Sydney Technical High School

Mathematics

YEAR 10 ASSESSMENT TASK 1

JUNE 2013

Instructions

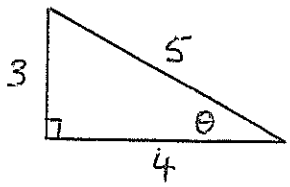
- Time allowed - **70 minutes**
- Show necessary working.
- Use a pen only and a ruler for straight lines.
- Marks shown are a guide and may need to be adjusted.
- Full marks may not be awarded for careless work or illegible answers.

Name_____

Teacher_____

Question 1 Trig.	Question 2 Sim. Eqns.	Question 3 Quad. Eqns.	Question 4 Prob.	Question 5 Consumer	Question 6 No. Plane	TOTAL
/14	/10	/17	/11	/10	/18	/80

QUESTION 1 (14 marks)

a) 

Find the value of:

i) $\tan\theta$ _____ ii) $\sin(90° - \theta)$ _____ 2

b) Evaluate $\tan 36°15'$ _____(2 dec. places) 1

c) Find θ, correct to the nearest minute, given that $\cos\theta = 0 \cdot 4$ _____ 1

d) In ΔPQR, $\angle R = 90°$ and $\cos P = {}^1/_2$. Draw a diagram and find:

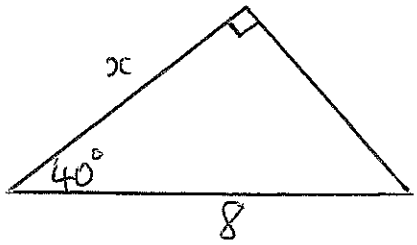
i) $\sin P$ _____ 1

ii) the size of angle Q_____ 1

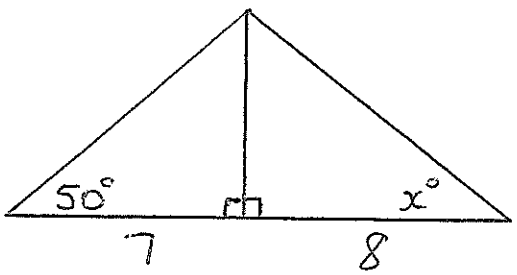
e) Find x, correct to 1 decimal place:

i) $\sin 42° = \dfrac{6.5}{x}$

Answer _____ 1

ii) 

Answer _____ 1

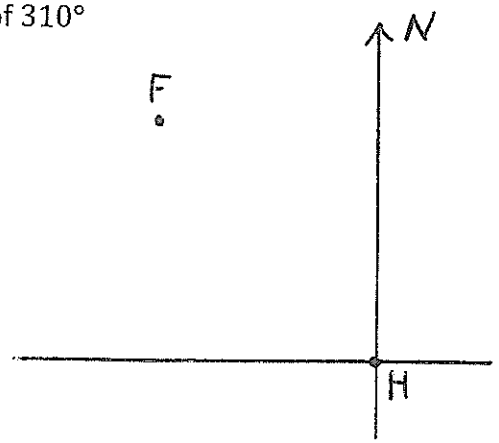
iii) 

Answer _____ 3

(care with rounding)

f) A boat travels from its harbour at H on a bearing of 310°

to its fishing grounds at F, 200 km away.



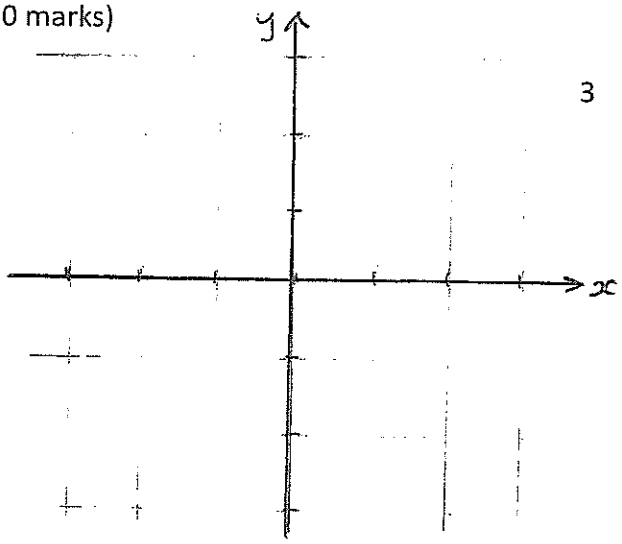
i) How far west of H is F? (nearest km) 2

ii) What is the bearing of H from F?

