Name.....

Teacher.....

2012
Preliminary Course
FINAL EXAMINATION

Mathematics

General Instructions

- Working Time - 2 hours.
- Write using a blue or black pen.
- Approved calculators may be used.
- All necessary working should be shown for every question.
- Begin each question on a fresh sheet of paper.

Total marks – 90

Section I *Pages 3–4* ***10 marks***

- *Attempt Questions 1–10*
- *Allow about 15 minutes for this section*

Section II *Pages 5–12* ***80 marks***

Attempt Questions 11–16

Allow about 1 hours 45 minutes for this section

Section I

Total marks (10)

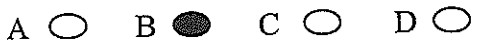
Attempt Questions 1-10

Allow about 15 minutes for this section

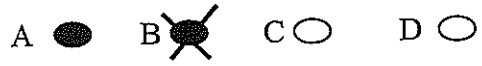
Use the multiple choice answer sheet.
Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample

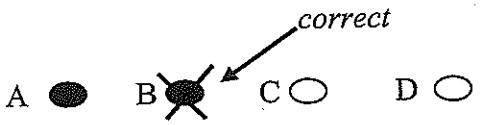
$2 + 4 = ?$ (A) 2 (B) 6 (C) 8 (D) 9



If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.



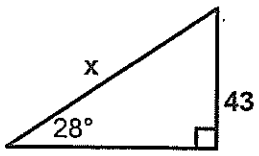
If you change your mind and have crossed out what you consider to be the correct answer, then indicate this by writing the word *correct* and drawing an arrow as follows:



1. The number 147.658 correct to two significant figures is

(a) 15 (b) 150 (c) 147.65 (d) 147.66

2. The value of x is given by



(a) $43 \times \cos 28°$ (b) $43 \times \sin 28°$

(c) $\dfrac{43}{\cos 28°}$ (d) $\dfrac{43}{\sin 28°}$

3. If $4y^2 - 12y + P = (2y + Q)^2$ then

(a) $P = 9 \quad Q = 3$ (b) $P = 9 \quad Q = -3$

(c) $P = -9 \quad Q = -3$ (d) $P = -9 \quad Q = -3$

4. The gradient of any line perpendicular to the line $3x - 2y + 12 = 0$ is

(a) $\frac{3}{2}$ (b) $-\frac{3}{2}$ (c) $\frac{2}{3}$ (d) $-\frac{2}{3}$

5. $\dfrac{2}{\sqrt{2}} + \dfrac{\sqrt{2}}{2} =$

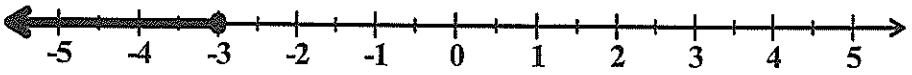
(a) 1 (b) 2 (c) $\dfrac{3\sqrt{2}}{2}$ (d) $\dfrac{2+\sqrt{2}}{2\sqrt{2}}$

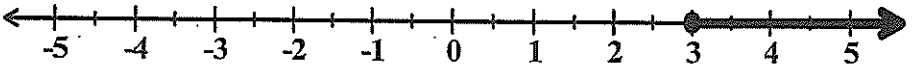
6. Simplify $\dfrac{x^2 - 1}{(x-1)^2}$

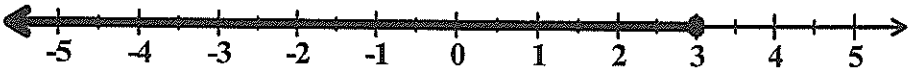
(a) 1 (b) $\dfrac{-1}{-2x+1}$ (c) $\dfrac{x+1}{x-1}$ (d) $\dfrac{x-1}{x+1}$

7. The solution to $1 - 2x \leq 7$ can be represented by

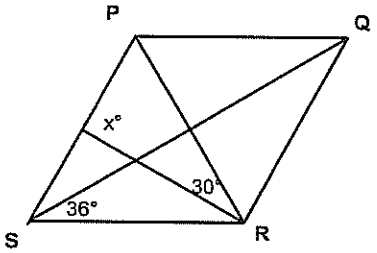
(a) 

(b) 

(c) 

(d) 

