Name:______________________________

Teacher/Class:______________________________

# SYDNEY TECHNICAL HIGH SCHOOL

# HSC ASSESSMENT TASK 1

DECEMBER 2005

# EXTENSION 1 MATHEMATICS

**Time Allowed :** **70 minutes**

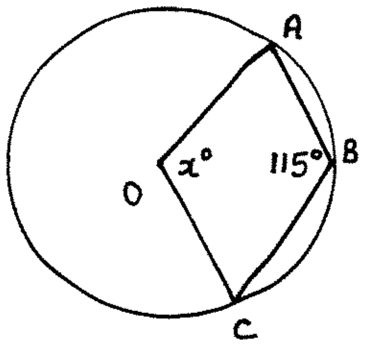
**Instructions:**

- Write your name and class at the top of each page
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start each question on a new page.
- Diagrams unless otherwise stated are not to scale.

| Question 1 | Question 2 | Question 3 | Question 4 | Question 5 | Question 6 | Total |
|---|---|---|---|---|---|---|
| /10 | /10 | /10 | /10 | /10 | /10 | /60 |

**QUESTION 1** (10 Marks)

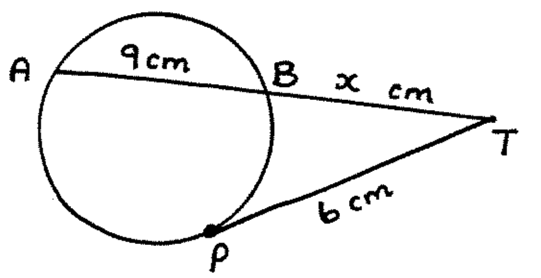
a)




A, B and C lie on the circumference of a circle centre 0.

Find the value of $x$, giving full reasons (2)

b)




PT is a tangent to the circle with secant AT.

Find the value of $x$ (2)

c) The sum of the first $n$ terms of a series is $S_n = 4n^2 + 7n$ (2)

i Find an expression for the $nth$ term of the series (2)

ii Show that the series is arithmetic (2)

iii Find the value of the sum $T_{11} + T_{12} + \ldots\ldots + T_{20}$ (2)

**QUESTION 2** **(10 Marks)** **Start a new page**

**a)** For the series $-5 + 10 + 25 + \ldots\ldots + 955$, **(4)**

**i** Find the number of terms

**ii** Hence, evaluate the sum of the series

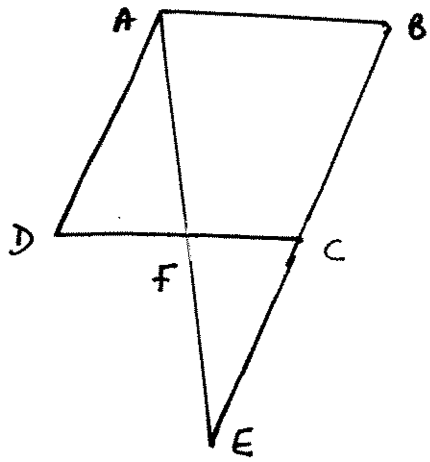
**b)** **i** Express $0.0\dot{4}\dot{5}$ as a geometric series and find the value of the first term and the common ratio. **(3)**

**ii** Hence, express $0.0\dot{4}\dot{5}$ as a simple fraction

**c)** Find the value of $\sum_{n=3}^{20} 2^n - 2n$ **(3)**

**QUESTION 3** (10 Marks) **Start a new page**

a)



