Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 12
Mathematics Extension 1

HSC Course
Assessment 2
TERM 1 2017

Time allowed: 90 minutes

General Instruction

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- BOSTES reference sheet is located at the end of the exam.

Section 1 Multiple Choice
Questions 1-5
5 Marks

Allow approximately 10 minutes for this section

Section II Questions 6 - 11
60 Marks

Allow approximately 80 minutes for this section

Section 1

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section
Use the Multiple Choice answer sheet for questions 1 – 5

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1. How many **turning points** does the curve $y = x^4 - 4x^3$ have?

   A. 0

   B. 1

   C. 2

   D. 3

---

2. Which of the following integrals is always equal to zero?

C

   A. $\int_0^1 f(x)\,dx$

   B. $\int_{-1}^1 f(x)\,dx$

   C. $\int_0^1 f(x)\,dx \; - \int_{-1}^0 f(x)\,dx$

   D. $\int_1^1 f(x)\,dx$

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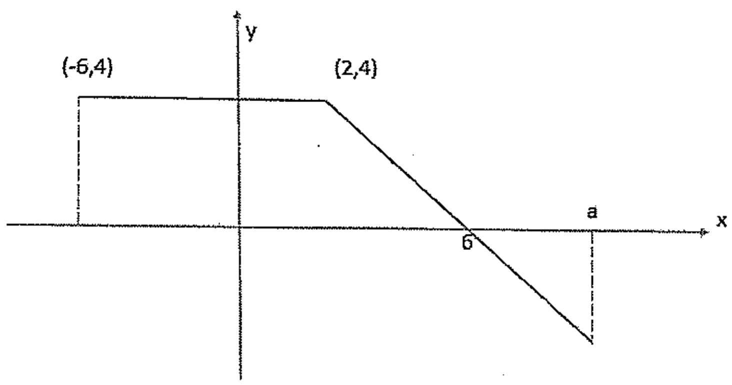
3. Using the graph of y = $f(x)$ below,

C



determine the value of a which satisfies the condition $\int_{-6}^{a} f(x)\,dx = 8$

   A. 8
   B. 10
   C. 12
   D. 14

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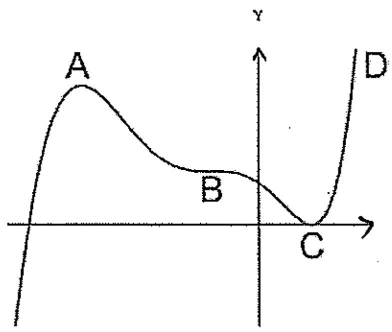
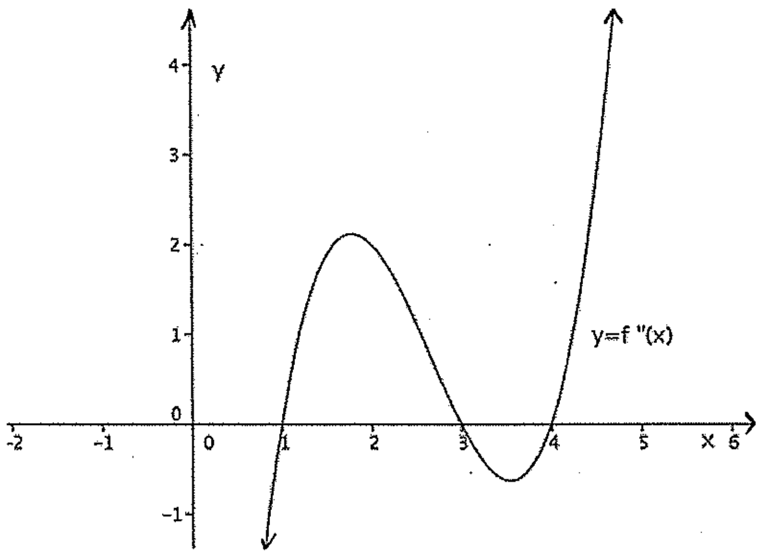
4. At which point on the graph of y= $f(x)$ shown below, is $f''(x)<0$ and $f'(x)=0$?



