

# SYDNEY TECHNICAL HIGH SCHOOL

( Established 1911 )



## YEAR 10 YEARLY EXAMINATION

2016

# Mathematics

### General Instructions

- Working time - 90 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in every question
- Diagrams are not drawn to scale
- Full marks may not be awarded for careless work or illegible writing
- All answers are to be in the answer booklet provided

### Total marks - 75

- Section A is multiple choice. Each question is worth 1 mark. Answer on the sheet provided in your answer booklet. Do not remove this page from the booklet.
- Section B is to be answered in your answer booklet. Start each question on a new page. Each question is worth 11 marks. Marks for each part are indicated.

Name : \_\_\_\_\_

Teacher : \_\_\_\_\_

## SECTION A

Answer the following multiple choice questions on the answer sheet provided in your answer booklet.

1.  $x \div 3 \times y + z$  is equal to

- (A)  $\frac{x}{3y} + z$       (B)  $\frac{xy}{3} + z$       (C)  $\frac{x}{3}(y + z)$       (D)  $\frac{x}{3y + z}$
- 

2. A coin is tossed three times. What is the probability that the side showing on the last toss is the same as that showing on the first toss?

- (A)  $\frac{1}{8}$       (B)  $\frac{1}{4}$       (C)  $\frac{3}{8}$       (D)  $\frac{1}{2}$
- 

3. The eye colours of a sample of children were recorded.

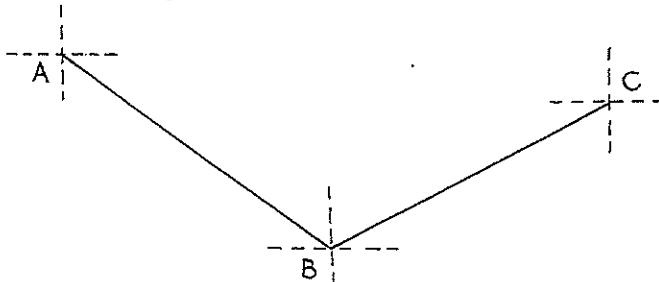
When analyzing this data, which of the following could be found?

- (A) Mode  
(B) Median  
(C) Mean  
(D) All of the above
- 

4.  $x^{\frac{4}{3}}$  is equivalent to

- (A)  $\sqrt[4]{x^3}$       (B)  $\sqrt[3]{x^4}$       (C)  $\frac{4}{x^3}$       (D)  $\frac{x^4}{3}$
- 

5.



Julie walks on a bearing of  $157^\circ$  from A to B and then on a bearing of  $038^\circ$  to C.

What is the bearing of B from C?

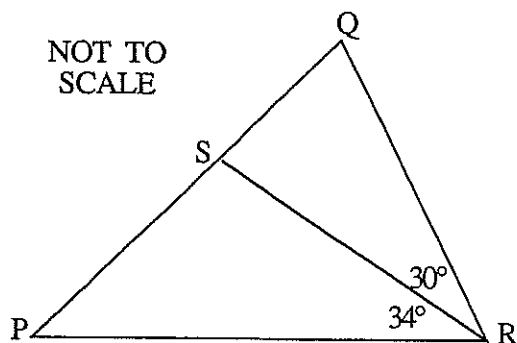
- (A)  $232^\circ$       (B)  $218^\circ$       (C)  $195^\circ$       (D)  $038^\circ$

6. The solutions of  $x^2 + 7x - 3 = 0$  are

- (A)  $x = \frac{-7 \pm \sqrt{37}}{2}$  (B)  $x = \frac{7 \pm \sqrt{37}}{2}$  (C)  $x = \frac{-7 \pm \sqrt{61}}{2}$  (D)  $x = \frac{7 \pm \sqrt{61}}{2}$
- 

7.

NOT TO  
SCALE

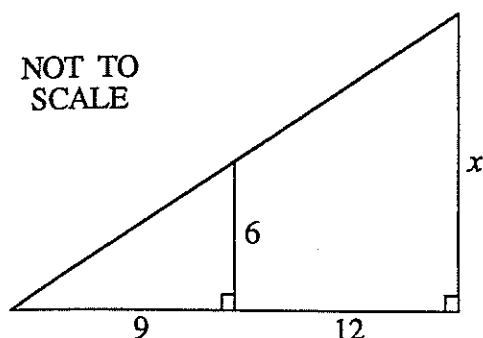


If  $PQ = PR$ , what is the size of  $\angle PSR$ ?

- (A)  $82^\circ$  (B)  $86^\circ$   
(C)  $94^\circ$  (D)  $112^\circ$
- 

8.