8. PQRS is a rhombus. Find the value of x



(a) 90° (b) 96° (c) 102° (d) 108°

9. Find the derivative of $f(x) = \frac{3x^2 - 2x + 1}{x}$

(a) $f'(x) = \frac{3x^2 - 1}{x}$ (b) $f'(x) = \frac{3x^2 - 1}{x^2}$

(c) $f'(x) = 3 - x^2$ (d) $f'(x) = 6x - 2$

10. $\left[\frac{x^{-1}}{3}\right]^{-2} =$

(a) $\frac{1}{9x^2}$ (b) $\frac{9}{x^2}$ (c) $\frac{x^2}{9}$ (d) $9x^2$

Question 11 (10 Marks) Use a Separate Sheet of paper **Marks**

(a) Find the value of $\frac{4.23}{\sqrt{6.14-1.78}}$, giving your answer correct to 2 decimal places. **1**

(b) If $s=\frac{a}{1-r}$ find s when $a=7, r=\frac{1}{3}$. **1**

(c) If $\sqrt{12}+\sqrt{27}=\sqrt{a}$, find the value of a. **2**

(d) Express $\frac{2}{5-2\sqrt{5}}$ as a fraction with a rational denominator. **2**

(e) Fully factorise the following expressions fully

(i) $16x^3y-2y^4$ **2**

(ii) $mx^2+my^2-nx^2-ny^2$ **2**

Question 12 (10 Marks) Use a Separate Sheet of paper **Marks**

(a) Express as a single algebraic fraction in simplest form: **2**

$$\frac{2x-3y}{4x} - \frac{x-3y}{x-1}$$

(b) Solve for x:

(i) $2(3x-4) - 3 = \frac{3x}{2} - 5$ **2**

(ii) $|2x-1| = 6$ **2**

(iii) $x^2 = 5x$ **2**

(c) Solve for x, giving your answers as exact values: **2**

$$2x^2 - 3x - 7 = 0$$

Question 13 (10 Marks) Use a Separate Sheet of paper **Marks**

(a) A function is defined by the rule $g(x)=\begin{cases} x+1, & \text{if } x\geq 1 \\ -1, & \text{if } -2<x<1 \\ 1-x, & \text{if } x\leq -2 \end{cases}$

Find

(i) $g(1)$ **1**

(ii) $g(-3)+g(0)$ **1**

(b) Sketch the graphs of the following, **stating the domain and range of each.**

(i) $y=\frac{2}{x}$ **2**

(ii) $x^2+y^2=25$ **2**

(iii) $3(x+2)-y=0$ **2**

(c) Show that the function $f(x)=\frac{1-x^2}{x}$ is an odd function. **2**

Question 14 (10 Marks) Use a Separate Sheet of paper **Marks**

(a) Find the **exact** value of the following :

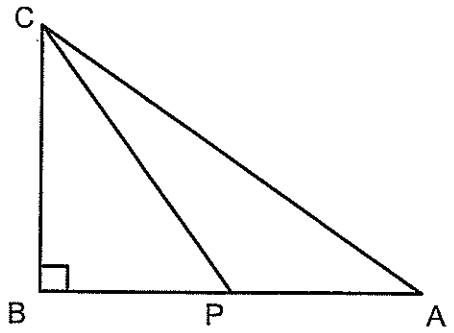
(i) $\cos 135^\circ$ **1**

(ii) $\cot 210^\circ$ **1**

(b) Solve for $0 \leq \theta \leq 360^\circ$

$2\sin^2\theta - 1 = 0$ **2**

(c) In ΔABC, $\angle B = 90^0$ and $\angle A = 31^0$. P is a point on AB such that $AP = 20cm$ and $\angle CPB = 68^0$.



(i) Show that $PC = \dfrac{20\sin 31^0}{\sin 37^0}$ **2**

(ii) Hence, find PB, correct to the nearest centimetre. **2**

(d) In $\Delta ABC, \sin C = \dfrac{2}{3}$, $BC = 12.6\,cm$ and $AC = 9.8\,cm$.

(i) Find an expression for the area of ΔABC. **1**

(ii) Hence, or otherwise, find the area of ΔABC. **1**

Question 15 (10 Marks) Use a Separate Sheet of paper **Marks**

The points A(2,0), B(8,4) ,C(4,6) and D (x_1, y_1) form the 4 vertices of a parallelogram ABCD.

