Name: ..... Maths Class: .....

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 PRELIMINARY COURSE

Extension 1 Mathematics

Task 1

May 2011

TIME ALLOWED: 70 minutes

Instructions:

- Write your name and class at the top of this page, and on all your answer sheets.
- Hand in your answers attached to the rear of this question sheet.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied at the time of marking
- ***Start each new question on a new page***

(FOR MARKERS USE ONLY)

1	2	3	4	5	6	TOTAL
/9	/9	/9	/9	/9	/9	/54

QUESTION 1 (9 Marks):

Marks

(a) Fully factorise and simplify: **2**

$$\frac{6^n - 2^n}{9^n - 3^n}$$

(b) Show that the expression $\frac{1}{3\sqrt{2}-2} - \frac{1}{3\sqrt{2}+2}$ is rational. **2**

(c) What is the Domain of the function given by $y = \frac{1}{\sqrt{x^2-1}}$? **2**

(d) If $\sin\alpha = x$ and $90^o \leq \alpha \leq 270^o$, find an expression for $\tan\alpha$ **3**

QUESTION 2 (9 Marks): *(Start a New Page)*

(a) On a set of Cartesian axes, neatly graph $|x| + |y| = 1$, showing all intercepts on the axes **2**

(b) (i) Sketch the graph of $y = \sqrt{9 - x^2}$ **2**

(ii) Give the Domain and Range of the function **2**

(iii) On your diagram, neatly shade the solution to $y < \sqrt{9 - x^2}$, carefully indicating whether boundaries are included or excluded from the solution. **3**

QUESTION 3 (9 marks): *(Start a New Page)*

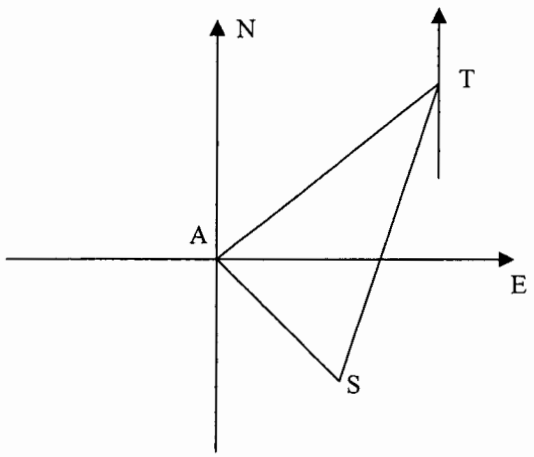
(a) Solve the two equations below simultaneously to find their point(s) of intersection: **3**

$$xy = 6$$

$$y = x - 5$$

(b) The diagram below represents the information given in the following problem:

A ship S sails from a port A on a bearing of $S18^oE$ at a speed of 10 km per hour.
A second ship, T, leaves at exactly the same time on a course of $N72^oE$.
After 2 hours the ship T sights S on a bearing of $S24^oW$



(i) Copy the diagram onto your answer page and put on it all the information given in the above question **2**

(ii) Find the size of angle ATS **1**

(ii) Find the distance between the two ships 2 hours after leaving port A (ie ST). (to the nearest km) **3**

You must show all necessary working to gain full marks

QUESTION 4 (9 Marks): *(Start a New Page)*

Marks

(a) (i) ***On the same set of axes***, sketch the graphs of $y = |2x - 1|$ and $y = 1 - 2x$ **3**

(ii) Use the diagram, or otherwise, to solve

$$|2x - 1| > 1 - 2x$$

for all real x.

