Student Name: .... Maths Teacher ....

SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT TASK 1
DECEMBER 2004

MATHEMATICS

Time allowed: 70 minutes

Instructions

* Write your details at the top of this page.
* Attempt all questions. All questions are worth equal marks.
* Answers are to be written on the paper provided.
* Do **not** divide your pages into two columns of working.
* You may write on the front and back of each page. Ask for more paper if required.
* Marks may not be awarded for careless or badly arranged working.
* Indicated marks are a guide and may be changed slightly if necessary.

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
/7	/7	/7	/7	/7	/7	/7	/7	/56

Question 1

3 a) A parabola has its focus at (0, 2) and its directrix is the line $y = -4$. Find

i) the coordinates of the vertex.

ii) the equation of the parabola.

4 b) The roots of the equation $x^2 - 8x + 10 = 0$ are α and β. Find

i) $\alpha + \beta$

ii) $\alpha\beta$

iii) $\alpha^2 + \beta^2$

Question 2 (Begin a new page)

4 a) A parabola has equation $y = x^2 - 4x - 21$. Find

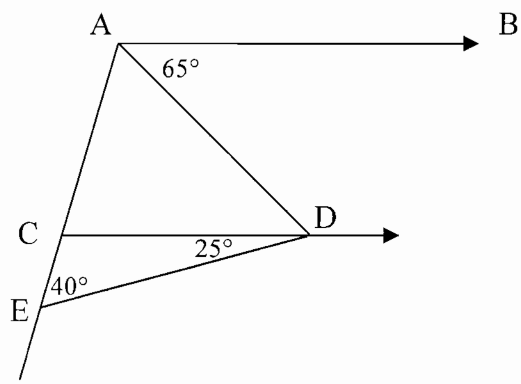
i) the equation of the axis of symmetry.

ii) the coordinates of the vertex.

iii) the x intercept/s.

iv) the values of x for which $x^2 - 4x - 21 < 0$.

3 b) Copy this diagram onto your page. AB || CD. Prove that triangle ACD is isosceles.

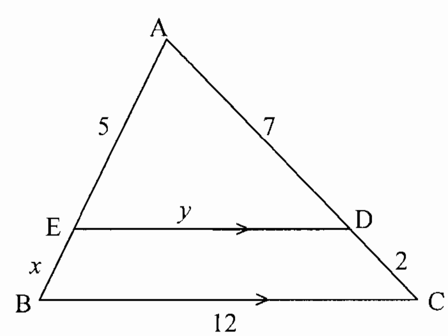


Question 3 (Begin a new page)

4 a) In this part **no** formal proofs are required but you must give full reasons for the statements you make.

i) Find the value of x giving reasons.

ii) Find the value of y giving reasons.



3 b) A parabola has equation $x^2 - 4y - 4x + 16 = 0$.

i) Write the equation in the form $(x-h)^2 = 4a(y-k)$.

ii) Find the coordinates of the vertex.

iii) Find the coordinates of the focus.

Question 4 (Begin a new page)

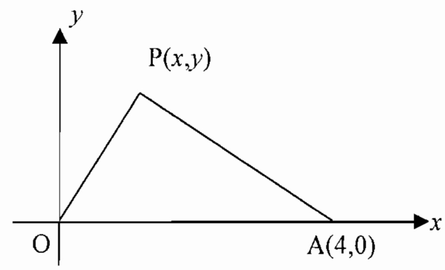
2 a) Solve for x: $x^4 - 6x^2 + 8 = 0$.

5 b) For the diagram at the right:

i) Write expressions for the gradients of AP and OP.

ii) If $\angle$OPA is always 90°, show that the equation of the locus of P represents a circle.

iii) State the centre and radius of the circle.