Answer _____ 1

QUESTION 2 - SIMULTANEOUS EQUATIONS (10 marks)

a) Find the simultaneous solution to $y = 2x$ and $y = -x - 3$ using their graphs. 3



b) Solve simultaneously $3x - 2y = 4$ and $x - y = 3$ using the substitution method. 2

c) Solve simultaneously $3x - 5y + 5 = 0$ and $5x - 6y - 8 = 0$ using the elimination method. 2

d) For tickets to a maths concert, x adults pay \$23 each and y children pay \$7 each. A total of \$1866 is collected from ticket sales and 150 tickets are sold.

Write and solve two equations in x and y to find the number of adults and the number of children attending the concert. 3

QUESTION 3 – QUADRATIC EQUATIONS (17 marks)

a) Solve each quadratic equation. Leave all answers in exact form. 10

i) $2x^2 = 50$ ii) $x^2 - 2x = 0$ iii) $x^2 - 2x - 8 = 0$

iv) $(2x-5)^2 = 47$

v) $\frac{3}{4-x} = x$

b) Use the quadratic formula to solve $2x^2 - x - 4 = 0$. Give your answer correct to 2 decimal places. 2

c) A son's age is x and his father's age is $(x + 30)$. The product of their ages is eight times the sum of their ages.

i) Write an equation in x to represent the above information. 2

ii) Solve this equation to find the son's age. 2

d) A quadratic equation $ax^2 + bx + c = 0$ has solution $x = \frac{3 \mp \sqrt{29}}{2}$. Find the value of c.

1

QUESTION 4 – PROBABILITY (11 marks)

a) Two regular dice are tossed and the sum of their scores is noted. Use a lattice, grid, diagram (or otherwise) to find:

i) the number of elements in the sample space? _____ 1

ii) the probability of obtaining a sum of 7 or 12? _____ 1

b) Two dice, numbered 0 to 5 on the faces, are tossed and the product of the two scores is recorded. What is the probability of recording a score of zero?

Answer _____ 1

c) A family has 3 children. By considering all possible arrangements, or a tree diagram, what is the probability of having: i) exactly 1 boy? Answer _____ 1

ii) at least 1 boy? Answer _____ 1

d) Four cards are marked with a 6, 7, 8 or 9 and placed in a hat. One card is drawn at random to give the tens-digit of a 2-digit number. The card is not returned to the hat. A second card is drawn to give the units-digit for the 2-digit number.

i) List all 2-digit numbers possible. 1

ii) What is the probability that a number formed is larger than 70? _____ 1

e) We say, with certainty, that the probability of obtaining Heads in a coin toss is $^1/_2$. However, you may toss a coin 100 times and obtain 77 Heads. Briefly explain the apparent contradiction. 1

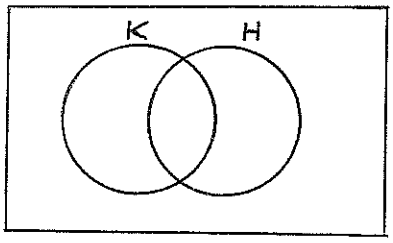
_____

_____

_____

f) In a class of 30 students, it is found that 17 students love K-POP, 20 love HIP-HOP, 9 love both and 2 love neither (they prefer Coco-Pops!)

i) Complete the Venn diagram with the appropriate numbers. 1



ii) One student is selected at random. What is the probability that the student loves K-POP or HIP-HOP, but not both? _____ 1

g) A drawer contains 5 pairs of socks, each pair a different colour, and arranged as 10 loose socks. Two socks are withdrawn at random. What is the probability that a matching pair of socks is withdrawn?

_____ 1

QUESTION 5 – CONSUMER (10 marks)

a) $2000 is invested for 15 years and earns 5% p.a. interest. Find the total interest earned (to the nearest dollar), if the interest is:

i) simple interest 1 ii) compound interest (annually) 2

b) The formula for depreciation is $A = P(1 - r)^n$.

A car, worth $30 000 when new, depreciates to $10 000 after 4 years. Find the annual percentage rate of depreciation. 2

Answer _____

c) A loan of $50 000 is repaid to the bank over a number of years. The bank charges reducible interest at the rate of 9% p.a.

At the end of each year, interest is first charged on the balance owing and then a repayment of $9935 is paid to the bank. What is the loan amount (balance) still owed to the bank after:

i) 1 year 1 ii) 3 years 2

iii) After 7 annual repayments, the loan is fully repaid. Find the equivalent annual percentage rate of flat (simple) interest for this loan (1 dec. place). 2

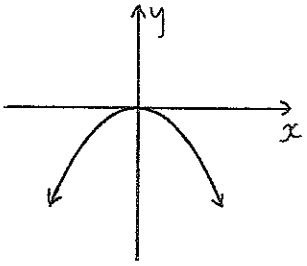
Answer _____

QUESTION 6 – NUMBER PLANE GRAPHS (18 marks)

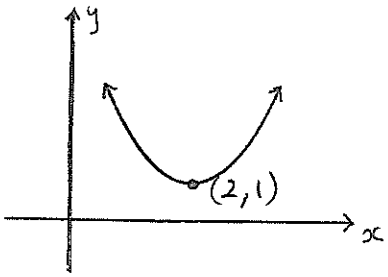
a) Each parabola shown is a result of shifting the basic $y = x^2$ parabola.