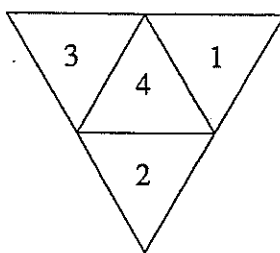
NOT TO  
SCALE



What is the value of  $x$ ?

- (A) 8 (B) 9  
(C) 14 (D) 18
- 

9. A die is made in the form of a tetrahedron (triangular pyramid). The net of the die is shown below.



The faces are numbered 1, 2, 3, and 4.

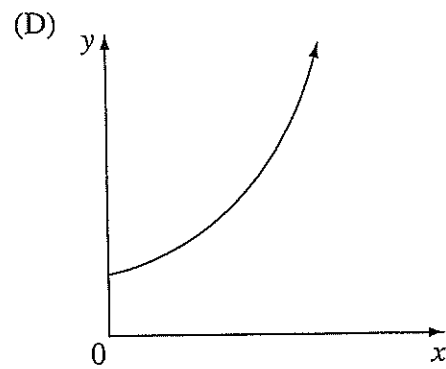
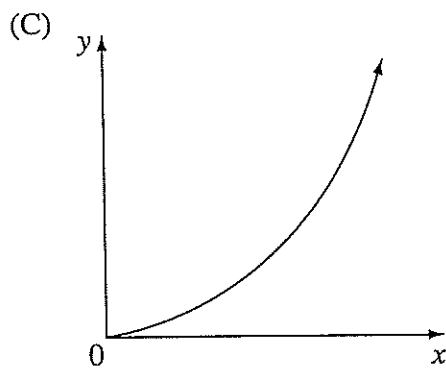
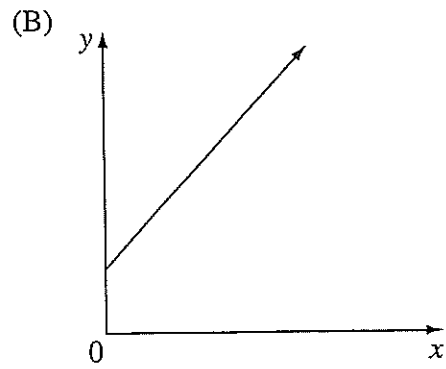
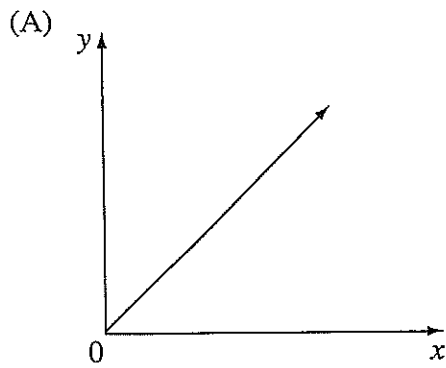
The die is rolled twice.

The number on the face that the die lands on is recorded each time.

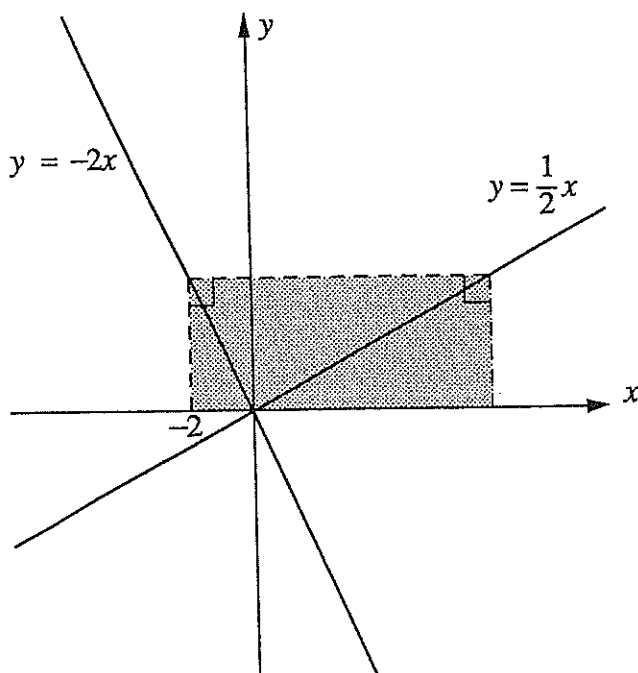
Find the probability that the sum of the two recorded numbers is 4.

- (A)  $\frac{3}{16}$  (B)  $\frac{1}{16}$  (C)  $\frac{1}{8}$  (D)  $\frac{1}{4}$

10. Which of the following graphs best represents the equation  $y = a^x$ , where  $a$  is a positive number greater than 1?