Question 5 (Begin a new page)

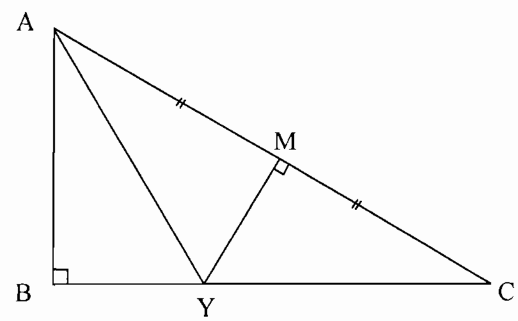
3 a) i) Sketch the graph of $(y-2)^2 = 8(x+1)$ showing clearly the coordinates of the vertex..

ii) Draw and label the directrix and write its equation on the sketch.

4 b) In ΔABC, $\angle$B = 90°. YM is the perpendicular bisector of AC. Copy the diagram onto your page.

i) Prove that $\Delta AYM \equiv \Delta CYM$.

ii) Suppose now that AY bisects $\angle$BAC. Find the size of $\angle$YCM (no reasons needed).



Question 6 (Begin a new page)

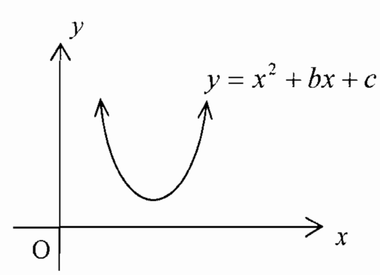
2 a) i) Write a quadratic equation with roots α and β if α + β = -2 and αβ = 6.

ii) Write a quadratic equation with roots k and $2k$.

3 b) By making a suitable substitution find all real values of x which satisfy the equation $\left(x^2-1\right)^2 - 3\left(x^2-1\right) = 0$.

2 c) i) State the condition, in terms of b & c, for the graph of $y = x^2 + bx + c$ to be entirely above the x axis.

ii) What two word description is used for quadratics of this type?



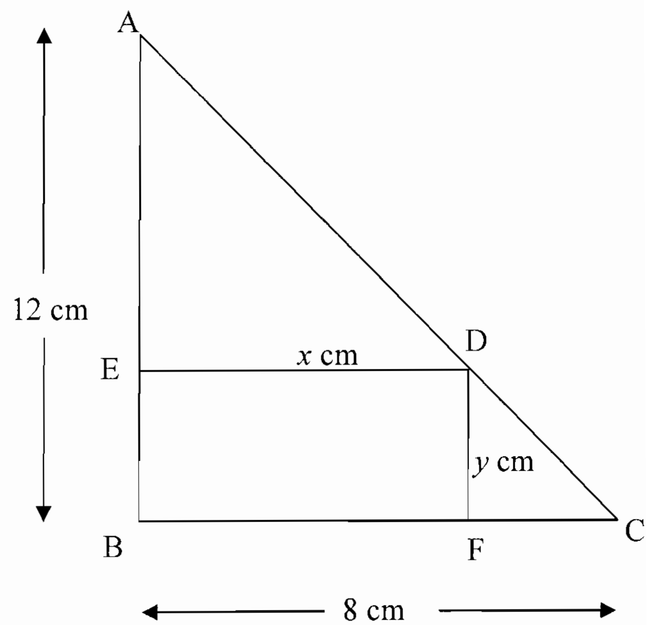
Question 7 (Begin a new page)

3 a) Find the value/s of k for which the equation $x^2 + 4x + k = 0$ has roots which are real and distinct (unequal).

4 b) Find possible values of m so that the line $y = mx - 9$ will be a tangent to the curve $y = x^2 - 2x$.

Question 8 (Begin a new page)

EDFB is a rectangle with sides x and y inscribed in the triangle ABC. Side AB is 12 cm in length and side BC is 8 cm in length.



1 a) Which test would be used to show that $\Delta ABC \mathrel{|||} \Delta DFC$?

2 b) Show that $\dfrac{8-x}{y} = \dfrac{8}{12}$ (give a reason).

2 c) Show that the area of the rectangle is $12x - \dfrac{3}{2}x^2$.

2 d) Use the theory of quadratic functions (not calculus) to find the value of x which makes the rectangle area a maximum and find this maximum area.

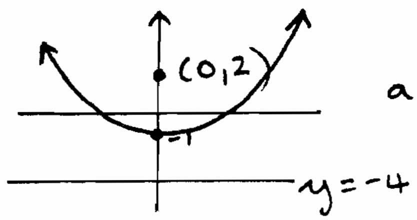
End of Questions.
Place this question paper on top of your answers to hand in.

