

Name: Solutions

Teacher/Class: \_\_\_\_\_

## SYDNEY TECHNICAL HIGH SCHOOL



### HSC ASSESSMENT TASK 1

DECEMBER 2009

## MATHEMATICS

**Time Allowed:** 70 minutes

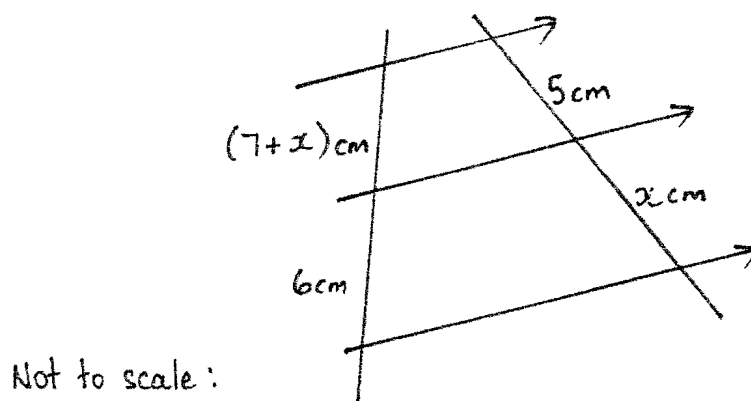
**Instructions:**

- Write your name and class at the top of each page
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start **each** question on a **new** page.
- Diagrams unless otherwise stated are **not** to scale

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Question 7	TOTAL
/8	8	/8	/8	/8	/8	/8	/56

**Question 1** (8 marks)

- a) i) Solve  $2x^2 - x = 1$  (3)  
ii) Hence, solve  $2x^2 - x - 1 > 0$
- b) What is the vertex of  $y = 3x^2 - 2x + 1$ ? (2)
- c) What is the focal length of  $x^2 = -12y$ ? (1)
- d) Find the value of  $x$  in : (2)

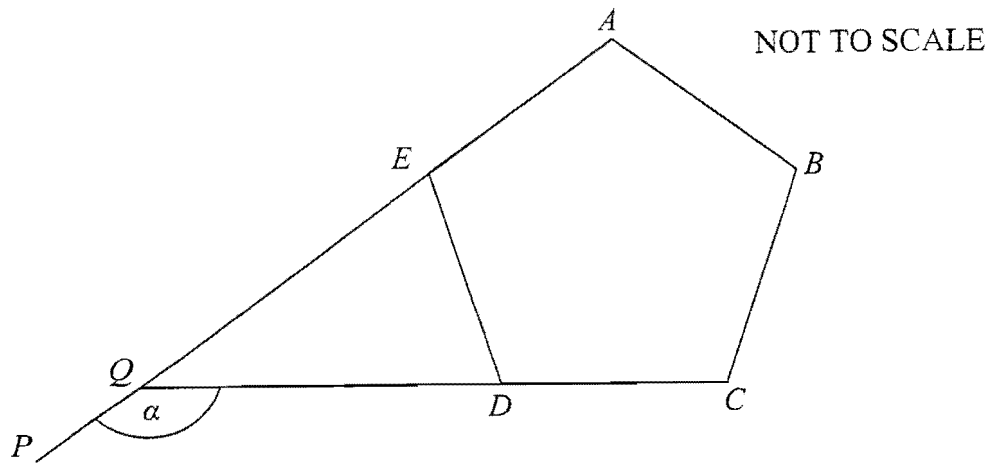


**Question 2** (8 marks) **Start a new page**

- a) The roots of  $5x^2 - 6x - 3 = 0$  are  $\alpha$  and  $\beta$  (3)
- i) Find the value of,  
(I)  $\alpha + \beta$   
(II)  $\alpha^2 \beta + \alpha \beta^2$
- ii) Write down a quadratic equation, in the form  $ax^2 + bx + c = 0$ , where (3)  
 $a, b$  and  $c$  are integers, whose roots are  $2\alpha$  and  $2\beta$ .
- b) Find the locus of a point  $P$  which moves so that it is always 4 units from the point  $(-1, 2)$  (2)

**Question 3** (8 marks) **Start a new page**

- a) ABCDE is a regular pentagon. The points P, Q, E and A are collinear.  
CD is produced to meet PA at Q.