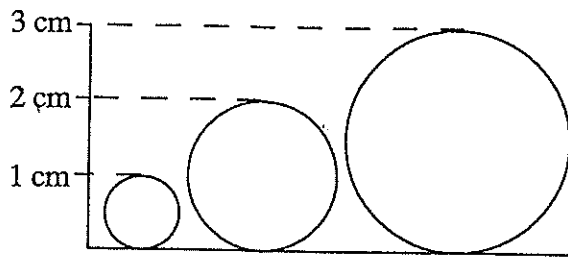
11.



Calculate the shaded area (in square units).

- (A) 16                      (B) 24  
(C) 32                      (D) 40

12.

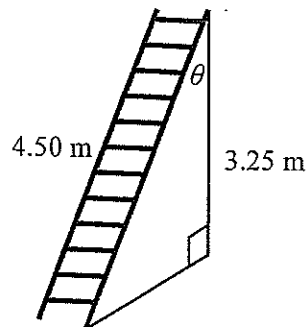


A factory makes three different sizes of solid chocolate balls that are spherical in shape.

A large ball contains the same amount of chocolate as

- (A) 1 small and 1 medium ball
- (B) 1 small and 2 medium balls
- (C) 3 small and 3 medium balls
- (D) 3 small and 4 medium balls

13. The top of a ladder that is 4.50 m long rests 3.25 m up a wall, as shown in the diagram below.

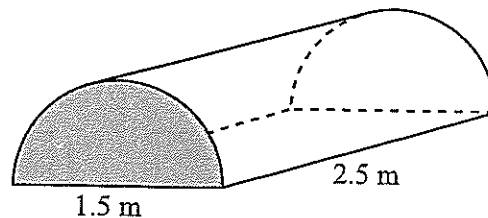


The angle,  $\theta$ , that the ladder makes with the wall is closest to

- A.  $36^\circ$
- B.  $44^\circ$
- C.  $46^\circ$
- D.  $50^\circ$

14. A tent with semicircular ends is the shape of half a cylinder.

The diameter of the ends is 1.5 m and the tent is 2.5 m long.



The total surface area (in  $\text{m}^2$ ) of the tent, including the base, is closest to

- A. 5.5
- B. 7.7
- C. 8.8
- D. 11.4

15.

The same class sat for tests in English, Mathematics and Science.

Esti's results are shown below:

TEST	CLASS MEAN	CLASS STANDARD DEVIATION	ESTI'S MARK
ENGLISH	75	5	80
MATHEMATICS	55	15	80
SCIENCE	60	10	80

In which test did Esti perform best, compared to the rest of her class?

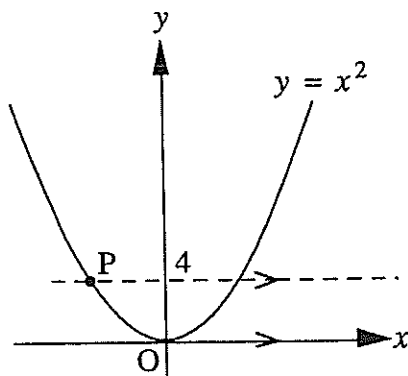
- (A) Science      (B) Maths      (C) English      (D) She performed as well in all 3 tests

16. A gardener used  $1000 \text{ m}^3$  of soil to re-surface a smooth, level field to a uniform depth. The area of the field is  $33\,000 \text{ m}^2$ .

The depth *in millimetres* of this soil is closest to

- (A) 0.03      (B) 3      (C) 30      (D) 33

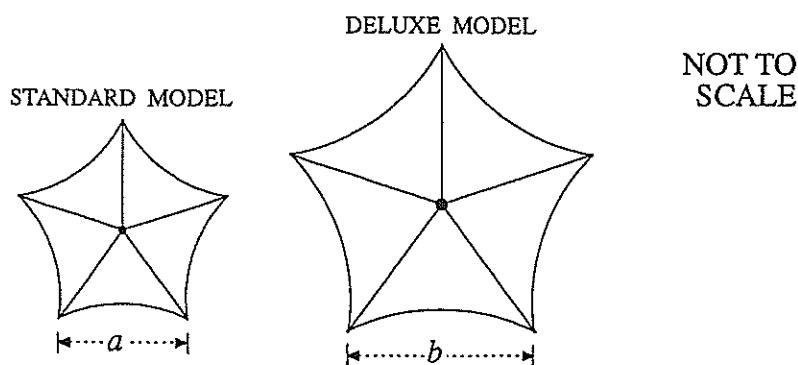
17.



