Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

Year 12

Mathematics

HSC Course
Assessment 2

March, 2016

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- A BOSTES Formulae booklet is provided at the rear of this Question Booklet, and may be removed at any time.

Section 1 Multiple Choice
Questions 1-5
5 Marks

Section II Questions 6- 9
66 Marks

SECTION I (5 Marks)

Choose the most appropriate answer from the choices, and fill in the circle on the multiple-choice answer sheet provided in your answer booklet

1 For what values of the function $y = x^3 - 6x^2$ is the curve increasing?

A. $x > 0$ **B.** $x < 0$ **C.** $x > 2$ **D.** $x < 0$ or $x > 4$

2 The value of $\int_{-1}^{5} 3x^2 \; dx$ is

A. 126 **B.** 124 **C.** 76 **D.** 74

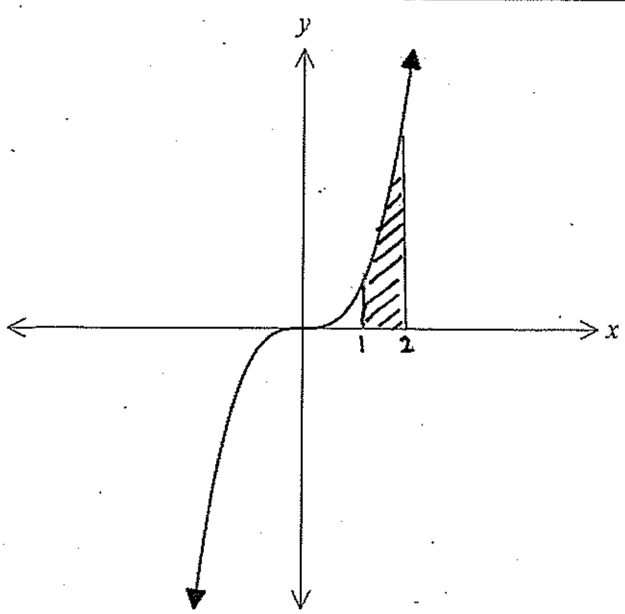
3. The *nth.* term of the series 2, 9, 28 …….. is given by:

A. $7n - 5$ **B.** $2n^2 + 1$ **C.** $(n+1)^3$ **D.** $n^3 + 1$

4 For a certain value $x = m$, it is known that $f'(m) = 0$ and $f''(m) = 0$
This means that at $x = m$ on the curve $y = f(x)$, there is:

A. a horizontal inflection point **B.** an oblique inflection point
C. a turning point **D.** we cannot be certain what there is

5.



In the diagram, at left, the curve shown is that of $y = x^3$

The shaded area is equal to $\mathbf{A} = \int_{1}^{2} f(x)dx$

The Integral given by $\int_{-1}^{2} f(x)dx =$

A. 0

B **A**

C. 2A

D. 3A

SECTION II

Start each new question on a new page

QUESTION 6 (17 Marks)

Marks

(a) Find

(i) $\frac{d}{dx}(2x^3 - 3x + 4)$ (ii) $\frac{d}{dx}\left(\frac{x-1}{x^2+2}\right)$ **3**

(b) Find

(i) $\int 4x^5 \, dx$ (ii) $\int \frac{4x^3+2x}{x} \, dx$ **2**

(c) Find the value of: $\int_0^3 (x+4)^2 \, dx$ **2**

(d) Find the limiting sum of the series $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \ldots\ldots$ **2**

(e) Find $\int \frac{dx}{\sqrt{x}}$ **2**

(f) (i) Find the equation of the tangent to the curve $y = x^2 + 5x - 1$ at the point where $x = 2$. **2**

