SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT

TASK 1

MATHEMATICS

December 2003

Time allowed: 70 minutes

Weighting 10%

Name:_____ Class:_____

Instructions

- Start each question on a new page
- Marks indicated are approximate only
- Answer all questions – marks may be deducted for poorly arranged work

leave out circled questions

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	**Total**

Question 1 (8 marks)

a) For the series -12 + 13 + 38 +.....

i) Find the value of the 12^{th} term (2)

ii) Find the sum of the first 12 terms (2)

iii) Which term has the value 963? (2)

b) Write down the coordinates of the focus for the parabola $x^2 = 12 - 12y$ (2)

Question 2 (8 marks)

a) The roots of $3x^2 - 5x - 4 = 0$ are α *and* β (6)

Find the value of,

i. $\alpha\beta$

ii $\alpha + \beta$

iii $\frac{1}{\alpha} + \frac{1}{\beta}$

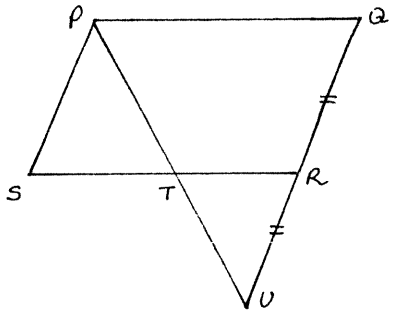
iv $\alpha^3\beta + \alpha\beta^3$

(b) The first two terms of a geometric series are 18 and 12 respectively. (2)

Find the limiting sum of the series.

Question 3 (8 marks)

a) In the diagram, PQRS is a parallelogram. Q R is produced to U so that Q R = R U (4)



Not to scale

i. Giving clear reasons, show that the triangles PST and URT are congruent.

ii Hence or otherwise, show that T is the midpoint of SR

(b) Write $0.\dot{1}\dot{0}\dot{2}$ as a geometric series, and state the value of a and r. Hence, write $0.\dot{1}\dot{0}\dot{2}$ as a fraction in its simplest form. (4)

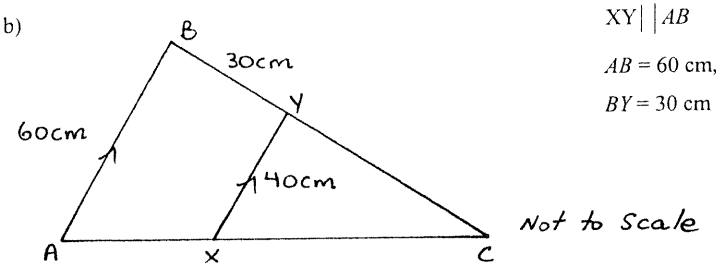
Question 4 (8 marks)

a) Evaluate $\sum_{n=4}^{20} 2n-5$ (3)

b) $XY \parallel AB$ (5)

$AB = 60$ cm, $XY = 40$ cm

$BY = 30$ cm



i Prove $\Delta ABC /// \Delta XYC$

ii Calculate the length CY

Question 5 (8 marks)

a) Solve for K if the zeros of the parabola $y = 2x^2 - 7x + K$ are not real. (3)

b) If $x+2,\ 2x-4,\ 4x+1$ are the first three terms in an Arithmetic sequence, find x. (2)

c) Find the values of P, Q and R if $3x^2 + 5x - 1 \equiv P(x+1)^2 + Q(x+1) + R$ (3)

Question 6 (8 marks)

a) A parabola has equation $x^2 = -12y$ (6)

i Find the coordinates of the vertex of the parabola

ii Write down the coordinates of the focus of the parabola

iii Find the equation of the tangent to the parabola at the point where $x = 6$.

iv Find the coordinates of M, the point where the tangent cuts the y axis

b) Write down the equation of the parabola whose focus is (1, 3) and directrix is x = 5. (2)

Question 7 (8 marks)

a) Consider the points A (-2, 1) B (4,1) and P (x, y) (6)

i Find expressions for the gradients of the two intervals PA and PB

ii Show that the locus of P is $x^2 - 2x + y^2 - 2y = 7$ if $\angle APB = 90°$

iii Show that the locus represents a circle and give its centre and radius.

b) Find the sum of the first ten terms of the series $1 - \sqrt{2} + 2.....$ (2)

Express your answer in simplest exact form with a rational denominator.