The coordinates of the point P are

- (A)  $(-2, 4)$       (B)  $(-4, 2)$   
(C)  $(-4, 16)$       (D)  $(-16, 4)$

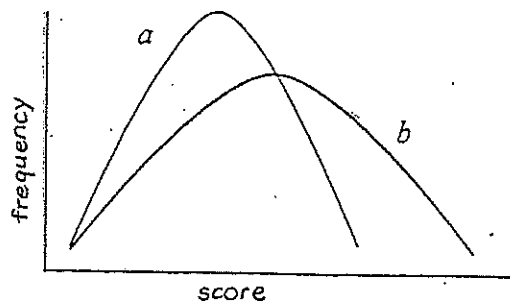
18.



The two umbrellas above are similar shapes with  $a : b = 4 : 5$ . The standard model requires  $1.44 \text{ m}^2$  of material. How much material is required for the deluxe model?

- (A)  $1.152 \text{ m}^2$       (B)  $1.8 \text{ m}^2$       (C)  $2.0736 \text{ m}^2$       (D)  $2.25 \text{ m}^2$

19. Both curves represent sets of scores.



- A. curve  $a$  represents a greater average and a greater standard deviation than  $b$   
 B. curve  $a$  represents a greater average and a smaller standard deviation than  $b$   
 C. curve  $a$  represents a smaller average and a greater standard deviation than  $b$   
 D. curve  $a$  represents a smaller average and a smaller standard deviation than  $b$

20. Cubes are to be stacked as shown below.



FIGURE 1  
(1 cube)

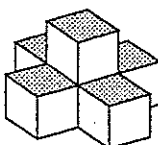


FIGURE 2  
(6 cubes)

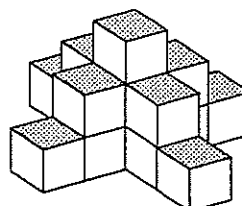


FIGURE 3  
(15 cubes)

Which of the following expressions gives the number of cubes in the  $n$ th figure?

- (A)  $2n - 1$       (B)  $5n - 4$       (C)  $2n^2 - n$       (D)  $2n^2 + 3n + 1$

## SECTION B

Answer the following questions in your answer booklet showing all necessary working.

### Question 1 (11 marks)

- a) Factorise  $6a^2 + 13a - 5$  1
- b) Find the value of  $x$  if  $\sqrt{3} + \sqrt{48} = \sqrt{x}$ . 1
- c) New Zealand has a 15% rate of GST. 1  
A camera sells for \$368 in New Zealand which includes GST.  
Find the amount of GST paid for the camera.
- d) Find the vertex of the parabola  $y = 6x + x^2$  1
- e) If  $x^5 = 5000$  find the value of  $x$  correct to 3 significant figures. 1
- f) Find, correct to the nearest degree, 2 values of  $\theta$  2  
that satisfy the equation  $3 \sin \theta = 1$ .
- g) Solve simultaneously for  $x$  and  $y$ ,  $3x - 2y = 14$  2  
 $x - 3y = 7$
- h) 4 red and 2 blue discs are placed in a container. 2  
Two discs are drawn out at random.  
Find the probability that the discs are;
- i) Both red.
- ii) Different colours.



**Question 2** (11 marks) - start a new page in your answer booklet.

a) If  $\sin \theta = \frac{2}{3}$  and  $\theta$  is acute, find the exact value of  $\tan \theta$ . 1

b) Solve for  $x$ ,  $x^2 - 2x - 8 = 0$  1

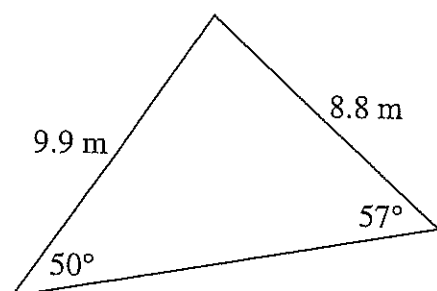
c) A set of 31 scores is displayed in the following stem-and-leaf plot. 3

0	2								
0	5	6	7	7	8	9			
1	0	0	0	0	1	4	4	4	
1	5	6	7						
2	3	3	4						
2	7	7	8	9					
3	0	4							
3	5	6							
4	1								
4	7								