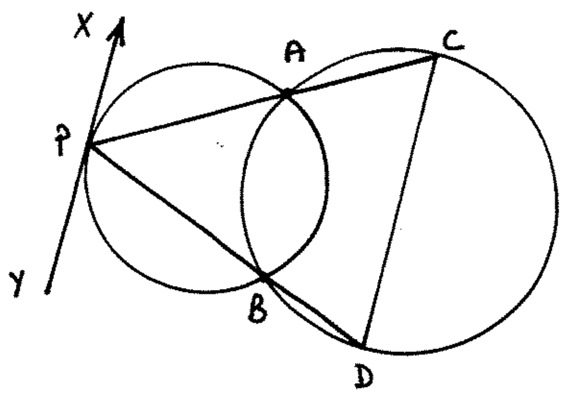

ABCD is a rhombus with BC produced to E, so that C is the midpoint of BE

i Prove that $\Delta ADF$ is similar to $\Delta EBA$ **(2)**

ii Prove that F is the midpoint of DC **(2)**

b) The limiting sum of an infinite geometric series is 48. If the common ratio is doubled, the limiting sum becomes $-24$, find the original series. **(3)**

c)



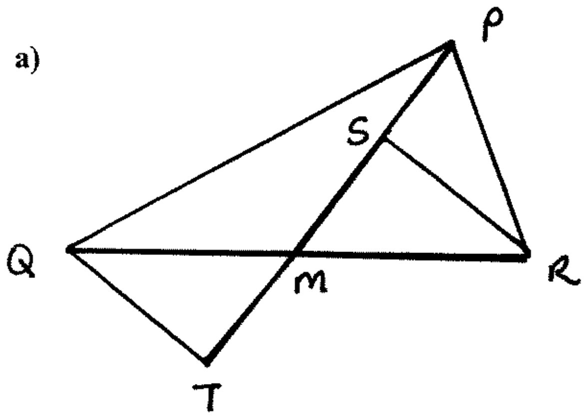

Copy this diagram into your booklet.

PAC and PBD are straight lines. **(3)**

Prove that the chord CD is parallel to the tangent at P.

**QUESTION 4** (10 Marks) **Start a new page**

a)




In the figure QT and RS are both perpendicular to PT

TS = 12cm and M is the midpoint of QR

Copy the diagram

i Prove that $\Delta QMT$ is congruent to $\Delta RMS$ **(3)**

ii Find the length of TM, giving reasons **(1)**

iii Prove, with reasons, that QTRS is a parallelogram **(1)**

b) Bart's father Homer deposits $100 into an account on each of his birthdays from his first to his eighteenth. The money earns 6% p.a. compounded quarterly.

i How much is in the account immediately after the second deposit? **(2)**

ii Find the balance of the account after the last payment on his eighteenth birthday. **(2)**

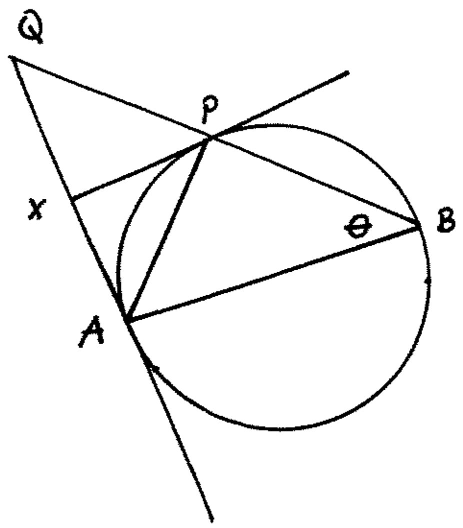
iii Bart leaves the balance in the account, without making any more deposits or withdrawals, for three years at the same conditions as above. How much is in the account at the end of this period? **(1)**

**QUESTION 5** (10 Marks) **Start a new page**

**a)** Use mathematical induction to show that for all positive integers $n \geq 1$,

$$\frac{1}{1\times 3} + \frac{1}{3\times 5} + \frac{1}{5\times 7} + \;\cdots\cdots\; + \frac{1}{(2n-1)\,(2n+1)} = \frac{n}{2n+1}$$ **(6)**

**b)**




In the diagram AB is a diameter of the circle.

The chord BP of the circle is produced to meet, at Q, the tangent to the circle at A.

