SYDNEY TECHNICAL HIGH SCHOOL

Mathematics Extension 1

YEAR 11 PRELIMINARY HSC
ASSESSMENT TASK 1
MAY 2009

General Instructions

- Working time allowed – 70 minutes
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown
- Start each question on a new page
- Attempt all questions

NAME :_____

TEACHER :_____

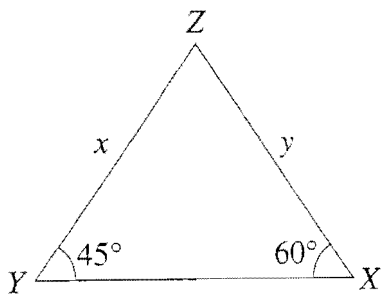
QUESTION 1	QUESTION 2	QUESTION 3	QUESTION 4	QUESTION 5	QUESTION 6	TOTAL

Question 1 (10 marks)

Marks

a) Simplify $\dfrac{1}{1-2x} - \dfrac{2x}{1-4x^2}$ 2

b)



NOT TO SCALE

In the diagram, XYZ is a triangle where $\angle ZYX = 45°$ and $\angle ZXY = 60°$. 2

Find the exact value for the ratio $\dfrac{x}{y}$.

c) Write $(\sqrt{6} + 2)^3$ in the form $a + b\sqrt{6}$ where a and b are integers. 2

d)

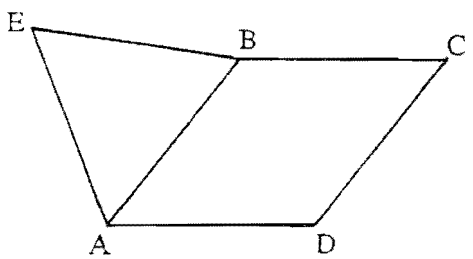


Figure not to scale.

$ABCD$ is a rhombus with $\angle CDA = 134°$.

ABE is an equilateral triangle.

i) Find the size of $\angle EAD$, giving reasons for your answer. 2

ii) Find the size of $\angle EDA$, giving reasons for your answer. 2

Question 2 (9 marks) Start a new page

Marks

a) Fully factorise $x^2 - 4ax + 4a^2 - 4b^2$ 2

b) If $\tan\theta = \frac{2}{5}$ and $\cos\theta < 0$ find the exact value of $\sin\theta$. 2

c) Find a value of x that satisfies the equation $\cos 2x = \sin 80°$ 1

d) Solve for x : $2^{-4x} \times 4^{3x-2} = 16$ 2

e) Draw a neat sketch of the function $y = g(x)$ 2

given that $g(x) = \begin{cases} 1-x\ , & x < 1 \\ \sqrt{x}\ , & x \geq 1 \end{cases}$

Question 3 (9 marks) Start a new page

a) Solve $2\cos 2\theta + 1 = 0$ for $0° \leq \theta \leq 360°$ 3

b) Prove that $\frac{1}{\sin x \cos x} - \frac{\cos x}{\sin x} = \tan x$ 2

c) Consider the function $y = \frac{x^2}{1-x^2}$. 4

i) State the equations of the vertical asymptotes.

ii) State the equation of the horizontal asymptote.

iii) Draw a neat sketch of $y = \frac{x^2}{1-x^2}$

Question 4 (9 marks) Start a new page Marks

a) Show that $8^{120} - 8^{119} - 8^{118}$ is divisible by 55. 1

b) Solve the equation $3\sin^2\theta = 2\cos\theta + 2$ for $0° \le \theta \le 360°$. 4

Give your answers correct to the nearest degree.

c) A regular polygon has an interior angle of $162°$. 1

How many sides does the polygon have ?

d) Solve simultaneously for x and y : 3

$25x^2 - y^2 = 84$ and $5x - y = 6$.

Question 5 (9 marks) Start a new page Marks

a) Solve $\frac{x}{x+1} \geq 2$ 3

b) Solve $8 \sin\theta \cos\theta \tan\theta = \operatorname{cosec}\theta$ for $0° \leq \theta \leq 360°$ 2

