# SYDNEY TECHNICAL HIGH SCHOOL

# MATHEMATICS

## YEAR 11 YEARLY EXAMINATION

## PRELIMINARY HSC ASSESSMENT TASK 3

## SEPTEMBER 2009

**General Instructions:**

- Working time allowed – 120 minutes
- Write using black or blue pen
- Approved calculators may be used
- All necessary working should be shown
- Start each question on a new page
- Attempt all questions
- Questions are of equal value
- Full marks may not be awarded if working is poorly set our or difficult to read

**Name:** _____ **Teacher:**_____

| Question 1 | Question 2 | Question 3 | Question 4 | Question 5 | Question 6 | Question 7 | Question 8 | TOTAL |
|---|---|---|---|---|---|---|---|---|
| /10 | /10 | /10 | /10 | /10 | /10 | /10 | /10 | /80 |

**Question 1 (10 marks)**

a) Evaluate $3^{2.5}$ correct to 3 significant figures 2

b) Factorise completely $ab - a - bx + x$ 2

c) Find integers $a$ and $b$ such that $\frac{1}{\sqrt{3}+2} = a\sqrt{3} + b$ 2

d) Solve $tan\theta = \frac{-1}{\sqrt{3}}$ for $0° \leq \theta \leq 360°$ 2

e) Write $1 - \frac{a-b}{a+b}$ as a single fraction in simplest form 2

**Question 2 (10 marks) Start a new page**

a) i) Sketch $y = 2 - (x + 1)^2$ clearly label the vertex and the y intercept 2

ii) State the range for $y = 2 - (x + 1)^2$ 1

b) The points A, B and C have coordinates (1,5), (6,0) and (5,7) respectively.
Plot these points on a number plane.

i) Show that the length of AB is $5\sqrt{2}$ units. 1

ii) Show that the triangle ABC is isosceles by finding the length of BC. 1

iii) Find the equation of the line AB. 2

iv) BA is produced to meet line $y = 7$ at P.
Find the coordinates of P. 1

v) Find the area of triangle PAC. 2

**Question 3 (10 marks) (Start a new page)**

a) Express $0.3 + 0.\dot{3}$ in the form $\frac{a}{b}$ where $a$ and $b$ are integers 2

b) Differentiate the following

i) $y = \frac{2}{\sqrt{x}}$ 1

ii) $y = \frac{3}{x-1}$ 2

iii) $y = \frac{2x+1}{1-x}$ 2

c) If $\alpha$ *and* $\beta$ are the roots of the quadratic equation $x^2 - 5x + 2 = 0$ find the value of 3

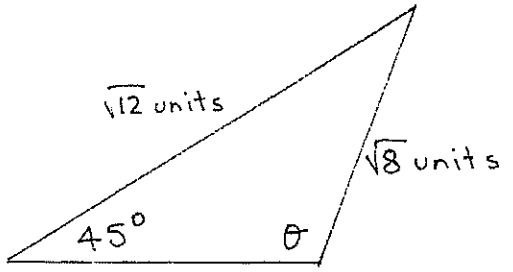
i) $\alpha + \beta$

ii) $\alpha\,\beta$

iii) $\alpha^2 + \beta^2$

**Question 4 (10 marks) (Start a new page)**

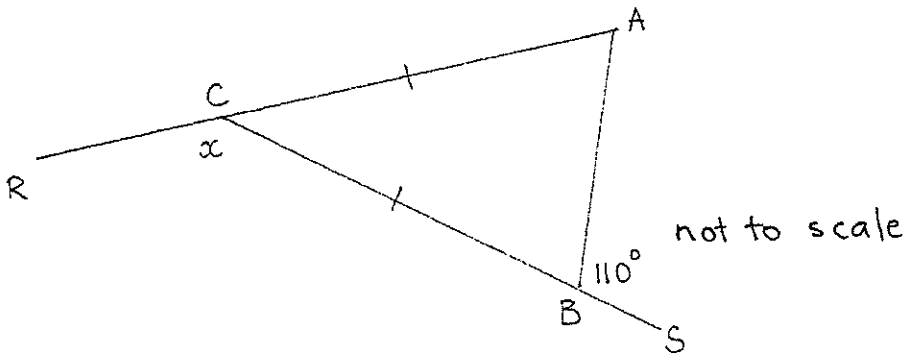
a)




