Name: ..... Maths Class: .....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11

Mathematics Extension 1

Preliminary Course

Assessment 1

May, 2017

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- BOSTES reference sheet is located at the end of the exam.

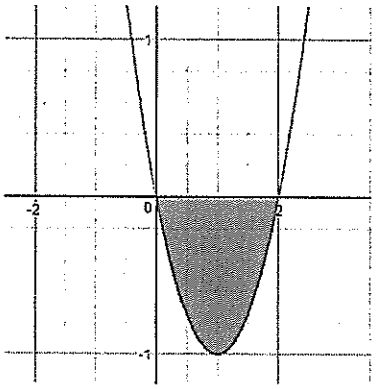
Section 1	Multiple Choice Questions 1-5 5 Marks
Section II	Questions 6-11 60 Marks

Section 1 - Multiple Choice – (5 marks)

Answer on the sheet provided

Allow approximately 10 minutes for this section

1. The diagram shows the graph of the function $y = x^2 - 2x$



Which pair of inequalities specify the shaded region?

A. $y \leq x^2 - 2x$ and $y \leq 0$

B. $y \leq x^2 - 2x$ and $y \geq 0$

C. $y \geq x^2 - 2x$ and $y \leq 0$

D. $y \geq x^2 - 2x$ and $y \geq 0$

2. The graph with equation $y = x^2$ is translated 2 units down and 3 units to the right. Which equation represents the resulting graph?

A. $y = (x - 3)^2 + 2$

B. $y = (x + 3)^2 + 2$

C. $y = (x + 3)^2 - 2$

D. $y = (x - 3)^2 - 2$

3. What is the domain of the function $f(x) = \sqrt{x+1} - \sqrt{x+2}$

   A. $x \leq -2$

   B. $x \geq -1$

   C. $-1 \leq x < -2$

   D. $x \leq -2$ or $x \geq -1$

4. What is $8^3 \times 6^{1/2} \div 32^{3/2}$ in simplest form?

   A. $4\sqrt{3}$

   B. $2\sqrt{3}$

   C. $3\sqrt{2}$

   D. $4\sqrt{2}$

5. What are the solutions of $3x^2 - 7x - 1 = 0$?

   A. $x = \frac{-7 \pm \sqrt{61}}{6}$

   B. $x = \frac{-7 \pm \sqrt{37}}{6}$

   C. $x = \frac{7 \pm \sqrt{61}}{6}$

   D. $x = \frac{7 \pm \sqrt{37}}{6}$

End of section 1

Section II

Answer questions in booklet provided.

Start each question on a new page

Allow approximately 80 minutes for this section

Question 6: (10 marks) **Marks**

a) Write $(1+\sqrt{7})^2$ in the form $a+b\sqrt{7}$ 2

b) Solve for x, $16^{4-x}=\frac{1}{8^x}$ 2

c) Factorise fully, $81-x^4$ 2

d) What is the centre and radius of $x^2+y^2+6x+8y-11=0$? 2

e) Write down the equations of horizontal and vertical asymptotes for 2

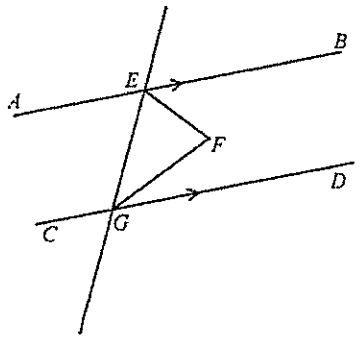
$y=\frac{x^2+1}{x^2-1}$

(Start each new question on a new page)

Question 7: (10 marks)

Marks

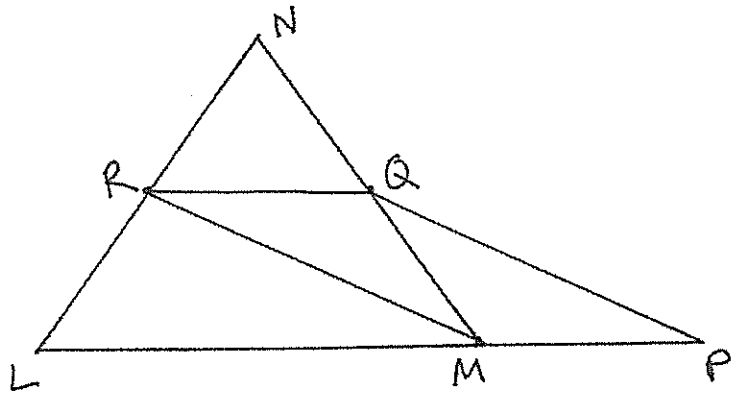
a) In the diagram below, AB || CD. EF bisects $\angle BEG$ and GF bisects $\angle EGD$. What is the size of $\angle EFG$? Give clear reasons. 3