A. A
B. B
C. C
D. D

---

5. The graph of $y=f''(x)$ is shown below.



Which of the following is true for the graph of $y=f(x)$?

A. At $x=1$ there is a maximum turning point.

B. At $x=1$ there is a minimum turning point.

C. At $x=2$ there is a maximum turning point.

D. At $x=2$ there is a minimum turning point.

End of Section 1

Section II

Attempt Questions 6 – 11
Allow about 1 hour and 20 minutes for this section.
Answer each question in your answer booklet STARTING EACH QUESTION ON A NEW PAGE.

In Questions 6 – 11 your responses should include all relevant mathematical reasoning and / or calculations.

Question 6 - 10 marks

a. Differentiate,

i. $\dfrac{2x+1}{2x-1}$ **2**

ii. $\dfrac{4}{3x}+\dfrac{3x}{4}$ **2**

b. Find,

i. $\int (4t+3)^{-3}\,dt$ **1**

ii. $\int \dfrac{5}{\sqrt{x}}\,dx$ **2**

c. Evaluate $\displaystyle\int_1^2 \left(x+\frac{1}{x}\right)^2 dx$ **3**

Question 7 - 10 marks

Begin this question on a NEW PAGE in your answer booklet.

a. Evaluate $\displaystyle\int_{-1}^{2} |2x-1|\,dx$ **2**

b. Find the primitive function of $\dfrac{x+1}{\sqrt[3]{x}}$ **2**

c. Find the value of k if, $k>1$ and $\displaystyle\int_1^k (3x^2-25)\,dx=24$ **3**

d. The region bounded by the curve $y=\sqrt{4-2x}$ and the coordinate axes, (in the first quadrant), is rotated about the y-axis.
Find the volume of the solid formed. **3**

Question 8 - 10 marks

Begin this question on a NEW PAGE in your answer booklet.

a. Prove by mathematical induction that:

$$\frac{1}{1(4)}+\frac{1}{4(7)}+\frac{1}{7(10)}+.......+\frac{1}{1(3n-2)(3n+1)}=\frac{n}{3n+1} \quad \text{for } n\geq 1$$ **4**

b. Given $\frac{dy}{dx}=x^3(2x-1)^2(3x+1)$, determine the nature of the stationary point at $x=\frac{1}{2}$ **2**

c. The gradient function of the curve $y=f(x)$ is given by $f'(x)=3x^2-4$.

i. Find $y=f''(x)$ **1**

ii. Find the values for x, for which the curve $y=f(x)$ is both increasing and concave down. **1**

iii. If the curve passes through the point (1, -2), find the equation of the curve. **2**

Question 9 - 10 marks

Begin this question on a NEW PAGE in your answer booklet.

a. Prove by mathematical induction that $2^{3n}-3^n$ is divisible by 5, if n is a positive integer. **3**

b.

i. Find $\frac{d}{dx}\left(2x\sqrt{x-3}\right)$ in its simplest form. **2**

ii. Hence, evaluate $\int_4^7 \frac{x-2}{\sqrt{x-3}}dx$ **2**

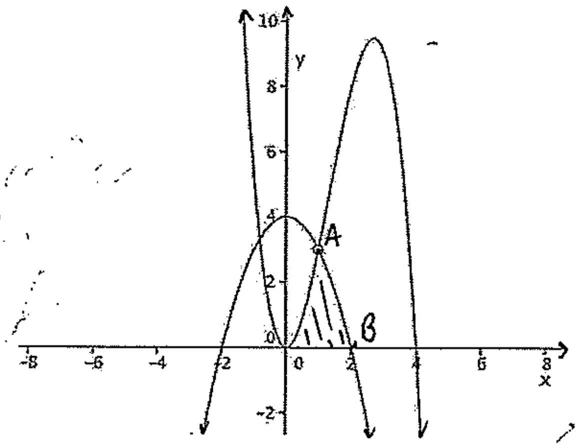
c. Given that $y=(3x+1)^2$

Find the value of k, so that, $2y-y'-x(y'-12)+k=0$ **3**

Question 10 - 10 marks

Begin this question on a NEW PAGE in your answer booklet.