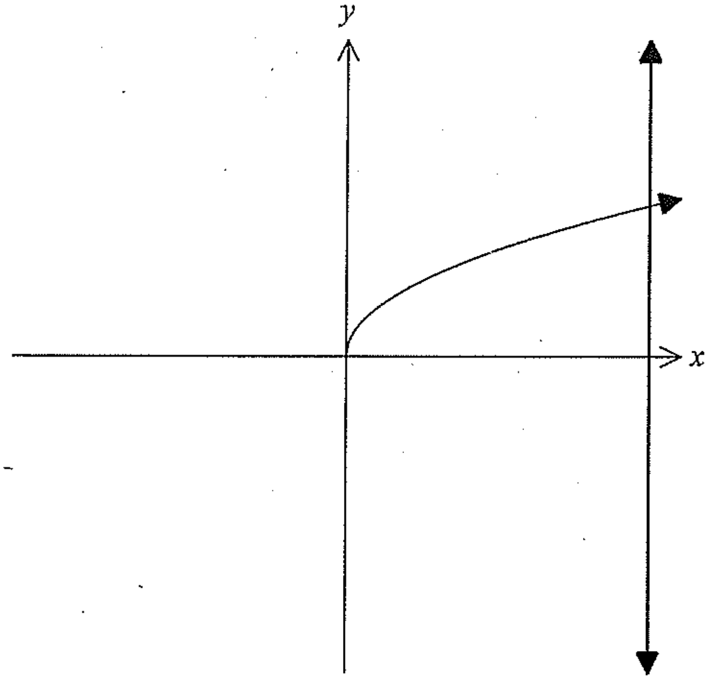
(ii) This tangent cuts the x-axis at T and the y-axis at G. Find the area of ΔOTG where O is the origin **2**

(g) If $\int_0^6 kx^2 dx = 144$, find the value of k **2**

QUESTION 7: (17 Marks) *(Start a New Page)*

Marks

(a)



(i) Find the area enclosed between the curve $y = \sqrt{x}$, the x-axis and the line $x = 4$ as shown above **2**

(ii) The area in part (i) is rotated about the x–axis. Find the volume of the solid so generated **2**

(b) A deposit of \$6500 is made into an account earning an interest rate of 6% pa which is compounded monthly.

(i) How much is the interest rate per month? **1**

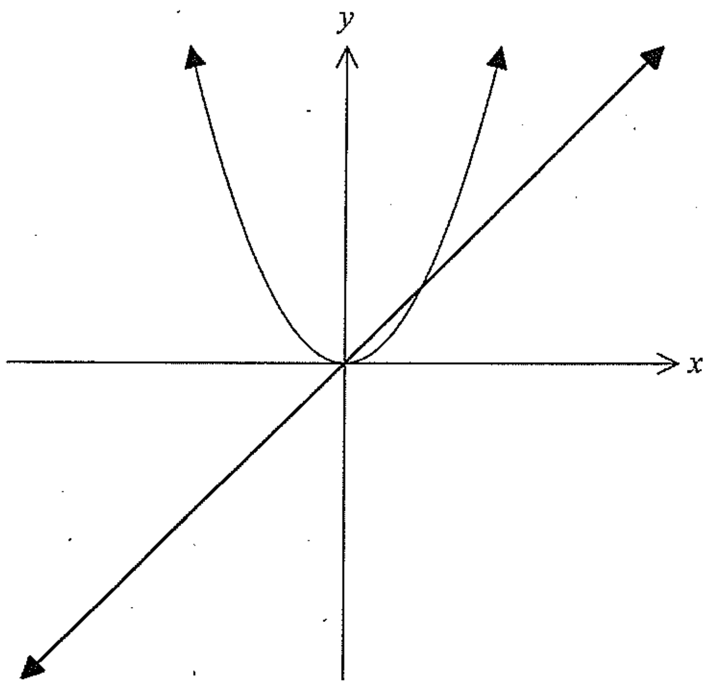
(ii) Calculate the interest earned on the deposit over 5 years, correct to the nearest dollar **2**

(iii) What would be the equivalent Simple rate of annual interest on the final amount? **1**
(Give your answer to the nearest whole number.)

QUESTION 7 continues overleaf.....)

QUESTION 7 continued...

(c)



(i) Find the area between the line $y = x$ and the curve $y = x^2$ as shown above **2**

(ii) Find the volume when this same area is revolved about the y-axis. **3**

(d) On January 1st each year an amount of \$10 000 is deposited into a sinking fund account.
The account earns 5% interest compounded annually.