c) Solve $y^{\frac{2}{3}} = 64$ 1

d) Simplify $\frac{(1-\tan x)^2 + (1+\tan x)^2}{(1+\cot x)^2 + (1-\cot x)^2}$ 3

Please turn over for question 6

Question 6 (9 marks) Start a new page Marks

a) 5

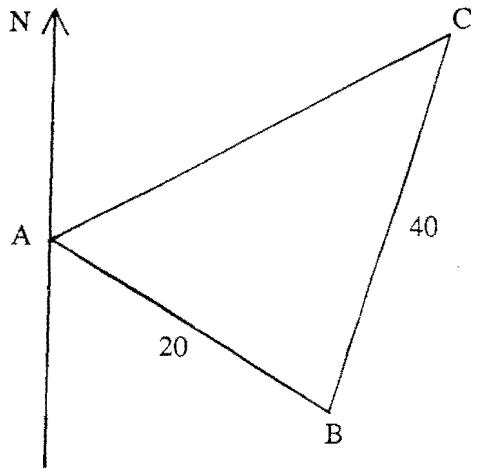


Figure not to scale

A ship sails 20 kilometres from point A on a bearing of $146°T$ to point B.

The ship then sails 40 kilometres from point B on a bearing of $015°T$ to point C.

i) Find the size of angle ABC.

ii) Find the distance between point A and point C correct to the nearest hundred metres.

iii) Find the bearing of point A from point C correct to the nearest degree.

b) i) Draw a neat sketch of the function $y = \dfrac{20}{|x|}$ 1

ii) Solve $\dfrac{20}{|x|} < 17 - 3x$ 3

End of Paper

YR11 EXT 1 SOLUTIONS MAY 2009

QUESTION 1

a) $\dfrac{1}{1-2x} - \dfrac{2x}{(1-2x)(1+2x)}$

$= \dfrac{1+2x-2x}{(1-2x)(1+2x)}$

$= \dfrac{1}{(1-2x)(1+2x)}$

b) $\dfrac{x}{\sin 60^\circ} = \dfrac{y}{\sin 45^\circ}$

$\therefore \dfrac{x}{y} = \dfrac{\sin 60^\circ}{\sin 45^\circ}$

$= \dfrac{\frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}}$

$= \dfrac{\sqrt{6}}{2}$

c) $(\sqrt{6}+2)^3$

$= 6\sqrt{6} + 36 + 12\sqrt{6} + 8$

$= 44 + 18\sqrt{6}$

d) i) $\angle BAD = 46^\circ$

(co-interior angles, $AB \parallel DC$)

$\angle EAB = 60^\circ$

(angle in equilateral triangle)

$\therefore \angle EAD = 60^\circ + 46^\circ$

$= 106^\circ$

ii) $\angle EDA = \angle AED$

(angles opposite equal sides are equal)

$\therefore 2x \angle EDA + 106^\circ = 180^\circ$

(angle sum of a triangle)

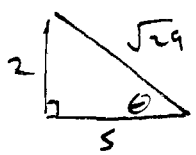
$\therefore \angle EDA = 37^\circ$

QUESTION 2

a) $x^2 - 4ax + 4a^2 - 4b^2$

$= (x-2a)^2 - 4b^2$

$= (x-2a+2b)(x-2a-2b)$

b) 

3rd quad

$\therefore \sin\theta = -\dfrac{2}{\sqrt{29}}$

c) $2x + 80 = 90$

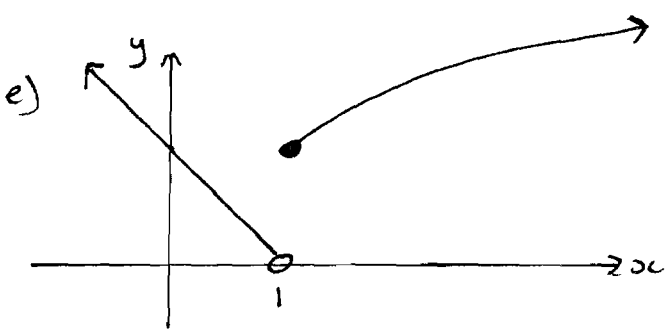
$x = 5^\circ$

d) $2^{-4x} \times 4^{3x-2} = 16$

$2^{-4x} \times 2^{6x-4} = 2^4$

$-4x + 6x - 4 = 4$

$x = 4$

e) 