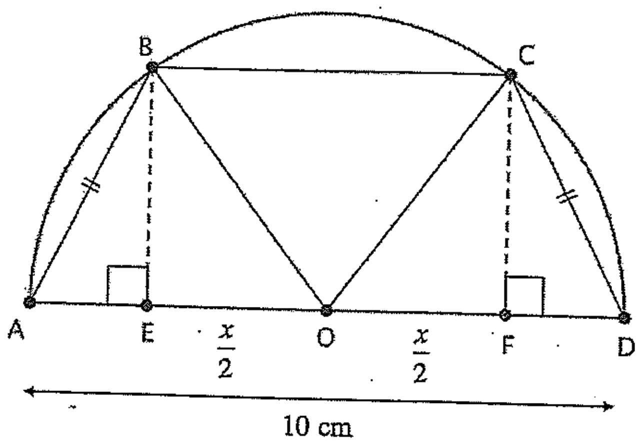
a. The graphs of $y = 4 - x^2$ and $y = x^2(4 - x)$ are shown below.



i. Show that the co-ordinates of A are (1, 3) **1**

ii. Calculate the shaded area OAB. **3**

b. An isosceles trapezium ABCD is drawn with its vertices lying on the circumference of a semicircle centre O and diameter 10cm.



i. If $EO = OF = \frac{x}{2}$ show that $BE = \frac{1}{2}\sqrt{100 - x^2}$ **2**

ii. Show that the area of the trapezium ABCD is given by: **1**

$$A = \frac{1}{4}(x + 10)\sqrt{100 - x^2}$$

iii. Hence, find the length of BC so that the area of the trapezium is a maximum. **3**

Question 11 - 10 marks

Begin this question on a NEW PAGE in your answer booklet.

A curve is defined by,

$$y=\frac{2x^2-x+2}{x}$$

i. For what value/s of x is the curve undefined? **1**

ii. Find the co-ordinates of any turning points and determine their nature. **3**

iii. Explain why the curve has no points of inflexion. **2**

iv. Sketch the curve, on one third of a page, showing all turning points and the equations of all asymptotes. **3**

v. Hence, solve the equation $\frac{2x^2-x+2}{x}-3=0$ **1**

END OF TASK

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REFERENCE SHEET

– Mathematics –

– Mathematics Extension 1 –

– Mathematics Extension 2 –

Mathematics

Factorisation

$a^2 - b^2 = (a+b)(a-b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Angle sum of a polygon

$S = (n-2) \times 180°$

Equation of a circle

$(x-h)^2 + (y-k)^2 = r^2$

Trigonometric ratios and identities

$\sin\theta = \dfrac{\text{opposite side}}{\text{hypotenuse}}$ $\quad \operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

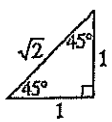
$\cos\theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}}$ $\quad \sec\theta = \dfrac{1}{\cos\theta}$

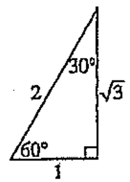
$\tan\theta = \dfrac{\text{opposite side}}{\text{adjacent side}}$ $\quad \tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\cot\theta = \dfrac{\cos\theta}{\sin\theta}$

$\sin^2\theta + \cos^2\theta = 1$

Exact ratios





Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule

$c^2 = a^2 + b^2 - 2ab\cos C$

Area of a triangle

$\text{Area} = \dfrac{1}{2}ab\sin C$

Distance between two points

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Perpendicular distance of a point from a line

$d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

Slope (gradient) of a line

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Point-gradient form of the equation of a line

$y - y_1 = m(x - x_1)$

***n*th term of an arithmetic series**

$T_n = a + (n-1)d$

Sum to *n* terms of an arithmetic series

$S_n = \dfrac{n}{2}[2a + (n-1)d]$ or $S_n = \dfrac{n}{2}(a + l)$

***n*th term of a geometric series**

$T_n = ar^{n-1}$

Sum to *n* terms of a geometric series

$S_n = \dfrac{a(r^n - 1)}{r - 1}$ or $S_n = \dfrac{a(1 - r^n)}{1 - r}$

Limiting sum of a geometric series

$S = \dfrac{a}{1-r}$

Compound interest

$A_n = P\left(1 + \dfrac{r}{100}\right)^n$

Mathematics (continued)

