

SYDNEY TECHNICAL HIGH SCHOOL



MATHEMATICS

Year 10

2012 Yearly

Total Time Allowed: 2 hours

Instructions:

- Only Board approved calculators may be used
- Write using blue or black pen.
- Attempt all questions.

Start each question on a new sheet of paper

Question 1

10 marks

a) Fully factorise the following:

i) $6x^2 + 5x - 4$ (1)

ii) $(2x + 1)^2 - 9$ (2)

iii) $(x - 2)^2 + 2x - 4$ (1)

b) Expand and simplify

i) $(2 - \sqrt{3})(\sqrt{3} + 2)$ (2)

ii) $(1 + 2\sqrt{2})^2$ (2)

c) Given $\log_5 3 = 0.68$ and $\log_5 4 = 0.86$, find:

i) $\log_5 12$ (1)

iii) $\log_5 0.75$ (1)

Question 2

10 marks

a) Solve for the unknowns in each of the following:

i) $3(2x - 1) + 4(2 - x) = 5x - 25$ (2)

ii) $\frac{3x-1}{5} - \frac{x+4}{3} = -1$ (2)

b) Write this expression so that each term is expressed as a power of x in non fractional form

$$\frac{12 - 5x^2 + \sqrt{x}}{x\sqrt{x}} \quad (2)$$

c) Forty-five balls, numbered 1 to 45, are placed in a barrel, and one ball is drawn at random. What is the probability that the number on the ball drawn is even? (1)

d) Solve for x and graph your solution on a number line:
 $4 \leq 3(2 - x)$ (3)

Question 3

10 marks

a) Solve the following equation leaving your answer in surd form:

i) $\left(x - \frac{1}{3}\right)^2 = \frac{5}{9}$ (2)

b) Solve the following equations using the quadratic formula if necessary, giving your answer correct to two decimal places:

i) $6x^2 = x + 3$ (2)

ii) $1 = \frac{2-x^2}{x}$ (2)

c) Find the angle subtended by a chord of length 6cm in a circle of radius 4cm, correct to the nearest degree (2)

d) Neesha's scores in Chemistry this semester were rather inconsistent:

100, 85, 55, 95, 75, 100.

For this population, which scores are within one standard deviation of the mean? (2)

Question 4

10 marks

a) Let A and B be the points (0,1) and (2,3) respectively.

i) Find the coordinates of the midpoint AB (1)

ii) Find the slope of AB (1)

iii) Find the equation of the perpendicular bisector of the line AB (2)

Question 4 (continued)

b) If $f(x) = 2x^2 - x + 3$ find:

i) $f(3)$ (1)

ii) $f(-2)$ (1)

iii) $f(0)$ (1)

iv) $3f(-2) + 2f(3) + \frac{1}{2}f(0)$ (1)

v) $f(p + 1)$ in simplest form (2)

Question 5

10 Marks

a) A bag contains two red balls, one black ball and one white ball. Andrew selects one ball from the bag and keeps it hidden. He then selects a second ball, also keeping it hidden.

i) Draw a tree diagram to show all the possible outcomes (1)

ii) Find the probability that both the selected balls are red (1)

iii) Find the probability that at least one of the selected balls is red (1)

iv) Andrew drops one of the selected balls and we can that it is red.
What is the probability that the ball that is still hidden is also red? (1)

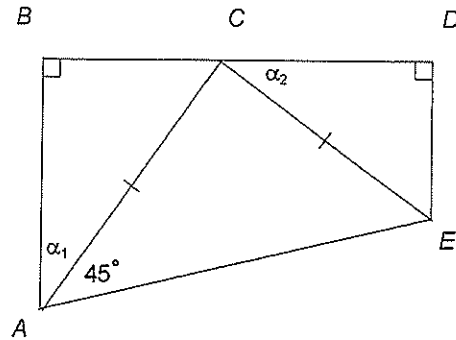
b) Out of forty students, 14 are taking English Composition and 29 are taking Chemistry.
There are five students completing both courses.

i) Represent the above information in a Venn diagram: (1)

ii) What is the probability that a randomly-chosen student from this group
is taking only the Chemistry class? (2)

Question 5 (continued)

c)



With a formal setting out, show that $\alpha_1 = \alpha_2$

(3)