Find the size of angle  $\alpha$  giving reasons. (2)

- b) Consider the parabola  $x^2 = 8y + 16$
- Find the co-ordinates of the vertex (2)
  - Find the co-ordinates of the focus (2)
  - Find the equation of the tangent to the parabola at the point  $(2, -\frac{3}{2})$  (2)

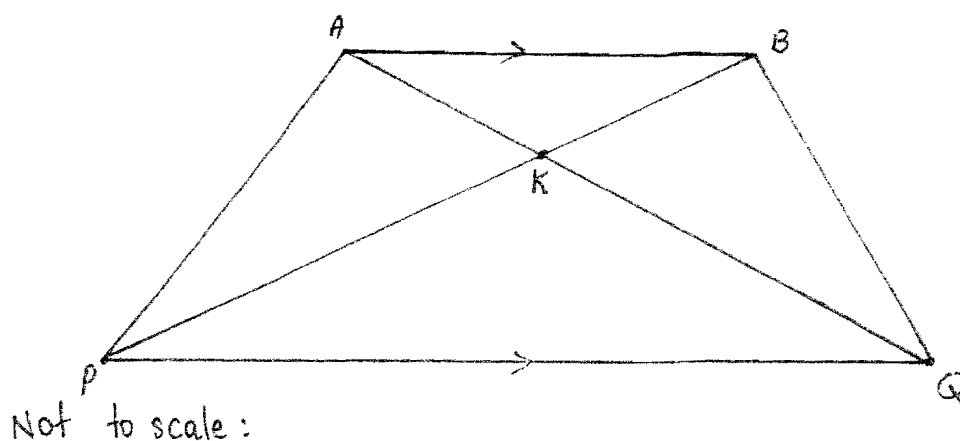
**Question 4** (8 marks) [Start a new page](#)

- a) Find the radius and the centre of the circle whose equation is:

$$x^2 - 4x + y^2 + 6y - 12 = 0$$

(2)

- b) In the diagram  $AB \parallel PQ$ ,  
AQ bisects PB at K and  
 $PK = KQ$



Copy the diagram onto your answer page, marking on it all relevant information.

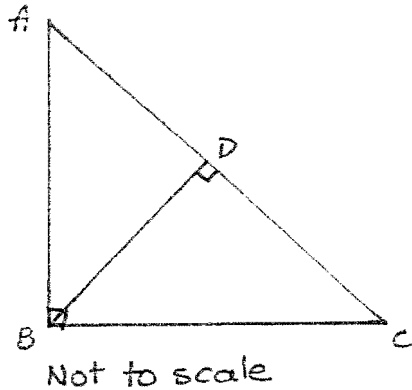
- i) Prove that  $\triangle AKB$  is isosceles (3)
- ii) Prove that  $\triangle AKP \equiv \triangle BKQ$  (2)
- iii) Hence, show that  $AP = BQ$  (1)

**Question 5** (8 marks) [Start a new page](#)

- a) Solve,  $4^x - 9(2^x) + 8 = 0$  (3)
- b) Find the value of A, B and C, given  
 $3x^2 + 4 \equiv A(x+2)^2 + B(x+2) + C$  (3)
- c) Sketch the parabola  $y^2 = 12x$  showing its focus and directrix (2)

**Question 6** (8 marks) **Start a new page**

- a) ABC is a triangle in which  $\angle ABC = 90^\circ$  and  $BD \perp AC$



- i) Show that  $\triangle ABD$  is similar to  $\triangle ACB$  (2)

- ii) If  $AB = 3\text{cm}$  and  $BC = 4\text{cm}$ , find the length of  $BD$ , with reasons (2)

- b) Consider the quadratic expression  $3kx^2 - 5x + 3k$

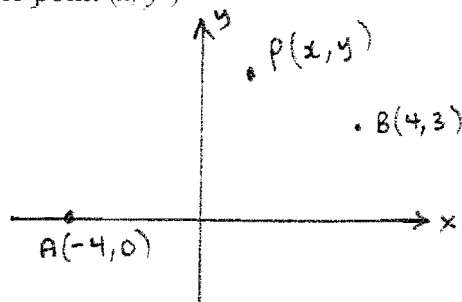
- i) Write down an expression for the discriminant of this quadratic. (1)

- ii) Hence, find the value/s of  $k$  for which  $3kx^2 - 5x + 3k$  is negative definite. (3)

**Question 7** (8 marks) **Start a new page**

- a) Let A and B be the fixed points  $(-4, 0)$  and  $(4, 3)$  respectively.