Write the equation for each new parabola.

i)

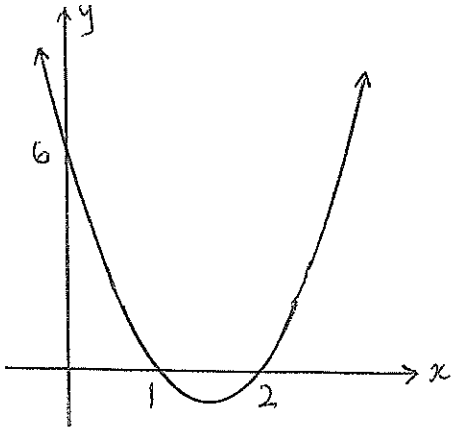


ii)



Answers _____ _____ 2

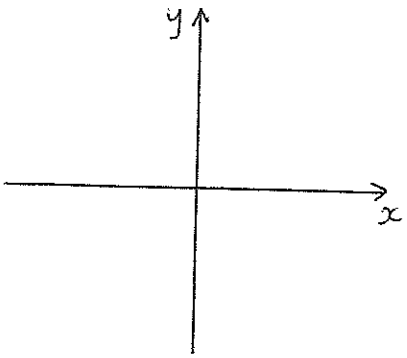
b) Find the equation of this parabola:



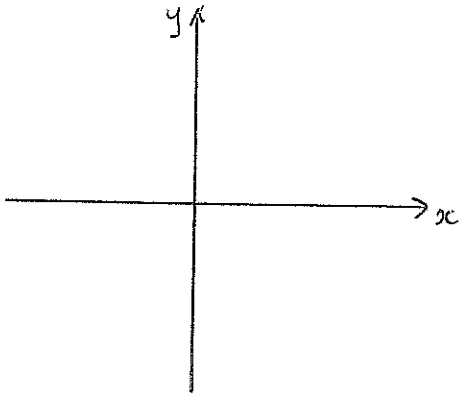
Answer _____ 2

c) Sketch each parabola. Clearly show intercepts on the axes and the coordinates of the vertex.

i) $y = (x+2)^2$ 2



ii) $y = 3x - x^2$ 2

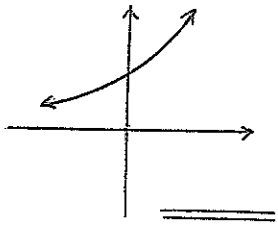


d) Find the maximum value of $6 + 2x - x^2$

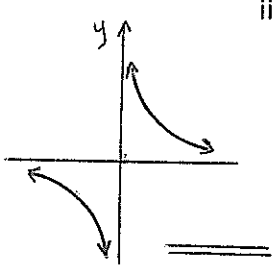
Answer _____ 2

e) Match each curve or line with the appropriate equation from A – J below: 2

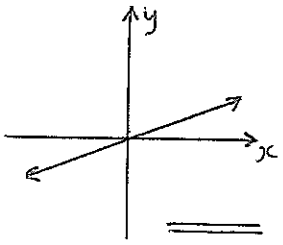
i)



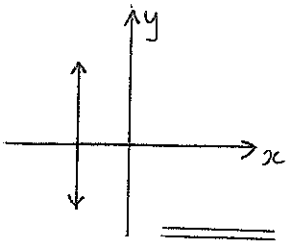
ii)



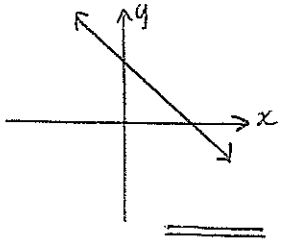
iii)



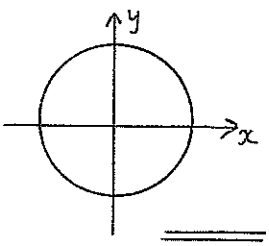
iv)



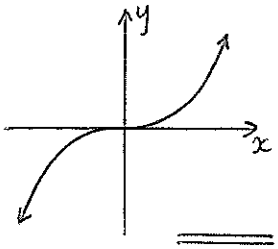
v)



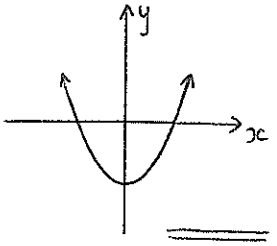
vi)



vii)