(i) Show that the amount in the fund after 2 years is given by $\$10\,000(1.05 + 1.05^2)$ **1**

(ii) Show that the amount in the account at the end of 10 years, just after the interest is earned, but before the 11th deposit is $\$21\,000(1.05^{10} - 1)$ **2**

(iii) Find the value of the amount in part (ii) to the nearest dollar. **1**

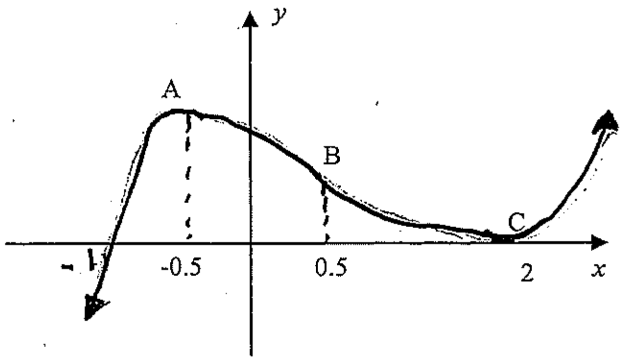
QUESTION 8: (17 Marks) *(Start a New Page)*

Marks

(a) (i) If $y = x^4 - 14x^3 + 36x^2 + 5$, find $\frac{d^2y}{dx^2}$ **1**

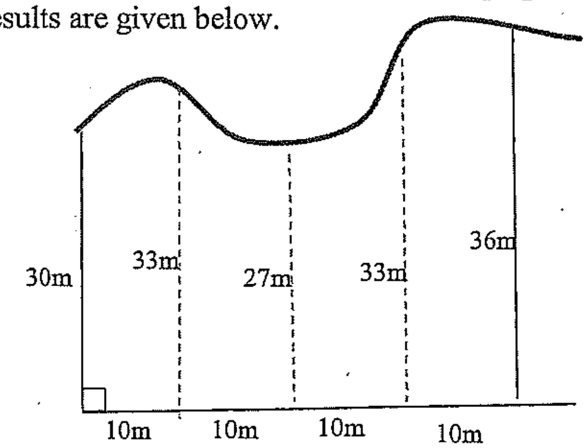
(ii) Find the range of values for x for which is concave up. **2**

(b) You are given the curve $y = f(x)$ shown below **2**



On a neat set of axes, draw a possible graph of $y = f'(x)$, showing the x-values of all 3 points A, B and C given to you above.

(c) A surveyor wants to estimate the land area enclosed by a paddock with 3 straight sides and fronting an irregular river. **3**
He takes 5 measurements at 10 metre intervals perpendicular to the side opposite the river, and his results are given below.

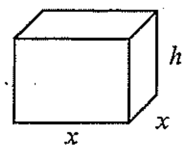


By using Simpson's Rule with 5 function values, give an estimation of the area enclosed.

QUESTION 8 continues overleaf.....)

QUESTION 8 continued....

(d) A sealed rectangular tin box, with a square base, is to have a volume of $64m^3$
The length of the base is x metres, as shown



(i) Express the height, h, in terms of x. **1**

(ii) Show that the total surface area of the box is given by
$SA = \frac{256}{x} + 2x^2$ **2**

(iii) Find the dimensions of the box, so as to use the least amount of tin. **3**

(e) Find the value of x for which $f'(x) = 0$ if $f(x) = x\sqrt{x+2}$ **3**

QUESTION 9: (15 Marks) *(Start a New Page)*

Marks

(a) Ezra invests \$50 000 into an account which earns 8% pa interest, compounded annually. He intends to withdraw \$M at the end of each year, immediately after the interest has been paid.
Ezra wishes to be able to do this for exactly 20 years, so that the account will have nothing left afterwards.