For these scores find

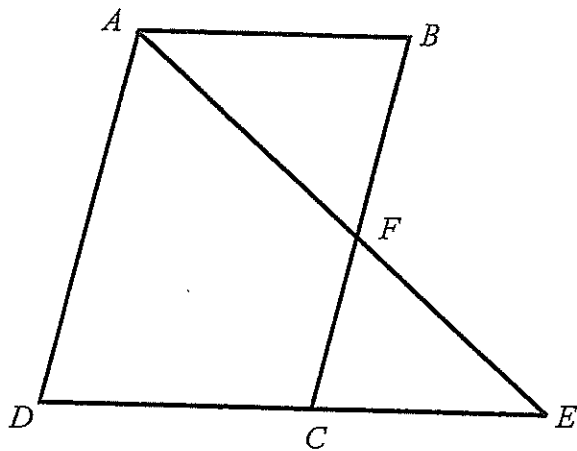
- i) the mode
- ii) the median
- iii) the interquartile range.

d) Find the area of the following triangle, correct to the nearest square metre. 2



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e)



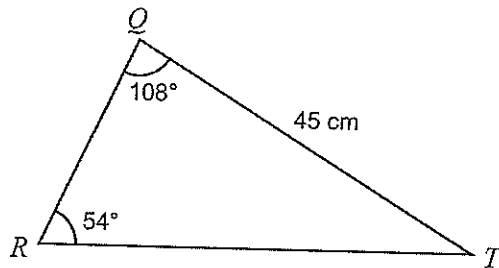
ABCD is a parallelogram. DC is produced to E. AE cuts BC at F.  
AD=16 cm , CE=9 cm and BF=10 cm.

Copy the diagram into your answer booklet and mark on it all the given information

- |     |  |   |
|-----|--|---|
| i)  | Prove that $\triangle ABF$ is similar to $\triangle ECF$ , giving clear reasons. | 3 |
| ii) | Find the length of AB.   | 1 |

**Question 3** (11 marks) - start a new page in your answer booklet.

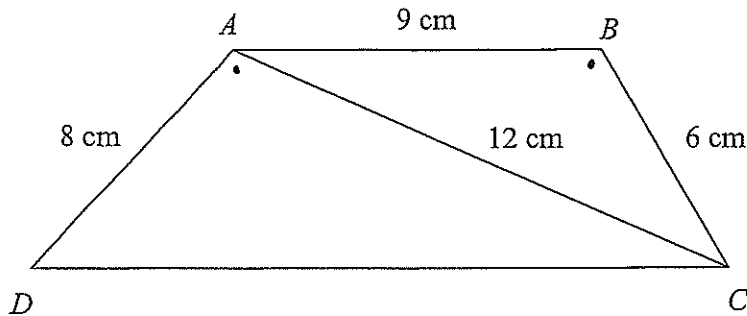
- a) Given the points  $A(-4,2)$  and  $B(2,-1)$  find,
- i) The gradient of AB 1
  - ii) The equation of the line perpendicular to AB that passes through the point  $B$  2
- b) On a number plane, sketch the region satisfied by both  $y \geq x^2 - 1$  and  $2x + y \geq 2$  2
- c) A 3D printer is used to make 2 similar models. 2  
If the smaller model has a height of 5 cm and a volume of  $75 \text{ cm}^3$   
find the volume of the larger model given that it has a height of 18 cm.  
Answer correct to the nearest cubic centimetre.
- d) Find the length of RT ,in centimetres correct to 1 decimal place. 2



- e) Find, correct to the nearest degree, the size of the largest angle in a triangle with side lengths 3 cm , 5 cm and 6 cm. 2

**Question 4** (11 marks) - start a new page in your answer booklet.

- a) In the diagram below,  $AB=9$  cm,  $BC=6$  cm,  $AD=8$  cm,  $AC=12$  cm and  $\angle ABC = \angle DAC$ .

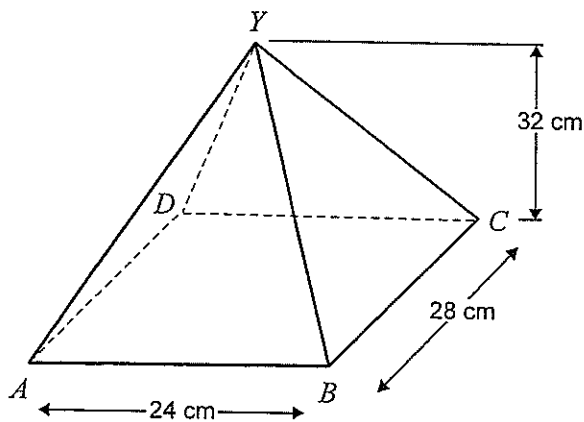


NOT TO SCALE

Copy this diagram neatly into your answer booklet.