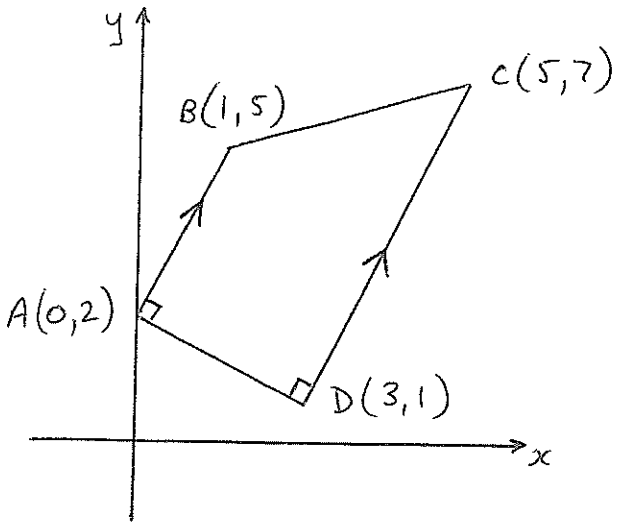
viii)



A. $y = \frac{x}{2}$ *B.* $y = \frac{2}{x}$ *C.* $x^2 - y^2 = 2$ *D.* $x + y - 2 = 0$ *E.* $y = 2^x$

F. $y = \frac{x^3}{2}$ *G.* $x - y + 2 = 0$ *H.* $x^2 + y^2 = 2$ *I.* $x + 2 = 0$ *J.* $x^2 = y + 1$

f) The trapezium $ABCD$ has vertices as shown, with $AB \parallel DC$. AD is perpendicular to both AB and DC.



i) Find the gradient of AD. 1

ii) Find the length of DC (in exact form) 1

iii) Find the area of $ABCD$. 3

iv) What are the coordinates of a new point M so that $MBCD$ forms a parallelogram? 1

END OF TEST (yay)

Sydney Technical High School

SOLUTIONS

Mathematics

YEAR 10 ASSESSMENT TASK 1

JUNE 2013

Instructions

- Time allowed - **70 minutes**
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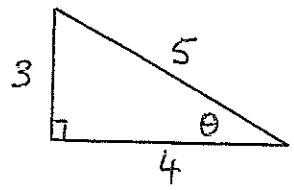
Name_____

Teacher_____

Question 1 Trig.	Question 2 Sim. Eqns.	Question 3 Quad. Eqns.	Question 4 Prob.	Question 5 Consumer	Question 6 No. Plane	TOTAL
/14	/10	/17	/11	/10	/18	/80

QUESTION 1 (14 marks)

a) 

Find the value of:

i) $\tan\theta$ $\frac{3}{4}$ ii) $\sin(90° - \theta)$ $\frac{4}{5}$ 2

b) Evaluate $\tan 36°15'$ 0·73 (2 dec. places) 1

c) Find θ, correct to the nearest minute, given that $\cos\theta = 0 \cdot 4$ $66°25'$ 1

d) In ΔPQR, $\angle R = 90°$ and $\cos P = \frac{1}{2}$. Draw a diagram and find:

i) $\sin P$ $\frac{\sqrt{3}}{2}$ 1

ii) the size of angle Q $30°$ 1

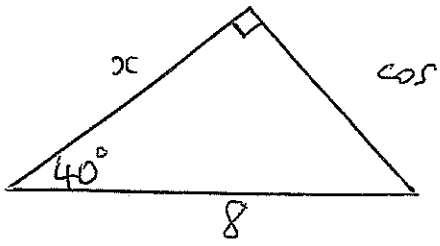
e) Find x, correct to 1 decimal place:

i) $\sin 42° = \frac{6.5}{x}$

$x = \frac{6.5}{\sin 42°}$

$= 9.7$

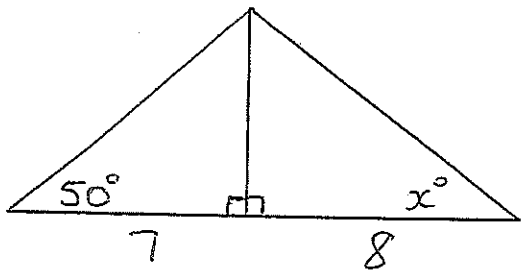
Answer 9.7 1

ii) 

$\cos 40° = \frac{x}{8}$

$x = 8\cos 40°$

Answer 6.1 1

iii) 