Question 6

10 marks

For the parabola $y = x^2 - 6x + 8$

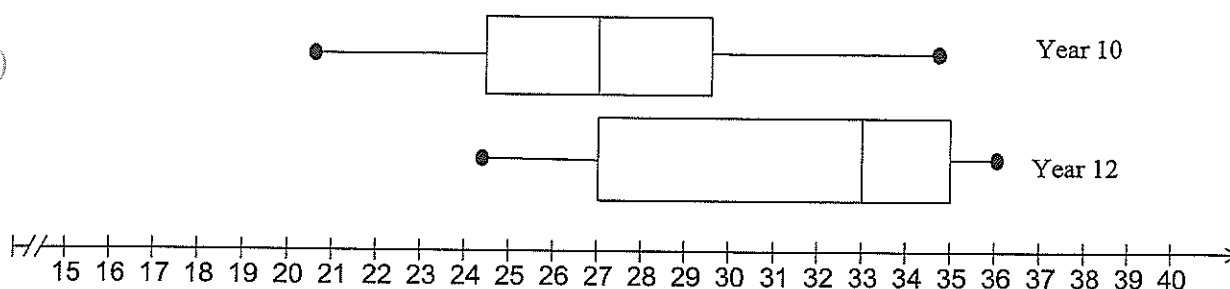
- i) State the equation of the axis of symmetry (2)
- ii) Find the co-ordinates of the vertex (2)
- iii) Find where the parabola cuts the x and y axes (2)
- iv) Draw a neat sketch labelling all the information in (i), (ii), (iii) (3)
- v) Use your graph to find values of x for which $x^2 - 6x + 8 \geq 0$ (1)

Question 7

10 marks

- a) A ship sailed off from port on a bearing of 68° T and travelled for 30 minutes at a speed of 24 kilometres per hour. It then changed bearing to 158° T and travelled for 45 minutes at the same speed. The weather changed quite suddenly and so the crew wanted to head straight back to port.
- Draw a $\frac{1}{2}$ page neat sketch of this situation and show all working. (1)
 - Show that the distance of the ship from the starting position is $\sqrt{468}$ kms. (2)
 - Travelling at the same speed, what bearing do they need to set to return to port? (2)

- b) The following box plots compare the time students in years 10 and 12 spend on homework over a period of 1 week.



- Determine the range, interquartile range and median for each group (3)
- Comment, giving reasons, on which year level spends the most time on homework. (2)

Question 8

10 marks

Interest rate % pa reducible	Approximate monthly repayment per \$1000 of loan over:						
	2 years \$	5 years \$	7 years \$	10 years \$	15 years \$	20 years \$	25 years \$
6.0	44.32	19.33	14.61	11.10	8.44	7.16	6.44
6.5	44.55	19.57	14.85	11.35	8.71	7.46	6.75
7.0	44.77	19.80	15.09	11.61	8.99	7.75	7.07
7.5	45.00	20.04	15.34	11.897	9.27	8.06	7.39
8.0	45.23	20.28	15.59	12.13	9.56	8.36	7.72
8.5	45.46	20.52	15.84	12.40	9.85	8.68	8.05
9.0	45.68	20.76	16.09	12.67	10.14	9.00	8.39

- a) Mary borrowed \$314 000 which she repaid in monthly instalments over 20 years at 6% pa.
- use the table to determine the size of the monthly repayments (2)
 - What is the total amount paid in 20 years? (1)
 - How much interest is paid in this time? (1)
 - What is the equivalent flat rate of interest? (2)
- b) A library now worth \$17 600 has been depreciating at a compounding rate of 9% pa for the last 8 years. What was its value 8 years ago? (2)
- c) A cricketer played 12 innings at an average of 42.5 runs per innings and then scored 13 and 65 in the next two innings. What is her average? (2)

Question 9

10 Marks

a) Pauline and Tony were told that they needed \$43 200 deposit before they could borrow enough money to buy their new home. At the time of their marriage, Pauline had saved \$10 900 while Tony had saved \$14 000. They both worked and although they paid rent of \$1120 each month, they were still able to save \$525 per week.

i) When they married, how much less than the deposit did they have? (1)

ii) How many weeks did it take to save the rest of the deposit? (1)

iii) If the deposit was 15% of the cost of buying the home (including all extra charges), what is this cost? (1)

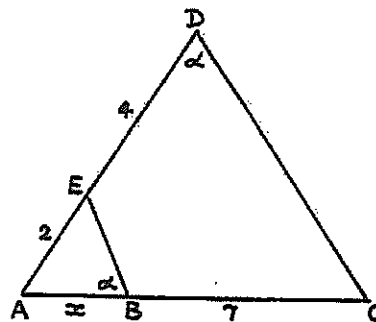
iv) How much money had to be borrowed? (1)

v) How much interest is charged for the first month if the interest rate is 7% pa reducible, with monthly repayments? (1)

b) From the diagram below:

i) prove that $\triangle AEB$ and $\triangle ACD$ are similar (2)

ii) hence find the exact value of x . (3)



