Name: .... **Maths Class:** ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics
Extension 1

HSC ASSESSMENT TASK ONE

December 2002

TIME ALLOWED: 60 minutes

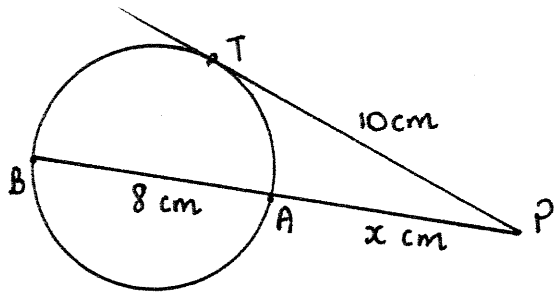
Instructions:

- Write your name and class at the top of this page.
- Start each question on a new page
- At the end of the examination this examination paper must be attached to the front of your answers.
- The marks for each question are indicated on the question sheet
- **ALL** questions should be attempted
- All necessary working must be shown. Marks will be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.

Q 1	Q 2	Q 3	Q 4	Q 5	Q 6	TOTAL
/7	/ 8	/ 8	/ 9	/ 7	/ 6	/ 45

QUESTION ONE (7 marks)

a) Find the value of x **(2)**



b) Evaluate $6 + 11 + 16 + \ldots\ldots + 426$ **(3)**

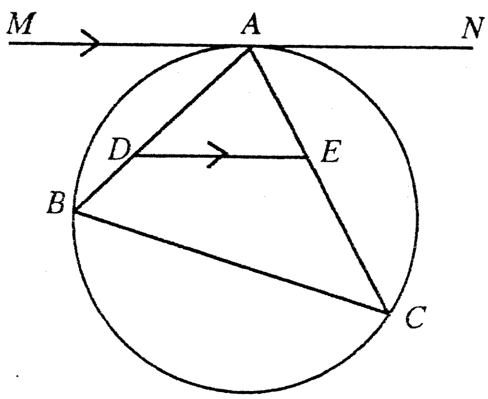
c) Find the value of $\sum_{n=4}^{7} n^2 + 2$ **(2)**

QUESTION TWO (8 marks)

a) If $S_n = 3n^2 + 2n$, find, **(4)**

i. The value of the second term
ii. The nth term

b) **(4)**



ABC is a triangle inscribed in a circle. MAN is the tangent to the circle at A. D is a point on AB and E is a point on AC such that DE // MAN.

i. Copy the diagram onto your answer page

ii. Explain why $\angle MAB = \angle ACB$.

iii. Hence show that BCED is a cyclic quadrilateral.

QUESTION THREE (8 marks)

a)The fourth term of a geometric sequence is 4 and the seventh term of the same sequence is 32 **(4)**

i. Find the value of the first term and the common ratio.

ii. Find the sum of the first 7 terms

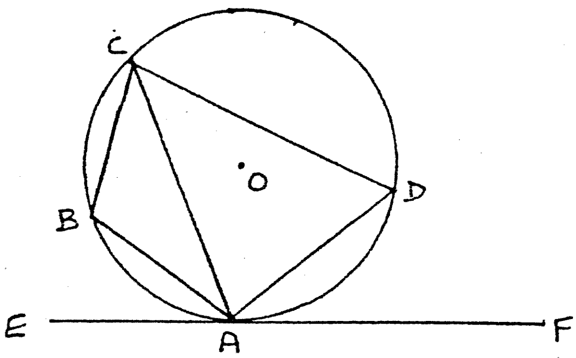
b) If $1^2 + 3^2 + \dots\dots\dots + (2k-1)^2 = \dfrac{k(2k-1)(2K+1)}{3}$ **(4)**

Prove that

$$1^2 + 3^2 + \dots\dots\dots + (2k-1)^2 + (2(K+1)-1)^2 = \frac{(K+1)(2K+1)(2K+3)}{3}$$

QUESTION FOUR (9 marks)

a) **(2)**



ABCD are points on the circumference of a circle with centre O.
EF is a tangent touching the circle at A.

$\angle$ CAE = 50^0

Find, giving reasons, $\angle$ABC

b) Prove, by mathematical induction, that $9^{n+2} - 4^n$ is divisible by 5, for Integers $n \geq 1$ **(4)**

c) A geometric series is given as **(3)**

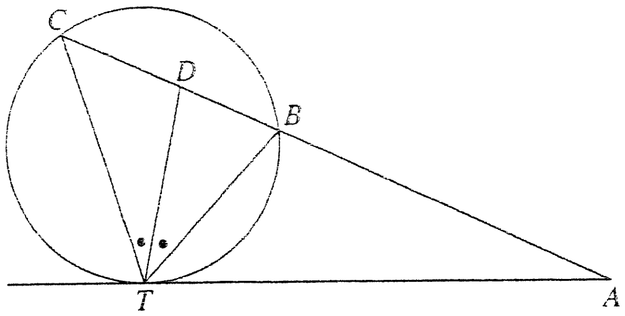
$1 + (2x+1) + (2x+1)^2 + \dots\dots$

i. For what values of x does the series have a limiting sum ?

ii. Is it possible for the series to have a limiting sum of -1. Explain.

