Name: ..... Maths Class: .....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11
Mathematics

Preliminary HSC Course

Yearly Exam

September, 2016

Time allowed: 2 hours

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided

Section 1	Multiple Choice Questions 1-10 10 Marks
Section II	Questions 11-18 72 Marks
Total	82 marks

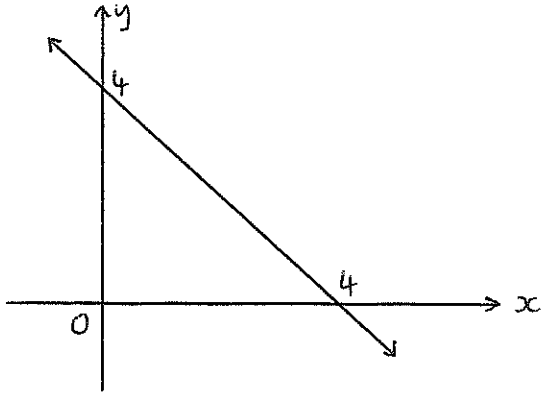
SECTION 1 -- MULTIPLE CHOICE (10 marks)

QUESTION 1

What is the gradient of a line parallel to the line $2x + 3y - 1 = 0$?

A. 2 B. -2 C. $\frac{3}{2}$ D. $-\frac{2}{3}$

QUESTION 2



The equation of the line above is: A. $x - y + 4 = 0$ B. $x + y - 4 = 0$

C. $x + y + 4 = 0$ D. $x - y - 4 = 0$

QUESTION 3

A function is given by $f(x) = \sqrt{9 - x^2}$. What is its natural domain?

A. $x < 3$ B. $x \leq 3$ C. $-3 \leq x \leq 3$ D. $-9 \leq x \leq 9$

QUESTION 4

The function in Question 3 above is:

A. even B. odd C. neither D. cannot be determined

QUESTION 5

What is the minimum value of $x^2 - 4x + 6$?

A. 2 B. 4 C. 6 D. 8

QUESTION 6

If $a^b = 5$, what is the value of $2a^{3b}$?

A. 30 B. 250 C. 500 D. 1000

QUESTION 7

If $3^{x-4} = 9^{2x}$, then $x = ?$

A. $\frac{3}{4}$ B. $\frac{4}{3}$ C. $-\frac{3}{4}$ D. $-\frac{4}{3}$

QUESTION 8

If $2x^2 - 12x + 11$ is expressed in the form $2(x-b)^2 + c$, what is the value of c?

A. -25 B. -7 C. 2 D. 29

QUESTION 9

$\frac{\sin(180°-\theta)}{\cos(90°-\theta)}$ simplifies to:

A. 1 B. 2 C. $\tan\theta$ D. $\cot\theta$

QUESTION 10

If $a > b$, which of the following is <u>always</u> true?

A. $a^2 > b^2$ B. $\frac{1}{a} < \frac{1}{b}$ C. $-a > -b$ D. $2^a > 2^b$

END OF SECTION 1

SECTION 2

QUESTION 11 (9 marks)

a) Evaluate $13.6 \sin 42°15'$ correct to 2 significant figures. 1

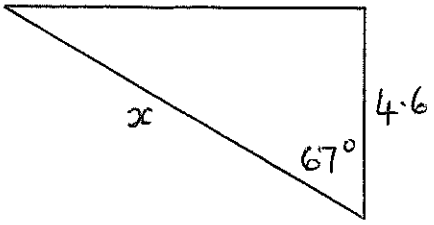
b) Expand and simplify $(2\sqrt{3}-1)(\sqrt{3}+4)$ 2

c) Write the exact value of cosec 60°. 1

d) Simplify $\frac{x-3}{x^2-4x+3}$ 1

e) Find the value of x, correct to 1 decimal place. 2

Not to Scale



f) Solve $(x+1)^2 = 5$, giving answers correct to 1 decimal place. 2

QUESTION 12 (9 marks) Start a new page.