- |     |  |   |
|-----|--|---|
| i)  | Prove that $\triangle ABC$ is similar to $\triangle CAD$ , giving clear reasons. | 2 |
| ii) | Hence, find the length of side CD.   | 1 |

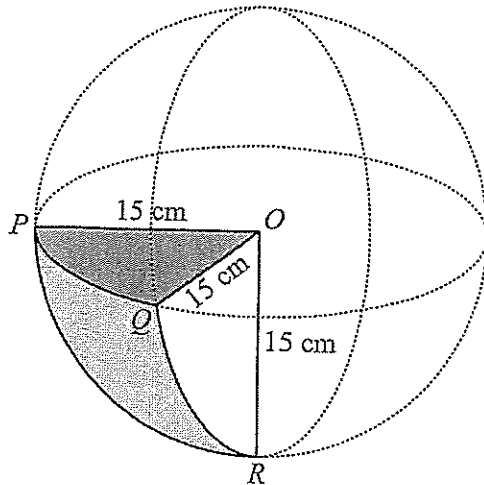
- |    |  |   |
|----|--|---|
| b) |  | 1 |
|----|--|---|



Find the volume of the right rectangular pyramid shown above

c)

2



The solid OPQR, as shown above, is one-eighth of a sphere of radius 15 cm. The point O is the centre of the sphere and the point P, Q and R are on the surface of the sphere.

$$\angle POQ = \angle QOR = \angle ROP = 90^\circ$$

Find the surface area of the solid OPQR, in  $\text{cm}^2$  correct to 1 decimal place.

d) Hugo paid \$7500 for a bike under a hire purchase agreement.

2

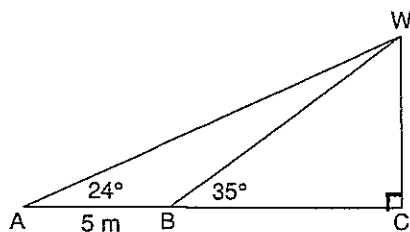
A flat interest rate of 12% per annum was charged.

He will fully repay the principal and the interest in 24 equal monthly instalments.

Determine the monthly instalment that Hugo will pay.

Give your answer in dollars, correct to the nearest cent.

e)



Using the information given in the diagram above, noting that  $AB = 5$  metres, find;

i) the size of  $\angle AWB$ 

1

ii) the length of CW in metres correct to 1 decimal place.

2

**Question 5** (11 marks) - start a new page in your answer booklet.

- a) Students studying at least one of the languages, Spanish and Japanese, attend a meeting. Of the 28 students present, 18 study Spanish and 22 study Japanese. 1

What is the probability that a randomly chosen student studies both languages ?

- b) Make  $R$  the subject of  $S = R - \sqrt{R^2 - y^2}$  2

- c) Solve the following equations simultaneously for  $x$  and  $y$  2  
 $x^2 + y^2 = 16$  and  $y = x^2 - 4$

d)

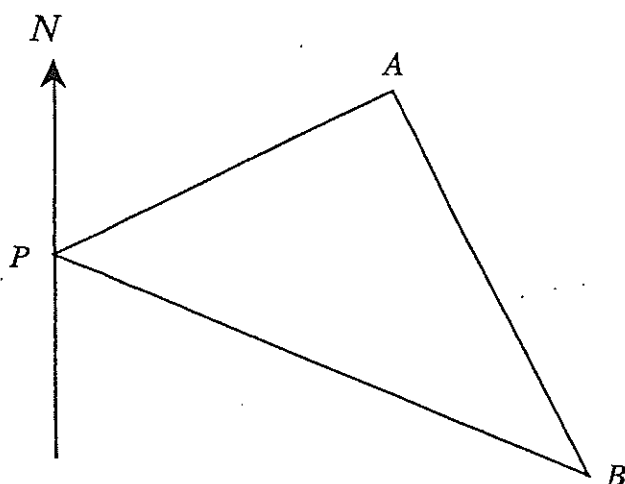


Figure not to scale.

Ship A leaves port P and sails for 20 kilometres on a bearing of  $055^\circ$ .