(b) Solve , for $0^o \leq \theta \leq 360^o$, **3**

$$4cos^2\theta - 3 = 0$$

(c) Prove that $\frac{\cos x}{1+\sin x} + \frac{1+\sin x}{\cos x} = \frac{2}{\cos x}$ **3**

QUESTION 5 (9 Marks): *(Start a New Page)*

(a) You are given the function $f(x) = \frac{2}{x^2-4}$

(i) Is the function odd, even, or neither? (Give reasons) **1**

(ii) What are the vertical asymptotes for *f(x)* ? **1**

(iii) Where does this curve cut the y-axis ? **1**

(iv) Sketch the curve $y = f(x)$, showing all important features and indicating the ***horizontal*** asymptote(s). **4**

(b) Find all solutions over the Real number system to: **2**

$$\frac{1}{x-1} < \frac{1}{x}$$

Show all necessary working

QUESTION 6 (9 Marks): *(Start a New Page)*

(a) Simplify $\frac{\cos(90^o - \theta)}{\sin(90^o + \theta)}$ **2**

(c) A function is defined by the following:

$$f(x) = \begin{cases} 3, & if \ x \leq -2 \\ -x, & if \ -2 < x < 0 \\ x^2, & if \ x \geq 0 \end{cases}$$

(i) Sketch $y = f(x)$ for $-4 \leq x \leq 2$ **3**

(ii) Find the value of $f\left(\frac{1}{2}\right) + f\left(-\frac{1}{2}\right)$ **1**

(iii) Find the value of $f(|a|)$ where a is real. **1**

(c) $g(x) = 2x + 4$ is a function for all real values of x. **2**

A second function, $f[x]$ is defined thus:

$$f[g(x)] = x$$

Find the function $f[x]$

END OF EXAMINATION PAPER

SOLUTIONS – Maths Extension 1

YEAR 11 – EXT

May 2011

(1) (a) $\dfrac{2^n(3^n-1)}{3^n(3^n-1)} = \begin{cases} 2^n/3^n \\ (2/3)^n \end{cases}$

1 mark for each factorisation = (2)

(b) $\dfrac{3\sqrt{2}+2}{14} - \dfrac{3\sqrt{2}-2}{14}$

1 mark for the rationalisation

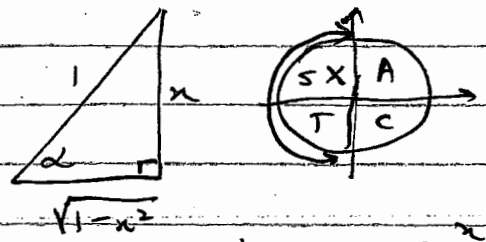
$= \begin{cases} 4/14 \\ 2/7. \end{cases}$ which is rational

1 mark for $4/14$ or $2/7$.

(c) $\mathcal{D}$: $x \geq 1$ or $x \leq -1$

(2) MARKS – subtract 1 if they have "=" signs.

(d)



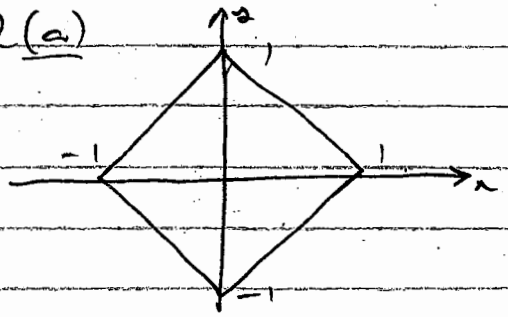
$\tan\alpha = -\dfrac{x}{\sqrt{1-x^2}}$

1 mark for $\sqrt{1-x^2}$

1 mark for the expression for $\tan\alpha$

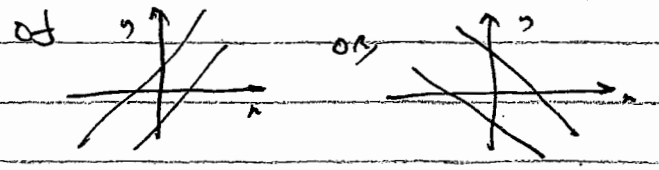
1 for the negative.