MARKING SCHEME.

STHS 2UNIT HSC TASK1 DEC 2004

Question 1

a)



$a = 3$

i) $\underline{\text{Vertex } (0,-1)}$

ii) $\underline{x^2 = 12(y+1)}$

a) i) ① mark

ii) ① for correct form: $X^2 = 4aY$
① for correct equation.

b) i) $\alpha + \beta = \frac{-b}{a} = \underline{8}$

ii) $\alpha\beta = \frac{c}{a} = \underline{10}$

iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= 8^2 - 2 \times 10$
$= \underline{44}$

b) i) ① mark
ii) ① mark

allow errors carried through

(iii) ② for correct answer
OR
① for correct formula

Question 2

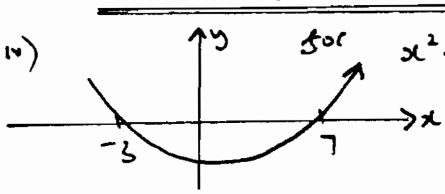
a) $y = x^2 - 4x - 21$

i) axis of sym $x = \frac{4}{2}$
$\therefore \underline{x = 2}$

ii) $\underline{\text{Vertex } (2,-25)}$

iii) $y = (x-7)(x+3)$
$\therefore$ $\underline{x \text{ intercepts } x = 7 \text{ and } -3}$

iv)

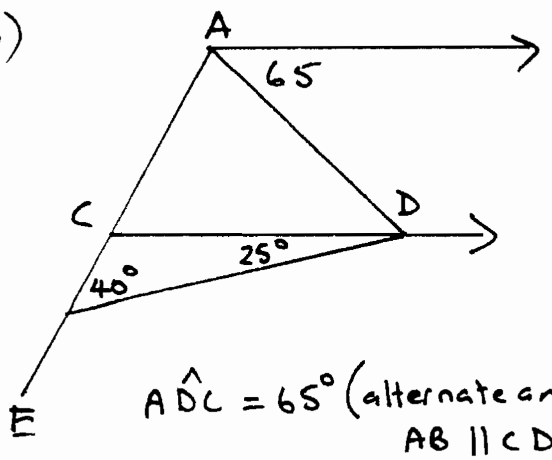


for $x^2 - 4x - 21 < 0$

$\therefore \underline{-3 < x < 7}$

① mark each part

Allow errors carried forward from i) to ii) and from iii) to iv) b)



$A\hat{D}C = 65°$ (alternate angles, $AB \parallel CD$) —— ① all correct

$A\hat{C}D = 65°$ (exterior angle of triangle) —— ① all correct

$\therefore \triangle ACD$ is isosceles (base angles equal) —— ① Correct reason – allow (2 angles equal) etc

Question 3

a) i) $\frac{5}{x} = \frac{7}{2}$ (ratio of intercepts, parallel lines)

$7x = 10$

$x = 10/7$

ii) $\frac{y}{12} = \frac{7}{9}$ (similar triangles corresponding sides in proportion)

$9y = 84$

$y = 9\frac{1}{3}$

a) i) and ii)
① for correct answer
① for reason which fits the equation – Reason must clearly identify the theorem used for the written equation.

—— DITTO

b) i) $x^2 - 4x = 4y - 16$

$x^2 - 4x + 4 = 4y - 16 + 4$

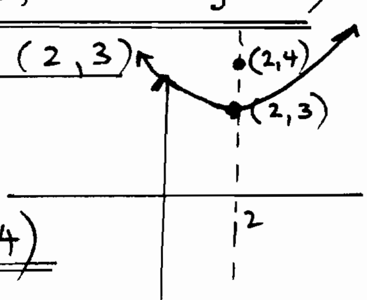
$(x-2)^2 = 4y - 12$

$(x-2)^2 = 4(y-3)$

ii) Vertex $(2,3)$

iii) $a = 1$

Focus $(2,4)$



b) ① for each part.

Allow errors carried from (i) to (ii) and (iii).

