Sydney Technical High School

Mathematics

YEAR 10 ASSESSMENT TASK 2

SEPTEMBER 2013

Instructions

- Time allowed – **70 minutes**.
- Show necessary working.
- Use a pen only and a ruler for straight lines.
- Marks shown are a guide and may need to be adjusted.
- Full marks may not be awarded for careless work or illegible answers.

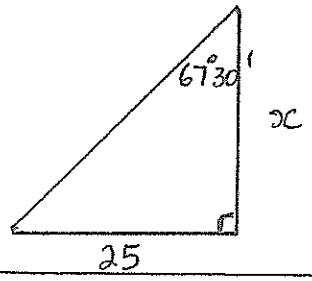
Name: _____

Teacher: _____

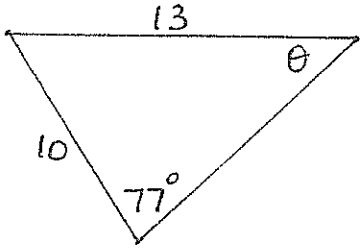
Question 1 Trigonometry	Question 2 Statistics	Question 3 Surface Area/Volume	Question 4 Number Plane	Question 5 Miscellaneous	Total
/15	/15	/16	/17	/17	/80

Question 1 : Trigonometry

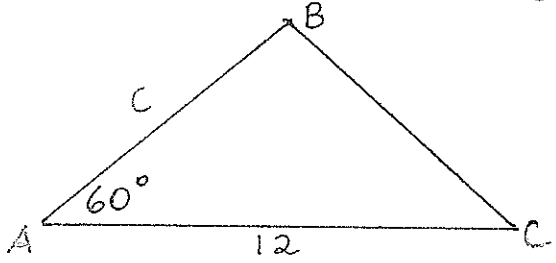
a) Find x correct to 1 decimal place 2



b) Find θ to the nearest minute 2



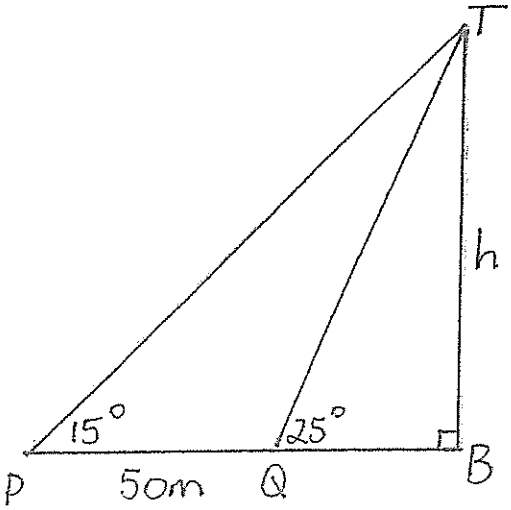
c) Find c if the area of the triangle is $30cm^2$ 2



d) A man is sitting in a boat at P, where the angle of elevation of the top T of a vertical cliff BT is 15°. He then rows 50 metres directly towards the cliff to Q, where the angle of elevation of T is 25°.

i. Show that $TQ = \frac{50sin15^\circ}{sin10^\circ}$ (2marks)

ii. Hence find the height of the cliff, correct to the nearest tenth of a metre.(2marks)

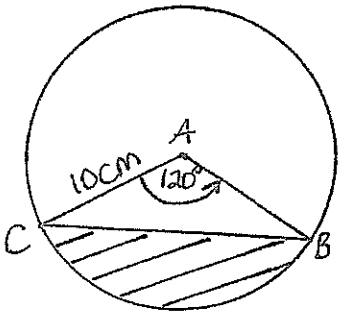


e) The sides of a triangular field have lengths 80m, 90m and 100m. Calculate the size of the largest angle the field makes correct to the nearest minute. 2

---

f) Given that A is the centre of the circle with dimensions shown, calculate correct to 1 decimal place.

i. The area of the sector ABC 1



ii. The area of the triangle ABC 1

iii. The area of the shaded segment 1

Question 2 : Statistics

a) **(6 marks)** The following data gives the ages of students in a karate class.