2 (a)

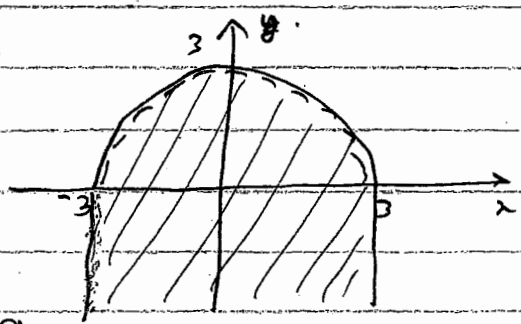


2 for the graph and its intercepts

1 MARK ONLY for either



(b)



(2) MARKS – 1 for the shape and 1 for intercepts. (1 ONLY if full circle).

(ii) $\mathcal{D}$: $-3 \leq x \leq 3$ $\quR$: $0 \leq y \leq 3$

1 MARK each.

(iii)

1 mark for shading inside the circle
1 mark for the extension
1 mark for dots and heavy line

(3)(a) $\begin{cases} y = 6/x \\ y = x - 5 \end{cases}$

(1) into (2) $6/x = x - 5$

$6 = x^2 - 5x$

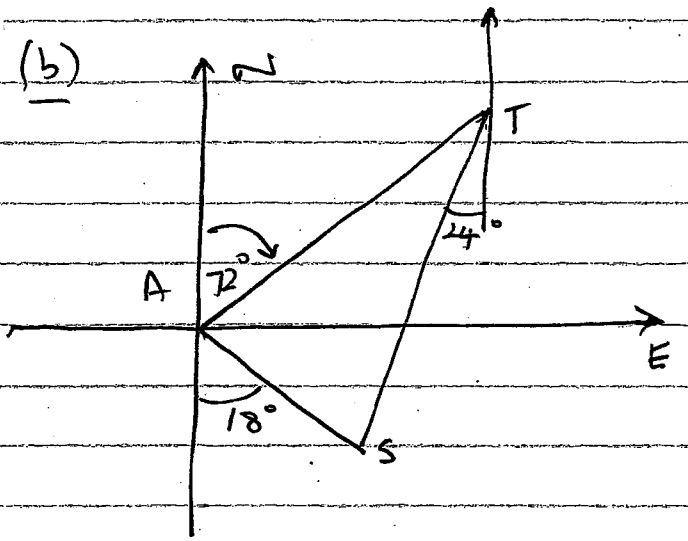
$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$\therefore x = 6$ or $x = -1$ ← 2 MARKS

$\therefore$ POINTS are $(6,1)$ and $(-1,-6)$ ← 1 MARK

(b)



All of these angles must be shown to get 2 MARKS

(no extra marks for $\angle TAS = 90°$ or $AS = 20$]

(i) $\angle ATS = (72 - 24)°$ ← 1 MARK

$= 48°$

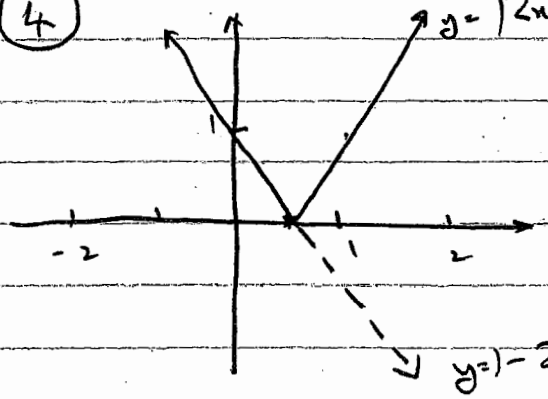
(ii) $AS = 20$ and $\angle TAS = 90°$ ① MARK for both

$\therefore ST/20 = 1/\cos 48$ ← 1 MARK

$\therefore ST = 29.88$ } 1 for either

≈ 30 km }

(4)



← 1 MARK.

