SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

HSC ASSESSMENT TASK 1 - 2008

Students' name: _____ Teacher's name: _____

Q1	Q2	Q3	Q4	Q5	Q6	Total
						/ 50

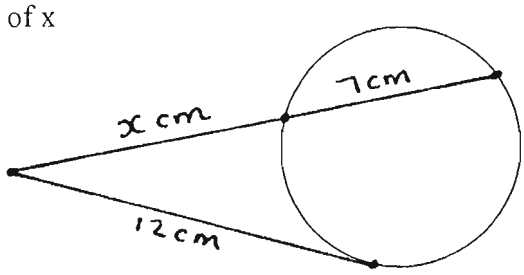
Time allowed: 70 minutes

- Attempt all questions
- Show all necessary working.
- Marks may not be awarded for insufficient working or poorly set out solutions.

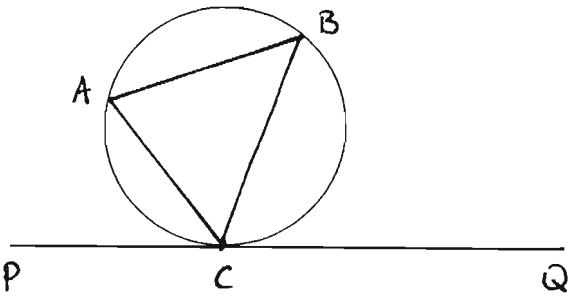
START EACH QUESTION ON A NEW PAGE

Question 1 (8 marks)

a) Find the value of x **2**



b) Explain why angle PCA equals angle CBA. **1**



c) i. Insert 6 terms between -10 and 165 so that the series is arithmetic. **2**

ii. If this pattern continues find the sum of the first 20 terms. **2**

d) Evaluate $\sum_{n=3}^{7} n^3$ **1**

Question 2 (start a new page) (8 marks)

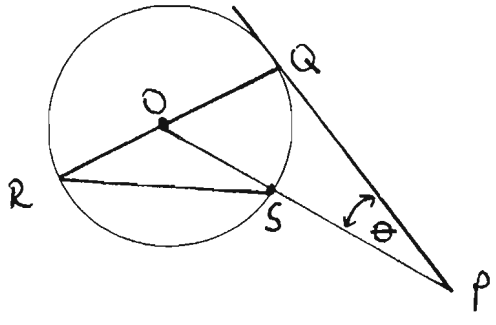
a) Given $S_n = 17n - 3n^2$, find an expression for the nth term. **3**

b) Which term of the series, $100 + 20 + 4 + \ldots$,

is the first term with a value of less than 10^{-4} **3**

c) Given QP is a tangent to a the circle centre O, **2**

Find, the size of angle ORS in terms of θ. Give reasons.



Question 3 (start a new page) (8 marks)

a) Given $\frac{1}{1\times2} + \frac{1}{2\times3} + \frac{1}{3\times4} + \cdots \frac{1}{k(k+1)} = \frac{k}{k+1}$ **3**

Show that $\sum_{n=1}^{K+1} \frac{1}{n(n+1)} = \frac{k+1}{k+2}$

b) P($2ap$, ap^2) is any point on the parabola $x^2 = 4ay$.
The line l passes through the focus, S, and is parallel to the tangent at P.

i. Find the equation of the line l **2**

ii. The line l intersects the x – axis at Q.
Write down the coordinates of Q. **1**

iii. Find the equation of the locus of the midpoint of QS. **2**

Question 4 (start a new page) (9 marks)

a) Consider the series, $x + 4x^2 + 16x^3 + \ldots$

i. For what value of x does the series have a limiting sum **1**

ii. Find the value of x given $S_\infty = \frac{3}{2}$ **2**

b) Tom borrows \$250 000 to buy a unit in Cronulla.
The rates are 6% p.a. monthly reducible interest and equal monthly repayments of \$1900 are payable at the end of each month.