$\tan 50 = \frac{m}{7}$

$m = 7\tan 50$ ①

$\tan x = \frac{7\tan 50}{8}$ ①

$= 46.2$

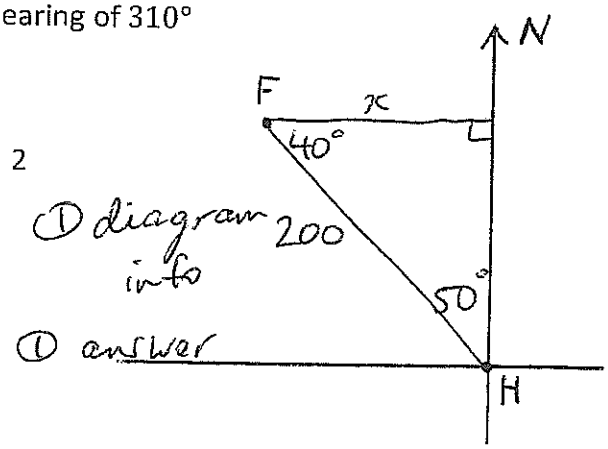
Answer 46.2 3

(care with rounding)

(2 marks for 46.1)

f) A boat travels from its harbour at H on a bearing of 310°

to its fishing grounds at F, 200 km away.



① diagram info

① answer

i) How far west of H is F? (nearest km) 2

$\sin 50 = \frac{x}{200}$

$x = 200 \sin 50$

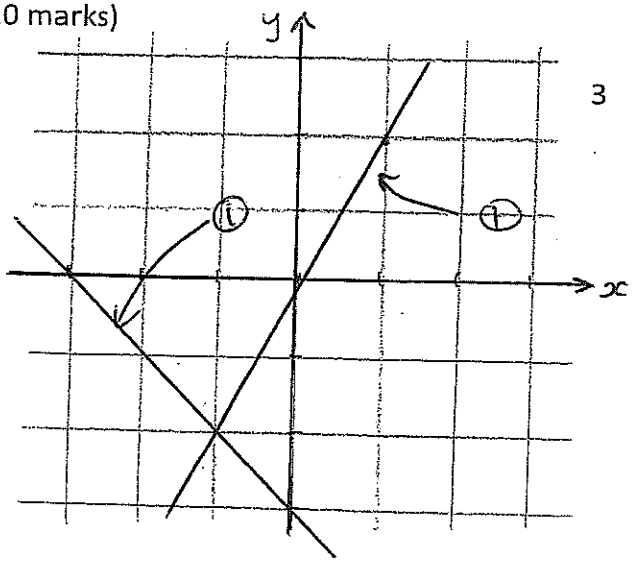
$= 153 \text{ km}$

ii) What is the bearing of H from F?

Answer 130° 1

QUESTION 2 - SIMULTANEOUS EQUATIONS (10 marks)

a) Find the simultaneous solution to $y = 2x$ and $y = -x - 3$ using their graphs. 3



$x = -1, y = -2$

or $(-1, -2)$ ①

b) Solve simultaneously $3x - 2y = 4$ and $x - y = 3$ using the substitution method. 2

$3x - 2y = 4$ — ① $\quad x - y = 3$ — ②

From ②: $x = y + 3$

Sub in ①: $3(y+3) - 2y = 4$

$3y + 9 - 2y = 4$

$\boxed{y = -5}$

1 mark method

1 answers.

Sub in ②: $x + 5 = 3$

$\boxed{x = -2}$

c) Solve simultaneously $3x - 5y + 5 = 0$ and $5x - 6y - 8 = 0$ using the elimination method. 2

$3x - 5y + 5 = 0$ —①

$5x - 6y - 8 = 0$ —②

1 mark correct method

① × 5 : $15x - 25y + 25 = 0$ —③

② × 3 : $15x - 18y - 24 = 0$ —④

1 mark answers

③ − ④ : $-7y + 49 = 0$

$\boxed{y = 7}$ Sub in ① : $3x - 35 + 5 = 0$

$\boxed{x = 10}$

d) For tickets to a maths concert, x adults pay \$23 each and y children pay \$7 each. A total of \$1866 is collected from ticket sales and 150 tickets are sold.

Write and solve two equations in x and y to find the number of adults and the number of children attending the concert. 3

$23x + 7y = 1866$ —① $x + y = 150$ —②

From ② : $y = 150 - x$ 1 mark equations

Sub in ① : $23x + 7(150 - x) = 1866$ 1 mark method

$16x = 816$ 1 mark answer

$x = 51$ (and $y = 99$)

∴ 51 adults, 99 children

QUESTION 3 – QUADRATIC EQUATIONS (17 marks)

a) Solve each quadratic equation. Leave all answers in exact form. 10

i) $2x^2 = 50$

$x^2 = 25$

$\therefore x = \pm 5$

ii) $x^2 - 2x = 0$

$x(x - 2) = 0$

$\therefore x = 0$ or 2

iii) $x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$

$\therefore x = 4, -2$

2 marks each

iv) $(2x-5)^2 = 47$

$2x-5 = \pm\sqrt{47}$

$2x = 5 \pm \sqrt{47}$

$\therefore x = \frac{5 \pm \sqrt{47}}{2}$

v) $\frac{3}{4-x} = x$

$3 = x(4-x)$

$3 = 4x - x^2$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$\therefore x = 1, 3$