← 1 MARK

$x > 1/2$ ← 1 MARK

= (3)

(b) $\cos\theta = \pm \sqrt{3}/2$ ← (1) MARK

$\theta = 30°, 150°, 210°, 330°$ ← (2) MARKS.) ONLY FOR 30°, OR FOR 30°, 150°

(c)

$$\text{LHS} = \frac{\cos x(1-\sin x)}{1-\sin^2 x} + \frac{(1+\sin x)\cos x}{\cos^2 x}$$

$$= \frac{\cos x(1-\sin x) + (1+\sin x)\cos x.}{\cos^2 x.}$$

$$= 2/\cos x$$

There are several methods. But for 3 marks, they cannot work down both sides (2 max.) and must show enough steps to explain what is happening

(5) (a) $f(x) = \frac{2}{x^2-4}$.

(i) $f(a) = \frac{2}{a^2-4}$ $f(-a) = \frac{2}{(-a)^2-4}$

$= \frac{2}{a^2-4}$

$= f(a)$

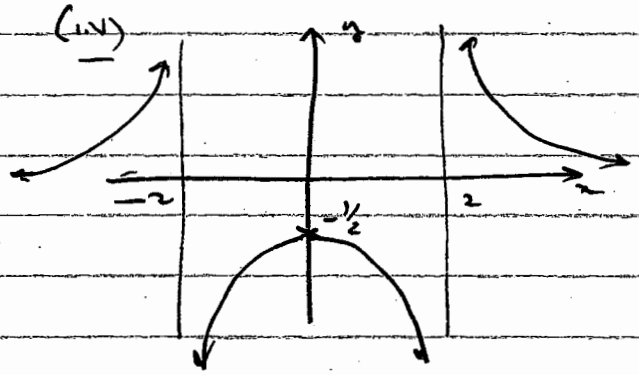
$\therefore$ EVEN.

} 1 mark but some explanation as to why should be given (may not be done as this)

(ii) $x \neq \pm 2$ ← 1 mark

(iii) cuts y-axis at $(0, -\frac{1}{2})$ ← 1 mark

(iv)



1 marks for each arm = (2)

(1) for the part between $-2 < x < 2$

(1) for indicating $y = 0$ as the asymptote.

(b) $\frac{1}{x-1} < \frac{1}{x}$

C.V. $x \neq 0$

$x \neq 1$.

$x - 1 = x$.

NO SOLN

$0 < x < 1$

} 1 for any acceptable method shown.

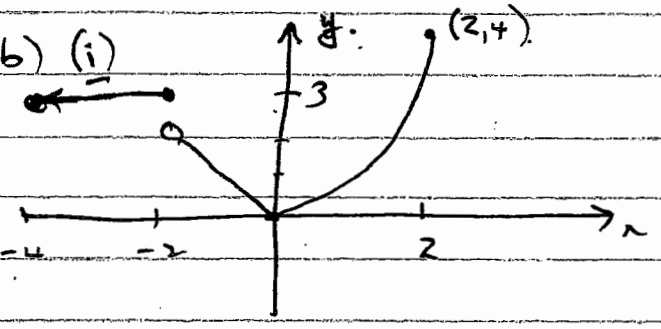
← 1 for solution

(6) (a) $\dfrac{\cos(90^\circ - \theta)}{\sin(90^\circ + \theta)} = \dfrac{\sin\theta}{\cos\theta}$

$= \tan\theta$

(2) 1 for each simplification.

(b) (i)



1 mark for each "part" = (3)

Subtract 1 more for no open circle

Subtract 1 if the graph doesn't "end" at both ends

(ii) $f(\frac{1}{2}) + f(-\frac{1}{2}) = \frac{1}{4} + \frac{1}{2}$

$= 3/4$

1 mark

(iii) Since $|a| \geq 0 \ \forall\, a$,

$f(|a|) = a^2$

1 mark

(c) $f[x] = \dfrac{x-4}{2}$

(2) marks

1 for division by 2

1 for subtraction of 4