The tangent to the circle at P meets AQ at X.

**i** If $\angle ABP = \theta$, show that $\angle XPQ = 90° - \theta$ **(3)**

**ii** Write down an expression for $\angle AQB$, in terms of $\theta$ **(1)**

**QUESTION 6** **(10 Marks) Start a new page**

Mr and Mrs Smith decide to borrow \$250 000 to buy a house. Interest is calculated monthly on the balance still owing, at a rate of 6% p.a. The loan is to be repaid at the end of 15 years with equal monthly instalments of \$**M**.

**i** Find an expression for the amount owing immediately after the second repayment. **(2)**

**ii** Show that the amount owing after the $nth$ repayment is

$250000\,(1.005)^{n} - 200\,M\,\left[1.005^{n} - 1\right]$ **(3)**

**iii** If the loan is fully repaid at the end of 15 years, calculate the value of **M.** **(2)**

**iv** How many months would it take to repay the loan if Mr and Mrs Smith repay \$2400 each month. **(3)**

**End of test**

Extension 1 Dec 2005

Answers

Question 1

a) reflex $\angle AOC = 230°$ (angle at the centre is twice the angle at the circumf. on arc AC) ①

$x + 230 = 360$ (revolution)

$x = 130$ * ①

b) $6^2 = x(x+9)$ ①

$36 = x^2 + 9x$

$x^2 + 9x - 36 = 0$

$(x+12)(x-3) = 0$

$x > 0 \therefore x = 3$ * ①

c) $T_n = S_n - S_{n-1}$

(i) $= 4n^2 + 7n - \left(4(n-1)^2 + 7(n-1)\right)$ ①

$= 8n + 3$ ①

Alternatively 11, 19, 27, .....

$T_n = a + (n-1)d$ ①

$= 11 + (n-1).8$

$= 8n + 3$ ①

(ii) Need to have found

$T_1 = 11 \quad T_2 = 19 \quad T_3 = 27$ ①

AP as $T_2 - T_1 = T_3 - T_2 = 8$ ①

iii) $T_{11} = 91 \quad T_{12} = 99 \quad ... \quad T_{20} = 163$

sum $= 91 + 99 + ... + 163$ ①

$= \frac{10}{2}(91 + 163)$

$= 1270$

① either line

Question 2

a) $-5 + 10 + 25 + ... + 955$

i. $955 = a + (n-1)d$

$955 = -5 + (n-1) \times 15$ ①

$960 = 15(n-1)$

$n - 1 = 64$

$n = 65$ * ① $\therefore$ 65 terms

ii. $S_n = \frac{n}{2}(a + l)$

$= \frac{65}{2}(-5 + 955)$ ①

$= 30875$ ①

b) $0.0\dot{4}\dot{5} = \frac{45}{1000} + \frac{45}{100000} + \frac{45}{10000000} + ..$

$a = \frac{45}{1000} \quad r = \frac{1}{100}$ ①

ii. $S_\infty = \frac{a}{1-r}$ ①

$= \frac{45}{1000} \div \frac{99}{100}$

$= \frac{1}{22}$ * ①

c) $\sum_{n=3}^{20} 2^n + \sum_{n=3}^{20} (-2n)$ or ①

$= 8 + 16 + .. + 2^{20} + (-6 - 8 - 10 ... - 40)$

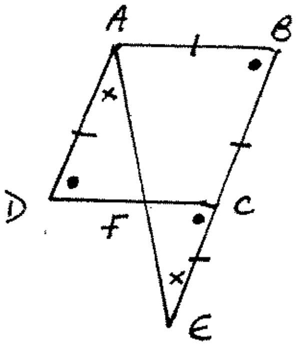
$= \frac{a(r^n - 1)}{r - 1} + \frac{n}{2}(a + l)$

$= \frac{8(2^{18} - 1)}{2 - 1}$ ① $+ \frac{18}{2}(-6 - 40)$ ①

$= 2096730$

Question 3

a) i.