i. How much does Tom owe immediately after the second repayment. **1**

ii. Show that after n months Tom owes
\$ ($380\,000 - 130\,000(1.005)^n$) **2**

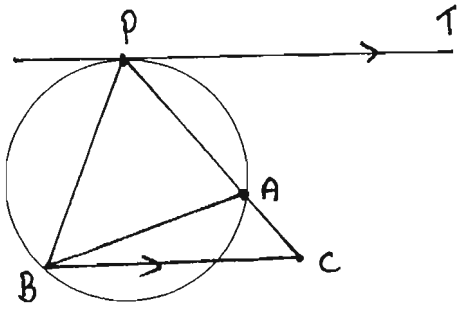
iii. Find the balance owing at the end of 5 years **1**

iv. If after 5 years the interest rate increases to 7.2% p.a., find the minimum monthly repayment needed to repay the loan in a further 20 years. **2**

Question 5 (start a new page) (8 marks)

a) Prove by mathematical induction that $13 \times 6^n + 2$ is divisible by 5 for every positive integer n. **4**

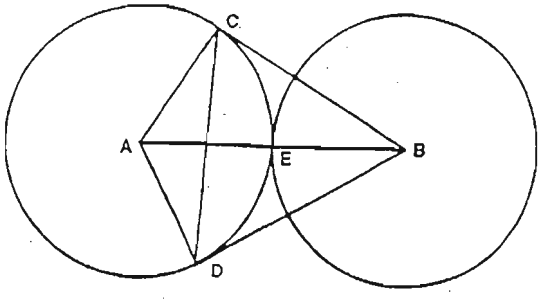
b) In the diagram A, P and B are points on the circle. The chord PA is produced to C and BC is parallel to the tangent at P.



i. Show that Angle PBA = Angle PCB **2**

ii. Deduce that $PB^2 = PA \times PC$ **2**

Question 6 (start a new page) (9 marks)



Two circles of equal radius and with centres at A and B respectively touch each other externally at E. BC and BD are tangents from B to the circle with centre A.

i. Copy the diagram **1**

ii. Show that BCAD is a cyclic quadrilateral. **2**

iii. Show that E is the centre of the circle which passes through B, C, A and D. **2**

iv. Show that angle CBA = angle DBA = 30^0 **2**

v. Show that triangle BCD is equilateral **2**

2008 HSC task I Extension I.

Question 1.

a)

$12^2 = x(x+7)$ ✓

$144 = x^2 + 7x$

$0 = x^2 + 7x - 144$

$0 = (x+16)(x-9)$

$x > 0 \quad \therefore x = 9.$ ✓

b) The angle between a tangent and a chord is equal to the angle in the alternate segment. ✓

c)

i. $-10, -, -, -, -, -, -, 165$ AP

$165 = -10 + 7d$ ✓

$175 = 7d$

$d = 25$ ✓

$\therefore$ 15, 40, 65, 90, 115, 140

ii. $S_{20} = {}^{20}/_{2}(-20 + 19 \times 25)$

$= 4550$ ✓✓

d) $\sum_1^5 = 3^3 + 4^3 + 5^3 + 6^3 + 7^3$

$= 775.$ ✓

Question 2.

a) $T_n = S_n - S_{n-1}$ ✓

$= 17n - 3n^2 - \left[17(n-1) - 3(n-1)^2\right]$

$= 17n - 3n^2 - \left[17n - 17 - 3n^2 + 6n - 3\right]$

$= 17n - 3n^2 - \left[-3n^2 + 17n + 6n - 20\right]$

$= 20 - 6n$ ✓✓

b) $100 + 20 + 4 + \ldots$ $\quad a = 100$

$r = 1/5$

$ar^{n-1} < 10^{-4}$ ✓ $\quad T_n = 10^{-4}$

$100 \times \left(\frac{1}{5}\right)^{n-1} < 10^{-4}$

$0.2^{n-1} < 0.000001$ ✓

LOGS (OR) Trial & error

$(n-1)\log 0.2 < \log 10^{-6}$ | $0.2^9 = 5.12 \times 10^{-7}$

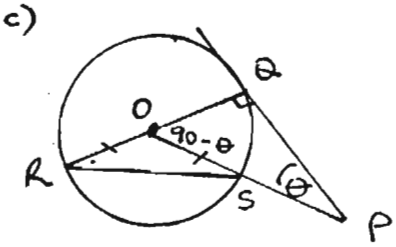
$n - 1 > 8.584\ldots$ | $0.2^8 = 2.56 \times 10^{-6}$