a) Solve $|3x-6| < 12$ 2

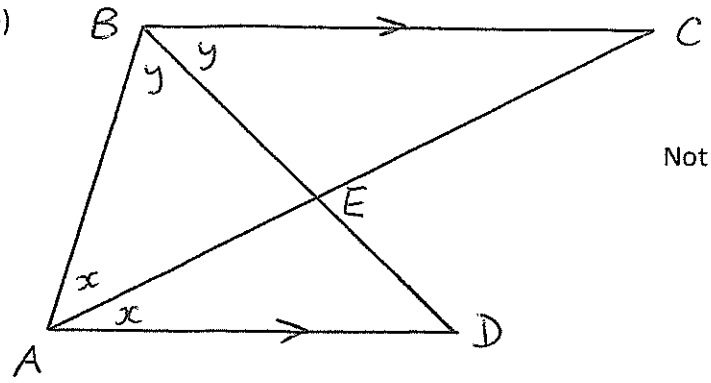
b) Find θ to the nearest degree if $\cos\theta = 0.4$ and $0° \leq \theta \leq 360°$. 1

c) Fully simplify $\frac{a+b}{\frac{1}{a}+\frac{1}{b}}$ 2

d) Find derivatives of : i) $y = 3x^2 - 4 + 7x$ 1

ii) $f(x) = \frac{4}{x^2}$ 1

e)



Not to Scale

$AD \parallel BC$. AC and BD intersect at E. $\angle BAD$ and $\angle ABC$ are bisected as shown.

Prove that $\angle BEA = 90°$. 2

QUESTION 13 (9 marks) Start a new page.

a) Factorise $x^3 - 27$. 1

b) Simplify $\sin\theta(1 + cot^2\,\theta)\tan\theta$. 2

c) Differentiate i) $(x^2 + 5)^4$ 1

ii) $x\sqrt{x}$ 1

d) Find the gradient of the curve $y = \frac{2x}{x+3}$ when $x = -2$. 2

e) The solutions of a quadratic equation are $x = \frac{1 \mp \sqrt{5}}{2}$. Write a quadratic equation with these solutions. 2

QUESTION 14 (9marks) Start a new page.

a) Find the coordinates of the vertex of the parabola $y = (x + 3)^2 + 4$. 1

b) Given $(x) = x^2 + \frac{x}{2}$, evaluate $f(2) + f'(2)$. 2

c) Solve for θ, given $0° \leq \theta \leq 360°$:

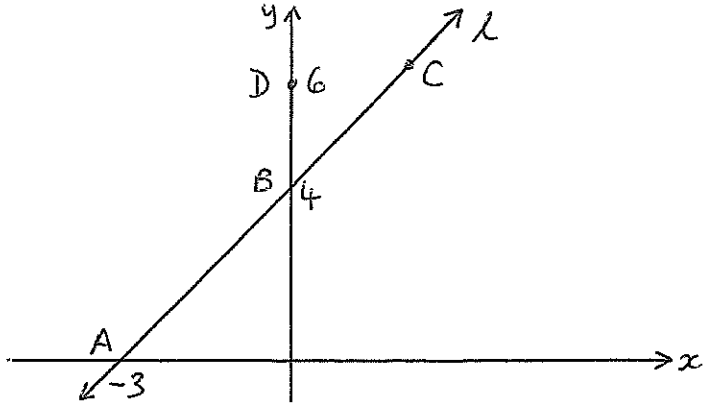
i) $(\sin\theta + 1)(\cos\theta - 1) = 0$ 2

ii) $3tan^2\theta - 1 = 0$ 2

d) If $\cos\theta = -\frac{2}{3}$ and $\sin\theta > 0$, find the exact value of $\tan\theta$. 1

e) Fully factorise $x^2 + 8x + 16 - y^2$. 1

QUESTION 15 (9 marks) Start a new page.