In $\Delta ADF$ and $\Delta EBA$

$\angle DAF = \angle BEA$ (alternate $\angle$'s $AD \parallel BE$) (1)

$\angle ADC = \angle ABE$ (opp $\angle$'s in Rhombus equal)

$\therefore \Delta ADF \parallel\!\mid \Delta EBA$ (AAA) (1)

ii. As C is the midpt of EB and $\Delta ADF \parallel\!\mid \Delta EBA$

$CE : BE = 1 : 2$ (1)

$AB = CD$ opp sides of rhombus =

$FC : AB = 1 : 2$ ratio of sides in $\parallel\!\mid$ $\Delta$'s

$\therefore FC = \frac{1}{2} AB$ (1)

$= \frac{1}{2} DC$ and F is the midpoint of DC

NB Can prove $\Delta ADF \equiv \Delta ECF$ (SAA) and then $DF = CF$

b) $\frac{a}{1-r} = 48$ and $\frac{a}{1-2r} = -24$ (1)

$a = 48(1-r)$ $a = -24(1-2r)$

$\therefore 48(1-r) = -24(1-2r)$

$48 - 48r = -24 + 48r$

$96r = 72$ (1)

$r = 3/4$ (0.75)

$\frac{a}{1-0.75} = 48$ $a = 12.$ (1)

$\therefore 12 + 9 + 6.75 + \dots$

)

$\angle YPB = \angle PAB$ (1) ($\angle$ between a tangent and a chord equals angle in the alternate segment)

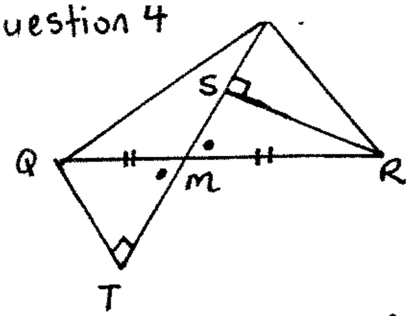
$\angle PAB = \angle BDC$ (1) (ext $\angle$ of cyclic quad equals opp interior $\angle$)

$\therefore \angle YPB = \angle BDC$ $(= \angle PAB)$

and $XY \parallel CD$ as the alternate angles are equal (1)

Question 4

a)




i.

$\angle QMT = \angle RMS$ (vertically opposite)

(1) $QM = RM$ (given M is the midpt of QR)

(1) $\angle QTM = \angle MSR$ ($= 90°$ given RS and $QT \perp P$)

$\therefore \Delta QMT \equiv \Delta RMS$ (AAS) (3)

ii. $TM = SM$ corresponding sides of congruent $\Delta$'s

$\therefore$ M is the midpt of ST (1)

$TM = 6$ cm

iii. M is the midpt of ST (part (ii))

M is the midpt of QR (given)

$\therefore$ QTRS is a parallelogram as the diagonals bisect each other (at M) (1)

b)

i. $A = 100(1.015)^4 + 100$ (1) (1)

ii. $100\left[1.015^{68} + 1.015^{64} + \dots + 1\right]$ (1)

$= 100 \times \left[\frac{1\left((1.015^4)^{18} - 1\right)}{1.015^4 - 1}\right]$

$= \$3130.78$ (1)

iii. $3130.78(1.015)^{12}$

$= \$3743.22$

either (1)