$n > 9.584\ldots$ | $\therefore n - 1 = 9$

$n = 10$ | $n = 10$

✓

c)



$\angle OQP = 90°$ (radius to tangent is 90°)

$\angle QOS = 90 - \theta$ (angle sum ΔOQP)

$\angle ROS = 90 + \theta$ (angles on a straight line)

$RO = OS$ radii

$\therefore \angle ORS = \angle OSR$ (equal angles opp equal sides)

$\angle ORS = \frac{90 - \theta}{2}$ ✓✓

Question 3

a) $\frac{1}{1\times2} + \frac{1}{2\times3} + \ldots + \frac{1}{K(K+1)} + \frac{1}{(K+1)(K+2)}$

$= \frac{K}{K+1} + \frac{1}{(K+1)(K+2)}$ ✓

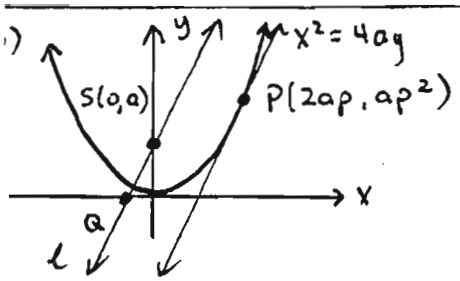
$= \frac{K(K+2) + 1}{(K+1)(K+2)}$ ✓

$= \frac{K^2 + 2K + 1}{(K+1)(K+2)}$

$= \frac{(K+1)^2}{(K+1)(K+2)}$

$= \frac{K+1}{K+2}$ as required ✓

)



I. $y = \frac{x^2}{4a}$

$y' = \frac{2x}{4a}$ at $x = 2ap$

$m_T = p$ as ℓ is // to tangent

$m_2 = p$ thru $(0, a)$ ✓

$\therefore \ell: \quad y - a = p(x - 0)$ ✓

$y = px + a$

II. Q: $y = 0 \qquad 0 = px + a$

$-\frac{a}{p} = x$

$\therefore \quad (-a/p, 0)$

III. Midpt $\left[\frac{-a/p + 0}{2}, \frac{0 + a}{2}\right]$ ✓

$\left[-a/2p, a/2\right]$

Locus of midpt $\Rightarrow \quad y = \frac{1}{2}a$ ✓

b) 6% p.a = 0.005 a month

I. $A_1 = 250\,000(1.005) - 1900$

$A_2 = 250\,000(1.005)^2 - 1900(1.005) - 1900$ ✓

II. $A_3 = 250\,000(1.005)^3 - 1900\left[1.005^2 + 1.005 + 1\right]$

$A_n = 250\,000(1.005)^n - 1900\left[1.005^{n-1} + 1.005^{n-2} \; \cdots + 1\right]$ ✓✓

$A_n = 250\,000(1.005)^n - 1900\left[1\frac{(1.005^n - 1)}{0.005}\right]$

$= 250\,000(1.005)^n - 380\,000(1.005^n - 1)$

$= 380\,000 - 130\,000(1.005)^n$

III. $n = 5 \times 12 = 60$

$A_{60} = 380\,000 - 130\,000(1.005)^{60}$

$= 204649.48$ ✓

IV.