Not to Scale

In the diagram above, line l cuts the x axis at A(-3,0) and the y axis at B(0,4). D has coordinates (0,6) and point C is on l.

a) Find the gradient of line l. 1

b) Show that line l has equation $4x - 3y + 12 = 0$. 1

c) B is the midpoint of AC. Find the coordinates of C. 1

d) Find the perpendicular distance from D to the line l. 1

e) Find the area of ΔBDC. 2

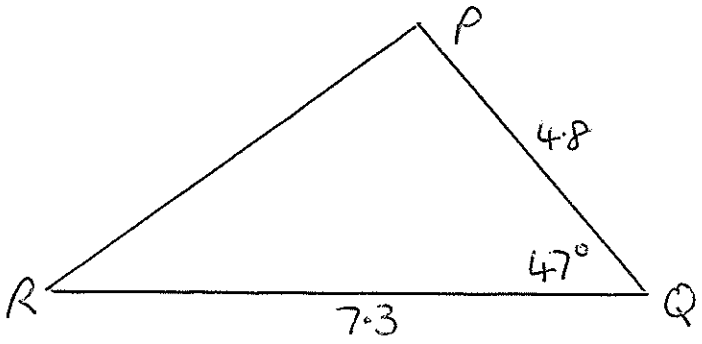
f) Find the equation of the perpendicular bisector of AB. Leave your answer in general form. 3

QUESTION 16 (9 marks) Start a new page.

a) Find $\frac{d}{dr}(\frac{4}{3}\pi r^3)$ 1

b) Find the point(s) on the curve $y = x^3 - 3x^2 + 3x$ where the tangent is horizontal. 2

c)

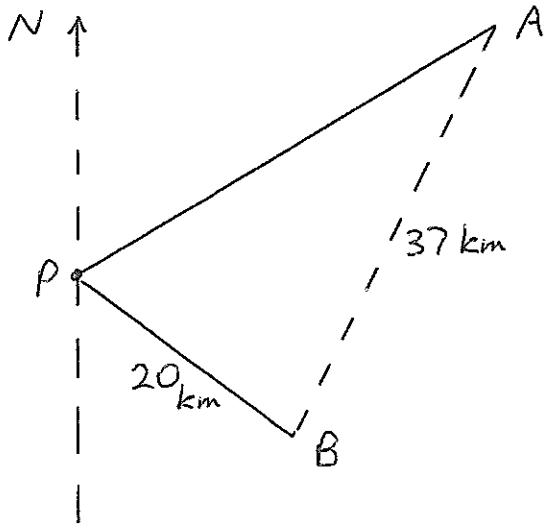


Not to Scale

i) Find the area of ΔPQR. 1

ii) Find the length of RP, correct to 1 decimal place. 2

d)



Not to Scale

Ship A leaves port P and sails on a compass bearing of $N50°E$. Ship B also leaves port P and sails 20 km on a compass bearing of $S55°E$. The two ships are now 37 km apart.

i) Find $\angle APB$. 1

ii) Find $\angle PAB$ to the nearest minute. 2

QUESTION 17 (9marks) Start a new page.

a) The interior angle of a regular polygon is 165°. How many sides does the polygon have? 1