21 14 13 16 16 15
13 10 13 18 12 14

Using the scores above, find the:

i. Mean _____

ii. Median _____

iii. Mode _____

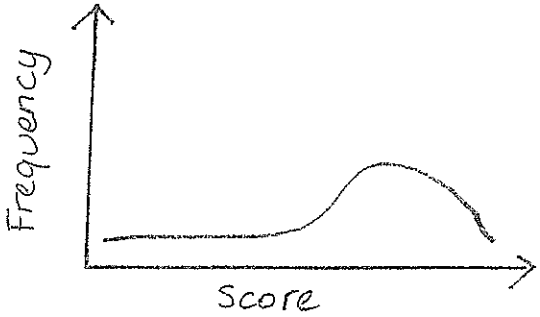
iv. Range _____

v. Standard Deviation _____
(correct to 3 decimal places)

vi. Interquartile range _____

---

b) With reference to the graph below, the scores in the distribution are either positively skewed, negatively skewed or symmetrical. Which one is it?



_____1

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c) If every score in a distribution is 5, what would be the standard deviation

(A) O (B) 1 (C) 2.5 (D) 5 _____1

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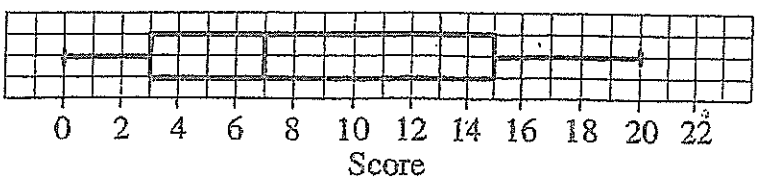
d) For the given box and whisker plot, find the inter-quartile range.



_____ 1

e) Use the mean and standard deviation to determine which Test is the better result.

	Mark	Mean	Standard Deviation
Test 1	70	88	12
Test 2	69	83	7

f) The results of the fastest thirty runners in the Masters Marathon are recorded below.

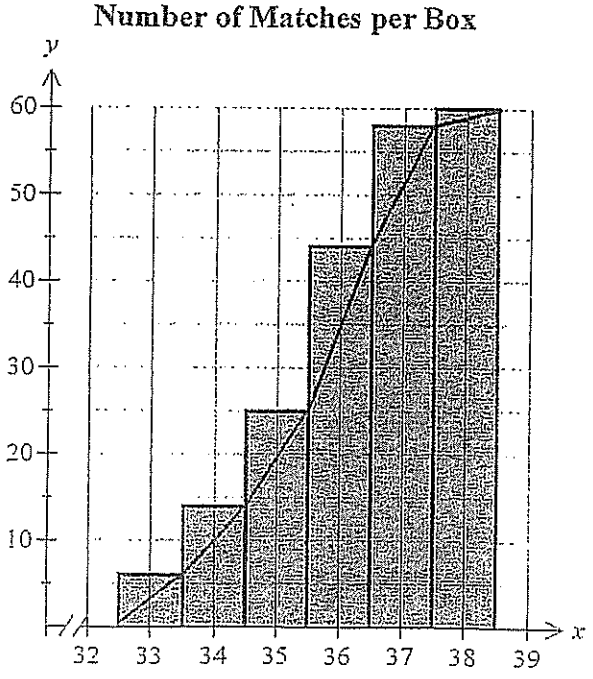
Class (minutes)	Class Centre	Number of Runners
141-145		1
146-150		2
151-155		1
156-160		k
161-165		11
166-170		9

i. Find the value of K _____ 1

ii. What is the mean time for these runners using class centres? 2

_____
(correct to 1 decimal place)

g) Class 10C of Montrose High School counted the numbers of matches per box. The results are displayed in the cumulative frequency histogram and polygon shown.