2 marks each

b) Use the quadratic formula to solve $2x^2 - x - 4 = 0$. Give your answer correct to 2 decimal places. 2

$x = \frac{1 \pm \sqrt{1-4\times2\times(-4)}}{4}$

1 mark formula

1 mark answers

$= \frac{1 \pm \sqrt{33}}{4}$ $= 1.69$ or -1.19

c) A son's age is x and his father's age is $(x+30)$. The product of their ages is eight times the sum of their ages.

i) Write an equation in x to represent the above information. 2

$x(x+30) = 8(x+x+30)$ ①①

or $x^2 + 14x - 240 = 0$ or equivalent

ii) Solve this equation to find the son's age. 2

$x^2 + 14x - 240 = 0$

$(x-10)(x+24) = 0$ ①

$\therefore x = 10$ or -24 (no solution)

$\therefore$ Son is 10 y.o. ①

d) A quadratic equation $ax^2 + bx + c = 0$ has solution $x = \frac{3 \mp \sqrt{29}}{2}$. Find the value of c. 1

$b = -3$, $a = 1$, $b^2 - 4ac = 29$

$\therefore 9 - 4c = 29$

$4c = -20$

$\therefore c = -5$ ①

QUESTION 4 – PROBABILITY (11 marks)

a) Two regular dice are tossed and the sum of their scores is noted. Use a lattice, grid, diagram (or otherwise) to find:

i) the number of elements in the sample space? 36 1

ii) the probability of obtaining a sum of 7 or 12? $\frac{7}{36}$ 1

43 61
34 16
52 66
25

b) Two dice, numbered 0 to 5 on the faces, are tossed and the product of the two scores is recorded. What is the probability of recording a score of zero?

00 10 20 30 40 50
01
02
03
04
05

Answer $\frac{11}{36}$ 1

c) A family has 3 children. By considering all possible arrangements, or a tree diagram, what is the probability of having: i) exactly 1 boy? Answer $\frac{3}{8}$ 1

ii) at least 1 boy? Answer $\frac{7}{8}$ 1

d) Four cards are marked with a 6, 7, 8 or 9 and placed in a hat. One card is drawn at random to give the tens-digit of a 2-digit number. The card is not returned to the hat. A second card is drawn to give the units-digit for the 2-digit number.

i) List all 2-digit numbers possible. 1

67 76 86 96
68 78 87 97
69 79 89 98

ii) What is the probability that a number formed is larger than 70? $\frac{9}{12}$ or $\frac{3}{4}$ 1

e) We say, with certainty, that the probability of obtaining Heads in a coin toss is $\frac{1}{2}$. However, you may toss a coin 100 times and obtain 77 Heads. Briefly explain the apparent contradiction. 1

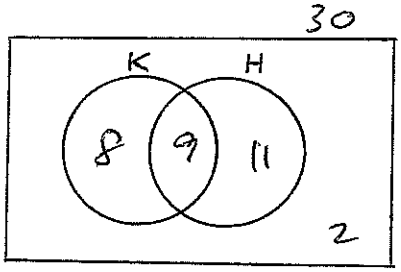
$\frac{1}{2}$ is based on very many tosses

OR 100 tosses is far too small

OR as the no. of tosses increases, prob. gets closer to $\frac{1}{2}$

f) In a class of 30 students, it is found that 17 students love K-POP, 20 love HIP-HOP, 9 love both and 2 love neither (they prefer Coco-Pops!)

i) Complete the Venn diagram with the appropriate numbers. 1



ii) One student is selected at random. What is the probability that the student loves K-POP or HIP-HOP, but not both? $\frac{19}{30}$ 1

g) A drawer contains 5 pairs of socks, each pair a different colour, and arranged as 10 loose socks. Two socks are withdrawn at random. What is the probability that a matching pair of socks is withdrawn? $\frac{1}{9}$ 1

first any 1 sock is drawn out

∴ chance of choosing a match sock is $\frac{1}{9}$

QUESTION 5 – CONSUMER (10 marks)

a) $2000 is invested for 15 years and earns 5% p.a. interest. Find the total interest earned (to the nearest dollar), if the interest is:

i) simple interest 1

$0.05 \times 2000 \times 15$

$= \$1500$

ii) compound interest (annually) 2

$A = 2000(1.05)^{15}$

$= 4157$ ①

$\therefore$ comp. int. $= \$2157$ ①

b) The formula for depreciation is $A = P(1-r)^n$.