b) Find the centre and radius of the circle $x^2 + y^2 - 4y - 1 = 0$. 2

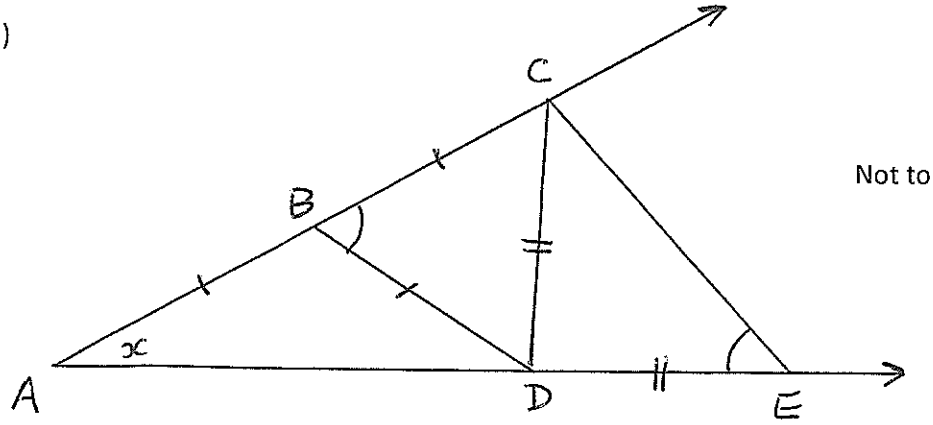
c) i) On the same axes, neatly sketch the functions $y = \frac{1}{x+2}$ and $y - x = 2$. Use a ruler, label any asymptotes and all x and y intercepts. 2

ii) Find the points of intersection of the two graphs. Show working. 2

iii) Find the equation of the normal to the curve $y = \frac{1}{x+2}$ at the point where it crosses the y axis. 2

QUESTION 18 (9 marks) Start a new page.

a)



Not to Scale

Rays AC and AE enclose isosceles triangles ABD, BCD and CDE as shown above.

i) If $\angle A = x$, find $\angle CBD$ in terms of x, giving reasons. 2

ii) Hence, find the size of $\angle DEC$. Reasons are not required. 1

b) Simplify $\lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$. Show full working. 2

c) Prove that $\sec\theta - \sin\theta \tan\theta = \cos\theta$. 2

d) Differentiate $y = x^2(x^3 - 1)^4$. Leave your answer in fully factored form. 2

END OF TEST

SOLUTIONS Yr 11 [redacted] 2 unit

1. D 2. B 3. C 4. A 5. A 6. B
7. D 8. B 9. A 10. D

(11) a) 9.1

b) $6+8\sqrt{3}-\sqrt{3}-4$ ← ①

$= 2+7\sqrt{3}$ ← ①

c) $\frac{2}{\sqrt{3}}$

d) $\frac{1}{x-1}$

e) $\cos 67° = \frac{4.6}{x}$ ← ①

$x = \frac{4.6}{\cos 67°}$ ①

$= 11.8$ (1 dec.)

f) $x+1 = \pm\sqrt{5}$ ← ①

$x = -1 \pm \sqrt{5}$

$= 1.2$ or -3.2 ①

(1 dec.)

(12) a) $3x-6<12$ or $-(3x-6)<12$

$3x<18$ $-3x+6<12$

$x<6$ $-3x<6$

$x>-2$ (must have)

∴ solution is $-2<x<6$

b) $\theta = 66°$ or $294°$ (need both)

c) $\frac{a+b}{\frac{b+a}{ab}} = a+b \times \frac{ab}{b+a}$ ①

$= ab$ ← ①

d) i) $y' = 6x+7$

ii) $f(x) = 4x^{-2}$

$f'(x) = -8x^{-3}$ ← ①

or $\frac{-8}{x^3}$

e) $2x+2y = 180°$ (co-interior angles parallel lines) ①

$\therefore x+y = 90°$

$\therefore \angle BEA = 90°$ (angle sum $\triangle BEA$) ①

(13) a) $(x-3)(x^2+3x+9)$

b) $\sin\theta \times \operatorname{cosec}^2\theta \times \tan\theta$

$= \sin\theta \times \frac{1}{\sin^2\theta} \times \frac{\sin\theta}{\cos\theta}$ ①

