinder}}{\text{cone}} = \frac{\pi r^2 \cdot 3h}{\frac{1}{3} \pi r^2 h}$$

$$= \underline{\underline{9 \text{ times}}}$$