Differentiation from first principles

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Derivatives

If $y=x^n$, then $\frac{dy}{dx}=nx^{n-1}$

If $y=uv$, then $\frac{dy}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}$

If $y=\frac{u}{v}$, then $\frac{dy}{dx}=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

If $y=F(u)$, then $\frac{dy}{dx}=F'(u)\frac{du}{dx}$

If $y=e^{f(x)}$, then $\frac{dy}{dx}=f'(x)e^{f(x)}$

If $y=\log_e f(x)=\ln f(x)$, then $\frac{dy}{dx}=\frac{f'(x)}{f(x)}$

If $y=\sin f(x)$, then $\frac{dy}{dx}=f'(x)\cos f(x)$

If $y=\cos f(x)$, then $\frac{dy}{dx}=-f'(x)\sin f(x)$

If $y=\tan f(x)$, then $\frac{dy}{dx}=f'(x)\sec^2 f(x)$

Solution of a quadratic equation

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

Sum and product of roots of a quadratic equation

$\alpha+\beta=-\frac{b}{a}$ $\qquad$ $\alpha\beta=\frac{c}{a}$

Equation of a parabola

$$(x-h)^2=\pm 4a(y-k)$$

Integrals

$$\int (ax+b)^n\,dx=\frac{(ax+b)^{n+1}}{a(n+1)}+C$$

$$\int e^{ax+b}\,dx=\frac{1}{a}e^{ax+b}+C$$

$$\int \frac{f'(x)}{f(x)}dx=\ln|f(x)|+C$$

$$\int \sin(ax+b)dx=-\frac{1}{a}\cos(ax+b)+C$$

$$\int \cos(ax+b)dx=\frac{1}{a}\sin(ax+b)+C$$

$$\int \sec^2(ax+b)dx=\frac{1}{a}\tan(ax+b)+C$$

Trapezoidal rule (one application)

$$\int_a^b f(x)dx=\frac{b-a}{2}\left[f(a)+f(b)\right]$$

Simpson's rule (one application)

$$\int_a^b f(x)dx=\frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

Logarithms – change of base

$$\log_a x=\frac{\log_b x}{\log_b a}$$

Angle measure

$180°=\pi$ radians

Length of an arc

$l=r\theta$

Area of a sector

Area $=\frac{1}{2}r^2\theta$

Mathematics Extension 1

Angle sum identities

$\sin(\theta+\phi)=\sin\theta\cos\phi+\cos\theta\sin\phi$

$\cos(\theta+\phi)=\cos\theta\cos\phi-\sin\theta\sin\phi$

$\tan(\theta+\phi)=\frac{\tan\theta+\tan\phi}{1-\tan\theta\tan\phi}$

***t* formulae**

If $t=\tan\frac{\theta}{2}$, then

$$\sin\theta=\frac{2t}{1+t^2}$$

$$\cos\theta=\frac{1-t^2}{1+t^2}$$

$$\tan\theta=\frac{2t}{1-t^2}$$

General solution of trigonometric equations

$\sin\theta=a,\quad \theta=n\pi+(-1)^n\sin^{-1}a$

$\cos\theta=a,\quad \theta=2n\pi\pm\cos^{-1}a$

$\tan\theta=a,\quad \theta=n\pi+\tan^{-1}a$

Division of an interval in a given ratio

$$\left(\frac{mx_2+nx_1}{m+n},\ \frac{my_2+ny_1}{m+n}\right)$$

Parametric representation of a parabola

For $x^2=4ay$,

$x=2at,\quad y=at^2$

At $(2at,\ at^2)$,

tangent: $y=tx-at^2$

normal: $x+ty=at^3+2at$

At $(x_1,\ y_1)$,

tangent: $xx_1=2a(y+y_1)$

normal: $y-y_1=-\frac{2a}{x_1}(x-x_1)$

Chord of contact from $(x_0,\ y_0)$: $xx_0=2a(y+y_0)$

Acceleration

$$\frac{d^2x}{dt^2}=\frac{dv}{dt}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$$