b) Solve simultaneously $x + 2y = 5$ and $2xy - x^2 = 3$ 3

c) The side LM of ΔLNM is produced to P so that MP=$\frac{1}{2}$LM. If Q and R are the midpoints of MN and LN respectively,

i) Copy the diagram onto your answer sheet.



ii) Prove that ΔRNQ is similar to ΔLNM. 2

iii) Prove that PQRM is a parallelogram. 2

(Start each new question on a new page)

Question 8: (10 marks)

Marks

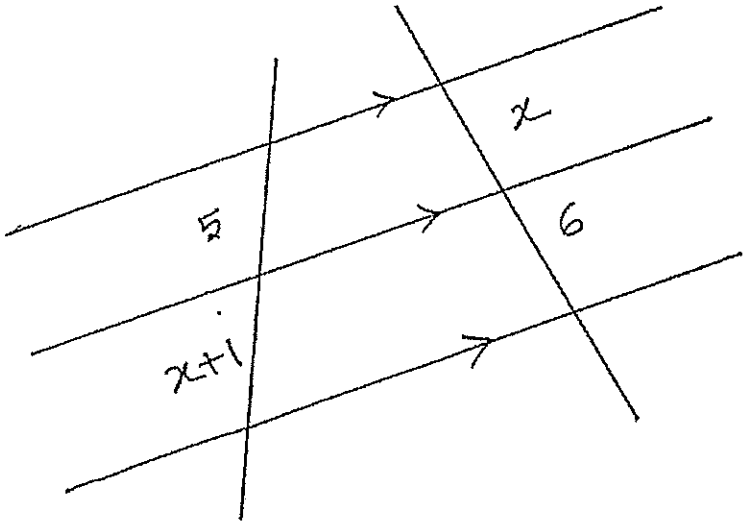
a) Solve $3x - 2\sqrt{x} - 8 = 0$ 3

b) A function is defined by $f(x) = \begin{cases} 2 - x^3, x \leq -5 \\ 2x + 1, -5 < x < 0 \\ x^2 - 9, x > 0 \end{cases}$ 2

Find the value of $2f(-5) + [f(1)]^2 + f(-1)$

c) Solve $2x - 1 \geq \frac{6}{x}$ 3

d) Find the value of x in: 2



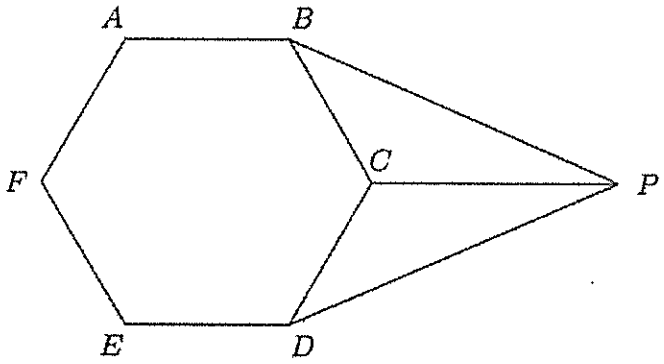
(Start each new question on a new page)

Question 9: (10 marks)

Marks

a) Fully factorise $x^6 - 26x^3 - 27$ 2

b) ABCDEF is a regular hexagon, and CP || AB.



i) Find the size of $\angle BCP$, giving reasons. 2

ii) Prove that $\Delta BCP \equiv \Delta DCP$. 2

c) i) On the same set of axes, sketch the graphs of 2
$y = |2x + 5|$ and $y = x + 4$

ii) Hence or otherwise, solve $|2x + 5| \leq x + 4$ 2

(Start each new question on a new page)

Question 10: (10 marks)