Question 10

10 Marks

- i) Solve $\log_x 16 = 2$ (1)
- ii) If $\log_p x = n$ and $\log_p y = t$, find $\log_p xy^2$ in terms of n and t (2)
- iii) Match the following graphs with the equations by listing the small letter of the equation with the capital letter of the graph: (4)

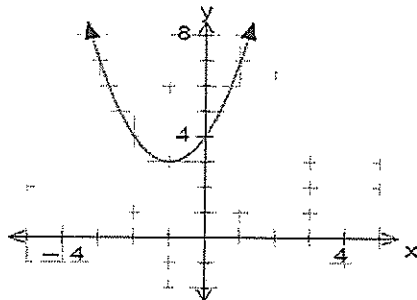
(a) $y = 2(x-1)^2 - 3$

(b) $y = (x+1)^2 + 3$

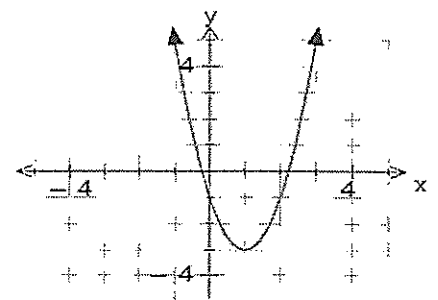
(c) $y = -(x-1)^2 + 2$

(d) $y = -2(x+1)^2 - 2$

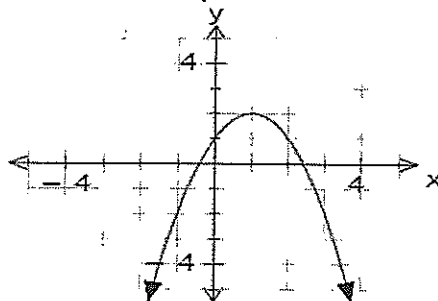
A



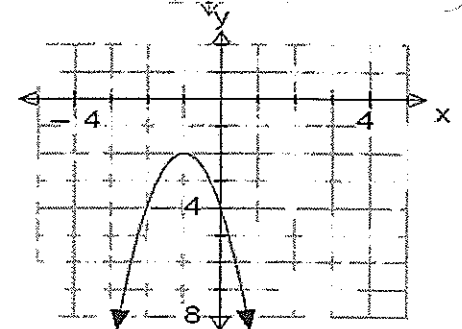
B



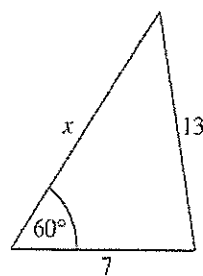
C



D



iv)



NOT TO SCALE

The diagram shows a triangle with sides 7 cm and 13 cm and x cm, and an angle of 60° as marked. Use the cosine rule to show that $x^2 - 7x = 120$, and hence find the exact value of x . (3)

-----End of Paper-----

Q1

$$(a)(i) (3x+4)(2x-1)$$

$$(ii) (2x+1-3)(2x+1+3) \\ = (2x-2)(2x+4) \\ = 4(x-1)(x+2)$$

$$(iii) (x-2)^2 + 2(x-2) \\ = (x-2+2)(x-2) \\ = x(x-2)$$

$$b)(i) 2\sqrt{3} + 4 - 3 - 2\sqrt{3} = 1$$

$$(ii) (1+2\sqrt{2})^2 = 1 + 4\sqrt{2} + 8 \\ = 9 + 4\sqrt{2}$$

$$\Rightarrow (i) 1.54$$

$$(ii) -0.18$$

Q2

$$(a)(i) 6x-3+8 \dots -4x = 5x-25 \\ 2x+5 = 5x-25 \\ -3x = -30 \\ x = 10$$

$$(ii) 9x-3-5x-20 = -15 \\ 4x = 8 \\ x = 2$$

$$(b) 12x^{3/2} - 5x^{1/2} + x^{-1}$$

$$(c) P(\text{even}) = 22/45$$

$$(d) 4 \leq 6-3x \\ -2 \leq -3x \\ 2 \geq 3x \\ x \leq 2/3$$

Q3

$$(a) x - 1/3 = \frac{\pm\sqrt{5}}{3}$$

$$(i) x = \frac{1 \pm \sqrt{5}}{3}$$

(b)

$$(i) 6x^2 - x + 3 = 0$$

$$x = \frac{1 \pm \sqrt{73}}{12}$$

$$x = 0.72, -0.63$$

$$(ii) x = 2 - x^2$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x = -2, 1$$