QUESTION 3

a) $2\cos 2\theta + 1 = 0$

$\cos 2\theta = -\frac{1}{2}$

$2\theta = 120^\circ, 240^\circ, 480^\circ, 600^\circ$

$\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$

b) $LHS = \dfrac{1}{\sin x \cos x} - \dfrac{\cos x}{\sin x}$

$= \dfrac{1 - \cos^2 x}{\sin x \cos x}$

$= \dfrac{\sin^2 x}{\sin x \cos x}$

$= \dfrac{\sin x}{\cos x}$

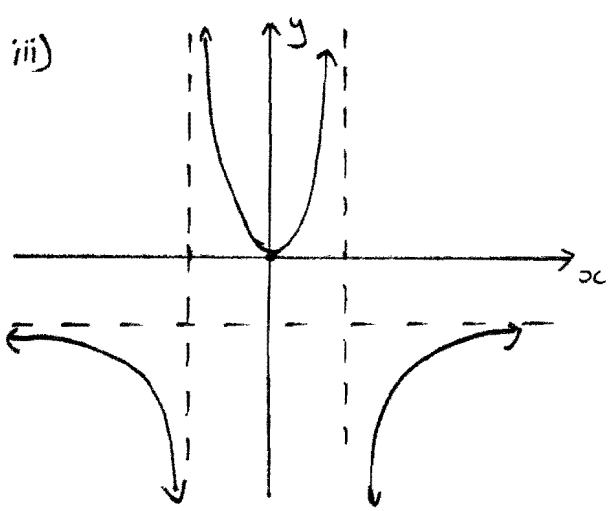
$= \tan x$

$= RHS$

c) i) $x = 1$ and $x = -1$

ii) $y = -1$

iii)



QUESTION 4

a) $8^{120} - 8^{119} - 8^{118}$

$= 8^{118}(8^2 - 8 - 1)$

$= 55 \times 8^{118}$

which is divisible by 55.

b) $3\sin^2\theta = 2\cos\theta + 2$

$3(1 - \cos^2\theta) = 2\cos\theta + 2$

$3\cos^2\theta + 2\cos\theta - 1 = 0$

$(3\cos\theta - 1)(\cos\theta + 1) = 0$

$\cos\theta = \frac{1}{3}, -1$

$\theta = 71^\circ, 289^\circ, 180^\circ$

c) exterior angle $= 180^\circ - 162^\circ$

$= 18^\circ$

$\therefore$ number of sides $= \dfrac{360}{18}$

$= 20$

d) $25x^2 - y^2 = 84 \quad -(1)$

$5x - y = 6$

$\therefore \quad y = 5x - 6$ sub. into (1)

$25x^2 - (5x - 6)^2 = 84$

$60x - 36 = 84$

$x = 2$

$\therefore y = 4$

QUESTION 5

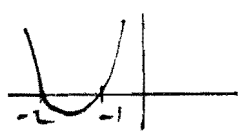
a) $\dfrac{x}{x+1} \geqslant 2$

$\dfrac{x(x+1)^2}{x+1} \geqslant 2(x+1)^2$

$x(x+1) \geqslant 2x^2 + 4x + 2$

$x^2 + 3x + 2 \leqslant 0$

$(x+2)(x+1) \leqslant 0$



$-2 \leqslant x < -1$

b) $8\,\text{Sin}\theta\,\text{Cos}\theta\,\text{Tan}\theta = \text{Cosec}\,\theta$

$8\,\text{Sin}\theta\,\text{Cos}\theta\,\dfrac{\text{Sin}\theta}{\text{Cos}\theta} = \dfrac{1}{\text{Sin}\theta}$

$8\,\text{Sin}^3\theta = 1$

$\text{Sin}^3\theta = \frac{1}{8}$

$\text{Sin}\theta = \frac{1}{2}$

$\theta = 30°,\ 150°$

) $y = \pm 512$

d)

$$\frac{1 - 2\tan x + \tan^2 x + 1 + 2\tan x + \tan^2 x}{1 + 2\,\text{Cot}x + \text{Cot}^2 x + 1 - 2\,\text{Cot}x + \text{Cot}^2 x}$$

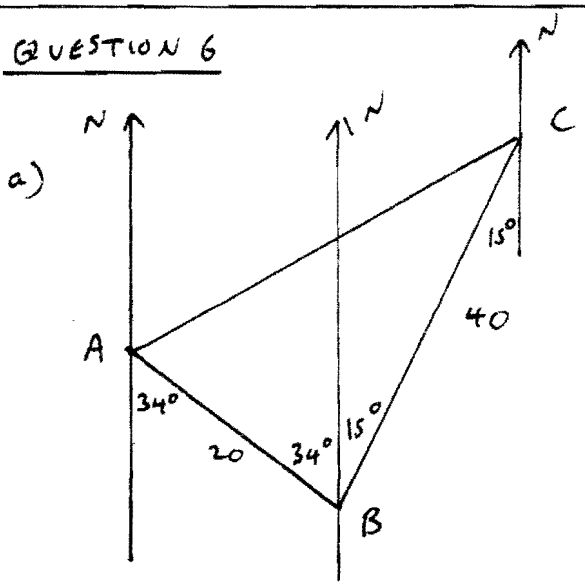
$$= \frac{2(1 + \tan^2 x)}{2(1 + \text{Cot}^2 x)}$$