$= \frac{1}{\cos\theta} = \sec\theta$. ①

c) i) $y' = 4(x^2+5)^3 \times 2x$

$= 8x(x^2+5)^3$

ii) $y = x^{3/2}$

$y' = \frac{3}{2}x^{1/2}$

or $\frac{3\sqrt{x}}{2}$

(13) d) $y' = \dfrac{2(x+3) - 1(2x)}{(x+3)^2}$

$= \dfrac{6}{(x+3)^2}$ ← ①

When $x = -2$, $m_T = 6$ ← ①

← ①

e) $a = 1, b = -1, c = -1$

$\therefore x^2 - x - 1 = 0$ ← must have ①

(14) a) $V(-3, 4)$

b) $f(2) = 5$

$f'(x) = 2x + \frac{1}{2}$

$\therefore f'(2) = 4\frac{1}{2}$ ← ①

$\therefore f(2) + f'(2) = 9\frac{1}{2}$ ← ①

c) i) $\sin\theta = -1$ or $\cos\theta = 1$

$\therefore \theta = 270°, 0°, 360°$

① ①

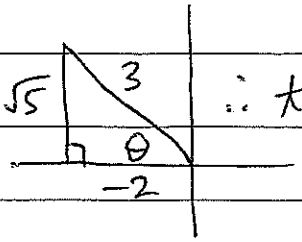
ii) $\tan^2\theta = \frac{1}{3}$ ← ①

$\therefore \tan\theta = \pm\frac{1}{\sqrt{3}}$

$\therefore \theta = 30°, 150°, 210°, 330°$

all 4 ①

d) 2nd quadrant



$\therefore \tan\theta = -\dfrac{\sqrt{5}}{2}$

e) $(x+4)^2 - y^2$

$= (x + 4 + y)(x + 4 - y)$

(15) a) $m_x = \frac{4}{3}$

b) use $B(0, 4)$

$\therefore y - 4 = \frac{4}{3}(x - 0)$

$\therefore 3y - 12 = 4x$

$\therefore 4x - 3y + 12 = 0$ as reqd.

c) $C(3, 8)$

d) $D(0, 6)$ and $4x - 3y + 12 = 0$

$\therefore$ p.d. $= \dfrac{|0 - 18 + 12|}{\sqrt{4^2 + 3^2}}$

$= \frac{6}{5}$ ← ①

e)

$B(0,4)$ $C(3,8) \Rightarrow BC = \sqrt{9 + 16}$

$= 5$ ← ①

$\therefore$ area $\triangle BDC = \frac{1}{2} \times 5 \times \frac{6}{5}$

$= 3u^2$ ← ①

f) $MP(AB) = (-1\frac{1}{2}, 2)$ ← ①

$m_{AB} = \frac{4}{3} \Rightarrow m_\perp = -\frac{3}{4}$ ← ①

$\therefore$ eqn is $y - 2 = -\frac{3}{4}(x + 1\frac{1}{2})$

$4y - 8 = -3x - 4\frac{1}{2}$

$\therefore 6x + 8y - 7 = 0$

← ①

(16) a) $4\pi r^2$

b) $\frac{dy}{dx} = 0$

$\therefore 3x^2 - 6x + 3 = 0$

$\therefore 3(x^2 - 2x + 1) = 0$ } ①

$3(x-1)^2 = 0$

$\therefore x = 1 \Rightarrow (1, 1)$ on curve ← ①

c) i) $A = \frac{1}{2} \times 7.3 \times 4.8 \times \sin 47°$

$\doteq 12.81 \text{ cm}^2$ ← ①

ii) $RP^2 = 7.3^2 + 4.8^2 - 2 \times 7.3 \times 4.8 \times \cos 47°$

$= 28.53555$

$\therefore RP \doteq 5.3$ (1 dec.) ← ①

d) i) 75°

ii) $\frac{20}{\sin A} = \frac{37}{\sin 75°}$ ← ①

$\therefore \sin A = \frac{20 \sin 75°}{37}$

$= 0.522\ldots$ ← ①

$\therefore \angle PAB = 31°28'$

(17) a) Ext. angle = 15°

$\therefore$ no. of sides $= \frac{360}{15}$

$= 24$ sides.