i. Find the mode 1

_____

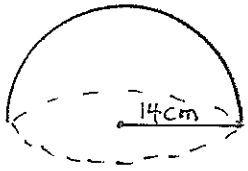
ii. Find the median 1

_____

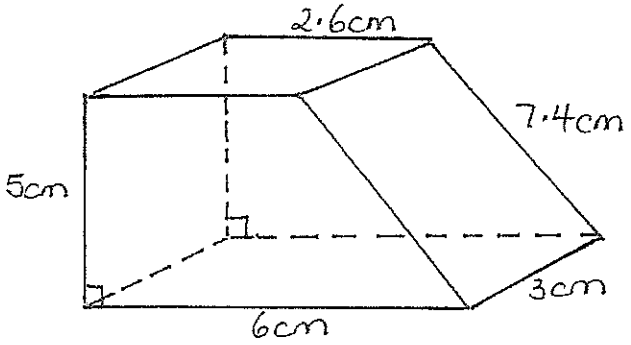
Question 3 : Surface Area/Volume

a) Find correct to 1 decimal place, the radius of a sphere whose volume is $1000cm^3$ 2

b) Find the surface area of the hemisphere below to the nearest cm^2 2



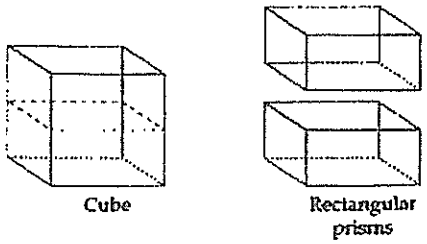
c) Find the volume of 2



d) Find the outside surface area of a cylinder without a lid with radius 4cm and height 2cm. Leave your answer in exact form. 2

e) Find the surface area of a cube whose volume is $512cm^3$ 2

f) A cube is cut into two rectangular prisms, as shown. 1
The surface area has now increased.



By what fraction has the surface area increased?

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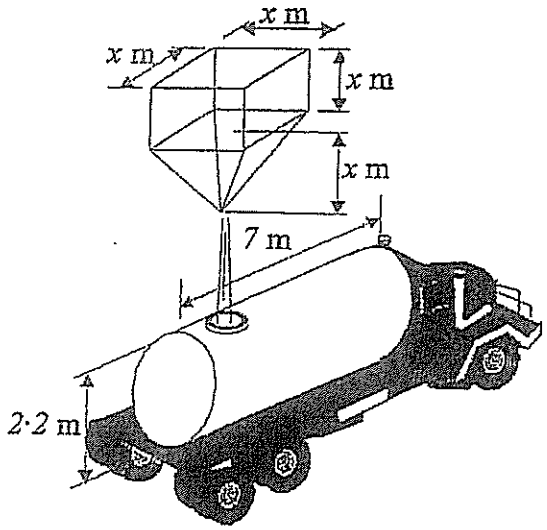
g) A 'hopper' is a hollow storage container. This hopper (as shown in the diagram below) is made by joining a cube and a square pyramid, each of height x m.
It is used to fill a cylindrical tank of diameter $2.2m$ and length $7m$.



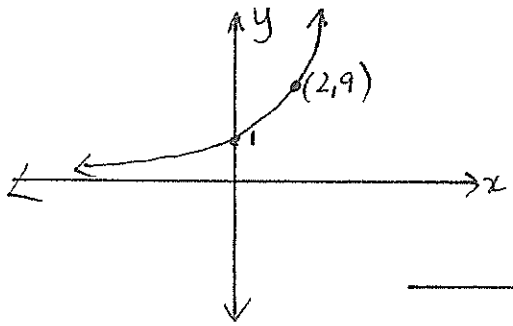
i. Find the volume of the cylindrical tank to 1 decimal place 2

ii. Show that the volume (V) of the hopper is given by $V = \frac{4}{3}x^3$ cubic metres. 1

iii. A full load in the hopper exactly fills the tank. Find the value of x 2

Question 4 : Number Plane Graphs

a) The equation of this graph is



A. $y = 2x + 1$ C. $y = 2^x$

B. $y = x^2 + 1$ D. $y = 3^x$

I. _____

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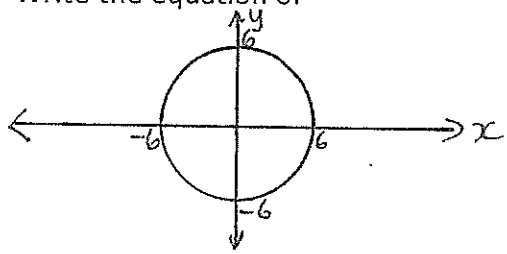
b) Write the equation of



