SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

YEAR 11 COMMON TEST

MAY 2008

Time allowed: 70 minutes

Instructions:

- Show necessary working in every question .
- Start each question on a new page .
- Attempt all questions .
- Marks shown are approximate and may be changed .
- Full marks may not be awarded for careless or badly arranged work .
- Your sketches must be neat. Use a ruler to draw axes. Label the axes .
- Approved calculators may be used .
- These questions are to handed in with your answers .

Name:_____

Class:_____

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Total
/8	/8	/8	/9	/9	/9	/51

QUESTION 1

a) Expand $(2x-1)^3$ **1**

b) Solve $8^{x+1} = \frac{1}{4\sqrt{2}}$ **2**

c) Fully factorise

i) $8x^3 + 1$ **1**

ii) $x^2 - xy - 2y^2$ **1**

iii) $y^2 - x^2 + 2x - 1$ **2**

d) Simplify $2^{n+3} - 2^{n+2}$ **1**

QUESTION 2

a) Simplify $\frac{\sqrt{3}-1}{\sqrt{3}+1} - \frac{\sqrt{3}+1}{\sqrt{3}-1}$ **2**

b) State the natural domain of $y = \frac{1}{\sqrt{4-x^2}}$ **2**

c) i) Sketch $y = x^2 - 3x - 4$ for $0 \leq x \leq 5$ **1**

ii) State the range for the above function for the given domain . **1**

d) Express $\frac{3^n + 3^n + 3^n}{9^n}$ in simplest index form . **2**

QUESTION 3

a) True or false: " $|a + b| = |a| + |b|$ for all values a and b".

Give an example to illustrate your response . **1**

b) Solve $\frac{x}{x^2-4} < 0$ **2**

c) i) Simplify $\frac{x+3}{x^2-9}$ **1**

ii) Hence sketch $y = \frac{x+3}{x^2-9}$

Clearly label the axes, any asymptotes, intercepts and critical points. **2**

d) Given that $\cos\theta = -\frac{2}{3}$ and $\tan\theta > 0$, find the exact value of $\sin\theta$. **2**

QUESTION 4

a) Solve $\frac{2}{|2x-1|} > \frac{1}{2}$ **3**

b) i) On the same axes, neatly sketch $y = 4 - x^2$ *and* $y = \frac{1}{4-x^2}$

Label axes, asymptotes and intercepts . **3**

ii) Find the x values of the points of intersection of the two curves above. **3**

QUESTION 5

a) Simplify $\dfrac{\cos(90°-\theta)}{\sin(90°+\theta)}$ **2**

b) Sketch $y = \dfrac{x+1}{|x+1|}$ **2**

c) i) Sketch $y = 2 - |x|$ **1**

ii) Hence or otherwise, solve $|1-x| + |x| > 2$ **2**

d) Prove that $\cot\alpha + \tan\alpha = \sec\alpha \ cosec\ \alpha$ **2**

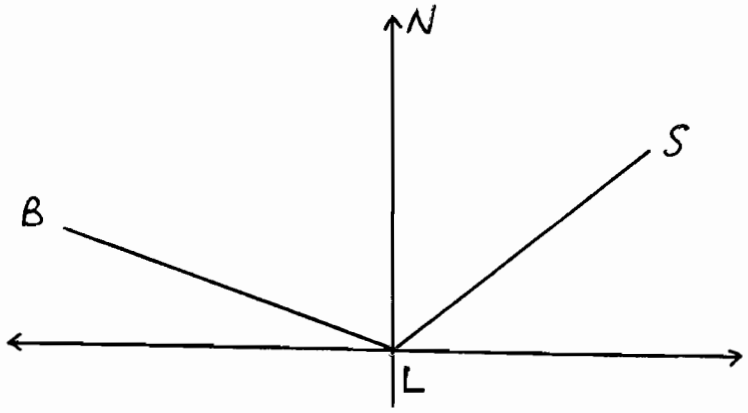
QUESTION 6

a) Solve for $0° \leq \theta \leq 360°$. Give answers correct to the nearest minute where applicable.

i) $\dfrac{3}{sin^2\theta} = 4$ **2**

ii) $3\sec^2\theta = 3 + 5\tan\theta$ **3**

b)



From a lighthouse L a ship S bears 053° T and is at a distance of 8 nautical miles.

