Integration

Trapezoidal & Simpson's Rules
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Definite Integrals – Even & Odd functions
Areas enclosed by the x-axis, y-axis
Volumes
Substitution Method

Trapezoidal & Simpson's Rules

Trapezoidal Rule	Simpson's Rule
$A \approx \frac{h}{2}\left(y_{1st} + y_{last} + 2(others)\right)$	$A \approx \frac{h}{3}\left(y_{1st} + y_{last} + 2(odd) + 4(even)\right)$ $y_1 = 1^{st}$ $A \approx \frac{h}{3}\left(y_{1st} + y_{last} + 4(odd) + 2(even)\right)$ $y_0 = 1^{st}$
For n number of strips, you must need n+1 number y values Height is the difference of 2 x values.	

Example 1

x	0	1	2	3	4
y	1	2	4	8	16

4 strips

By Trapezoidal Rule

$$\begin{aligned} A &\approx \frac{1}{2}\left(1+16+2(2+4+8)\right) \\ &\approx \frac{1}{2}\left(1+16+28\right) \\ &\approx 22\tfrac{1}{2} \end{aligned}$$

By Simpson's Rule

$$\begin{aligned} A &\approx \frac{1}{3}\left(1+16+2(4)+4(2+8)\right) \\ &\approx \frac{1}{3}\left(1+16+8+40\right) \\ &\approx 21\tfrac{2}{3} \end{aligned}$$

Indefinite Integrals

$\int x^n \, dx$	$= \dfrac{x^{n+1}}{n+1} + c$	$n \neq -1$
$\int \dfrac{1}{x} \, dx$	$= \ln\lvert x \rvert + c$	
$\int (ax+b)^n \, dx$	$= \dfrac{(ax+b)^{n+1}}{a(n+1)} + c$	
$\int \dfrac{f'(x)}{f(x)} \, dx$	$= \ln\lvert f(x) \rvert + c$	

Definite Integrals

$\int_a^b f(x) \, dx$	$= F(b) - F(a)$	
$\int x^n \, dx$	$= \left[\dfrac{x^{n+1}}{n+1}\right]_a^b$ $= \dfrac{b^{n+1}}{n+1} - \dfrac{a^{n+1}}{n+1}$	$n \neq -1$
$\int_0^a f(x) \, dx$	$= \int_0^a f(a-x) \, dx$	
$\int_{-a}^a f(x) \, dx$	$= 2\int_0^a f(x) \, dx$	if $f(x)$ is an EVEN function
$\int_{-a}^a f(x) \, dx$	$= 0$	if $f(x)$ is an ODD function

Areas enclosed by the x-axis, y-axis

Areas above the x-axis will give a positive result

$$\text{Area} = \int_a^b f(x)\ \text{dx}$$

Areas below the x-axis will give a negative result. Take the Absolute value.

$$\text{Area} = \left|\int_a^b f(x)\right|\ \text{dx}$$

When finding areas for both above and below, separate them into different Areas.

To find the areas between a curve and the y-axis, we change the subject of the equation of the curve to x. $x = f(y)$

$$\text{Area} = \int_a^b f(y)\ \text{dy} \quad \text{OR} \quad = \int_a^b x\ \text{dy}$$

Similarly to the x-axis, Areas to the right are Positive
Areas to the left are Negative

Volumes

Rotation about the x-axis

$$\text{Volume} = \pi\int_a^b y^2\ \text{dx}$$

Rotation about the y-axis

$$\text{Volume} = \pi\int_a^b x^2\ \text{dy}$$

Substitution Method

- Let u =
- Find $\frac{du}{dx}$
- Find dx or du

Example 1

Find $\int_2^3 x(x^2-3)\ \text{dx}$

Let u $= x^2 - 3$

$\frac{du}{dx} = 2\text{x}$

$du = 2\text{x dx}$

$x = 3;\quad u = 3^2 - 3$

$u = 6$

$x = 2;\quad u = 2^2 - 3$

$u = 1$

$$\begin{aligned}\int_2^3 x(x^2-3)\ \text{dx} &= \tfrac{1}{2}\int_2^3 (x^2-3)\,2\text{x dx}\\ &= \tfrac{1}{2}\int_1^6 u\ \text{du}\\ &= \tfrac{1}{2}\left[\tfrac{u^2}{2}\right]_1^6\\ &= \tfrac{1}{2}\left[\tfrac{6^2}{2} - \tfrac{1^2}{2}\right]\\ &= 8\tfrac{3}{4}\end{aligned}$$