1

_____

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c) Given $y = ax^2 + C$ and the table of values below,
Find the value of the co-efficient a

x	0	1	2
y	4	9	24

1

_____

---

d) Consider the graph of the equation $y = -\frac{1}{x}$

I. In which two quadrants does the curve lie? _____ 1

II. Does the graph have any asymptotes? _____ 1
If so, write down the equation/s.

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e) For the graph $y = 2^{-x}$

I. Describe what happens to the y values as the x valves increase 1

_____

II. Where does the curve cut the y axis? 1

_____

f) For the parabola $y = -2x^2 + 5x - 3$

I. Find the axis of symmetry

1

II. Hence find the co-ordinates of its vertex

2

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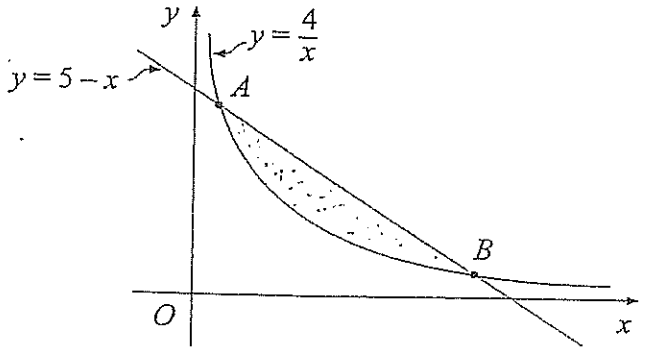
g)



NOT TO SCALE

The diagram shows the graphs of $y = \frac{4}{x}$ and $y = 5 - x$.

The graphs intersect at the point A and B as shown.

Find the x coordinates of the points A and B. 2

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h) The point $(k, -3)$ lies on the line $3x - 5y - 21 = 0$. Find the value of k 1

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i) $(6, -2)$ is the midpoint of $P\ (x, y,)$ and Q (1,4). Find the co-ordinates of P. 2

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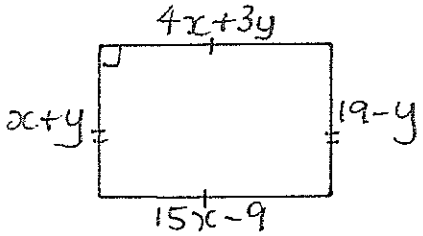
j) Find the equation of the line through (2,1) and perpendicular to $y = \frac{1}{3}x - 5$ 2

Question 5 : Miscellaneous

a) Simplify $(\sqrt{6}-2\sqrt{2})^2$ 2

b) Solve $10x^2 - 53x + 36 = 0$ 2

c) Solve x and y for the diagram below 3



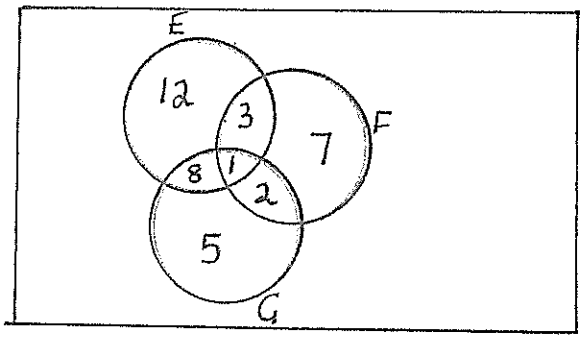
d) Simplify $\frac{1}{3a+6} + \frac{1}{a^2-4}$ 2

e) Michael bought an LED television priced at \$3600.
He makes 24 monthly payments of \$159.75.

i. How much does Michael pay for the television? 1

ii. What is the rate of simple interest charged per year? 2

f)



If the sets E, F and G in the diagram represent the languages spoken at a back packer's hotel (E=English, F=French and G=German), what is the probability that a backpacker selected at random from the hotel speaks:

i. English? _____ 1

ii. French but not English? _____ 1

iii. German and English? _____ 1

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g) Find the values of p and q such that 2

$$\frac{\sqrt{5}}{\sqrt{5}-2} = p + q\sqrt{5}$$

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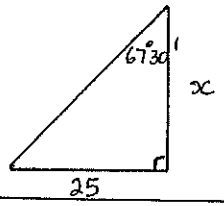
Name: MORRIS