A car, worth $30 000 when new, depreciates to $10 000 after 4 years. Find the annual percentage rate of depreciation. 2

$10000 = 30000(1-r)^4$

$(1-r)^4 = \frac{1}{3}$

$1 - r = \sqrt[4]{\frac{1}{3}}$ ①

$\therefore r \doteq 0.24$

Answer 24% ①

c) A loan of $50 000 is repaid to the bank over a number of years. The bank charges reducible interest at the rate of 9% p.a.

At the end of each year, interest is first charged on the balance owing and then a repayment of $9935 is paid to the bank. What is the loan amount (balance) still owed to the bank after:

i) 1 year 1

$50000 \times 1.09 - 9935$

$= \$44565$

ii) 3 years 2

$44565 \times 1.09 - 9935$

① → $= \$38640.85$

ECF?

$38640.85 \times 1.09 - 9935$

① → $= \$32183.53$

iii) After 7 annual repayments, the loan is fully repaid. Find the equivalent annual percentage rate of flat (simple) interest for this loan (1 dec. place). 2

Total interest paid $= 7 \times 9935 - 50000$

$= \$19545$ ← ①

Using $PRN = 19545$

$50000 \times R \times 7 = 19545$

$\therefore R = \frac{19545}{350000}$

$\doteq 0.056$

① ✓

Answer 5.6%

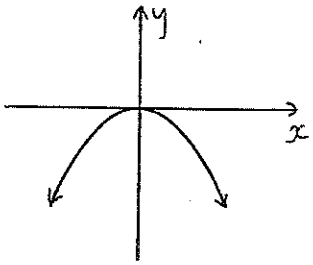
(+ accept 5.5%)

QUESTION 6 – NUMBER PLANE GRAPHS (18 marks)

a) Each parabola shown is a result of shifting the basic $y = x^2$ parabola.

Write the equation for each new parabola.

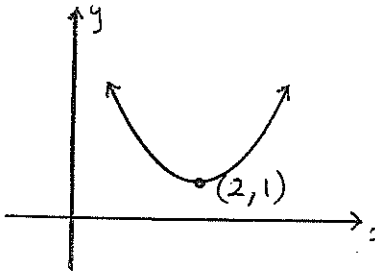
i)



(1)

Answers $y = -x^2$

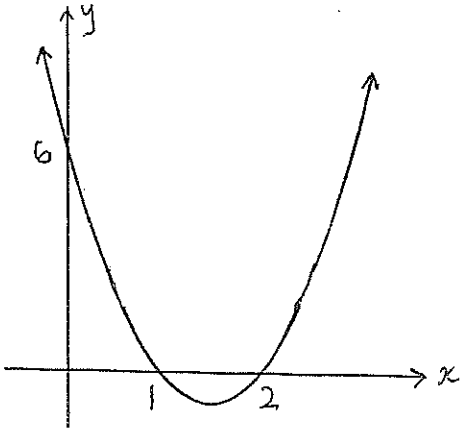
ii)



(1)

$y = (x-2)^2 + 1$ 2

b) Find the equation of this parabola:



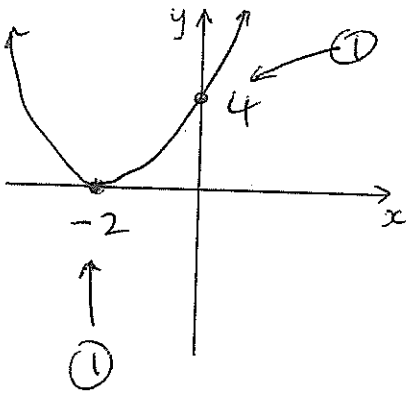
① ①

Answer $y = 3(x-1)(x-2)$ 2

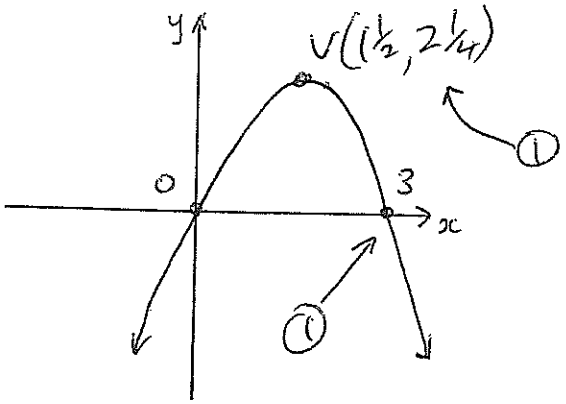
or $y = 3x^2 - 9x + 6$

c) Sketch each parabola. Clearly show intercepts on the axes and the coordinates of the vertex.