$$= \frac{2\,\text{Sec}^2 x}{2\,\text{Cosec}^2 x}$$

$$= \text{Tan}^2 x$$

QUESTION 6

a)



i) $\angle ABC = 49°$

ii) $AC^2 = 20^2 + 40^2 - 2 \times 20 \times 40 \times \text{Cos}\,49°$

$AC = 30.8$ km

iii) $\dfrac{\text{Sin}C}{20} = \dfrac{\text{Sin}\,49°}{30.8}$

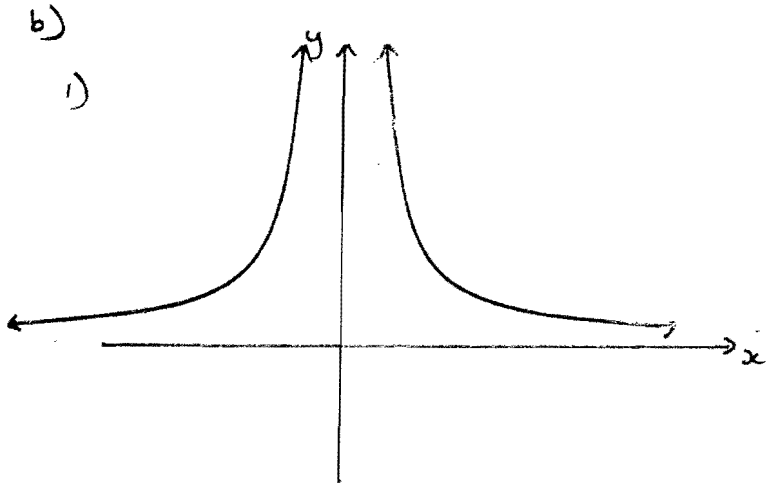
$\angle C = 29°$

$\therefore$ bearing $= 180° + 15° + 29°$

$= 224°$

b)

1)



11) $\frac{20}{|x|} < 17 - 3x$

Solve $\frac{20}{|x|} = 17 - 3x$

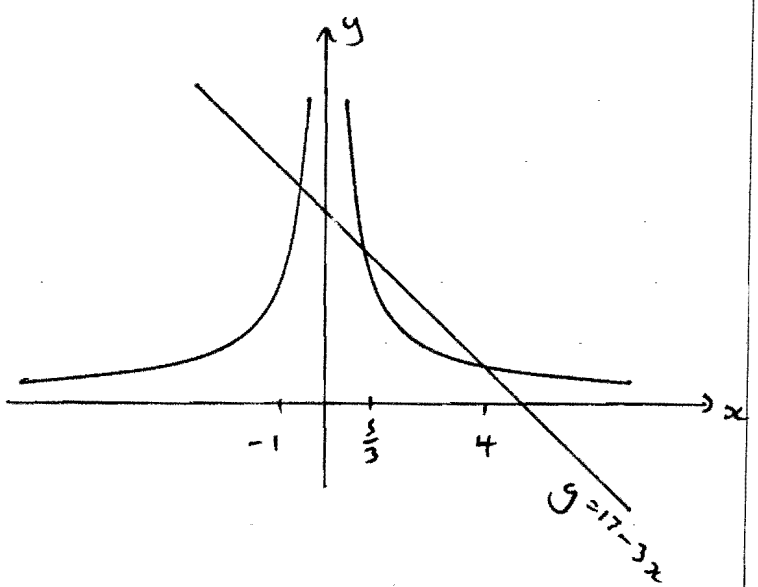
$\frac{20}{x} = 17 - 3x$ or $\frac{20}{-x} = 17 - 3x$

$20 = 17x - 3x^2$ $\quad$ $20 = -17x + 3x^2$

$3x^2 - 17x + 20 = 0$ $\quad$ $3x^2 - 17x - 20 = 0$

$(3x - 5)(x - 4) = 0$ $\quad$ $(3x - 20)(x + 1) = 0$

$x = \frac{5}{3}, 4$ $\quad$ $x = \frac{20}{3}, -1$



$\therefore \quad x < -1, \quad \frac{5}{3} < x < 4$