Simple harmonic motion

$x=b+a\cos(nt+\alpha)$

$\ddot{x}=-n^2(x-b)$

Further integrals

$$\int\frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\frac{x}{a}+C$$

$$\int\frac{1}{a^2+x^2}dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

Sum and product of roots of a cubic equation

$\alpha+\beta+\gamma=-\frac{b}{a}$

$\alpha\beta+\alpha\gamma+\beta\gamma=\frac{c}{a}$

$\alpha\beta\gamma=-\frac{d}{a}$

Estimation of roots of a polynomial equation

Newton's method

$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

Binomial theorem

$$(a+b)^n=\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

2017 - Term 1

Extension one Mathematics

Part A :

1. B 2. D 3. D 4. A 5. D

Question 6

a. i. $y' = \dfrac{(2x-1)(2) - (2x+1)(2)}{(2x-1)^2}$

$= \dfrac{4x - 2 - 4x - 2}{(2x-1)^2}$

$= \dfrac{-4}{(2x-1)^2}$

ii. $y' = \dfrac{-4}{3x^2} + \dfrac{3}{4}$

b. i. $\int (4t+3)^{-3}\,dt = \dfrac{(4t+3)^{-2}}{-2\times 4} + c$

$= \dfrac{-1}{8}(4t+3)^{-2} + C$

ii. $\int 5x^{-1/2}\,dx = \dfrac{5x^{1/2}}{1/2}$

$= 10\sqrt{x} + C$

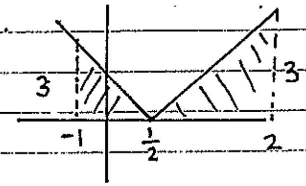
c. $\int_1^2 x^2 + 2 + x^{-2}\,dx$

$= \left[\dfrac{x^3}{3} + 2x - x^{-1}\right]_1^2$

$= \left[\dfrac{8}{3} + 4 - \dfrac{1}{2} - \left(\dfrac{1}{3} + 2 - 1\right)\right]$

$= 4\tfrac{5}{6}$

Question 7

a. 

$\int_{-1}^{2} |2x-1|\,dx$

$= \dfrac{1}{2}\times 3\times 1\dfrac{1}{2} + \dfrac{1}{2}\times 3\times 1\dfrac{1}{2}$

$= 4\dfrac{1}{2}.$

b. $\dfrac{x+1}{\sqrt[3]{x}} = x^{2/3} + x^{-1/3}$

$\therefore y = \dfrac{3}{5}x^{5/3} + \dfrac{3}{2}x^{2/3} + c$

c. $\int_1^k (3x^2 - 25) = 24$

$\left[x^3 - 25x\right]_1^k = 24$

$k^3 - 25k - (1-25) = 24$

$k^3 - 25k + 24 = 24$

$k(k-5)(k+5) = 0$

$k > 1$

$\therefore k = 5$

d. $V_y = \pi\int x^2\,dy$

$V_y = \pi\int_0^2 \dfrac{(4-y^2)^2}{4}\,dy$

$= \dfrac{\pi}{4}\int_0^2 16 - 8y^2 + y^4\,dy$

$= \dfrac{\pi}{4}\left[16y - \dfrac{8}{3}y^3 + \dfrac{1}{5}y^5\right]_0^2$

$V = \dfrac{\pi}{4}\left[32 - \dfrac{8}{3}\times 8 + \dfrac{32}{5} - 0\right]$

$= \dfrac{64\pi}{15}\ u^3$

Question 8

a) Test $n=1$

$LHS = \frac{1}{1(4)} = \frac{1}{4}$ $\qquad RHS = \frac{1}{3(1)+1} = \frac{1}{4}$

$\therefore$ true for $n=1$

Assume true for $n=K$

ie $\frac{1}{1(4)} + \frac{1}{4(7)} + \dots + \frac{1}{(3K-2)(3K+1)} = \frac{K}{3K+1}$

prove true for $n=K+1$

aim to prove:

$\frac{1}{1(4)} + \dots + \frac{1}{(3K-2)(3K+1)} + \frac{1}{(3(K+1)-2)(3(K+1)+1)} = \frac{K+1}{3(K+1)+1}$

$LHS = \frac{K}{3K+1} + \frac{1}{(3K+3-2)(3K+3+1)}$ using Assumpt.