3

Use the sine rule to find the value of $\theta$, where $\theta$ is obtuse

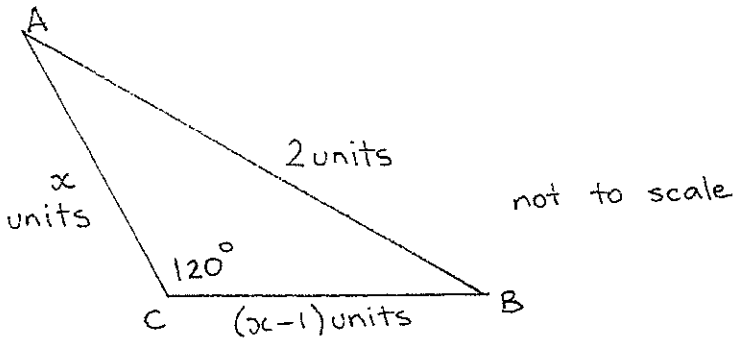
b)




In the diagram above $AC = BC$, $A\hat{B}S = 110°$ and $R\hat{C}B = x$.

Copy the diagram onto your answer sheets. Find the value of $x$ giving reasons. 2

c)




In the diagram above, ABC is a triangle in which $AC = x$ units, $BC = (x - 1)$ units and $AB = 2$ units, $ACB = 120°$ .

Find the exact value of $x$ 3

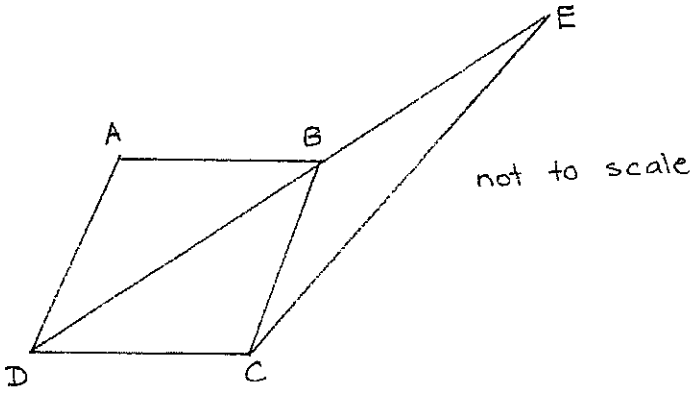
d) Solve the pair of simultaneous equations

$3x - y = 15$

$x + 2y = -2$ 2

**Question 5 (10 marks) (Start a new page)**

a)




In the diagram above, $ABCD$ is a rhombus.

$DB$ is produced to E such that $D\hat{E}C = 10°, B\hat{D}C = 30°$

Copy the diagram showing the above information.

Calculate the size of $B\hat{C}E$ giving reasons 3

b) Solve $2cos^2x - 3cosx - 2 = 0$ for $-180° \leq x \leq 180°$ 3

c) Prove $\frac{1}{cot^2\theta} + 1 = sec^2\theta$ 2

d) Prove that $5x - 12y + 52 = 0$ is a tangent to the circle with centre the origin and radius 4 2

**Question 6 (10 marks) (Start a new page)**

a) i) On the same diagram sketch $y = x - 4$ and $xy = 5$ (use a ruler for your axes) Label each function clearly 2

ii) Find the points of intersection of these graphs 2

b) A piecemeal function is defined as $f(x) = \begin{cases} 1 - x, x < 0 \\ x^2 + 2, x \geq 0 \end{cases}$