(a) Draw a number plane and plot the points A, B & C. **1**
(USE A RULER)

(b) Find the gradient of line AB **1**

(c) Show that the equation of the line l parallel to AB and passing through C is $2x-3y+10=0$ **2**

(d) Find the point $D(x_1, y_1)$ and mark this point on your diagram. **2**

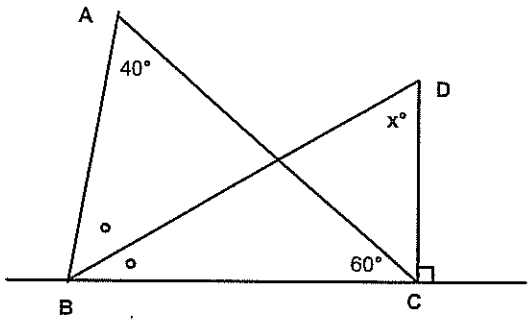
(e) Find the angle θ to the **nearest degree** that the line AB makes with the positive x-axis **2**

(f) Find the perpendicular distance between the line l and A. **2**

Question 16 (10 Marks) Use a Separate Sheet of paper **Marks**

(a) Find the size of an interior angle of a regular nonagon. (9 sided polygon). **2**

(b) In the figure, BD bisects $\angle ABC$, DC is perpendicular to BC, $\angle ACB = 60°$ $\angle BAC = 40°$ $\angle BDC = x°$.



(i) Draw a neat sketch of the diagram. **1**

(ii) Calculate x giving reasons for each step in your calculation. **2**

(c) If $f(x) = 5x - x^2$, find $\dfrac{f(x+h) - f(x)}{h}$ **2**

(d) Find the point on the curve $y = 3x^2 - 5x - 4$ where the tangent is parallel to the line $y - x - 3 = 0$ **3**

Question 17 (10 Marks) Use a Separate Sheet of paper **Marks**

(a) Find the derivative of the following: (You do not need to simplify your answers after finding the derivative.)

(i) $x^4 - 3x^3 + 2$ **1**

(ii) $\sqrt{x^3}$ **1**

(iii) $\frac{1}{3x^4}$ **1**

(b) Find $f'(2)$ for $f(x) = \left(3x^2 - 5x\right)^5$. **2**

(c) Given $y = \frac{x^2 - 1}{x^2 + 1}$ find $\frac{dy}{dx}$ **2**

(d) Find the equation of the normal to the curve $y = 2x^3 - 4x^2$ at the point (1, -2) **3**

Question 18 (10 Marks) Use a Separate Sheet of paper **Marks**

(a) Show the region of the number plane where the following hold simultaneously: **3**

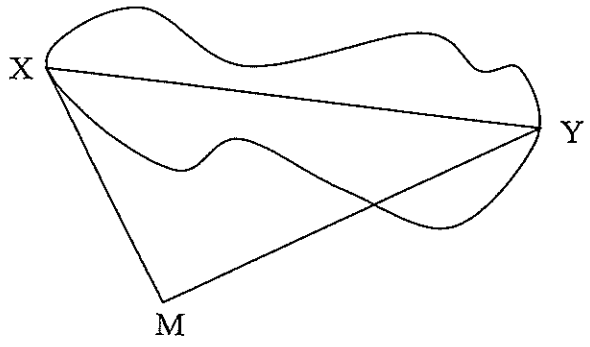
$$(x-2)^2 + y^2 \leq 4$$

$$y \leq x-2$$

and $y \leq 0$

(b) Prove that $\sec\theta + \tan\theta = \dfrac{1+\sin\theta}{\cos\theta}$. **2**

(c) A surveyor takes bearings and measurements from a point M to determine the distance across a lake XY.



NOT TO SCALE

X is 1200 m from M on a bearing of 340° and Y is 1500 m from M on a bearing of 060°.

i) Copy the diagram and mark the relevant information on it. **1**

ii) Find the distance XY across the lake to the nearest metre. **2**

iii) Hence or otherwise find the bearing of Y from X? **2**

End of Examination

☺

Sydney Technical High School

Preliminary Examination
Mathematics 2012

Multiple Choice Answer Sheet

Name _____

Teacher _____

Completely fill the response oval representing the most correct answer.