(c)



$$\sin \theta = 3/4$$

$$\theta = 48.59^\circ$$

$$\therefore \text{angle} = 97^\circ$$

$$(d) \bar{x} = 85 \quad s.d. = 16.07$$

$$101 \rightarrow 68.93$$

$$100, 85, 95, 75 \text{ and } 100$$

04

(a) (i) mid pt $\left(\frac{2+0}{2}, \frac{3+1}{2}\right)$
 $= (1, 2)$

(ii) $m = 1$ $\left(\frac{3-1}{2-0} = 1\right)$

(iii) $y - 2 = -1(x - 1)$
 $y = -x + 3$

(b) (i) $f(3) = 18$

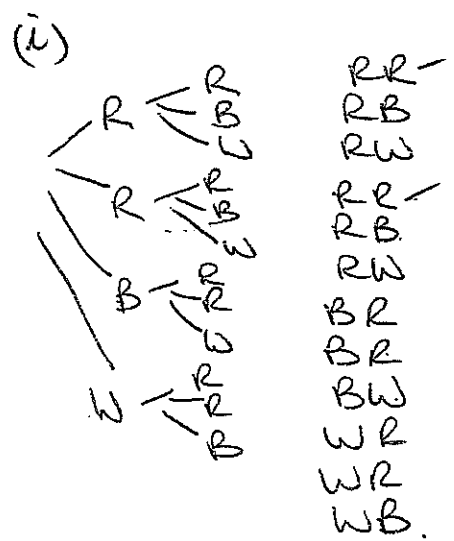
(ii) $f(-2) = 3$

(iii) $f(0) = 3$

(iv) $= 76.5$

(v) $2p^2 + 3p + 4$

05 2R 1B 1W

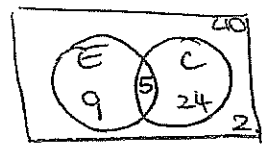


(ii) $P(RR) = 2/12 = 1/6$

(iii) $P(\text{at least 1R}) = 5/6$

(iv) $R \begin{matrix} \swarrow R \\ \searrow W \\ \quad B \end{matrix} = 1/3$

(b)



(i) 2

(iii) $P(\text{chem}) = 24/40 = 3/5$

(c) $\angle BCA = 90^\circ - \alpha_1$

$\angle BCA + \angle ACE + \angle DCE = 180^\circ$

$90 - \alpha_1 + 90^\circ + \alpha_2 = 180$

$180 - \alpha_1 + \alpha_2 = 180^\circ$

$\alpha_1 = \alpha_2$

06

$y = x^2 - 6x + 8$

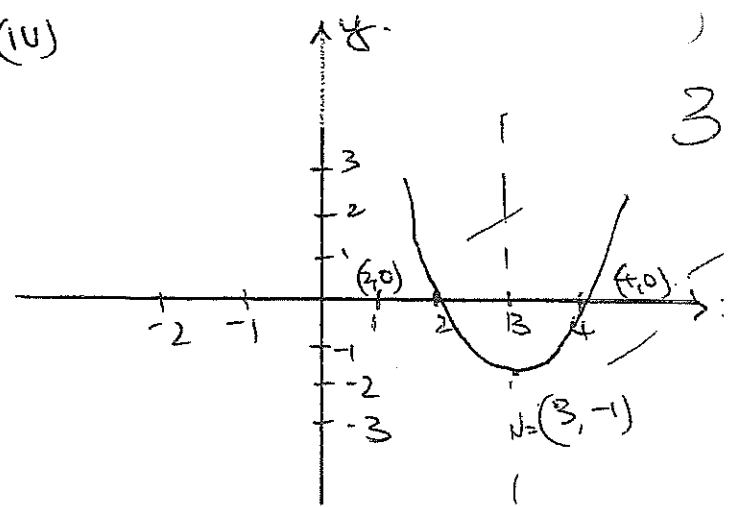
(i) $x = -b/2a$
 $= 6/2$
 $x = 3$

(ii) $x = 3$ $y = -1$ $(3, -1)$

(iii) at $x = 0$ $y = 8$ $(0, 8)$
 at $y = 0$ $(x-4)(x-2) = 0$
 $x = 4, 2$

$(4, 0)$ $(2, 0)$

(iv)