i) Sketch the function 2

ii) Find the value of $f(-2) + f(0) + f(3)$ 1

iii) For what value(s) of $x$ is this function not differentiable 1

c) Find the value of $k$ if the equation $x^2 - 6x + 4k = 0$ has real roots 2

**Question 7 (10 marks) (Start a new page)**

a) Differentiate $y = x^2(1 - 2x)^3$ 2

b) Find $\lim_{x \to 2}\left(\frac{x^2 - 4}{x - 2}\right)$ 1

c) If $f(x) = x^2 - 3x + 5$, for what value of $x$ is $f'(x) = 9$ 2

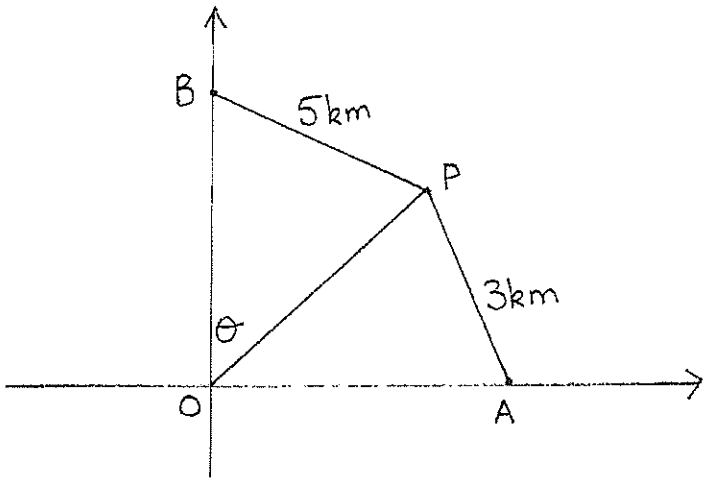
d) The curve $y = ax + \frac{b}{x^2}$ cuts the $x$ axis at $x = 2$, and the gradient of the tangent to the curve at this point is 3. Find the values of $a$ and $b$. 3

e) By first substituting $m$ for $x^2$, solve the equation $x^4 - 10x^2 + 9 = 0$ for $x$ 2

**Question 8 (10 Marks) (Start a new page)**

a) For what values of a will $ax^2 + 5x + a$ be positive definite 2

b)




In the diagram above, A is a point due east of $O$ and $B$ is a point due north of $O$. $P$ is on a bearing of 120° from $B$ and 345° from $A$. $AP$ is 3 km and $BP$ is 5 km. The bearing of P from O is $\theta$.

i) Find the size of $O\hat{B}P$ and $O\hat{A}P$ 2

ii) Prove that $OP = \frac{5\sqrt{3}}{2sin\theta}$ 2

iii) Prove that $OP = \frac{3sin75°}{cos\theta}$ 2

iv) Using parts ii) and iii) find $\theta$ to the nearest degree 2

# S.T.H.S YR11 2unit Prelim HSC TASK 3 Sept 2009

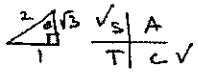
## Question 1

a) $3^{2.5} = \underline{15.6}$ (3 sig fig)

b) $ab - a - bx + x$
$= a(b-1) - x(b-1)$
$= \underline{(b-1)(a-x)}$

c) $\frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2} = \frac{\sqrt{3}-2}{3-4}$
$= -\sqrt{3}+2$
$\therefore \underline{a=-1 \quad b=2}$

d) $\tan\theta = -\frac{1}{\sqrt{3}}$

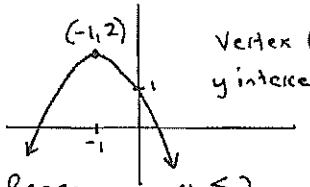



acute $\theta = 30°$
$\therefore \underline{\theta = 150°, 330°}$

e) $1 - \frac{a-b}{a+b} = \frac{(a+b)-(a-b)}{a+b}$
$= \underline{\frac{2b}{a+b}}$

## Question 2

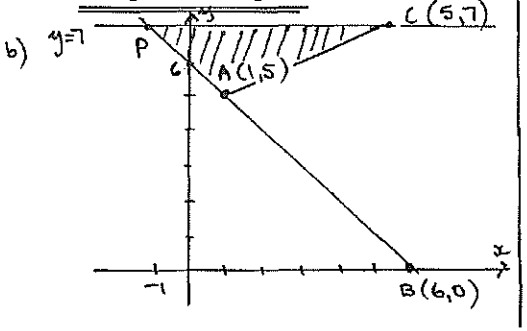
a) i)




Vertex $(-1,2)$
y intercept $y=1$

ii) Range $\underline{y \leq 2}$

b) $y=7$




i) $AB = \sqrt{(6-1)^2 + (0-5)^2}$
$= \sqrt{25+25}$
$= \sqrt{50}$
$= \underline{5\sqrt{2} \text{ units}}$

ii) $BC = \sqrt{(6-5)^2 + (0-7)^2}$
$= \sqrt{1+49}$
$= \sqrt{50}$
$= \underline{5\sqrt{2}}$

$\underline{AB = BC \therefore \triangle ABC \text{ is isosceles}}$

iii) $m_{AB} = \frac{5-0}{1-6} = \frac{5}{-5} = -1$

eqn AB: $y - 5 = -1(x-1)$
$\underline{y = -x+6}$

iv) $\underline{P(-1,7)}$

v) area $\triangle PAC = \frac{6 \times 2}{2}$
$= \underline{6 \text{ units}^2}$

## Question 3

a) $0.3 + 0.3$
$\frac{3}{10} + \frac{1}{3} = \frac{9+10}{30}$
$= \underline{19/30}$