1. A ○ B ● C ○ D ○
2. A ○ B ○ C ○ D ●
3. A ○ B ● C ○ D ○
4. A ○ B ○ C ○ D ●
5. A ○ B ○ C ● D ○
6. A ○ B ○ C ● D ○
7. A ● B ○ C ○ D ○
8. A ○ B ● C ○ D ○
9. A ○ B ● C ○ D ○
10. A ○ B ○ C ○ D ●

Question 11

a) 2·03 (2 dp)

b) $S = \frac{a}{1-r}$

$S = \frac{7}{1-\frac{1}{3}}$

$S = 10\frac{1}{2}$

c) $\sqrt{12} + \sqrt{27} = \sqrt{a}$

$2\sqrt{3} + 3\sqrt{3} = \sqrt{a}$

$5\sqrt{3} = \sqrt{a}$

$\sqrt{25} \times \sqrt{3} = \sqrt{a}$

$a = 75$

d) $\frac{2}{5-2\sqrt{5}} \times \frac{5+2\sqrt{5}}{5+2\sqrt{5}}$

$= \frac{10+4\sqrt{5}}{25-20}$

$= \frac{10+4\sqrt{5}}{5}$

e) i) $2y(8x^3 - y^3) = 2y(2x-y)(4x^2+2xy+y^2)$

ii) $mx^2+my^2-nx^2-ny^2 = m(x^2+y^2) - n(x^2+y^2)$

$= (m-n)(x^2+y^2)$

Question 12

a) $\frac{2x-3y}{4x} - \frac{x-3y}{x-1}$

$= \frac{(x-1)(2x-3y) - (x-3y)4x}{4x(x-1)}$

$= \frac{2x^2-3xy-2x+3y-4x^2+12xy}{4x(x-1)}$

$= \frac{-2x^2+9xy-2x+3y}{4x(x-1)}$

b.i) $2(3m-4)-3 = \frac{3m}{2} - 5$

$4(3m-4)-6 = 3m-10$

$12m-16-6 = 3m-10$

$9m = 12$

$m = \frac{12}{9} \to \frac{4}{3} \to 1\frac{1}{3}$

b.ii) $|2x-1| = 6$

$2x-1 = 6$ $\quad$ $2x-1 = -6$

$2x = 7$ $\quad$ $2x = -5$

$x = 3\frac{1}{2}$ $\quad$ $x = -\frac{5}{2}$

b.iii) $x^2 = 5x$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0 \quad x = 5$

c) $2x^2 - 3x - 7 = 0$

$a = 2$

$b = -3$

$c = -7$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{3 \pm \sqrt{3^2-4\times2\times-7}}{2\times2}$

$x = \frac{3 \pm \sqrt{9+56}}{4}$

$x = \frac{3 \pm \sqrt{65}}{4}$

Question 13

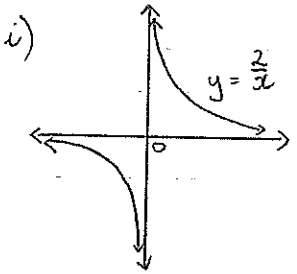
a i) $g(1) = 2$

ii) $g(-3) + g(0)$

$= 4 + -1$

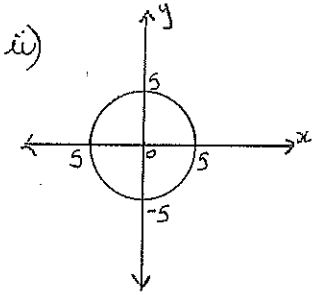
$= 3$

b) i)



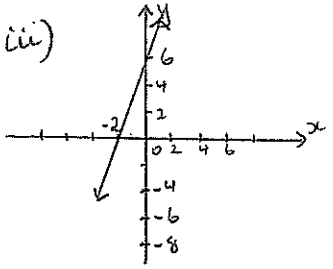
Domain : all reals $x \neq 0$
Range : all reals $y \neq 0$

ii)



Domain : $-5 \leq x \leq 5$
Range : $-5 \leq y \leq 5$

iii)