P is the variable point  $(x, y)$



- i) Write down expressions for  $PA^2$  and  $PB^2$  in terms of  $x$  and  $y$  (2)

- ii) Hence, or otherwise, find the locus of the point  $P$ , such that,  $PA = 2PB$  (3)


- b) Find the value/s of  $k$  for which the quadratic equation,  $(k - 3)x^2 - 3kx + 25 = 0$  has one root double the other root. (3)

**End of paper**

## Answers

### Question 1

a)  $2x^2 - x - 1 = 0$   
 $(2x+1)(x-1) = 0$   
 $x = -1/2, x = 1$

ii)   $x < -1/2, x > 1$

b)  $x = \frac{2}{6}$  Vertex  $(\frac{1}{3}, \frac{2}{3})$

c)  $x^2 = -12y$   
 - focal length 3 units

d)  $\frac{x+7}{6} = \frac{5}{x}$   
 $x^2 + 7x = 30 \quad x > 0$   
 $(x+10)(x-3) = 0$   
 $\therefore x = 3$

### Question 2.

a)  $\alpha + \beta = -\frac{b}{a}$   
 $= \frac{6}{5}$

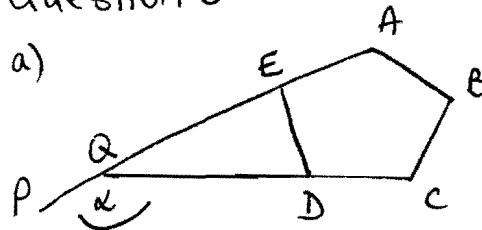
ii)  $\alpha\beta(\alpha + \beta) = \frac{c}{a} \times \frac{6}{5}$   
 $= \frac{-3}{5} \times \frac{6}{5}$   
 $= \frac{-18}{25}$

ii.  $x^2 - (2\alpha + 2\beta)x + 2\alpha \cdot 2\beta = 0$   
 $x^2 - 2(\alpha + \beta)x + 4\alpha\beta = 0$   
 $x^2 - 2(\frac{6}{5})x + 4(\frac{-3}{5}) = 0$   
 $5x^2 - 12x - 12 = 0$

b)  $\sqrt{(x+1)^2 + (y-2)^2} = 4$

$\therefore (x+1)^2 + (y-2)^2 = 16.$

### Question 3



$\angle AED = \angle CDE = 108^\circ$

(angles of a regular pentagon)

$\angle QED = \angle EDQ = 72^\circ$

(straight line equals  $180^\circ$ )

$\alpha = \angle QED + \angle EDQ$  (exterior  $\angle$  of  $\triangle QED$ )  
 $\alpha = 144^\circ$

b)  $x^2 = 8(y+2)$  ②  $a = 2$

i.  $V(0, -2)$  ②

ii.  $S(0, 0)$  ②



iii.  $8y = x^2 - 16$

$y = \frac{x^2}{8} - 2$

$dy/dx = 2x/8$  at  $x = 2$

$MT = 1/2$

$\therefore y + 3/2 = 1/2(x - 2)$

$2y + 3 = x - 2$

$2y = x - 5.$  ②

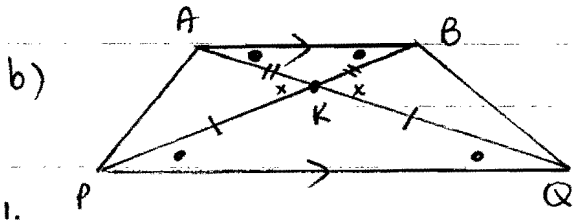
#### Question 4

$$a) \quad x^2 - 4x + y^2 + 6y = 12$$

$$\quad \quad +4 \quad \quad +9 \quad \quad +13$$

$$(x-2)^2 + (y+3)^2 = 25$$

$$C = (2, -3) \quad r = 5$$



i.  $\angle KPQ = \angle KQP$  (equal angles opposite equal sides,  $PK = QK$ )

$$\angle RBA = \angle KQP \text{ (alt } \angle\text{'s } AB \parallel PQ)$$

$$\angle BAK = \angle KQP \text{ (alt } \angle\text{'s } AB \parallel PQ)$$

$$\therefore \triangle AKB \text{ is isosceles (2 equal } \angle\text{'s)}$$

ii. In  $\triangle AKP$  and  $\triangle BKQ$

$$PK = QK \quad (\text{given})$$

$$AK = BK \quad (\text{equal sides opposite equal } \angle\text{'s, } \triangle AKB \text{ isosceles})$$