Question 8 (8 marks)

a) In an arithmetic series the eight term is 32 and the sum of the first ten terms is 400. (3)

Find,

i the value of the common difference

ii the value of the first term

b) At the beginning of each year Xena the Warrior princess invests $1200 in a superannuation fund, on which she is paid 8.5% pa interest, compounded annually. (5)

Find,

i the amount of interest earned in the first year

ii the total of her investments at the end of 25 years.

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2unit HSC Ass one – 2003 – Answers

Question 1

a) $a = -12$ $d = 25$

i. $T_{12} = -12 + 11 \times 25$ – ① or formula

$= 263$ – ① Ans

ii. $S_n = \frac{n}{2}(a + l)$ ① formula

$= \frac{12}{2}(-12 + 263)$ ① Ans

$= 1506$

iii. $963 = a + (n-1)d$

$963 = -12 + (n-1) \times 25$ – ①

$975 = 25n - 25$

$1000 = 25n$

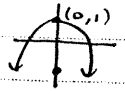
$n = 40$ – ①

$\therefore$ 40^{th} term has value 963

b) $x^2 = -12y + 12$

$x^2 = -12(y - 1)$ – ①

$V = (0, 1)$ $a = 3$



$S = (0, -2)$ – ①

Question 2.

a) $a = 3$ $b = -5$ $c = -4$

i. $\alpha\beta = c/a = -4/3$ ①

ii. $\alpha + \beta = -b/a = 5/3$ ①

iii. $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta}$ – ①

$= 5/3 \div -4/3$

$= -5/4$ – ①

iv. $\alpha^3\beta + \alpha\beta^3 = \alpha\beta(\alpha^2 + \beta^2)$

$= \alpha\beta((\alpha + \beta)^2 - 2\alpha\beta)$ – ①

$= -\frac{4}{3}\left[\left(\frac{5}{3}\right)^2 - 2 \times -\frac{4}{3}\right]$ – ①

$= -7\frac{7}{27}$

Q2 (b) $18 + 12 + \ldots$

GP $a = 18$ $r = 12/18$

$= 2/3$ – ①

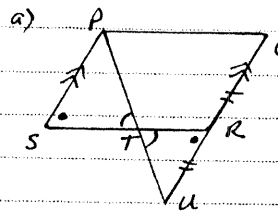
$S_\infty = \frac{a}{1 - r}$

$= \frac{18}{1 - 2/3}$

$= 54$ – ①

Question 3.

a)



i. $PS \parallel QR$ opp sides of Parallelogram equal – ①

$PS = QR$ opp sides of Parallelogram equal.

In ΔPST and ΔURT

$PS = UR$ $(= QR)$

$\angle PST = \angle URT$ (alt $\angle$'s $PS \parallel RU$) – ①

$\angle PTS = \angle UTR$ (vert opp) – ①

$\therefore \Delta PST \equiv \Delta URT$ (AAS)

ii. $ST = TR$ corresp. sides of $\equiv \Delta$'s $\therefore$ T is the Midpoint of SR – ①

b) $0.\dot{1}\dot{0}\dot{2} = \frac{102}{1000} + \frac{102}{1000\,000} + \ldots$

$\therefore a = \frac{102}{1000}$ ① $r = \frac{1}{1000}$ ①

$S_\infty = \frac{a}{1 - r}$ ①

$= \frac{102}{1000} \div \frac{999}{1000}$

$= \frac{102}{999}$

$= 34/333$ – ①

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Question 4

a) $\sum_{n=4}^{20} 2n - 5$

$n = 4$ $n = 5$ $n = 6$ ... $n = 20$

$3 + 5 + 7 + \ldots + 35$ ①

AP $a = 3$ $d = 2$ $n = 20 - 4 + 1$

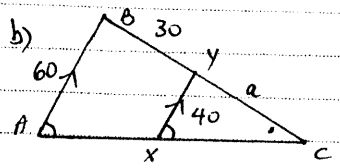
$= 17$ ①

$S_n = \frac{n}{2}(a + l)$

$= \frac{17}{2}(3 + 35)$

$= 323$ ①

b)



