Name: ..... Maths Class: .....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics

HSC Course

Assessment 1

November, 2015

Time allowed: 70 minutes

General Instructions:

- Marks for each question are indicated on the question
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A set of Reference Formulae is provided. You are not to write on this booklet and it must be returned at the end of the examination

Section 1 Multiple Choice
Questions 1-5
5 Marks

Section II Questions 6-10
60 Marks

SECTION I MULTIPLE CHOICE

Use the multiple choice answer sheet provided, and fill in the circle corresponding with the most appropriate answer.

Allow about 7 minutes for this section.

Question 1

In the first week of the snow season 5 cm of snow falls. In each following weeks the snowfalls increase by 2 cm, so in the second week there is 7 cm, in the third week there is 9 cm. How much snow falls in the 11th week?

(A) 23 cm

(B) 25 cm

(C) 27 cm

(D) 29 cm

Question 2

Owen starts on a salary of $30 000 with an annual increment of $1550. What is the total amount Owen would earn in eleven years of employment?

(A) $45 000

(B) $276 830

(C) $415 250

(D) $495 000

Question 3

The centre and radius of the circle with equation $x^2 + y^2 - 6y = 4x$ are

(A) Centre (0,3) radius 2

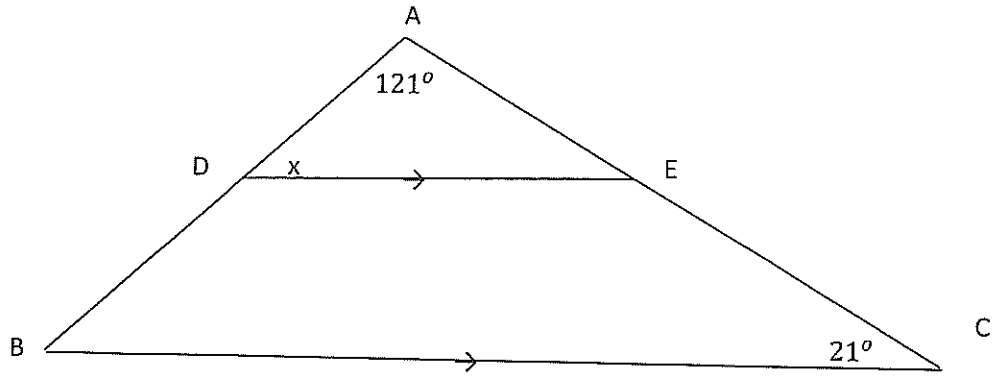
(B) Centre (0,9) radius 2

(C) Centre (2,3) radius $\sqrt{13}$

(D) Centre (2,3) radius 13

Question 4

The value of x in the following diagram is:-



(A) 38°

(B) 21°

(C) 121°

(D) 40°

Question 5

Which of the following series is <u>not</u> geometric:

(A) $\frac{3}{5}+\frac{2}{5}+\frac{4}{15}+\ldots$

(B) $\frac{7}{8}-\frac{21}{32}+\frac{63}{128}-\ldots$

(C) $\frac{6}{7}+\frac{4}{7}+\frac{18}{21}+\ldots$

(D) $\frac{5}{6}+\frac{2}{3}+\frac{8}{15}+\ldots$

SECTION II -

Use the answer booklet provided, and start each new question on a new page.

Allow about 63 minutes for this section.

QUESTION 6: (12 marks)

a) What is the vertex of the parabola $y = 2x^2 + 4x + 1$ (2)

b) Find the equation of the parabola with a focus S (-2,6) and vertex, V (-2,4) (2)

c) Solve $m - \frac{5}{m} = 4$ (2)

d) Find the sum of the first 21 terms of the sequence:

$3 + 7 + 11 + \ldots\ldots\ldots$ (2)

e) Write $0.61\dot{7}$ as an infinite GP and hence as a fraction in simplest form (2)

f) Find the locus of a point P which moves so that it is always 2 units from the point (-1,2).

Give your answer in algebraic form. (2)

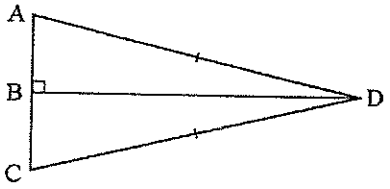
QUESTION 7: (12 marks) (Start a new page)

a) If α and β are the roots of $2x^2 - 3x + 4 = 0$, determine the value of

i) $\propto + \beta$

ii) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$ (3)

b) You are given a triangle which has two equal sides AD and CD. BD is perpendicular to AC (as shown in the diagram below).