Marks

a) State the domain and range of $y = 5 - |x - 1|$ 2

b) Solve $|2x + 5| + |x - 1| = 5$ 3

c) Consider the function $f(x) = \frac{3}{x^2 - 9}$

i) Is the function odd, even or neither? Give reasons 1

ii) What are the vertical and horizontal asymptotes for f(x)? 2

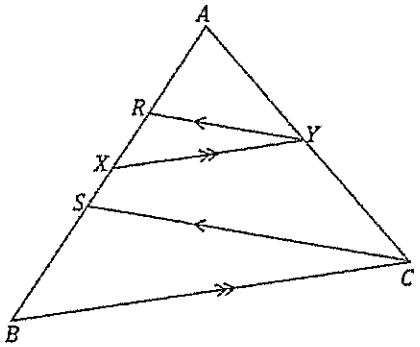
iii) Sketch the curve $y = f(x)$ neatly on at least one third of a page, showing all intercepts and asymptotes. 2

(Start each new question on a new page)

Question 11: (10 marks)

Marks

a) Given that in ΔABC, XY||BC and RY||SC 3



Prove $\frac{AX}{XB} = \frac{AR}{RS}$

b) i)Factorise $x^3 - 6x^2 + 9x$ 2

ii) Hence solve $x^3 - 6x^2 + 9x \leq 0$ 2

c) Express the following as a single fraction with rational denominator when $x = 3\sqrt{3} + 2$. 3

$\frac{1}{x-1} + \frac{1}{x+1} - \frac{2}{x^2-1}$

End of Assessment Task

1. C $y \geq x^2 - 2x$ and $y \leq 0$

2. D $y = (x-3)^2 - 2$

3. B $x \geq -1$

4. A $4\sqrt{3}$

5. C $x = \frac{7 \pm \sqrt{37}}{6}$

6. a) $(1+\sqrt{7})^2 = 1 + 2\sqrt{7} + 7 = 8 + 2\sqrt{7}$

$a = 8, \quad b = 2$

b) $16^{4-x} = \frac{1}{8^x}$

$2^{16-4x} = 2^{-3x}$

$16 - 4x = -3x$

$-x = -16$

$\therefore x = 16$

c) $81 - x^4$

$= 9^2 - (x^2)^2$

$= (9 - x^2)(9 + x^2)$

$= (3^2 - x^2)(3^2 + x^2)$

$= (3-x)(3+x)(9+x^2)$

d) $x^2 + y^2 + 6x + 8y - 11 = 0$

$x^2 + 6x + 9 + y^2 + 8y + 16 = 11 + 9 + 16$

$(x+3)^2 + (y+4)^2 = 36$ c: $(-3, -4)$ r: 6 units

e) $y = \frac{x^2+1}{(x-1)(x+1)}$

vertical asymptote: $x = \pm 1$

horizontal asymptote: $y = 1$

7. a) Let $\angle BEF = \alpha$ and $\angle FGD = \beta$

$\therefore \angle FEG = \alpha$ (EF bisects $\angle BEG$) Similarly, $\angle FGE = \beta$

$2\alpha + 2\beta = 180^\circ$ (cointerior angles, $AB \parallel CD$)

$\alpha + \beta = 90^\circ$

$\angle EFG + \alpha + \beta = 180^\circ$ (angle sum of $\triangle EFG$)

$\therefore \angle EFG = 90^\circ$

b) $x + 2y = 5 \Rightarrow y = \frac{5-x}{2}$

$2xy - x^2 = 3$

$2x\left(\frac{5-x}{2}\right) - x^2 = 3$

$2x^2 - 5x + 3 = 0$

$(2x-3)(x-1) = 0$

$\therefore x = \frac{3}{2}, 1$

when $x = \frac{3}{2}$ $y = \frac{7}{4}$

when $x = 1$ $y = \frac{5-1}{2} = 2$

$\therefore y = \frac{7}{4}, 2$

c) ii) In $\triangle RNQ$ and $\triangle LNM$

$\angle LNM$ is common

$\frac{RN}{LN} = \frac{1}{2}$ (given)

$\frac{QN}{MN} = \frac{1}{2}$ (given) $\therefore \frac{RN}{LN} = \frac{QN}{MN}$

$\therefore \triangle RNQ \parallel\parallel \triangle LNM$ (since two pairs of sides in equal ratio and included angles are equal)

iii) $\frac{RQ}{LM} = \frac{1}{2}$ (matching sides in similar triangles)