(i) If, after the second withdrawal, Ezra has A_2 in his account, show that **2**

$$A_2 = \$(58\,320 - 2.08M)$$

(ii) Write an expression for A_n which is the amount remaining in his account after the $nth.$ withdrawal **1**

(iii) Write an expression for A_{20}, and hence calculate the value of M (to the nearest dollar) which will make this process cease after 20 withdrawals. **3**

(b) (i) Using calculus for the curve $y = x^2(3 - x)$ find all stationary points and indicate their nature. **4**

(ii) Sketch the curve on a neat set of axes, showing all important features, including intercepts with the axes. **2**

(c) If $y = (x^2 + 1)^4$, show that $\frac{d^2y}{dx^2} = 8(x^2 + 1)^2(7x^2 + 1)$ **3**

END OF EXAMINATION PAPER

2 UNIT SOLUTIONS

1/ D 2/ A 3/ D 4/ D 5/ B

QUESTION 6:

(a) (i) $6x^2 - 3$ (ii) $\frac{(x^2+2)1 - (x-1)2x}{(x^2+2)^2}$

$= \frac{2x - x^2 + 2}{(x^2+2)^2}$

(b) (i) $\frac{2}{3}x^6 + k$ (ii) $\frac{4}{3}x^3 + 2x + k$

(c) $\int_0^3 x^2 + 8x + 16\,dx = \left[\frac{1}{3}x^3 + 4x^2 + 16x\right]_0^3$

$= 9 + 36 + 48$

$= 93$

(d) $S_\infty = \frac{a}{1-r}$

$= \frac{1}{5/4}$

$= \frac{4}{5}$

(e) $\int x^{-\frac{1}{2}}\,dx = \begin{cases} 2x^{\frac{1}{2}} + k \\ 2\sqrt{x} + k \end{cases}$

(f) (i) $\frac{dy}{dx} = 2x + 5$

At $(2, 13)$, $m_T = 9$

Equation is: $y - 13 = 9(x-2)$

$y = 9x - 5$

(ii) At T, $y = 0$ $\therefore$ T is $(\frac{5}{9}, 0)$

At G $x = 0$ $\therefore$ G is $(0, -5)$

$\therefore$ Area $\triangle OTG$ is $\frac{25}{18}\,u^2$

(g) $\int_0^6 kx^2\,dx = \left[\frac{1}{3}kx^3\right]_0^6$

$= 72k$

$\therefore k = 2$

QUESTION 7:

(a) (i) $A = \int_0^4 \sqrt{x}\,dx$

$= \left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4$

$= \frac{16}{3}\,u^2$

(ii) $VOL = \pi\int_0^4 y^2\,dy$

$= \pi\int_0^4 x\,dx$

$= \pi\left[\frac{1}{2}x^2\right]_0^4$

$= 8\pi\,u^3$

(b) (i) Interest rate $= 0.5\%$ monthly

(ii) $A = 6500(1.005)^{60}$

$= 8767.53$

$\therefore$ Interest $= \$2268$

(iii) $SI = P \times R \times N$

$\therefore R = \frac{2268}{6500} \times \frac{1}{5}$

$= 0.069$

$= 7\%$

(c) (i) $A = \int_0^1 x - x^2\,dx$

$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$

$= \frac{1}{6}\,u^2$

(ii) $VOL_1 = \pi\int_0^1 y^2\,dy$

$= \pi\left[\frac{1}{3}y^3\right]_0^1$

$= \frac{\pi}{3}\,u^3$

$VOL_2 = \pi\int_0^1 y\,dy$

$= \pi\left[\frac{1}{2}y^2\right]_0^1$

$= \frac{\pi}{2}\,u^3$

$\therefore VOL_{req} = \frac{\pi}{6}\,u^3$.

(d) (i) $A_1 = 10\,000(1.05)$

$A_2 = 10\,000(1.05) + 10\,000(1.05)^2$

$= 10\,000\,(1.05 + 1.05^2)$

(ii) $A_{10} = 10\,000\,(1.05 + 1.05^2 + \ldots + 1.05^{10})$

$= 10{,}000\left[\frac{1.05(1.05^{10} - 1)}{0.05}\right]$

$= 210{,}000\,(1.05^{10} - 1)$

(iii) $\approx 132{,}068$.