i) $y = (x+2)^2$ 2



ii) $y = 3x - x^2$ 2



d) Find the maximum value of $6 + 2x - x^2$

$x = \frac{-b}{2a}$

$= \frac{-2}{-2}$ 1

$= 1$

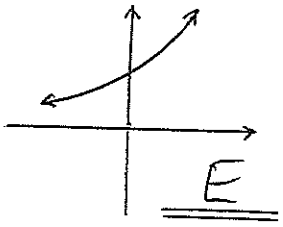
max $= 6 + 2 - 1$

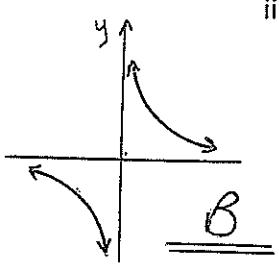
$= 7$ 1

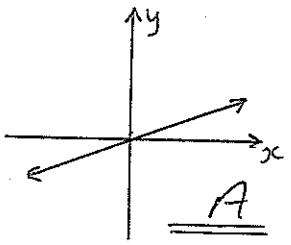
Answer 7 2

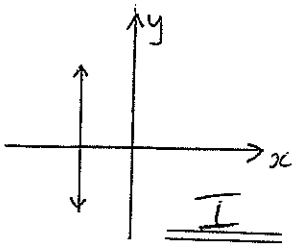
1 mark for (1, 7)

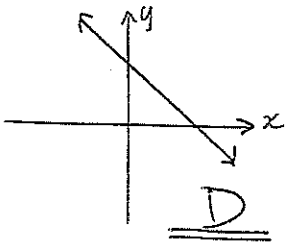
e) Match each curve or line with the appropriate equation from A – J below: 2

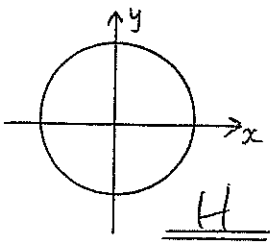
i) 

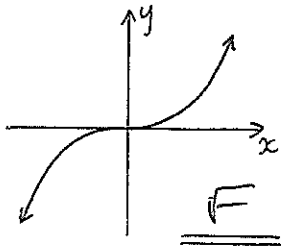
ii) 

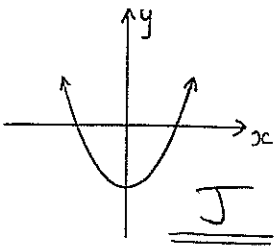
iii) 

iv) 

v) 

vi) 

vii) 

viii) 

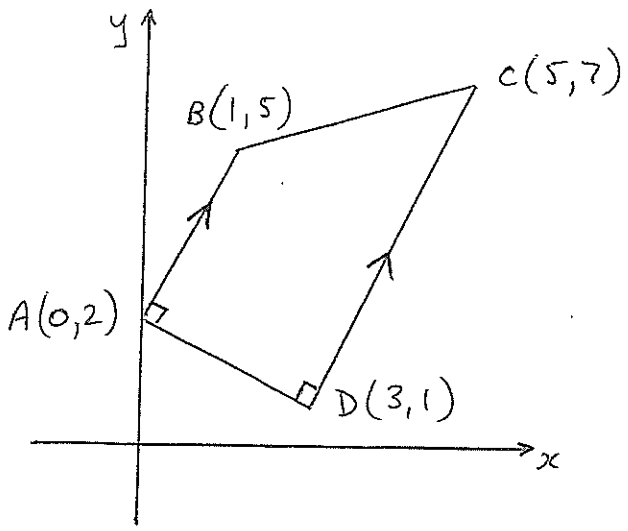
A. $y = \frac{x}{2}$ B. $y = \frac{2}{x}$ C. $x^2 - y^2 = 2$ D. $x + y - 2 = 0$ E. $y = 2^x$

F. $y = \frac{x^3}{2}$ G. $x - y + 2 = 0$ H. $x^2 + y^2 = 2$ I. $x + 2 = 0$ J. $x^2 = y + 1$

1 mark for 4 correct
2 marks for 8 correct

P. (

f) The trapezium $ABCD$ has vertices as shown, with $AB \parallel DC$. AD is perpendicular to both AB and DC.



i) Find the gradient of AD. 1

$$m_{AD} = \frac{2-1}{0-3}$$

$$= -\frac{1}{3}$$

ii) Find the length of DC (in exact form) 1

$$DC = \sqrt{4+36}$$

$$= \sqrt{40} \quad \text{or} \quad 2\sqrt{10}$$

iii) Find the area of $ABCD$. 3

① ①

Need $AB = \sqrt{10}$, $AD = \sqrt{10}$

$\therefore$ area $ABCD = \frac{1}{2} \times \sqrt{10}\left(\sqrt{10} + 2\sqrt{10}\right)$

$$= \frac{\sqrt{10}}{2}\left(3\sqrt{10}\right)$$

$$= 15 \text{ u}^2$$ ①

iv) What are the coordinates of a new point M so that $MBCD$ forms a parallelogram? 1

M is $(-1,-1)$

END OF TEST (yay)