b) i) $y = \frac{2}{\sqrt{x}} = 2x^{-1/2}$
$\frac{dy}{dx} = 2 \times -\frac{1}{2} x^{-3/2}$
$= \underline{\frac{-1}{\sqrt{x^3}}}$

ii) $y = 3(x-1)^{-1}$
$\frac{dy}{dx} = 3 \times -1 (x-1)^{-2}$
$= \underline{\frac{-3}{(x-1)^2}}$

iii) $u = 2x+1 \quad v = 1-x$
$u' = 2 \quad v' = -1$
$\frac{dy}{dx} = \frac{2(1-x) - -1(2x+1)}{(1-x)^2}$
$= \frac{2-2x+2x+1}{(1-x)^2}$
$= \underline{\frac{3}{(1-x)^2}}$

c) i) $\alpha + \beta = -\frac{b}{a} = \underline{5}$

ii) $\alpha\beta = \frac{c}{a} = \underline{2}$

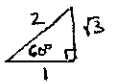
iii) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$
$= 25 - 4$
$= \underline{21}$

## Question 4

a) $\frac{\sin\theta}{\sqrt{12}} = \frac{\sin 45°}{\sqrt{8}}$

$\sin\theta = \sqrt{12} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{8}}$

$\sin\theta = \frac{2\sqrt{3}}{\sqrt{2} \cdot 2\sqrt{2}}$

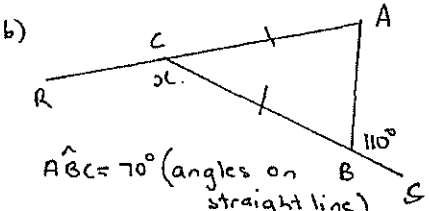



$\sin\theta = \frac{\sqrt{3}}{2}$

$\therefore$ acute $\theta = 60°$

$\underline{\text{obtuse } \theta = 120°}$

b)




$A\hat{B}C = 70°$ (angles on straight line)

$x = 140°$ (exterior angle of isosceles triangle)

c) $2^2 = x^2 + (x-1)^2 - 2x(x-1)\cos 120°$
$4 = x^2 + x^2 - 2x + 1 - 2x(x-1)(-\frac{1}{2})$
$4 = 2x^2 - 2x + 1 + x^2 - x$
$0 = 3x^2 - 3x - 3$
$0 = x^2 - x - 1$
$x = \frac{1 \pm \sqrt{1 - 4 \times 1 \times -1}}{2}$

$x = \frac{1 \pm \sqrt{5}}{2}$ since $x > 0$

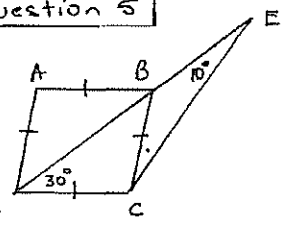
$\therefore x = \frac{1+\sqrt{5}}{2}$ units only

d) $3x - y = 15$ — ①
$x + 2y = -2$ — ②

① × 2 $\quad 6x - 2y = 30$
$\quad x + 2y = -2$ (+)
$\quad 7x = 28$
$\therefore \underline{x = 4 \quad y = -3}$

## Question 5

a)




$D\hat{C}E = 140°$ (angle sum $\triangle DCE$)
$D\hat{C}B = 120°$ (angle sum isosceles $\triangle DBC$)
$\therefore \underline{B\hat{C}E = 20°}$

b) $2\cos^2 x - 3\cos x - 2 = 0$
$(2\cos x + 1)(\cos x - 2) = 0$
$\cos x = -\frac{1}{2} \qquad \cos x = 2$ no sol.

| S | A |
|---|---|
| T | C |

acute $x = 60°$
$\therefore \underline{x = 120°, -120°}$

c) LHS $= \dfrac{1}{\cot^2\theta} + 1$

$= \dfrac{1+\cot^2\theta}{\cot^2\theta}$

$= \dfrac{\operatorname{cosec}^2\theta}{\cot^2\theta}$

$= \dfrac{1}{\sin^2\theta} \div \dfrac{\cos^2\theta}{\sin^2\theta}$

$= \dfrac{1}{\cos^2\theta}$

$= \sec^2\theta$

= RHS

d) find perp dist $(0,0)$ to $5x - 12y + 52 = 0$

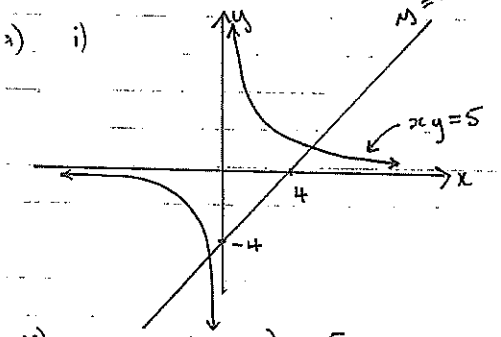
$p = \left| \dfrac{5\times0 - 12\times0 + 52}{\sqrt{25+144}} \right|$