$= \frac{K}{3K+1} + \frac{1}{(3K+1)(3K+4)}$

$= \frac{K(3K+1)+1}{(3K+1)(3K+4)}$

$= \frac{3K^2+K+1}{(3K+1)(3K+4)}$

$= \frac{(3K+1)(K+1)}{(3K+1)(3K+4)}$

$= \frac{K+1}{3(K+1)+1}$

$= RHS$

$\therefore$ If the statement is true for $n=K$ it is also true for $n=K+1$. As it is true for $n=1$ it is also true for $n=2,3,4$ etc. Hence, by Mathematical induction True all $n \geq 1$.

b) $dy/dx = x^3(2x-1)^2(3x+1)$

stat pts: $x = 0, \frac{1}{2}, -\frac{1}{3}$

test $x = \frac{1}{2}$

x	$\frac{1}{4}$	$\frac{1}{2}$	1
$\frac{dy}{dx}$	$\frac{1}{16} \times \frac{1}{4} \times \frac{13}{4}$ >0	0	$1 \times 1 \times 4$ >0
	/	—	/

$\therefore$ horizontal point of inflexion at $x = \frac{1}{2}$

c) ~~Increasing $y'>0$~~

i) $f''(x) = 6x$

ii) Increasing $y'>0$

$3x^2 - 4 > 0$

$3(x - 2/\sqrt{3})(x + 2/\sqrt{3}) > 0$

$x < \frac{-2}{\sqrt{3}}, \quad x > \frac{2}{\sqrt{3}}$

concave down $y'' < 0$

$6x < 0$

$x < 0$

$\therefore$ Both Increasing AND ∩

when $x < \frac{-2}{\sqrt{3}}$.

iii) $f'(x) = 3x^2 - 4$

$f(x) = x^3 - 4x + C$

$(1,-2)$

$-2 = 1^3 - 4(1) + C$

$C = 1$

$\therefore f(x) = x^3 - 4x + 1$.

Question 9

a)

Test $n=1$

$2^{3\times1} - 3^1$

$= 8 - 3$

$= 5$ which is divisible by 5

$\therefore$ true for $n=1$

Assume true for $n=K$

ie $2^{3K} - 3^K = 5M$ where M is a positive integer.

Prove true for $n=K+1$

$2^{3(K+1)} - 3^{(K+1)}$

$= 2^{3K} \cdot 2^3 - 3 \cdot 3^K$

$= 8[5M + 3^K] - 3 \cdot 3^K$ using the assumption

$= 40M + 8 \cdot 3^K - 3 \cdot 3^K$

$= 40M + 5 \cdot 3^K$

$= 5(8M + 3^K)$

Which is divisible by 5

$\therefore$ If the statement is true for $n=K$ it is also true for $n=K+1$, As it is true for $n=1$ it is also true for $n=2,3,4$ etc. Hence by Mathematical Induction true for all positive integer n.

b) $\frac{d}{dx}\left(2x\sqrt{x-3}\right)$

product rule

$= 2\sqrt{x-3} + \frac{2x}{2\sqrt{x-3}}$

$= 2\sqrt{x-3} + \frac{x}{\sqrt{x-3}}$

$= \frac{2(x-3) + x}{\sqrt{x-3}}$

$= \frac{3x - \cancel{3}6}{\sqrt{x-3}}$

ii) Now

$\frac{x - \cancel{3}2}{\sqrt{x-3}} = \frac{1}{3}\frac{d}{dx}\left(2x\sqrt{x-3}\right)$

$\therefore \int_4^7 \frac{x-2}{\sqrt{x-3}}\,dx = \int \frac{1}{3}\frac{d}{dx} 2x\sqrt{x-3}\,dx$