Question 5

est $n=1$

$\text{LHS} = \frac{1}{1\times 3} = \frac{1}{3}$ $\quad$ $\text{RHS} = \frac{1}{2\times 1+1} = \frac{1}{3} = \text{LHS}$ ①

the statement is true for $n=1$

ssume true for $n=K$

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\dots+\frac{1}{(2K-1)(2K+1)}=\frac{K}{2K+1}$$ ①

ove true for $n=K+1$

m to prove

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\dots+\frac{1}{(2K-1)(2K+1)}+\frac{1}{(2K+1)(2K+3)}=\frac{K+1}{2K+3}$$

$$= \frac{K}{2K+1}+\frac{1}{(2K+1)(2K+3)}$$ ①

$$= \frac{K(2K+3)+1}{(2K+1)(2K+3)}$$

$$= \frac{2K^2+3K+1}{(2K+1)(2K+3)}$$ ①

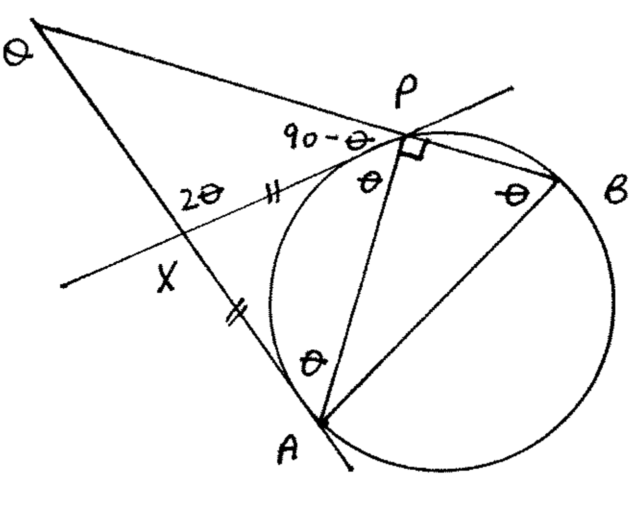
$$= \frac{(2K+1)(K+1)}{(2K+1)(2K+3)}$$

$$= \frac{K+1}{2K+3}$$ ①

RHS as required

∴ If true for $n=K$ then also true for $n=K+1$. As true for $n=1$ also true for $n=2,3,4$ etc. Hence true all +ve integer $n$. ①

b)




$\angle APB = 90^\circ$ ($\angle$ in the semi circle) ①

$\angle XPA = \angle PBA = \theta$ (angle between a tangent and a chord = $\angle$ in the alternate segment) ①

$\therefore \angle QPX = 90-\theta$ (straight line) ①

(ii) $\angle AQB = 90-\theta$ ①

Q6

$\$250\,000 \quad r = 0.005 \quad n = 15 \times 12 = 180$

i. $A_1 = 250\,000(1.005) - m$ ①

$A_2 = A_1(1.005) - m$

$= 250\,000(1.005)^2 - m(1.005) - m$ ①

ii. $A_3 = A_2(1.005) - m$

$= 250\,000(1.005)^3 - m(1.005)^2 - m(1.005) - m$

$A_n = 250\,000(1.005)^n - m\left[1.005^{n-1} + 1.005^{n-2} + \ldots + 1\right]$ ①

$= 250000(1.005)^n - m\left[\frac{1(1.005^n - 1)}{0.005}\right]$ ①

$= 250\,000(1.005)^n - 200m\left[1.005^n - 1\right]$ ①

iii. $A_n = 0$ when $n = 180$

$0 = 250\,000(1.005)^{180} - 200m\left[1.005^{180} - 1\right]$

$200m\left[1.005^{180} - 1\right] = 250\,000(1.005)^{180}$

$m = \frac{250\,000(1.005)^{180}}{200(1.005^{180} - 1)}$

$= \$2109.64$ ①

iv.

$0 = 250\,000(1.005)^n - 200[2400][1.005^n - 1]$ ① either

$0 = 250\,000(1.005)^n - 480\,000(1.005^n - 1)$

$0 = 480\,000 - 230\,000(1.005)^n$

$230\,000(1.005)^n = 480\,000$ ① either

$1.005^n = 2.0869\ldots$

$n \cdot \log 1.005 = \log 2.0869\ldots$

$n = \log 2.0869\ldots \div \log 1.005$

$= 147.908\ldots$ ① either

$\therefore$ need 148 months.

Trial & error fine if answer = 148