$p = \left| \dfrac{52}{13} \right| = 4$

since perp dist is equal to radius $\therefore$ tangent

## Question 6

a) i)




ii) $x(x-4) = 5$

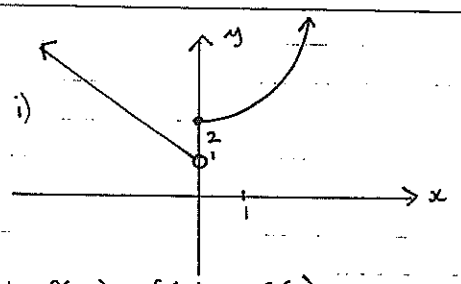
$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ $\quad x = -1$

$\therefore$ pts $(5, 1)$ & $(-1, -5)$

b)




i)

ii) $f(-2) + f(0) + f(3)$

$= 3 + 2 + 11$

$= 16$

iii) $x = 0$

c) Real roots $\Delta \geq 0$

$\Delta = (-6)^2 - 4\times1\times4k$

$\Delta = 36 - 16k$

$\therefore 36 - 16k \geq 0$

$-16k \geq -36$

$k \leq 2\frac{1}{4}$

## Question 7

a) $u = x^2$ $\quad v = (1-2x)^3$

$u' = 2x$ $\quad v' = 3\times-2(1-2x)^2$

$\dfrac{dy}{dx} = 2x(1-2x)^3 - 6x^2(1-2x)^2$

b) $\lim\limits_{x\to2} \dfrac{(x-2)(x+2)}{(x-2)} = 4$

c) $f'(x) = 2x - 3$

$2x - 3 = 9$

$2x = 12$

$x = 6$

d) cut $x$ axis at 2 $\therefore$ $(2,0)$ lies on curve

$0 = 2a + \frac{b}{4}$ — ①

$\dfrac{dy}{dx} = a - 2bx^{-3} = a - \dfrac{2b}{x^3}$

$m = 3$ $x = 2$ $\therefore$ $3 = a - \dfrac{2b}{8}$ — ②

$8a + b = 0$

$8a - 2b = 24$

$-3b = 24$

$b = -8$ $\quad a = 1$

e) $x^4 - 10x^2 + 9 = 0$

$m^2 - 10m + 9 = 0$

$(m-9)(m-1) = 0$

$m = 9$ $\quad m = 1$

$x^2 = 9$ $\quad x^2 = 1$

$x = \pm3$, $\quad x = \pm1$

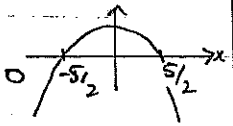
## Question 8

a) +ve def $\quad a > 0$, $\Delta < 0$

$\Delta = 25 - 4a^2$

$25 - 4a^2 < 0$

$(5-2a)(5+2a) < 0$

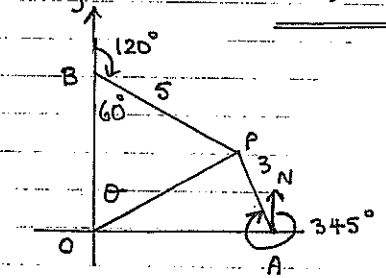



$a < -\frac{5}{2}$, $\quad a > 5/2$

but since $a > 0$

only solution $a > 5/2$

b)




i) $O\hat{B}P = 60°$

$O\hat{A}P = 75°$

ii) $\dfrac{OP}{\sin60°} = \dfrac{5}{\sin\theta}$

$OP = \dfrac{5\sin60°}{\sin\theta}$

$OP = \dfrac{5\sqrt{3}}{2\sin\theta}$

iii) $\dfrac{OP}{\sin75°} = \dfrac{3}{\sin(90-\theta)}$

$OP = \dfrac{3\sin75°}{\cos\theta}$

iv) $\dfrac{5\sqrt{3}}{2\sin\theta} = \dfrac{3\sin75°}{\cos\theta}$

$\dfrac{5\sqrt{3}}{6\sin75} = \dfrac{\sin\theta}{\cos\theta}$

$\therefore \tan\theta = \dfrac{5\sqrt{3}}{6\sin75°}$

$\therefore \theta = 56°$