$$\angle AKP = \angle BKQ \text{ (vertically opposite)}$$

$$\therefore \triangle AKP \equiv \triangle BKQ \text{ (SAS)}$$

iii.  $AP = BQ$ , corresponding sides in congruent triangles

#### Question 5

a) let  $m = 2^x \therefore m^2 - 9m + 8 = 0$

$$(m-8)(m-1) = 0$$

$$m = 8 \text{ or } m = 1$$

$$\text{i.e. } 2^x = 8 \text{ or } 2^x = 1$$

$$\text{and } x = 3 \text{ or } x = 0$$

b)  $3x^2 + 4 = Ax^2 + 4Ax + 4A + Bx + 2B + C$

Equating coeff

$$\boxed{x^2} \quad 3 = A \quad \underline{A = 3}$$

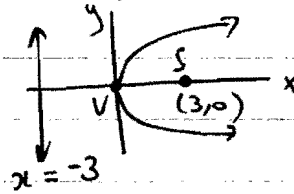
$$\boxed{x} \quad 0 = 4A + B$$

$$B = -12 \quad \underline{B = -12}$$

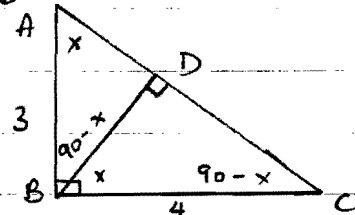
$$\boxed{x^0} \quad 4 = 4A + 2B + C$$

$$4 = 12 + -24 + C \quad \underline{C = 16}$$

c) Sideways  $v = (0,0) \quad a = 3$



#### Question 6

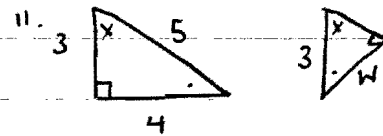


i. In  $\triangle ABD$  and  $\triangle ACB$

$$\angle ADB = \angle ABC \text{ (given } 90^\circ)$$

$$\angle A \text{ is common}$$

$$\therefore \triangle ABD \sim \triangle ACB \text{ (equiangular)}$$



$$\frac{w}{4} = \frac{3}{5} \quad (\text{ratio of corresp sides in } \sim \triangle\text{'s})$$

$$\therefore 5w = 12$$

$$w = 2.4 \therefore \underline{BD = 2.4 \text{ cm}}$$

b)  $3kx^2 - 5x + 3k = 0$

i.  $\Delta = b^2 - 4ac$

$$= 25 - 4(3k)(3k)$$

$$\Delta = 25 - 36k^2$$

ii.  $3k < 0$  and  $\Delta < 0$

$$k < 0 \quad (5-6k)(5+6k) < 0$$

$$\therefore \text{Sol}^n: k < -5/6 \text{ only.}$$

A i

### Question 7

$$a) i. PA = \sqrt{(x+4)^2 + (y-0)^2}$$

$$PA^2 = (x+4)^2 + y^2$$

$$PB^2 = (x-4)^2 + (y-3)^2$$

$$ii. PA = 2PB$$

$$PA^2 = 4PB^2$$

$$x^2 + 8x + 16 + y^2 = 4[x^2 - 8x + 16 + y^2 - 6y + 9]$$

$$x^2 + 8x + 16 + y^2 = 4x^2 - 32x + 64 + 4y^2 - 24y + 36$$

$$3x^2 - 40x + 48 + 3y^2 - 24y + 36 = 0$$

$$3x^2 - 40x + 3y^2 - 24y + 84 = 0$$

b) let the roots be  $\alpha$  and  $2\alpha$

$$\alpha + 2\alpha = -b/a$$

$$\alpha \times 2\alpha = c/a$$

$$3\alpha = \frac{3k}{k-3}$$

$$2\alpha^2 = \frac{25}{k-3}$$

②

$$\alpha = \frac{k}{k-3} \quad \text{--- ①}$$

Sub ① into ②

$$2 \left[ \frac{k}{k-3} \right]^2 = \frac{25}{k-3} \quad (k \neq 3)$$

$$2 \cdot \frac{k^2}{(k-3)^2} = \frac{25}{k-3} \quad \times (k-3)$$

$$\frac{2k^2}{k-3} = 25$$

$$2k^2 = 25(k-3)$$

$$2k^2 - 25k + 75 = 0$$

$$(2k-15)(k-5) = 0$$

$\therefore k = 15/2$  and  $k = 5$ .