QUESTION 8:

(a) (i) $\frac{dy}{dx} = 4x^3 - 42x^2 + 72x$

$\frac{d^2y}{dx^2} = 12x^2 - 84x + 72$

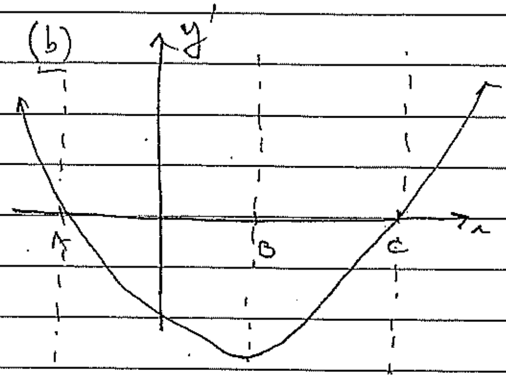
(ii) for concave up $\frac{d^2y}{dx^2} > 0$

$\therefore x^2 - 7x + 6 > 0$

$\therefore (x-6)(x-1) > 0$

$\therefore x > 6$ or $x < 1$

(b)



(c) $A_1 \approx \frac{1}{3} \times 10 \times [30 + 27 + 4 \times 33]$

$= 630 \text{ m}^2$

$A_2 = \frac{1}{3} \times 10 \times [27 + 36 + 4 \times 33]$

$= 650 \text{ m}^2$

$\therefore$ TOTAL AREA is 1280 m^2

(d) (i) $h = 64/x^2$ (ii) $SA = 4xh + 2x^2$

$= 256/x + 2x^2$

(iii) $dSA/dx = -256/x^2 + 4x$

$d^2SA/dx^2 = \frac{512}{x^3} + 4$

At min, $dSA/dx = 0 \Rightarrow x^3 = 64$

$\therefore \begin{cases} x = 4 \\ h = 4 \\ SA = 96 \text{ m}^2 \\ SA'' > 0 \Rightarrow \text{MIN} \end{cases}$

8 (e). $f(x) = x\sqrt{x+2}$

$f'(x) = (x+2)^{1/2} \cdot 1 + x \cdot \frac{1}{2}(x+2)^{-1/2}$

$= \frac{2x + 4 + x}{2\sqrt{x+2}}$

$= \frac{3x+4}{2\sqrt{x+2}}$

At $f'(x) = 0$,

$x = -4/3$

QUESTION 9:

(a) $A_1 = 50000(1.08) - M$

$A_2 = (50000(1.08) - M)(1.08) - M$

$= 50000(1.08)^2 - M[1 + 1.08]$

$= 58320 - 2.08M$

(ii) $A_n = 50000(1.08)^n - M(1 + 1.08 + \dots + 1.08^{n-1})$

(iii) If $A_{20} = 0$

then $50000(1.08)^{20} = M\left[\dfrac{1\cdot(1.08^{20}-1)}{0.08}\right]$

$\therefore M = \dfrac{50000(1.08)^{20}}{58.26}$

$\approx 4{,}000.13$

$\approx \$4{,}000$.

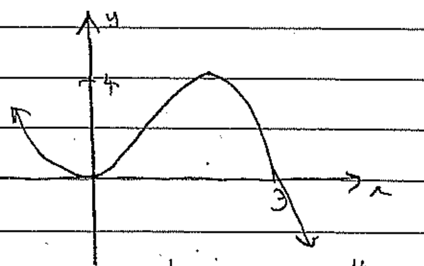
(b) $y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2$ At S.P's $\frac{dy}{dx} = 0$

$\frac{d^2y}{dx^2} = 6 - 6x$ $3x(x-2) = 0$

$\therefore \begin{cases} x = 0 \\ y = 0 \\ y'' > 0 \end{cases}$ or $\begin{cases} x = 2 \\ y = 4 \\ y'' < 0 \end{cases}$

$\Rightarrow$ MIN S.P at $(0,0)$ MAX at $(2,4)$



(c) $\frac{d}{dx}(x^2+1)^4 = 8x(x^2+1)^3$

$\frac{d^2y}{dx^2} = (x^2+1)^3 8 + 8x\cdot 3\cdot 2x(x^2+1)^2$

$= 8(x^2+1)^2\left[(x^2+1) + 6x^2\right]$

$= 8(x^2+1)^2(7x^2+1)$