From L a boat B bears 293° T at a distance of 6 nautical miles.

i) Draw the diagram above, marking the information supplied.

ii) Find the distance of ship S from boat B. Give your answer as a surd . **2**

iii) Find the bearing of ship S from boat B. Give your answer to the nearest degree . **2**

EXTENSION SOLUTIONS (Corrected Version)

1 a) $8x^3 - 12x^2 + 6x - 1$

b) $(2^3)^{x+1} = \dfrac{1}{2^2 \cdot 2^{1/2}}$

$2^{3x+3} = 2^{-5/2}$

$3x + 3 = -2\frac{1}{2}$

$3x = -5\frac{1}{2}$

$x = -1\frac{5}{6}$

c) i) $(2x+1)(4x^2 - 2x + 1)$

ii) $(x+y)(x-2y)$

iii) $y^2 - (x^2 - 2x + 1)$

$= y^2 - (x-1)^2$

$= (y + x - 1)(y - x + 1)$

d) $2 \cdot 2^{n+2} - 2^{n+2} = 2^{n+2}$

2 a) $\dfrac{(\sqrt{3}-1)^2 - (\sqrt{3}+1)^2}{3-1}$

$= \dfrac{3 - 2\sqrt{3} + 1 - (3 + 2\sqrt{3} + 1)}{2}$

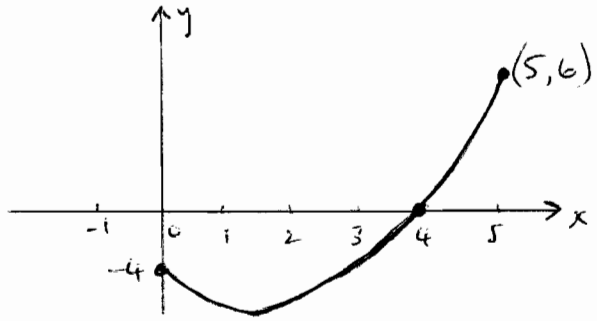
$= -\dfrac{4\sqrt{3}}{2}$

$= -2\sqrt{3}$

b) $4 - x^2 > 0$

$\therefore -2 < x < 2$

c) i) $y = (x-4)(x+1)$



ii) Vertex at $x = 1\frac{1}{2}$, $y = -6\frac{1}{4}$

$\therefore R: -6\frac{1}{4} \le y \le 6$

d) $\dfrac{3 \times 3^n}{(3^2)^n} = \dfrac{3^{n+1}}{3^{2n}}$

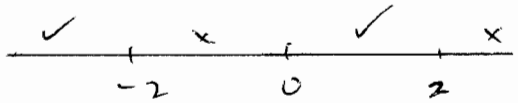
$= 3^{n+1-2n}$

$= 3^{1-n}$

3 a) False, eg. $|-3+4| \neq |-3| + |4|$

b) $\dfrac{x}{x^2-4} \times (x^2-4)^2 < 0 \times (x^2-4)^2$

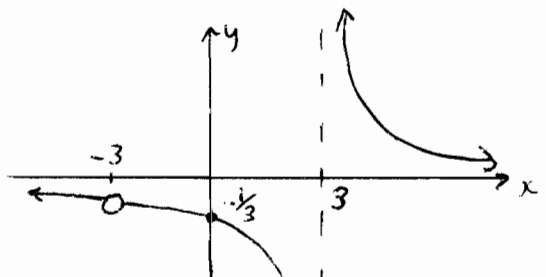
$x(x+2)(x-2) < 0$



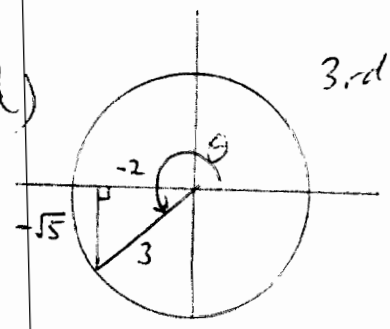
$\therefore x < -2$ or $0 < x < 2$

c) i) $\dfrac{x+3}{(x+3)(x-3)} = \dfrac{1}{x-3}$

ii)



d) 3rd quadrant



$\therefore \sin\theta = -\frac{\sqrt{5}}{3}$

4 a) $|2x-1|$ is positive and $x \neq \frac{1}{2}$

$\therefore \quad 2 > \frac{1}{2}|2x-1|$

$\therefore |2x-1| < 4$

$\therefore \quad 2x-1 < 4$ or $-(2x-1) < 4$

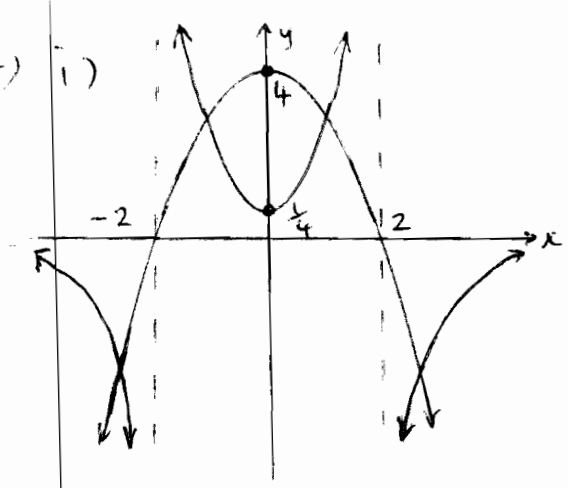
$x < 2\frac{1}{2}$ $\qquad -2x+1 < 4$

$-2x < 3$

$x > -1\frac{1}{2}$

$\therefore -1\frac{1}{2} < x < 2\frac{1}{2}, \quad x \neq \frac{1}{2}$

b) i)