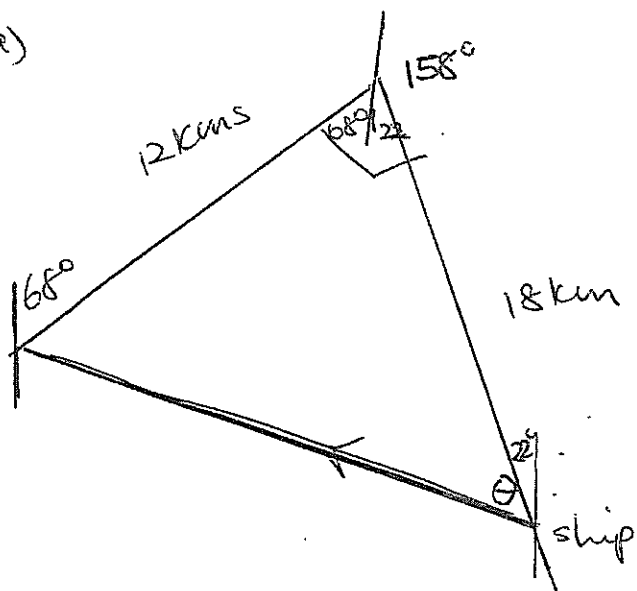


(v) $x < 2$; $x > 4$

$x = 3$

Q1

(a)



$$(ii) 12^2 + 18^2 = x^2$$

$$x = \sqrt{468}$$

$$(iii) \tan \theta = 18/12$$

$$\theta = 33^\circ 41'$$

$$\text{Bearing} = 304^\circ 19'$$

b) (i) Year 10

$$\text{Range} = 35 - 20.5$$

$$= 14.5$$

$$\text{IQR} = 29.5 - 24.5$$

$$= 5$$

$$\text{med} = 27$$

Yr 12

$$\text{Range} = 36 - 24.5$$

$$= 11.5$$

$$\text{IQR} = 35 - 27$$

$$= 8$$

$$\text{med} = 33$$

i) yr 12's on average spend more time on homework. 50% spend 8 hrs whereas 50% yr 10's spend 4 hrs

Q8

(a)

$$(i) 7.16 \times 314 = 2,248.24$$

$$(ii) 2248.24 \times 20 \times 12 = 539,577.60$$

$$(iii) \text{Int} = 225,577.60$$

$$(iv) I = PNR/100$$

$$R = 3.592\%$$

$$(b) V = P(1+r)^n$$

$$17600 = P(1-0.09)^8$$

$$P = \$37,426.70$$

$$(c) \bar{x} = 42.5$$

$$\text{Total} = (510 + 13 + 65) / 14$$

$$= 42$$

Q9

$$(i) \$43,200 - 24,900 = \$18,300$$

$$(ii) 35 \text{ weeks}$$

$$(iii) \$43,200 = 15\%$$

$$\$288,000 = 100\%$$

$$(iv) \$244,800$$

$$(v) \$244,800 + \text{Int.}$$

$$\text{Int} = \$142,800$$

Q9

(b) $\triangle AEB \parallel \triangle ACD$.

$\angle EBA = \angle ADC$ given (A)

$\angle EAB = \angle DAC$ common (A)

$\angle AEB = \angle DCA$ (angle sum of \triangle)
 $= 180^\circ$ (A)

$\therefore \triangle AEB \parallel \triangle ACD$ (AAA)

(ii) $\frac{AE}{AB} = \frac{AC}{AD}$

$$\frac{2}{x} = \frac{x+7}{6}$$

$$x^2 + 7x - 12 = 0.$$

$$x = \frac{-7 \pm \sqrt{49+48}}{2}$$

$$\therefore x = \frac{-7 + \sqrt{97}}{2} \quad x \neq \frac{-7 - \sqrt{97}}{2}$$

Q10

(i) $x^2 = 16$
 $x = 4$

(ii) $\log_p xy^2 = n+2t$

(iii) (A) (b)

(B) (a)

(C) (c)

(D) (d)

(iv)

$$13^2 = x^2 + 7^2 - 2x7 \cos 60$$

$$169 = x^2 + 49 - \frac{14x}{2}$$

$$169 = x^2 + 49 - 7x$$

$$x^2 - 7x - 120 = 0$$

$$x = \frac{7 \pm \sqrt{49+480}}{2}$$

$$x = \frac{7 \pm \sqrt{529}}{2}$$

$$x = \frac{7 \pm 23}{2}$$

$$x = 15$$

$$x \neq \frac{7-23}{2}$$