Use this figure and congruence only to prove the statement:

"If a triangle has two equal sides, then the triangle has two equal angles opposite those sides." (3)

c) Consider the parabola $x^2 = 8y + 16$

i) Find the co-ordinates of the vertex (1)

ii) Find the co-ordinates of the focus (1)

iii) Find the equation of the tangent to the parabola at the point $(2, -3/2)$ (2)

d) Sketch the parabola $y^2 = 12x$ showing its focus and directrix (2)

QUESTION 8: (12 marks) (Start a new page)

a) Find the equation of the circle with ends of diameter C (-3,-2) and D (1,4) (2)

b) The 5th term of an arithmetic series is 14 and the 10th term is 59.

Find the first term and the common difference of the series. (3)

c) Find the value of A, B and C, given
$3x^2 + 4 \equiv A(x+2)^2 + B(x+2) + C$ (3)

d) The numbers a, 1 and b are in Geometric Progression, while $\frac{1}{6}$, a and b are in Arithmetic Progression.

(i) Write down the <u>two</u> equations linking a and b (1)

(ii) By solving these two equations simultaneously, find a and b. (3)

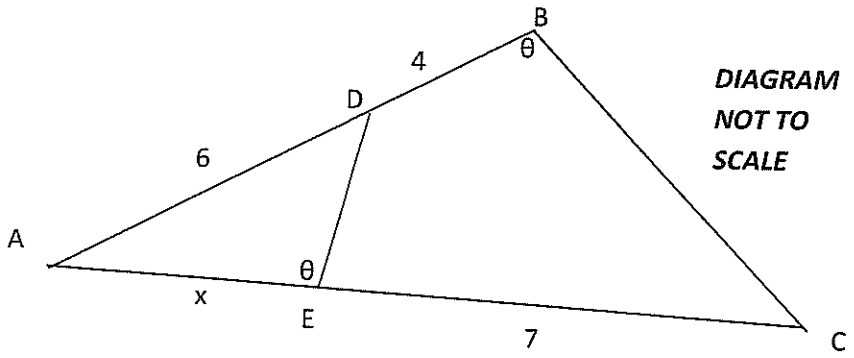
QUESTION 9: (12 marks) (Start a new page)

a) Consider the quadratic expression $3kx^2 - 5x + 3k$

i) Write down an expression for the discriminant of this quadratic (1)

ii) Hence, find the value(s) of k for which $y = 3kx^2 - 5x + 3k$ is negative definite. (3)

b) In the following diagram, the angles AED and ABC are equal.



i) Prove that the two triangles ADE and ACB are similar (2)

ii) Find the exact value of x (2)

c) Using a suitable substitution, reduce this equation to a quadratic, and hence solve for x:

$$(x^2 - 2x)^2 + 7(x^2 - 2x) + 6 = 0$$ (4)

QUESTION 10: (12 marks) (Start a new page)

a) Find the values of x for which the series:

$1 + (x-3)^2 + (x-3)^4$ ..... has a limiting sum (3)

b) If $\propto$ and β are the roots of the equation $x^2 + 5x + 7 = 0$,

(i) Find the values of $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ (2)

(ii) Find the value of $(\alpha - \beta)^2(\alpha + \beta)^2$ (1)

(iii) Find the quadratic equation whose roots are $(\alpha - \beta)^2$ and $(\alpha + \beta)^2$ (1)

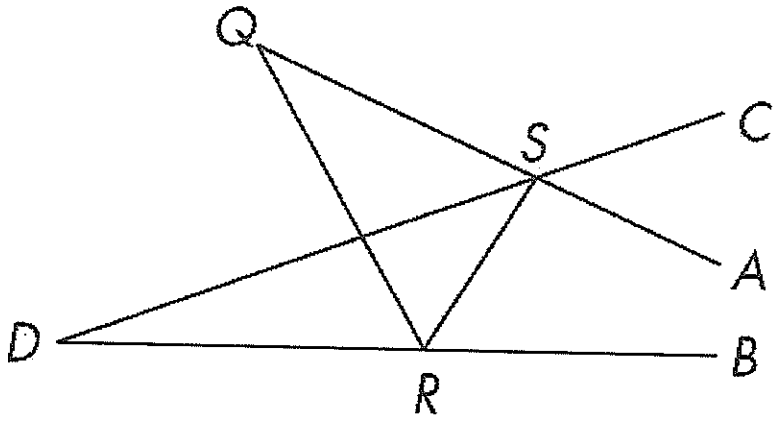
QUESTION 10 continues overleaf.....