i. In ΔABC & ΔXYC

$\angle C$ is common ①

$\angle BAC = \angle YXC$ (corresp $\angle$'s $AB \parallel XY$) ①

$\therefore \Delta ABC \,|||\, \Delta XYC$ (all $\angle$'s =) ①

ii. $\frac{60}{40} = \frac{30 + a}{a}$ ratio of sides in $|||\Delta$'s ①

$6a = 120 + 4a$

$2a = 120$

$a = 60$

$\therefore CY = 60$ ①

Question 5

a) Not real $\Delta < 0$ ① $a = 2$ $b = -7$ $c = K$

$b^2 - 4ac < 0$

$49 - 4 \times 2 \times K < 0$ ①

$49 - 8K < 0$

$-8K < -49$

$8K > 49$

$K > 49/8$ ①

b) $x + 2,\ 2x - 4,\ 4x + 1,\ \ldots$

AP $\rightarrow$ $T_2 - T_1 = T_3 - T_2$ ①

$2x - 4 - (x + 2) = (4x + 1) - (2x$

$2x - 4 - x - 2 = 4x + 1 - 2x + 1$

$x - 6 = 2x + 5$

$x = -11$ ①

c) $3x^2 + 5x - 1 \equiv P(x+1)^2 + Q(x+1) + R$

RHS $= P(x^2 + 2x + 1) + Qx + Q + R$

equating co-effients.

* $3 = P$ ①

* $5 = 2P + Q$

$5 = 6 + Q$ $Q = -1$ ①

* $-1 = P + Q + R$

$-1 = 3 - 1 + R$

$-1 = 2 + R$

$R = -3$ ①

$\therefore P = 3,\ Q = -1,\ R = -3$

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Question 6

a) $x^2 = -12y$

i. $V = (0,0)$ ①

ii. $S = (0,-3)$ ①

iii. $y = \frac{x^2}{-12}$

$y' = \frac{2x}{-12}$ at $x=6$ $y=-3$ ①

$m_T = -1$ ①

∴ equation

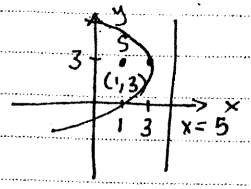
$y + 3 = -1(x-6)$ —①

$y + 3 = -x + 6$

$x + y - 3 = 0$

iv. $x = 0$ $y = 3$ ①

b)



Vertex $(3,3)$

$a = 2$

$(y-y_1)^2 = 4a(x-x_1)$

$(y-3)^2 = -8(x-3)$

① ↑① Neg.

Question 7

a) i. $m_{PA} = \frac{y-1}{x+2}$ ① $\quad m_{PB} = \frac{y-1}{x-4}$ ①

ii. $\frac{y-1}{x+2} \times \frac{y-1}{x-4} = -1$ ①

$\frac{y^2-2y+1}{x^2-2x-8} = -1$

$y^2 - 2y + 1 = -x^2 + 2x + 8$

$x^2 - 2x + y^2 - 2y = 7$ ①

iii. $x^2 - 2x + 1 + y^2 - 2y + 1 = 7 + 2$

$(x-1)^2 + (y-1)^2 = 9$

$C = (1,1)$ ① $\quad r = 3$ ①

b) $1 - \sqrt{2} + 2 \ldots\ldots$

GP $a = 1$ $\quad r = -\sqrt{2}$

$n = 10$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1\left((-\sqrt{2})^{10} - 1\right)}{-\sqrt{2} - 1}$

$= \frac{1(32 - 1)}{-\sqrt{2} - 1}$

$= \frac{31}{-\sqrt{2} - 1}$

$= \frac{-31}{\sqrt{2}+1} \times \frac{\sqrt{2}-1}{\sqrt{2}-1}$

$= \frac{-31\sqrt{2} + 31}{1}$

$= -31\sqrt{2} + 31.$

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Question 8

a)

$T_8 = a + 7d = 32$

$S_{10} = \frac{10}{2}(2a + 9d) = 400$

$2a + 9d = 80$ ①

$2a + 14d = 64$

$\therefore 5d = -16$

$d = \frac{-16}{5}$ ①

$a + 7 \times -16/5 = 32$

$a = 54\,2/5$ ①

b)

i. $A = 1200(1.085)^1$

$= 1302$ ①

∴ Interest = \$102 ①

ii. $A_1 = 1200(1.085)^{25}$

$A_2 = 1200(1.085)^{24}$

$A_3 = 1200(1.085)^{23}$

$\vdots$

$A_{25} = 1200(1.085)^1$ } ①

Total $= 1200\left(1.085 + 1.085^2 + \ldots 1.085^{25}\right)$

$= 1200 \times \frac{a(r^n - 1)}{r - 1}$ $\quad a = 1.085$, $r = 1.085$, $n = 25$ ① – setting up G.P.

$= 1200 \times \frac{1.085\left(1.085^{25} - 1\right)}{0.085}$

$=$ \$10 2425.47 —①