ii) $4 - x^2 = \frac{1}{4-x^2}$

$(4-x^2)^2 = 1$

$16 - 8x^2 + x^4 = 1$

$x^4 - 8x^2 + 15 = 0$

$(x^2-5)(x^2-3) = 0$

5 a) $\frac{\sin\theta}{\sin(90-\theta)} = \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

b) $y = \frac{x+1}{|x+1|}$

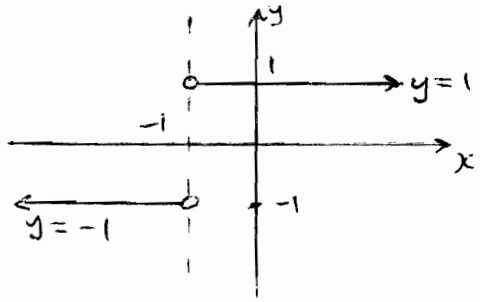
$= \frac{x+1}{x+1}$ for $x+1 > 0 \; (x > -1)$

$\therefore y = 1$ for $x > -1$

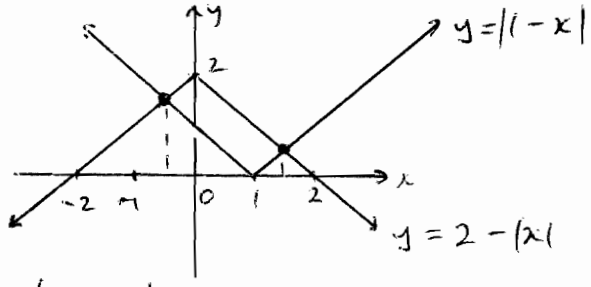
or $y = \frac{x+1}{|x+1|}$

$= \frac{x+1}{-(x+1)}$ for $x+1 < 0 \; (x < -1)$

$\therefore y = -1$ for $x < -1$



c) i) $y = -|x| + 2$



ii) $|1-x| > 2 - |x|$

$\therefore x < -\frac{1}{2}$ or $x > 1\frac{1}{2}$

d) LHS $= \frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}$

$= \frac{\cos^2 x + \sin^2 x}{\sin x \cos x}$

$= \frac{1}{\sin x \cos x}$

$= \frac{1}{\quad} \times \frac{1}{\quad}$

(6)

a) i) $3 = 4\sin^2\theta$

$\sin^2\theta = \frac{3}{4}$

$\therefore \sin\theta = \pm\frac{\sqrt{3}}{2}$

$\therefore \theta = 60°$ (all 4 quads)

$= \underline{60°, 120°, 240°, 300°}$

ii) $3(1+\tan^2\theta) = 3 + 5\tan\theta$

$3 + 3\tan^2\theta - 5\tan\theta - 3 = 0$

$3\tan^2\theta - 5\tan\theta = 0$

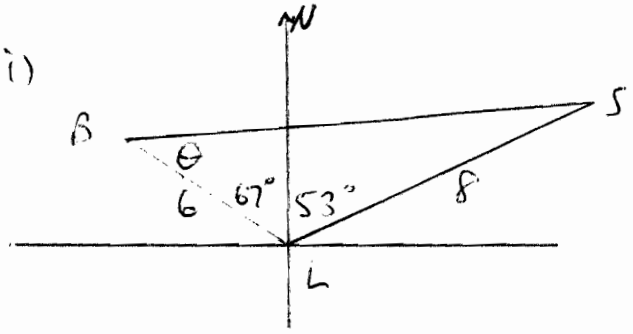
$\tan\theta(3\tan\theta - 5) = 0$

$\therefore \tan\theta = 0$ or $\frac{5}{3}$

$\therefore \theta = 0°, 180°, 360°$ or $59°2'$ (1st, 3rd quads)

$\therefore \underline{\theta = 0°, 180°, 360°, 59°2', 239°2'}$

b) i)



ii) $BS^2 = 6^2 + 8^2 - 2\times6\times8\times\cos120°$

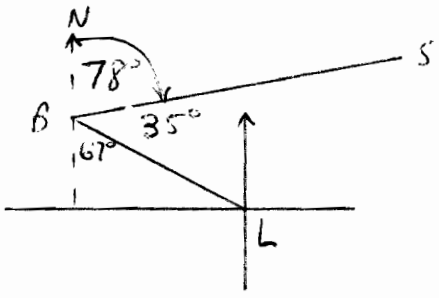
$= 100 - 96(-\frac{1}{2})$

$= 148$

$\therefore \underline{BS = \sqrt{148}}$ n.m.

iii) $\frac{8}{\sin\theta} = \frac{\sqrt{148}}{\sin120°}$

$\therefore \theta \doteqdot 35°$



$\therefore$ bearing of S from B is $\underline{078°\ T}$ or $N\ 78°\ E$