Teacher: _____

Question 1 Trigonometry	Question 2 Statistics	Question 3 Surface Area/Volume	Question 4 Number Plane	Question 5 Miscellaneous	Total
/15	/15	/16	/17	/17	/80

Question 1 : Trigonometry

a) Find x correct to 1 decimal place 2

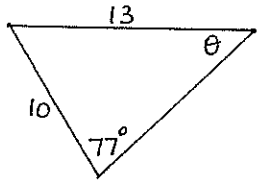


$\frac{25}{x} = \tan 67°30'$

$x = \frac{25}{\tan 67°30'}$

$x = 10.4$

b) Find θ to the nearest minute 2

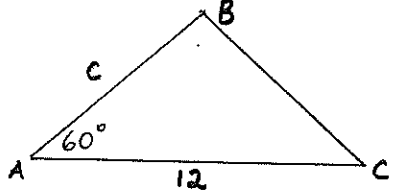


$\frac{\sin\theta}{10} = \frac{\sin 77}{13}$

$\sin\theta = \frac{10\sin 77}{13}$

$\theta = 48°33'$

c) Find c if the area of the triangle is $30cm^2$ 2



$30 = \frac{1}{2} 12c \sin 60$

$5 = c\sin 60$

$c = \frac{5}{\sin 60}$

$c = 5.77$

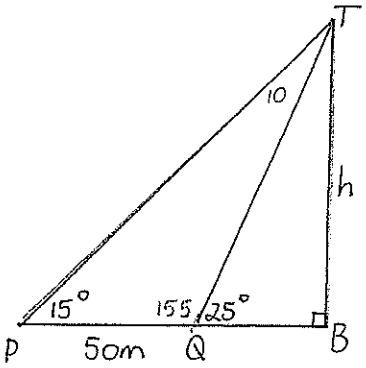
d) A man is sitting in a boat at P, where the angle of elevation of the top T of a vertical cliff BT is 15°. He then rows 50 metres directly towards the cliff to Q, where the angle of elevation of T is 25°.

i. Show that $TQ = \frac{50\sin 15°}{\sin 10°}$ (2marks)

$\frac{TQ}{\sin 15} = \frac{50}{\sin 10}$

$TQ = \frac{50\sin 15}{\sin 10}$

ii. Hence find the height of the cliff, correct to the nearest tenth of a metre.(2marks)

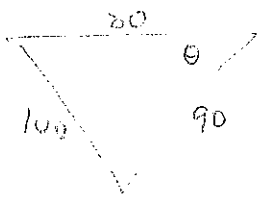


$\frac{h}{TQ} = \sin 25$

$h = TQ \sin 25$

$h = 31.5$

e) The sides of a triangular field have lengths 80m, 90m and 100m. 2
Calculate the size of the largest angle the field makes correct to the nearest minute



$\cos\theta = \frac{80^2 + 90^2 - 100^2}{2 \times 80 \times 90} = \frac{4500}{14400}$

$\theta = 71^\circ 47'$

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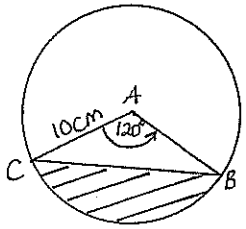
f) Given that A is the centre of the circle with dimensions shown, calculate correct to 1 decimal place.



i. The area of the sector ABC 1

$A_{sector} = \frac{120}{360} \times \pi \times 10^2$

$= 104.72$

ii. The area of the triangle ABC 1

$A = \frac{1}{2} \times 10 \times 10 \times \sin 120$

$A = 43.301 \text{ cm}^2$

iii. The area of the shaded segment 1

$A_{seg} = 104.72 - 43.301$

$= 61.419$

Question 2 : Statistics

a) **(6 marks)** The following data gives the ages of students in a karate class.