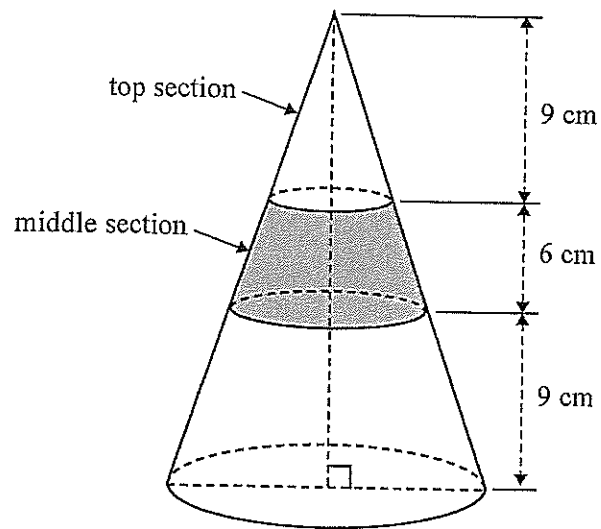
Ship B leaves port P and sails for 27 kilometres on a bearing of  $115^\circ$ .

Copy the diagram into your examination booklet and indicate the given information.

- i) Find  $\angle APB$  1  
 ii) Show that the distance between the two ships is 24.3 kilometres. (correct to 1 decimal place) 1  
 iii) Hence, find the bearing of ship B from ship A, giving your answer correct to the nearest degree 2

e) The middle section of a cone is shaded, as shown in the diagram below.

2



If the surface area of the unshaded top section of the cone is  $180 \text{ cm}^2$ ,  
Find the surface area of the middle section of the cone.

**End of Paper**

4  
6

C

C



## 2016 YEARLY SOLUTIONS

SECTION A

1. B

11. D

2. D

12. C

3. A

13. B

4. B

14. D

5. B

15. A

6. C

16. C

7. C

17. A

C

8. C

18. D

9. A

19. D

10. D

20. C

SECTION BQuestion 1

a)  $(3a-1)(2a+5)$

c)  $5.49$

$$\begin{aligned} \text{b) } \sqrt{3} + \sqrt{48} &= \sqrt{3} + 4\sqrt{3} \\ &= 5\sqrt{3} \\ &= \sqrt{75} \end{aligned}$$

f)  $\theta = 19^\circ, 161^\circ$

other answers possible

$\therefore x = 75$

g)  $x = 4, y = -1$

c)  $115\% \equiv \$368$

h) i)  $\frac{2}{3} \times \frac{3}{5} = \frac{2}{5}$

$\therefore 15\% \equiv \$48$

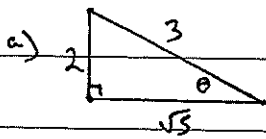
$\therefore \text{GST} = \$48$

ii)  $\frac{2}{3} \times \frac{3}{5} + \frac{1}{3} \times \frac{4}{5}$

$= \frac{8}{15}$

d)  $(-3, -9)$

Question 2



$$\tan \theta = \frac{2}{\sqrt{5}}$$

b)  $(x-4)(x+2) = 0$

$$\therefore x = 4, -2$$

c) i) 10

ii) 15

iii) 18

d)  $A = \frac{1}{2} \times 9.9 \times 8.8 \times \sin 73^\circ$   
 $= 42 \text{ m}^2$

e) i) In  $\triangle ABF$  and  $\triangle ECF$

$\angle ABF = \angle ECF$  (alternate angles,  $AB \parallel DE$ )

$\angle AFB = \angle CFE$  (vertically opposite angles)

$\therefore \triangle ABF \parallel \triangle ECF$  (equiangular)

ii)  $\frac{AB}{9} = \frac{10}{6}$

$$AB = 15 \text{ cm}$$

Question 3

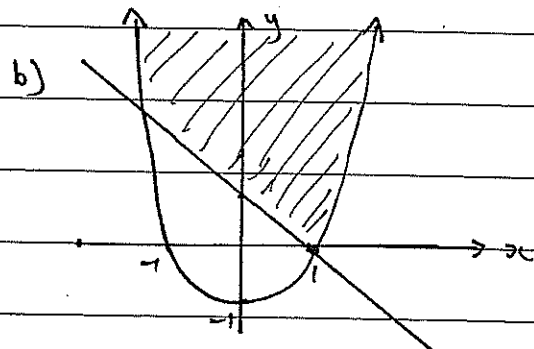
a) i)  $m = \frac{2-1}{-4-2}$   
 $= -\frac{1}{2}$