QUESTION FIVE (7 marks)

a) **(3)**



TA is a tangent to a circle. Line *ABDC* intersects the circle at *B* and *C*.
Line *TD* bisects angle *BTC*.

Prove $AT = AD$

b) Kermit invests \$2000 at the beginning of each year into an investment account earning 6% p.a. compounded monthly. **(4)**

Kermit begins his investment on January 1st 2002

i. What is the value of the first investment at the end of 2032?

ii. If Kermit makes his last investment on January 1st 2032, how much is in the account when he withdraws it all on December 31st 2032, immediately after the interest for the month has been added?

QUESTION SIX (6 marks)

(6)

Bert and Ernie have a small business account earning 9% p.a. compounded monthly

Into this account they invest the companies profits of \$5000 at the start of each month. At the end of each month, immediately after the interest has been paid, Bert and Ernie withdraw \$M for the coming month's expenses.

i. How much is in the account , immediately before the first withdrawal?

ii. Show that the amount in the account immediately after the second withdrawal is,

$$A_2 = 5000(1.0075^2 + 1.0075) - M(1.0075 + 1)$$

iii. Bert and Ernie hope to have saved \$100 000 by the end of three years, (immediately after the withdrawal for the coming months expenses) How much can they afford to withdraw for expenses each month?

Question One

a) $10^2 = x(x+8)$ 1

$x^2 + 8x - 100 = 0$

$x = \frac{-8 \pm \sqrt{64 - 4 \times 1 \times -100}}{2}$

$= \frac{-8 \pm \sqrt{464}}{2}$

$= \frac{-8 \pm \sqrt{29}.4}{2}$

$= -4 + 2\sqrt{29},\ -4 - 2\sqrt{29}$

as $x > 0$

$x = -4 + 2\sqrt{29}$ 1

b) $6 + 11 + 16 + \ldots + 426$

AP $a = 6$ $d = 5$ $T_n = 426$ 1

$426 = 6 + (n-1)5$

$420 = 5n - 5$

$425 = 5n$

$n = 85$ 1

$\therefore S_n = \frac{n}{2}(a + l)$

$= \frac{85}{2}(6 + 426)$

$= 18360$ 1

c) $\sum_{n=4}^{7} n^2 + 2$

1

$= (4^2+2) + (5^2+2) + (6^2+2) + (7^2+2)$

$= 134$ 1

Question two

a) $S_n = 3n^2 + 2n$

i. $T_2 = S_2 - S_1$ 1

$= (3 \times 4 + 4) - (3 \times 1 + 2)$

$= 11$ 1

ii. $5, 11, 17, \ldots$

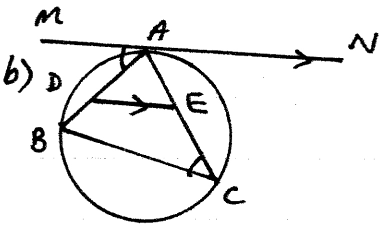
AP $a = 5$ $d = 6$

$\therefore T_n = a + (n-1)d$ 1

$= 5 + (n-1)6$

$= 6n - 1$ 1

b)



ii. $\angle MAB = \angle ACB$ 1

The angle between a tangent and a chord is equal to the angle in the alternate segment. 1

iii. $\angle MAB = \angle ADE$ (alt $\angle$'s, $MN \parallel DE$)

$\angle BDE = 180° - \angle ADE$ ($\angle$'s on a st. Line add to $180°$) 1

However, $\angle BDE = 180° - \angle ACB$

(as $\angle MAB = \angle ACB = \angle ADE$)

$\therefore \angle BDE + \angle ACB = 180°$

and BCED is a cyclic quad as opp. angles are supplementary 1

Question three:

a) i. $T_4 = ar^3 = 4$ — ①

$T_7 = ar^6 = 32$ — ②

② ÷ ①

$r^3 = 8$

$\therefore r = 2$ ✓①

$a = \frac{1}{2}$ ✓①

ii. $n=7$ $\quad S_n = \frac{a(r^n - 1)}{r-1}$

$= \frac{\frac{1}{2}(2^7 - 1)}{1}$ ①✓

$= 63.5$ ①✓

b)

$LHS = 1^2 + 3^2 + \ldots + (2K-1)^2 + (2K+1)^2$

$= \frac{K(2K-1)(2K+1)}{3} + (2K+1)^2$ ①

$= \frac{K(2K-1)(2K+1)}{3} + \frac{3}{3}(2K+1)^2$

$\frac{(2K+1)}{3}\left[K(2K-1) + 3(2K+1)\right]$ ①

$= \frac{(2K+1)}{3}\left[2K^2 + 5K + 3\right]$

$= \frac{(2K+1)(K+1)(2K+3)}{3}$ ①

$= \frac{(K+1)(2K+1)(2K+3)}{3}$

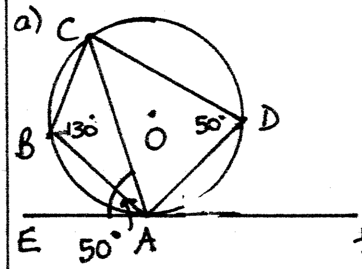
①

$= RHS$

setting out of LHS = RHS.