21 14 13 16 16 15
13 10 13 18 12 14

10 12 13 13 13 14 14 15 16 16 18 21

Using the scores above, find the:

i. Mean 14.58

ii. Median 14

iii. Mode 13

iv. Range 21 − 10 = 11

v. Standard Deviation 2.783 (correct to 3 dec pl)

vi. Interquartile range 16 − 13 = 3

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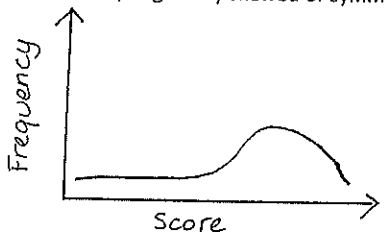
b) With reference to the graph below, the scores in the distribution are either positively skewed, negatively skewed or symmetrical. Which one is it?



negatively skewed 1

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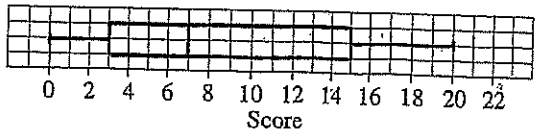
c) If every score in a distribution is 5, what would be the standard deviation

(A) 0 (B) 1 (C) 2.5 (D) 5 A 1

---

d) For the given box and whisker plot, find the inter-quartile range.



15 − 3 = 12 1

e) Use the mean and standard deviation to determine which Test is the better result.

	Mark	Mean	Standard Deviation
Test 1	70	88	12
Test 2	69	83	7

Test 1

f) The results of the fastest thirty runners in the Masters Marathon are recorded below.

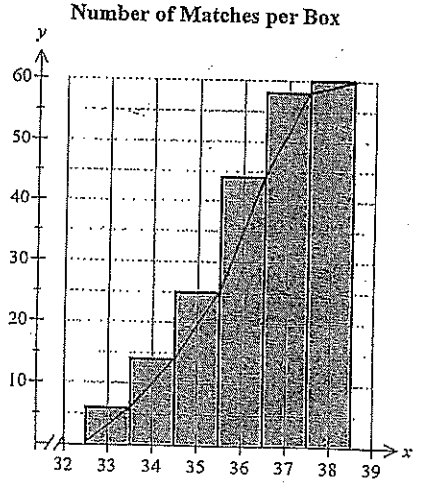
Class (minutes)	Class Centre	Number of Runners
141-145	143	1
146-150	148	2
151-155	153	1
156-160	158	k
161-165	163	11
166-170	168	9

i. Find the value of K 6 1

ii. What is the mean time for these runners using class centres? 2

161.5
(correct to 1 decimal place)

g) Class 10C of Montrose High School counted the numbers of matches per box. The results are displayed in the cumulative frequency histogram and polygon shown.



i. Find the mode 1

36

ii. Find the median 1

36

33 6
34 8
35 11

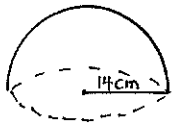
Question 3 : Surface Area/Volume

a) Find correct to 1 decimal place, the radius of a sphere whose volume is $1000cm^3$ 2

$V = \frac{4}{3}\pi r^3$

$1000 = \frac{4}{3}\pi r^3$ $r = 6.2$

b) Find the surface area of the hemisphere below to the nearest cm^2 2

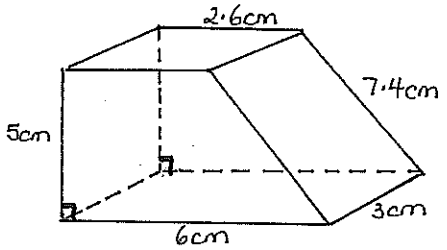


$SA = 2\pi r^2 + \pi r^2 = 3\pi r^2$

$= 3 \times \pi \times 14^2$

$= 1847.25648 cm^2$

c) Find the volume of 2



$V = \frac{1}{2} \times 5(2.6+6) \times 3$

$V = 64.5 cm^3$

d) Find the surface area of a cylinder with radius 4cm and height 2cm. 2
Leave your answer in exact form.

outside without a lid

$SA = \pi r^2 + 2\pi rh$

$= \pi \times 16 + 2 \times \pi \times 4 \times 2$

$= 32\pi$

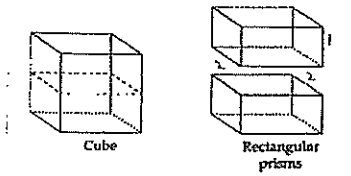
e) Find the surface area of a cube whose volume is $512cm^3$ 2

$s = 8$

$SA = 8 \times 8 \times 6$

$= 384 cm^2$

f) A cube is cut into two rectangular prisms, as shown. 1
The surface area has now increased.