ii)  $m_{\perp} = 2 \quad (2, -1)$

$$y - y_1 = m(x - x_1)$$

$$y + 1 = 2(x - 2)$$

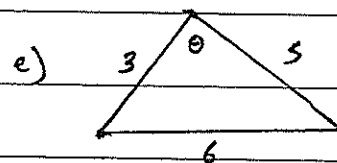
$$y = 2x - 5$$



c)  $\frac{V}{75} = \frac{18^3}{5^3}$   
 $V = 3499 \text{ cm}^3$

d)  $\frac{RT}{\sin 108^\circ} = \frac{45}{\sin 54^\circ}$

$$RT = 52.9 \text{ cm}$$



$$\cos \theta = \frac{3^2 + 5^2 - 6^2}{2 \times 3 \times 5}$$

$$\therefore \theta = 94^\circ \text{ (largest angle)}$$

Question 4a) i) In  $\triangle ABC$  and  $\triangle CAD$ 

$$\angle ABC = \angle DAC \text{ (given)}$$

$$\frac{AD}{BC} = \frac{8}{6} = \frac{4}{3}$$

$$\frac{AC}{AB} = \frac{12}{9} = \frac{4}{3}$$

$\therefore \triangle ABC \parallel \triangle CAD$  (sides about  
equal angle in equal ratio)

e) i)  $\angle AWB = 11^\circ$ 

$$ii) \frac{WB}{\sin 24^\circ} = \frac{5}{\sin 11^\circ}$$

$$WB = \frac{5 \sin 24^\circ}{\sin 11^\circ}$$

and

$$\sin 35^\circ = \frac{CW}{WB}$$

$$\therefore CW = \frac{5 \sin 24^\circ \sin 35^\circ}{\sin 11^\circ}$$

$$b) V = \frac{1}{3} \times 24 \times 28 \times 32$$

$$= 7168 \text{ cm}^3$$

$$= 6.1 \text{ m}$$

$$c) A = \frac{1}{8} \times 4 \times \pi \times 15^2 + 3 \times \frac{1}{4} \times \pi \times 15^2$$

$$= 883.6 \text{ cm}^2$$

d) Amount to be repaid

$$= 7500 + 2 \times 0.12 \times 7500$$

$$= \$9300$$

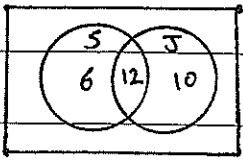
 $\therefore$  monthly repayment

$$= \frac{\$9300}{24}$$

$$= \$387.50$$

Question 5

a)



$$P = \frac{12}{28}$$

$$= \frac{3}{7}$$

b)

$$\sqrt{R^2 - y^2} = R - S$$

$$R^2 - y^2 = (R - S)^2$$

$$R^2 - y^2 = R^2 - 2RS + S^2$$

$$2RS = y^2 + S^2$$

$$R = \frac{y^2 + S^2}{2S}$$

c)  $x^2 + y^2 = 16$ ,  $y + 4 = x^2$

$$\therefore y + 4 + y^2 = 16$$

$$y^2 + y - 12 = 0$$

$$(y + 4)(y - 3) = 0$$

$$y = -4, 3$$

when  $y = -4$   $x = 0$

$$y = 3 \quad x^2 = 7$$

$$x = \pm\sqrt{7}$$

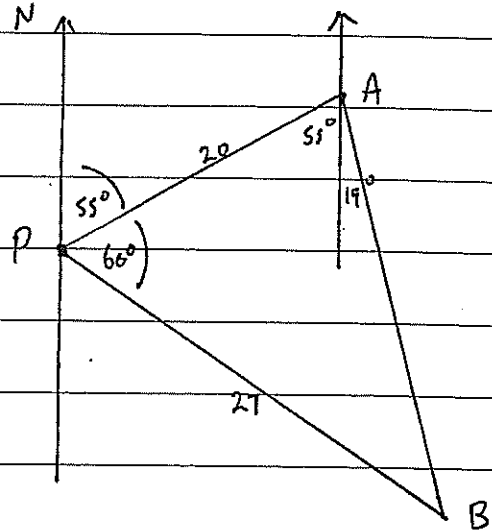
$\therefore$  Solutions are

$$x = 0, y = -4$$

$$x = \sqrt{7}, y = 3$$

$$x = -\sqrt{7}, y = 3$$

d)



i)  $\angle APB = 115^\circ - 55^\circ$   
 $= 60^\circ$

ii)  $AB^2 = 20^2 + 27^2 - 2 \times 20 \times 27 \times \cos 60^\circ$

$$\therefore AB = 24.3 \text{ km}$$

iii)  $\frac{\sin A}{27} = \frac{\sin 60^\circ}{24.3}$

$$\therefore A = 74^\circ$$

$$\therefore \text{bearing} = 180^\circ - 19^\circ$$

$$= 161^\circ$$

e) Similar solids  $\therefore \frac{A}{180} = \frac{15^2}{9^2}$

$$A = 500$$

$$\therefore \text{shaded area} = 500 - 180$$

$$= 320 \text{ cm}^2$$