Question four:

a)



$\angle CAE = 50°$ (given)

$\angle CAE = \angle ADC$ ($\angle$ between a tangent & a chord = 's $\angle$ in the alt. segment) ①

$\therefore \angle ADC = 50°$

$\angle ADC + \angle ABC = 180°$

(opp $\angle$'s in a cyclic Quad ADCB) ①

$\therefore \angle ABC = 130°$

b) $9^{n+2} - 4^n$ is ÷ by 5

Test $n=1$

$9^3 - 4^1 = 725$

$= 5 \times 145$ which is ÷ by 5

$\therefore$ true for $n=1$ ①

Assume true for $n=K$

ie $9^{K+2} - 4^K = 5M$ where M is an inte ①

Prove true for $n=K+1$

$9^{K+1+2} - 4^{K+1}$

$= 9.9^{K+2} - 4.4^K$ ①

$= 9(5M + 4^K) - 4.4^K$

$= 45M + 9.4^K - 4.4^K$

$= 45M + 5.4^K$

$= 5(9M + 4^K)$ which is ÷ by 5

as $9M + 4^K$ is an integer ①

$\therefore$ true for $n = K+1$

If true $n=K$ also true for $n=K+1$

As true $n=1$ also true for $n = 1+1 = 2, 3, 4 \ldots$

Hence by M.I. true all +ve integer n

Q4 c)

$1 + (2x+1) + (2x+1)^2 + \dots\dots$

i. $-1 < r < 1$

$-1 < 2x+1 < 1$

$-2 < 2x < 0$

$-1 < x < 0$ (1)

ii. $S_\infty = \dfrac{a}{1-r}$

$-1 = \dfrac{1}{1-(2x+1)}$

$-1 = \dfrac{1}{-2x}$

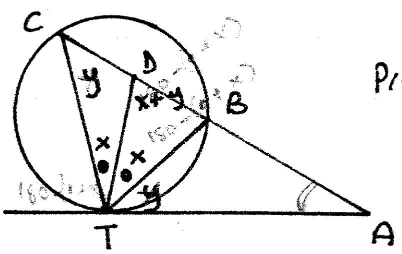
$2x = 1$

$x = \frac{1}{2}$ (1) but as $-1 < x < 0$

$x \neq \frac{1}{2}$

$\therefore S_\infty \neq -1$ (1)

Question five



Prove $AT = AD$

let $\angle CTD = x$

$\therefore \angle DTB = x$ (given)

let $\angle BTA = y$

$\therefore \angle TCA = y$ (alt. segment theorem)

$\angle TDA = \angle DTC + \angle TCD$

(exterior $\angle$ of ΔTDC)

$\therefore \angle TDA = x + y$

$\angle DTA = \angle DTB + \angle BTA$

$= x + y$

$\therefore \angle TDA = \angle DTA \quad (x+y)$

$\therefore AD = AT$ (sides opp. equal angles)

b) 6% p.a $= \frac{1}{2}\%$ p. month

Annual Inv $\rightarrow$ Comp monthly

i. $2000(1.005)^{\cancel{30}\,31 \times 12}$ (1)

$= \cancel{\$12\,045.15}$

$\$12788.07$

ii. $2000(1.005)^{30 \times 12}$

$+ \ 2000(1.005)^{29 \times 12}$ (1)

$+ \ 2000(1.005)^{28 \times 12}$

$\vdots$

$2000(1.005)^{1 \times 12}$

$\therefore$ Total $= 2000\left[1.005^{12} + 1.005^{24} + \dots 1.005^{31}\right.$

$\overrightarrow{GP}$ $a = 1.005^{12}$

$r = 1.005^{12}$

$n = \cancel{30}\ 31$

$= 2000 \times \dfrac{1.005^{12}\left[(1.005^{12})^{\cancel{30}\,31} - 1\right]}{1.005^{12} - 1}$ (1)

$= \cancel{\$172910.0466\dots}$

$= \cancel{\$172910}$ (nearest \$) (1)

$\$185698$

Question six

$r = 1.0075$

i. $5000(1.0075)$ 1

$= 5037.50$

ii. $A_1 = 5000(1.0075) - m$

$A_2 = [A_1 + 5000](1.0075) - m$ 1

$= 5000(1.0075)^2 - m(1.0075) + 5000(1.0075) - m$ 1

$= 5000[1.0075^2 + 1.0075] - m[1.0075 + 1]$

iii. $A_n = 5000[1.0075^n + 1.0075^{n-1} + \dots + 1.0075] - m[1.0075^{n-1} + 1.0075^{n-2} \dots + 1]$

$n = 36 \qquad A_n = 100\,000$

$$100\,000 = 5000 \times \frac{1.0075(1.0075^{36} - 1)}{1.0075 - 1} \;(1) - m\left[\frac{1 \times (1.0075^{36} - 1)}{1.0075 - 1}\right] \;(1)$$

$m = \$2607.53$ 1