Question 4

a) Let $u = x^2$

$u^2 - 6u + 8 = 0$

$(u-4)(u-2) = 0$

$u = 4 \qquad u = 2$ — ① to here

$x^2 = 4 \qquad x^2 = 2$

$x = \pm 2 \qquad x = \pm\sqrt{2}$

① for this operation as long as 1 value of u has been correctly solved for x.

b) i) $m_{OP} = \dfrac{y}{x} \qquad m_{PA} = \dfrac{y}{x-4}$

i) ① both must be correct

ii) $\dfrac{y}{x} \cdot \dfrac{y}{(x-4)} = -1$ ①

ii) → for correct statement allow E.C.F. from (i).

$y^2 = -x(x-4)$

$y^2 = -x^2 + 4x$

$4 - 4x + x^2 + y^2 = 4$

i.e. $(x-2)^2 + y^2 = 4$ which is a circle

① for rearranging into standard circle – even if incorrect eqn.

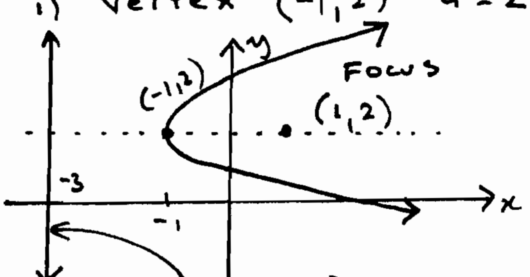
(iii) centre $(2,0)$

radius 2

① each. Allow if correctly deduced from an incorrect equation in (ii).

Question 5

a) i) vertex $(-1,2)$ $a = 2$



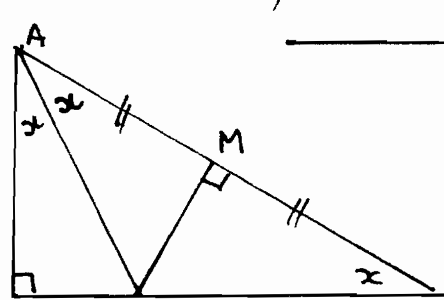
① for correct shape

① for correct vertex

ii) $x = -3$ (directrix)

① for correct directrix & eqn.

b)



QUESTION 5 (cont)

i) In Δ's AYM, CYM

AM = MC (M is midpt of AC)

$A\hat{M}Y = C\hat{M}Y$ (supplementary angles and $YM \perp AC$ $\therefore$ both $90°$)

MY is common

$\therefore \Delta AYM \equiv \Delta CYM$ (SAS)

③ for correct proof ie ① for each correct line – ignore conclusion.

ii) Let $B\hat{A}Y = x$ $\therefore Y\hat{A}M = x$ (AY bisects $B\hat{A}C$)

and $M\hat{C}Y = x$ (corsp. angles in congruent triangles)

$\therefore 3x = 90$

$Y\hat{C}M = x = 30°$ ——— ① for correct value ($30°$)($\pi/6$) no working needed. Allow if degree sign missing.

Question 6

a) i) $x^2+2x+6=0$ ——— ①

ii) $(x-k)(x-2k)=0$ ——— ① No need to expand.

b) $u = x^2 - 1$

$u^2 - 3u = 0$ ——— ① Simplified equation

$u(u-3)=0$

$u=0$ $\quad$ $u=3$ ——— ① Correct solutions

$x^2-1=0$ $\quad$ $x^2-1=3$

$x^2=1$ $\quad$ $x^2=4$

$x=\pm1$ $\quad$ $x=\pm2$ ——— ① 1 value of u correctly followed through.

c)

i) $\therefore (\Delta < 0)$ $\quad$ $b^2-4c<0$ ——— ① Must be in terms of b & c.

ii) positive definite ——— ① Allow mis spelling

Question 7

a) roots real and unequal

$\therefore \quad \Delta > 0$ — ① for rule

$4^2 - 4 \times 1 \times k > 0$ — ① for correctly finding Δ (even if inequality is wrong)

$16 - 4k > 0$

$16 > 4k$

$4 > k$

$\therefore \quad \underline{\underline{k < 4}}$ — ① Correct solution of <u>their</u> inequality.

b) tangent if <u>one</u> solution when solved simultaneously — ① for either of these

$$\begin{cases} y = mx - 9 \\ y = x^2 - 2x \end{cases}$$

$mx - 9 = x^2 - 2x$ — ① initial substitution correct.