SA of cube $6x^2$
SA of 2 prisms $8x^2$
Increase by
$\frac{8x^2-6x^2}{6x^2} = \frac{2x^2}{6x^2} = \frac{1}{3}$

By what fraction has the surface area increased?

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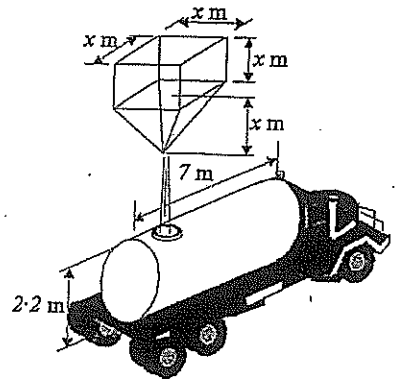
g) A 'hopper' is a hollow storage container. This hopper (as shown in the diagram below) is made by joining a cube and a square pyramid, each of height x m.
It is used to fill a cylindrical tank of diameter $2.2m$ and length $7m$.



i. Find the volume of the cylindrical tank to 1 decimal place 2
$V = \pi \times 1.1^2 \times 7$
$V = 26.609$

ii. Show that the volume (V) of the hopper is given by $V = \frac{4}{3}x^3$ 1
cubic metres.
$V = x \times x \times x + \frac{1}{3} \times x \times x \times x$
$V = x^3 + \frac{1}{3}x^3$
$V = \frac{4}{3}x^3$

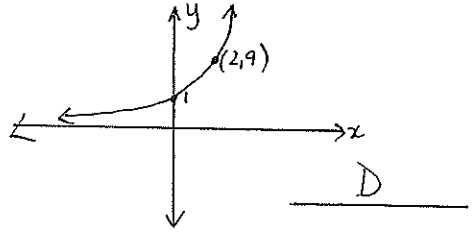
iii. A full load in the hopper exactly fills the tank. Find the value of x 2
$\frac{4}{3}x^3 = 26.609$
$x^3 = 19.95675$
$x = 2.712$

Question 4 : Number Plane Graphs

a) The equation of this graph is

A. $y = 2x + 1$ C. $y = 2^x$

B. $y = x^2 + 1$ D. $y = 3^x$



D

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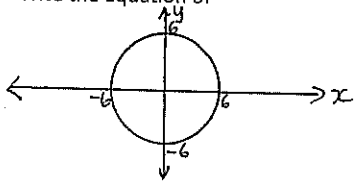
b) Write the equation of 1



$x^2 + y^2 = 36$

---

c) Given $y = ax^2 + C$ and the table of values below, 1
Find the value of the co-efficient a

x	0	1	2
y	4	9	24

$y = ax^2 + 4$
$9 = a + 4$

$a = 5$

---

d) Consider the graph of the equation $y = -\frac{1}{x}$

I. In which two quadrants does the curve lie? 2nd and 4th 1

II. Does the graph have any asymptotes? Yes $x = 0$ $y = 0$ 1
If so, write down the equation/s.

---

e) For the graph $y = 2^{-x}$

I. Describe what happens to the y values as the x valves increase 1
$y \to 0$

II. Where does the curve cut the y axis? 1
$(0,1)$

f) For the parabola $y = -2x^2 + 5x - 3$

I. Find the axis of symmetry $x = \frac{-5}{-4} = \frac{5}{4}$ 1

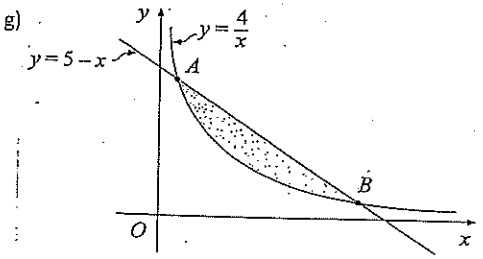
II. Hence find the co-ordinates of its **vertex**

Max when $x = 5/4$ $y = \frac{1}{8}$

$(5/4, \frac{1}{8})$ 2

---

g)



NOT TO SCALE

The diagram shows the graphs of $y = \frac{4}{x}$ and $y = 5 - x$.