Domain : all reals
Range : all reals

c) $f(x) = \frac{1-x^2}{x}$

$f(-x) = \frac{1-(-1)^2}{(-x)}$

$= -\left(\frac{1-x^2}{x}\right) \quad = -f(x)$ $\quad \therefore$ odd function

Question 14

a i) $\cos 135° = \frac{-1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$

ii) $\cot 210° = \sqrt{3}$

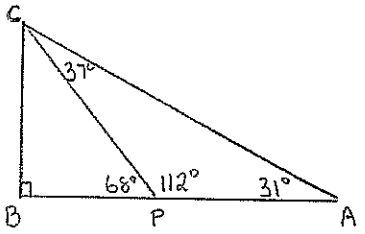
b) $2\sin^2\theta - 1 = 0$

$\sin^2\theta = \frac{1}{2}$

$\sin\theta = \pm\sqrt{\frac{1}{2}}$

$\theta = 45°, 135°, 225°, 315°$

c)



$\frac{CP}{\sin 31°} = \frac{20}{\sin 37}$

$CP = \frac{20 \times \sin 31°}{\sin 37°}$

$CP = 17.12$

c ii) $\cos 68° = \frac{BP}{17.12}$

$BP = 17.12 \cos 68°$

$BP = 6.41$

≈ 6 cm

d) i) $\sin C = \frac{2}{3}$

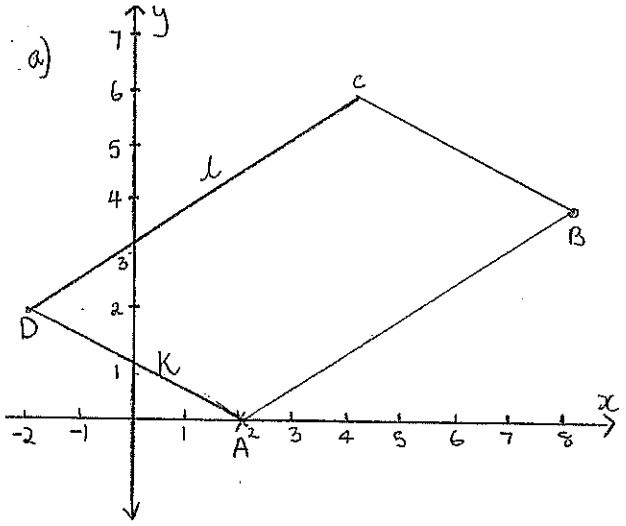
$\text{Area} = \frac{1}{2}ab\sin C$

$= \frac{1}{2} \times 9.8 \times 12.6 \times \frac{2}{3}$

d ii) Area $= 41.16$ units^2.

Question 15

a)



b) $M_{AB} = \frac{4-0}{8-2} = \frac{2}{3}$

c) $m_1 = m_2$ parallel

$m_1 = \frac{2}{3}$ (4, 6)

$y - 6 = \frac{2}{3}(x-4)$

$3y - 18 = 2x - 8$

$0 = 2x - 3y + 10$

d) D(-2, 2)

and plot on diagram

e) $\tan\theta = m$

$\tan\theta = \frac{2}{3}$

$\theta = 34°$

f) $d = \frac{|ax + by + c|}{\sqrt{a^2 + b^2}}$

$a = 2 \quad b = -3 \quad c = 10 \quad x = 2 \quad y$

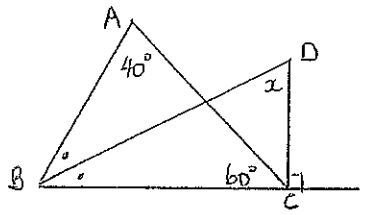
$d = \frac{|2 \times 2 - 3 \times 0 + 10|}{\sqrt{2^2 - (-3)^2}}$

$d = \frac{14}{\sqrt{13}}$ or $\frac{14\sqrt{13}}{13}$

Question 16

a) $(n-2) \times 180 =$
$(7 \times 180) \div 9$
$= 140°$ interior angle.

b)