$0 = x^2 - 2x - mx + 9$

$0 = x^2 - x(2+m) + 9$

$\therefore \quad \Delta = 0$ for this quadratic

$(2+m)^2 - 4 \times 1 \times 9 = 0$ — ① Correct inequality (even if from an incorrect quadratic eqn.)

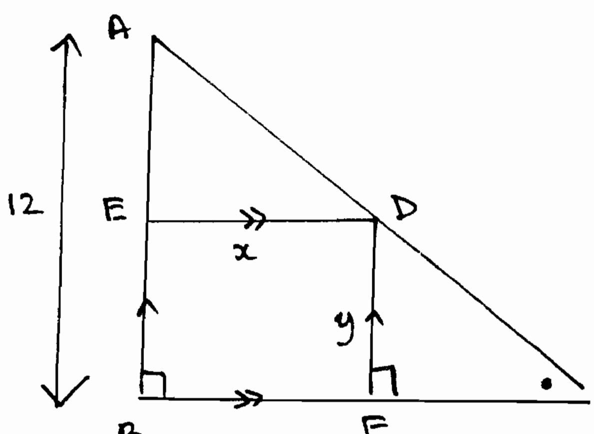
$4 + 4m + m^2 - 36 = 0$

$m^2 + 4m - 32 = 0$

$(m+8)(m-4) = 0$

If $\underline{\underline{m = -8 \quad m = 4}}$ line is tangent to curve. — ① correct solution of <u>their</u> inequality.

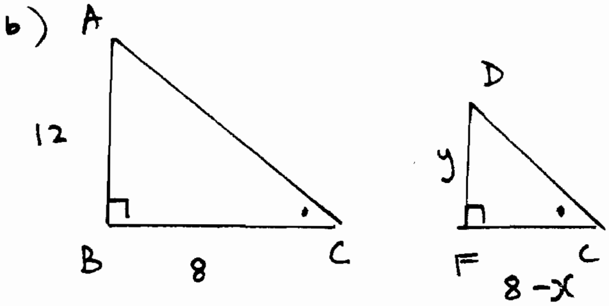
Question 8



QUESTION 8 (cont)

a) $\Delta ABC \,|||\, \Delta DFC$ (equiangular) —— ①

b)



$$\frac{8}{8-y} = \frac{12}{y} \quad \text{(corsp sides similar triangles)}$$

$$8y = 12(8-x)$$

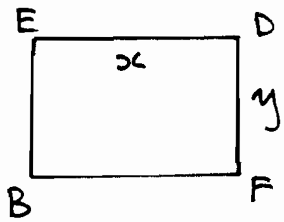
$$\frac{8}{12} = \frac{8-x}{y}$$

① $FC = 8-x$ must be stated or indicated (eg. on a diagram) etc.

① for suitable reason.

No marks for writing the given statement.

c)



$$8y = 12(8-x)$$

$$\therefore y = \frac{12(8-x)}{8} \text{ —— ①}$$

$$y = \frac{3(8-x)}{2}$$

Both must be correct. No E.C.F. allowed.

$$\therefore \text{area rectangle} = x \times 3\frac{(8-x)}{2} \text{ —— ①}$$

$$= 12x - \frac{3x^2}{2}$$

d) Let

$$A = -\frac{3x^2}{2} + 12x$$

$$\text{axis of sym. } x = \frac{-12}{2 \times -\frac{3}{2}}$$

$$x = 4 \text{ —— ①}$$

$\therefore$ max area is

$$= 12 \times 4 - \frac{3}{2}(4)^2$$

$$= 24 \text{ cm}^2 \text{ —— ① for correct value (24)}$$

→ Alternatively

$$A = -\frac{3}{2}(x^2 - 8x + 16) + 24$$

max of 24 when $x = 4$