$0 = 204649.48(1.006)^{240} - m\left[\frac{1.006^{240} - 1}{0.006}\right]$

$m\left[\frac{1.006^{240} - 1}{0.006}\right] = 204649.48(1.006)^{240}$

$m = \$1611.32.$

Question 4.

a) $x + 4x^2 + 16x^3 + \cdots\cdots$

I. $-1 < r < 1$

$-1 < 4x < 1$

$-1/4 < x < 1/4$ ✓

II. $\frac{x}{1 - 4x} = \frac{3}{2}$ ✓

$2x = 3 - 12x$

$14x = 3$

$x = \frac{3}{14}$ ✓

Question 5

a) Test $n=1$

$13 \times 6^1 + 2 = 80$

$= 5 \times 16$ ∴ divisible by 5

Assume true for $n=k$ ✓

ie $13 \times 6^k + 2 = 5M$ where M is a +ve integer

Prove true for $n=k+1$

$13 \times 6^{k+1} + 2$

$= 6 \times 13 \times 6^k + 2$ ✓

$= 6[5m - 2] + 2$

$= 30m - 12 + 2$

$= 30m - 10$ ✓

$= 5[6m - 2]$ ✓

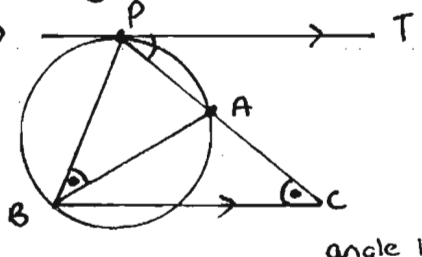
which is ÷ by 5 ∴ true $n=k+1$.

If true $n=k$ also true $n=k+1$

As true $n=1$ also true $n=2, 3, 4 \ldots$.

Hence by M.I true all positive integer n.

b)



I. $\angle PBA = \angle TPC$ (angle between a tangent and a chord equals angle in alternate segment)

$\therefore BCP = \angle TPC$ (alternate angles PT ∥ BC)

$\therefore \angle PBA = \angle PCB$. ✓✓

II. In $\triangle PBA$ and $\triangle PCB$

$\angle P$ is common

$\angle PBA = \angle PCB$ (part i)

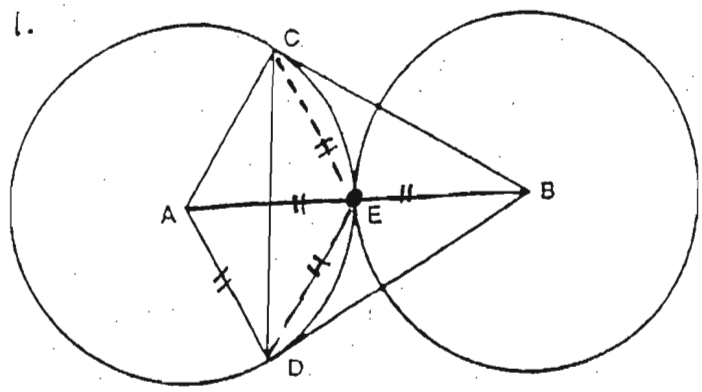
$\therefore \triangle PBA \equiv \triangle PCB$ (equiangular ✓)

$\therefore \frac{PB}{PC} = \frac{PA}{PB}$ (ratio of corresp sides)

$\therefore PB^2 = PA \times PC$ ✓

Question 6

I.



✓

II. $\angle ACB = 90°$ (radius to tangent)

$\angle ADB = 90°$ (radius to tangent)

Now $\angle ACB + \angle ADB = 180°$ ✓✓

and BCAD is cyclic (opposite angles supplementary)

III. As $\angle ACB = 90°$

AB is a diameter (angle in semi circle) ✓

As AE = EB

radii of equal circles

E is the midpoint of AB ✓

and the centre of circle B, C, A, D.

IV. As E is the centre of circle

$\therefore CE = EB = ED$

$AE = AD$ radii centre A.

$\therefore AE = ED = AD$

and $\angle DAE = 60°$

$\angle ADB = 90°$ (radii to tangent)

$\angle DBA = 30°$ (angle sum $\triangle ABD$)

Likewise $\angle ABC = 30°$

$\therefore \angle DBA = \angle ABC = 30°$ ✓✓

V.

CB = DB tangents from external point equal

$\therefore \angle BCD = \angle BDC$ (equal angles opposite equal sides)

$\angle CBD = 60°$ (part iv)

$\therefore \angle BCD = \angle BDC = 60°$ (angle sum $\triangle BCD$)

$\therefore \triangle BCD$ is equilateral.