$\angle DCA = 30°$ ($\angle$ sum of straight line)
$\angle ABC = 80°$ ($\angle$ sum of $\Delta = 180°$)
$\angle DAC = 40°$ (given)
$x = 180° - 90° - 40°$ ($\angle$ sum of $\Delta BDC = 180°$)
$x = 50°$

c) $f(x) = 5x - x^2$
$f(x+h) = 5(x+h) - (x+h)^2$
$= 5x + 5h - x^2 - 2xh - h^2$

$$\frac{f(x+h) - f(x)}{h}$$

$$= \frac{5x + 5h - x^2 - 2xh - h^2 - (5x - x^2)}{h}$$

$$= \frac{5h - 2xh - h^2}{h}$$

$$= \frac{h(5 - 2x - h)}{h}$$

$$= 5 - 2x - h$$

d) $y - x - 3 = 0$.
$y = x + 3$
$m = 1$

$y = 3x^2 - 5x - 4$
$\frac{dy}{dx} = 6x - 5$

$6x - 5 = 1$
$6x = 6$
$x = 1$
$y = -6$

$(1, -6)$

Question 17

ai) $y = x^4 - 3x^3 + 2$
$\frac{dy}{dx} = 4x^3 - 9x^2$

aii) $y = \sqrt{x^3} \rightarrow x^{3/2}$
$\frac{dy}{dx} = \frac{3}{2} x^{\frac{1}{2}}$

aiii) $y = \frac{1}{3x^4} \rightarrow \frac{1}{3} x^{-4}$
$\frac{dy}{dx} = -\frac{4}{3} x^{-5}$

b) $f(x) = (3x^2 - 5x)^5$
$f'(x) = 5(3x^2 - 5x)^4 \times (6x - 5)$
$= 5(6x - 5)(3x^2 - 5x)^4$

When $x = 2$
$= 5(6 \times 2 - 5)(3 \times 2^2 - 5 \times 2)^4$
$= 560$

c) $y = \frac{x^2 - 1}{x^2 + 1}$

$u = x^2 - 1$ $\quad$ $v = x^2 + 1$
$\frac{du}{dx} = 2x$ $\quad$ $\frac{dv}{dx} = 2x$

$$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{(x^2+1)2x - (x^2-1)2x}{(x^2+1)^2}$$

$$= \frac{2x^3 + 2x - 2x^3 + 2x}{(x^2+1)^2} = \frac{4x}{(x^2+1)^2}$$

d) $y = 2x^3 - 4x^2$

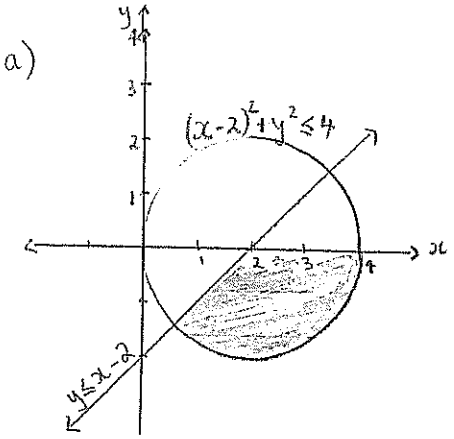
$\frac{dy}{dx} = 6x^2 - 8x$

When $x = 1$
$m = -2$
$m_2 = \frac{1}{2}$

$y - y_1 = m(x - x_1)$
$y + 2 = \frac{1}{2}(x - 1)$
$2y + 4 = x - 1$
$0 = x - 2y - 5$

Question 18:

a)



b) $\sec\theta + \tan\theta = \dfrac{1+\sin\theta}{\cos\theta}$

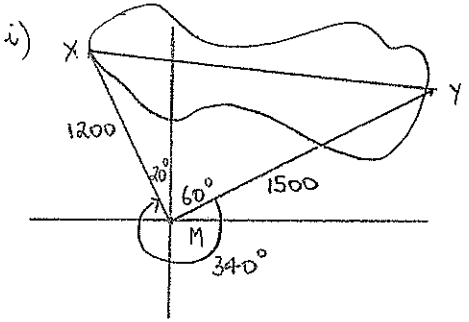
$L.H.S = \sec\theta + \tan\theta$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1+\sin\theta}{\cos\theta}$

$= R.H.S$

c) i)



c ii) $c^2 = a^2 + b^2 - 2bc\cos A$

$(XY)^2 = 1200^2 + 1500^2 - 2\times1200\times1500\times\cos 80^\circ$

$XY = 1750.68\ m$

c iii) $\dfrac{\sin\theta}{1500} = \dfrac{\sin B}{1750.68}$

$\theta = 57^\circ 33'$

$= 58^\circ$

$180^\circ - (20^\circ + 58^\circ)$

$= 102^\circ$

Bearing of Y from X is 102° T.