b) $x^2 + y^2 - 4y + 4 = 1 + 4$

$x^2 + (y-2)^2 = 5$ ← ①

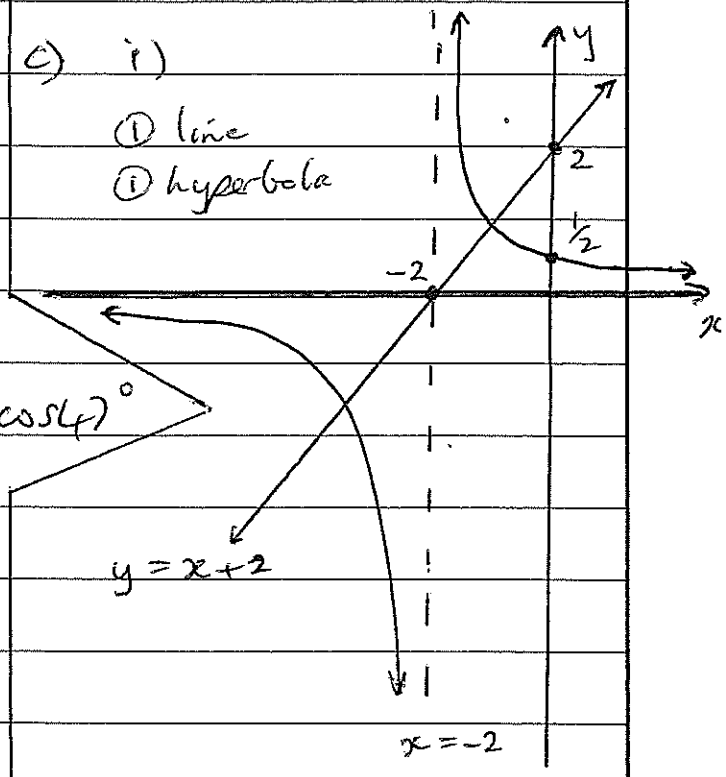
$\therefore$ centre is $(0, 2)$

radius is $\sqrt{5}$ u } ①

c) i)

① line

① hyperbola



ii) $\frac{1}{x+2} = x + 2$

$(x+2)^2 = 1$

$x^2 + 4x + 3 = 0$ } ①

$(x+3)(x+1) = 0$

$\therefore x = -3, -1$

$\therefore$ pts. of intersection are $(-3, -1)$ and $(-1, 1)$

① ← ①

(17) c) iii) $y = (x+2)^{-1}$

$y' = -1(x+2)^{-2} \times 1$

$= \frac{-1}{(x+2)^2}$

When $x = 0$, $y' = m_T = -\frac{1}{4}$ ①

$\therefore m_N = 4$ and use $(0, \frac{1}{2})$

$\therefore$ eqn. of normal is

$y - \frac{1}{2} = 4(x-0)$

$\therefore y = 4x + \frac{1}{2}$ ①

(or $8x - 2y + 1 = 0$)

(18) a) i) $\angle BDA = x$ (base angles isosceles $\triangle ABD$) ①

$\therefore \angle CBD = 2x$ (exterior angle $\triangle ABD$) ①

ii) 45°

b) $= \lim_{h \to 0} \frac{3(x^2 + 2xh + h^2) - 3x^2}{h}$

$= \lim_{h \to 0} \frac{3x^2 + 6xh + 3h^2 - 3x^2}{h}$

$= \lim_{h \to 0} \frac{h(6x + 3h)}{h}$

$= 6x$ ① method

① answer

c) LHS $= \frac{1}{\cos\theta} - \sin\theta \times \frac{\sin\theta}{\cos\theta}$

$= \frac{1 - \sin^2\theta}{\cos\theta}$ ①

$= \frac{\cos^2\theta}{\cos\theta}$

$= \cos\theta$

$=$ RHS. ①

d) $y' = 2x(x^3-1)^4$ ①

$+ 4(x^3-1)^3 \times 3x^2 \times x^2$

$= 2x(x^3-1)^4 + 12x^4(x^3-1)^3$

$= 2x(x^3-1)^3 [x^3 - 1 + 6x^3]$

$= 2x(x^3-1)^3(7x^3-1)$. ①