$= \frac{2}{3}\left[x\sqrt{x-3}\right]_4^7$

$= \frac{2}{3}\left[7\sqrt{4} - 4\sqrt{1}\right]$

$= \frac{2}{3}\left[14 - 4\right]$

$= \frac{20}{3}$

c. $y = (3x+1)^2$ $\qquad y' = 6(3x+1)$

$y'' = 18$

$2(3x+1)^2 - 6(3x+1) - x(18x+6-12)+k = 0$

$18x^2 + 12x + 2 - 6x + 12x + K = 0$

$\therefore k = 4$

Question 10

a)
$y = 4 - x^2$ and $y = x^2(4-x)$

i) Sub in $x = 1$

$\therefore y = 4 - 1^2 = 3$ and $y = 1^2(4-1) = 3$

$\therefore (1,3)$ satisfies both equation and $A = (1,3)$

ii) Shaded Area

$A_1 = \int_0^1 4x^2 - x^3 \, dx$

$= \left[\frac{4x^3}{3} - \frac{x^4}{4}\right]_0^1$

$= 4/3 - 1/4 - (0)$

$= \frac{13}{12}$

$A_2 = \int_1^2 4 - x^2 \, dx$

$= \left[4x - \frac{x^3}{3}\right]_1^2$

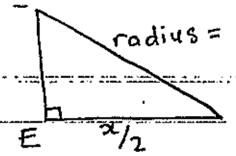
$= 8 - 8/3 - (4 - 1/3)$

$= 5/3$

$\therefore$ Area $= \frac{13}{12} + \frac{5}{3}$

$= \frac{11}{4} u^2$

b)



$\therefore 5^2 = BE^2 + (x/2)^2$

$25 = BE^2 + \frac{x^2}{4}$

$100 = 4.BE^2 + x^2$

$4.BE^2 = 100 - x^2$

$BE^2 = \frac{100 - x^2}{4}$

$BE = \frac{1}{2}\sqrt{100 - x^2}$

ii.

$A = \frac{1}{2}h(a+b)$

$= \frac{1}{2}\left[\frac{1}{2}\sqrt{100-x^2}\right]\left[x + 10\right]$

$= \frac{1}{4}(x+10)\sqrt{100-x^2}$

iii. Finding dA/dx

$u = \frac{x+10}{4}$ $\quad v = \sqrt{100 - x^2}$

$u' = 1/4$ $\quad v' = \frac{1}{2}(100-x^2)^{-\frac{1}{2}}(-2x)$

$= \frac{-x}{\sqrt{100-x^2}}$

$\frac{dA}{dx} = \frac{\sqrt{100-x^2}}{4} - \frac{x(x+10)}{4\sqrt{100-x^2}}$

For max $dA/dx = 0$

$\frac{\sqrt{100-x^2}}{4} = \frac{x(x+10)}{4\sqrt{100-x^2}}$

$100 - x^2 = x^2 + 10x$

$2x^2 + 10x - 100 = 0$

$x^2 + 5x - 50 = 0$

$(x+10)(x-5) = 0$

$x > 0 \quad \therefore x = 5$

Test x	4	5	6
dA/dx	0.76	0	-1
	/	—	\

(MAX) $\therefore BE = 5$ cm

Question 11

$y = \frac{2x^2 - x + 2}{x}$

$y = \frac{2x^2}{x} - \frac{x}{x} + \frac{2}{x}$

$= 2x - 1 + \frac{2}{x}$

I. undefined when $x = 0$

II. $\frac{dy}{dx} = 2 - \frac{2}{x^2}$

stat points $dy/dx = 0$

$2 = \frac{2}{x^2}$

$x^2 = 1$

$x = \pm 1$

$\therefore$ points $(1,3)$ and $(-1,-5)$

Nature:

$\frac{d^2y}{dx^2} = \frac{4}{x^3}$

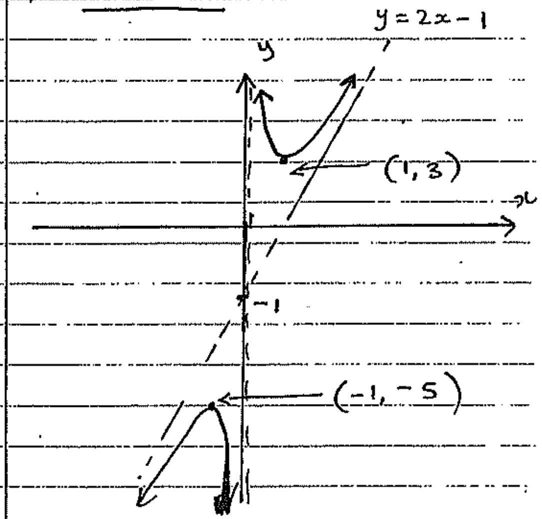
at $x = 1$ $\quad y'' = 4$
$> 0 \therefore$ MIN
$\cup$ $\quad (1,3)$

at $x = -1$ $\quad y'' = -4$
< 0 MAX
$\cap$ $\quad (-1,-5)$

III. $y'' = \frac{4}{x^3} \neq 0$ with $x \neq 0$

$\therefore$ No points of inflexion.

IV. Sketch



V. $\frac{2x^2 - x + 2}{x} - 3 = 0$

$\therefore \frac{2x^2 - x + 2}{x} = 3$

(this is where graph crosses horizontal line $y = 3$)

$\therefore$ at $x = 1$.

Solutions to multiple Choice

1) $y' = 4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$x = 0 \qquad x = 3$

test ↑ ↑

HPI Turning pt.

$\therefore$ only <u>ONE</u> Turning pt

(B)

(2)

A – can't decide

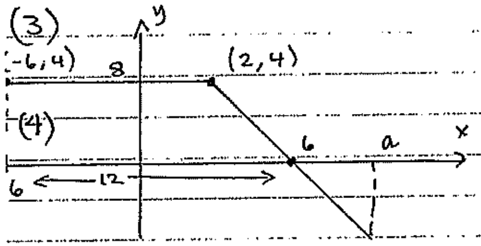
B – only if $f(x)$ is odd

C – only sometimes

D – zero as zero width

rule $\int_a^a f(x)\,dx = 0$

(3)



Trap = 40

equation Line

$y = 6 - x$

$\therefore \int_6^a 6 - x\,dx = -32$

$-32 = 6x - \frac{x^2}{2}\Big]_6^a$

$-32 = \left[6a - \frac{a^2}{2} - 36 + 18\right]$

$-14 = 6a - \frac{a^2}{2}$

test A 8 x

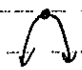
B 10 x

C 12 x

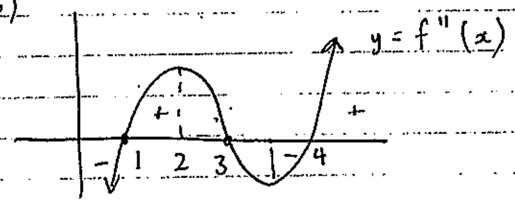
D 14 ✓ $\therefore$ (D)

(4) $f''(x) < 0$ concave down

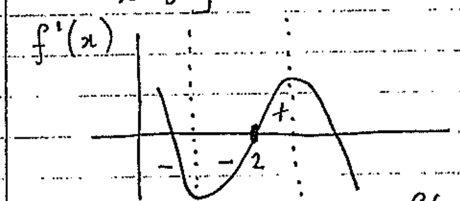
$f'(x) = 0$ stat point

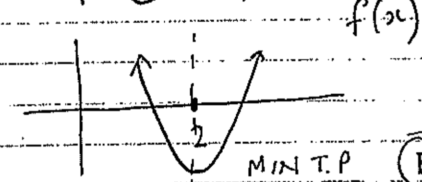
$\therefore$  = (A)

(5)



$\left.\begin{array}{l} x = 1 \\ x = 3 \end{array}\right]$ $f''(x) = 0$ $\therefore$ inflex.





MIN T.P at $x = 2$

(D)