The graphs intersect at the point A and B as shown.

Find the x coordinates of the points A and B. 2

$\frac{4}{x} = 5 - x$

$4 = 5x - x^2$

$x^2 - 5x + 4 = 0$

$(x-4)(x-1) = 0$

$x = 4, 1$

---

h) The point $(k, -3)$ lies on the line $3x - 5y - 21 = 0$. Find the value of k 1

$3k + 15 = 21$

$3k = 6$

$k = 2$

---

i) $(6, -2)$ is the midpoint of $P(x, y,)$ and Q (1,4). Find the co-ordinates of P. 2

$\frac{x+1}{2} = 6$ $\quad \frac{y+4}{2} = -2$

$x + 1 = 12$ $\quad y + 4 = -4$

$x = 11$ $\quad y = -8$

$P(11, -8)$

---

j) Find the equation of the line through (2,1) and perpendicular to $y = \frac{1}{3}x - 5$ 2

$m_\perp = -3$

$y - 1 = -3(x - 2)$

$y - 1 = -3x + 6$

$3x + y - 7 = 0$

Question 5 : Miscellaneous

a) Simplify $(\sqrt{6} - 2\sqrt{2})^2$ 2

$= 6 - 4\sqrt{12} + 8$

$= 14 - 8\sqrt{3}$

---

b) Solve $10x^2 - 53x + 36 = 0$ 2

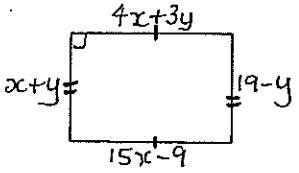
$x = \frac{53 \pm \sqrt{-53^2 - 4.10.36}}{20}$

$x = \frac{53 \pm 37}{20}$

$x = 4.5, 0.8$

---

c) Solve x and y for the diagram below 3



$4x + 3y = 15x - 9$

$11x - 3y = 9$ -①

$x + y = 19 - y$

$x + 2y = 19$ -②

$11x - 3y = 9$

$11x + 22y = 209$

$25y = 200$

$y = 8$

$x + 16 = 19$

$x = 3$

---

d) Simplify $\frac{1}{3a+6} + \frac{1}{a^2-4}$ 2

$\frac{1}{3(a+2)} + \frac{1}{(a-2)(a+2)}$

$= \frac{a - 2 + 3}{3(a+2)(a-2)}$

$= \frac{a+1}{3(a+2)(a-2)}$

---

e) Michael bought an LED television priced at $3600.
He makes 24 monthly payments of $159.75.

i. How much does Michael pay for the television? 1

$= 24 \times 159.75$

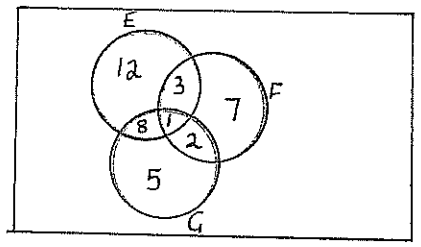
$= \$3834$

ii. What is the rate of simple interest charged per year? 2

$117 = 3600 \times r \times 1$

$r = 3.25\%$

f)



If the sets E, F and G in the diagram represent the languages spoken at a back packer's hotel (E=English, F=French and G=German), what is the probability that a backpacker selected at random from the hotel speaks:

i. English? $\frac{24}{38} = \frac{12}{19}$ 1

ii. French but not English? $\frac{9}{38}$ 1

iii. German and English? $\frac{9}{38}$ 1

---

g) Find the values of p and q such that 2

$$\frac{\sqrt{5}}{\sqrt{5}-2} = p + q\sqrt{5}$$

$$= \frac{\sqrt{5}}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$$

$$= \frac{\sqrt{5}(\sqrt{5}+2)}{5-4}$$

$$= 5 + 2\sqrt{5}$$

$p = 5 \quad q = 2$