QUESTION 10 continued.....

c) In the figure below,

$\angle QRD = \angle SRB$, $\angle RSD = \angle ASC$ and

DRB, DSC, QSA are straight lines.



Let $\angle SRB = x$ *and* $\angle ASC = y$

i) Copy or trace the diagram onto your answer sheet

ii) Explaining all steps, show that $\angle SDR = x - y$ (2)

iii) Hence or otherwise, show $\angle RQS = 2\angle SDR$ *(with reasons)* (2)

d) Show that the equation $a^2x^2 + x^2 + ax + 2 = 0$ has no real roots for all values of a. (1)

SOLUTIONS

MULTIPLE CHOICE

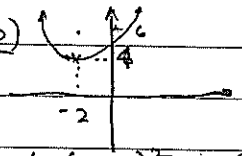
1/ B 2/ C 3/ C 4/ A 5/ C

SECTION 2

QUESTION 6:

(a) $y = 2(x+1)^2 - 1$

$\therefore$ V: $(-1, 1)$

(b) 

$a = 2$

$\therefore (x+2)^2 = 8(y-4)$

(c) $m^2 - 5 = 4m$

$(m-5)(m+1) = 0$

$m = 5$ or $m = -1$

(d) $S_{21} = \frac{21}{2}[6 + 20 \times 4]$

$= 903$

(e) $0.61 + 0.007 + 0.0007 + \ldots$

$= \frac{61}{100} + \frac{0.007}{0.9}$

$= \frac{61}{100} + \frac{7}{900}$

$= 139/225$

(f) $(x+1)^2 + (y-2)^2 = 4$

QUESTION 7:

(a) (i) $\alpha + \beta = 3/2$

$\alpha\beta = 2$

(ii) $\alpha/\beta + \beta/\alpha = \frac{\alpha^2 + \beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= -7/8$

(b) In $\triangle ABC$ and $\triangle CBD$,

$AC = CD$ (given)

$\angle ABC = \angle CBD = 90°$

BD is common

$\therefore \triangle ABC \equiv \triangle CBD$ (RHS)

$\therefore \angle BAD = \angle BCD$ (corresponding angles in congruent triangles)

(c) $x^2 = 4.2(y+2)$

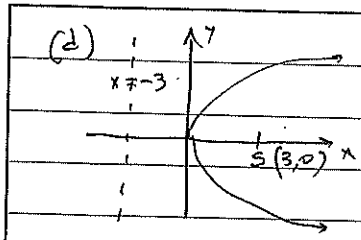
(i) V: $(0, -2)$ (ii) S$(0, a)$

(iii) $\frac{dy}{dx} = x/4$

at $(2, -3/2)$ $m_T = 1/2$

equation is $y + 3/2 = 1/2(x-2)$

$2y = x - 5$

(d) 

QUESTION 8:

(a) $M = (-1, 1)$ $r = \sqrt{13}$

Equation is $(x+1)^2 + (y-1)^2 = 13$

(b) $a + 4d = 14$

$a + 9d = 59$

$5d = 45$

$d = 9$

$a = -22$

(c) $x = -2$, $\therefore c = 16$

Coefficient of x^2: $A = 3$

constants $4A + 2B + C = 4$

$2B = -24$

$B = -12$

(d) $a, 1, b$ is in GP

$1/a = b/1$

$\therefore b = 1/a$

$1/6, a, b$ are in AP

$\therefore a - 1/6 = b - a$

$\therefore 2a = b + 1/6$

$\therefore 2a = 1/a + 1/6$

$\therefore 12a^2 = 6 + a$

$(3a+2)(4a-3) = 0$

$\therefore a = -2/3$, $b = -3/2$ or $a = 3/4$, $b = 4/3$

QUESTION 9:

(a)(i) $\Delta = 25 - 36k^2$

(ii) For neg definite,

$\Delta < 0$ and $k < 0$

$\therefore 36k^2 > 25$

$k^2 > 25/36$

$k < -5/6$ or $k > 5/6$

↑ NOT A SOL^N since $k < 0$

(b) In $\triangle ADE$ and $\triangle ACB$

$\angle DAE = \angle CAB$ (common)

$\angle AED = \angle ABC = \theta$ (given)

$\therefore \triangle ADE \,|||\, \triangle ACB$

(ii) $AE/AD = AB/AC$

$\therefore x/6 = 10/(x+7)$

$\therefore x^2 + 7x = 60$

$(x+12)(x-5) = 0$

$x = 5$

9 (c) Let $u = x^2 - 2x$

$\therefore u^2 + 7u + 6 = 0$

$(u+6)(u+1) = 0$

$\therefore u = -6$ or $u = -1$

$\therefore x^2 - 2x + 6 = 0$ or $x^2 - 2x + 1 = 0$

NO SOLN $(x-1)^2 = 0$

$\therefore x = 1$

QUESTION 10:

(a) For a limiting sum $|r| < 1$

$\therefore (x-3)^2 < 1$

$\therefore x^2 - 6x + 9 < 1$

$(x-2)(x-4) < 0$

$\therefore 2 < x < 4$

[NOTE: $x = 3$ is trivial]

(b) (i) $\alpha + \beta = -5$ $\alpha\beta = 7$

$(\alpha+\beta)^2 = 25$

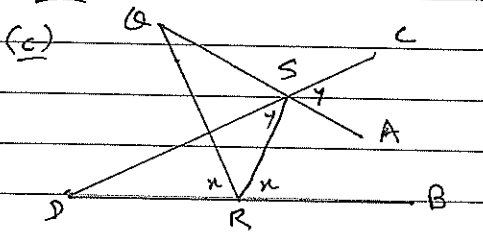
$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

$= -3$

(ii) -75

(iii) $x^2 - 22x - 75 = 0$

(c)



(ii) $\angle SPR + y = \angle SRB$

(exterior angle of $\triangle SDR$)

$\therefore \angle SPR = x - y$

(iii) $\angle RQS + \angle QRS = 180 - 2y$

(exterior angle of $\triangle QRS$)

and $\angle QRS = 180 - 2x$

$\therefore \angle RQS = 180 - 2y - (180 - 2x)$

$= 2x - 2y$

$= 2 \times \angle SPR$.

(d) For no real roots,

$\Delta < 0$

$\therefore a^2 - 8(a^2+1) \leq 0$

$\therefore -7a^2 - 8 < 0$

which is always true

since $a^2 > 0$

2 UNIT MARKERS' COMMENTS – December 2015

QUESTION 7:

(a)(ii) a lot of people when simplifying $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ had no denominator (ie they multiplied throughout by αβ

(b) RHS is used when there is a right angle and a hypotenuse. SAS means the angle has to be included. (it wasn't)

(c) You can only get the focus when you find out a.

QUESTION 8:

(a) Common problem: Not squaring the radius to fit into the standard equation of a circle

(c) A number of students tried substituting in various values of x to come up with3 equations. It may have been better to expand and equate coefficients.

(d) A significant number wrote b – 1 = 1 – a, which is an AP. Otherwise quite well done for a difficult 2 unit question.

QUESTION 9:

(a) (ii) When solving $k^2 > \frac{25}{36}$, if they got that far, many did not consider the gtwo cases $k < -\frac{5}{6}$ and $k > \frac{5}{6}$, and even cases when they did this, they did not realise negative definite means k < 0.

(b) (i) Proofs should always start with " In Δ ADE and ΔACB"

QUESTION 10:

(a) For a sequence to have a limiting sum, $|r| < 1$ which in this case means $(x-3)^2 < 1$. This then leads to the solution $2 < x < 4$

(b)(iv) Please note that the solution includes " = 0". ie $x^2 - 22x - 75 = 0$

(c) When quoting reasons like "angle sum of a triangle", you need to NAME the triangle, because in this case there are 6 of them.

(d) State a reason why Δ<0.

ie "Required to show that Δ<0 which then leads to $-7a^2 - 8 < 0$

which you should justify as always being true " because....."