$\frac{MP}{LM} = \frac{1}{2}$ (given) $\therefore RQ = MP$

$\angle NQR = \angle QML$ (matching angles in similar triangles)

$\therefore RQ \parallel MP$ (since $\angle NQR = \angle QML$, corresponding angles are equal)

$\therefore PQRM$ is a parallelogram (one pair of opposite sides are equal and parallel)

8. a) $3x - 2\sqrt{x} - 8 = 0$

Let $\sqrt{x} = a$

$3a^2 - 2a - 8 = 0$

$(3a+4)(a-2) = 0$

$a = -\frac{4}{3}, 2$

$\therefore \sqrt{x} = -\frac{4}{3}, 2$

Since $\sqrt{x} > 0$

$x = 4$ only solution

b) $f(-5) = 2 - (-5)^3$

$= 127$

$f(1) = 1^2 - 9 = 8$

$f(-1) = 2(-1) + 1 = -1$

$2f(-5) + [f(1)]^2 + f(-1)$

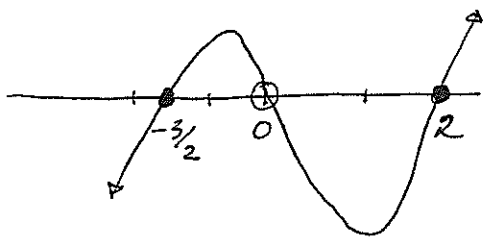
$= 2 \times 127 + 64 - 1 = \underline{\underline{317}}$

c) $2x - 1 \geq \frac{6}{x}$, $x \neq 0$

$x^2(2x-1) \geq 6x$

$2x^3 - x^2 - 6x \geq 0$

$x(2x+3)(x-2) \geq 0$



$\therefore -\frac{3}{2} \leq x < 0$ or $x \geq 2$

d) $\frac{5}{x+1} = \frac{x}{6}$

$30 = x^2 + x$

$x^2 + x - 30 = 0$

$(x+6)(x-5) = 0$

$\therefore x = -6, 5$

Since $x > 0$

$x = 5$ only

9. a) $x^6 - 26x^3 - 27$

Let $x^3 = a$

$a^2 - 26a - 27$

$= (a-27)(a+1)$

$= (x^3 - 3^3)(x^3 + 1^3)$

$= (x-3)(x^2+3x+9)(x+1)(x^2-x+1)$

b) i) $\angle ABC = \frac{(6-2)\times 180}{6} = 120^\circ$ (angle in hexagon)

$\therefore \angle BCP = 120^\circ$ (Alternate angles, $CP \parallel AB$)

ii) In $\triangle BCP$ and $\triangle DCP$

$\angle BCP = 120^\circ$ (Proved in part (i))

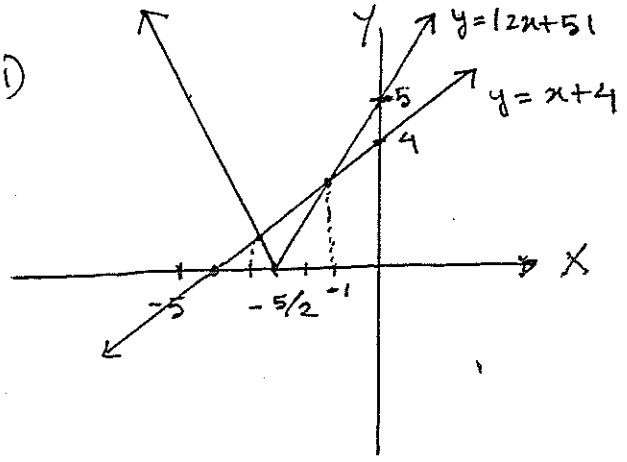
$\angle DCP = 120^\circ$ (Alternate angles, $ED \parallel CP$)

$BC = DC$ (sides of a regular hexagon)

CP is common

$\therefore \triangle BCP \equiv \triangle DCP$ (SAS)

c) i)



$2x+5 = x+4$

$x = -1$

$2x+5 = -x-4$

$x = -3$

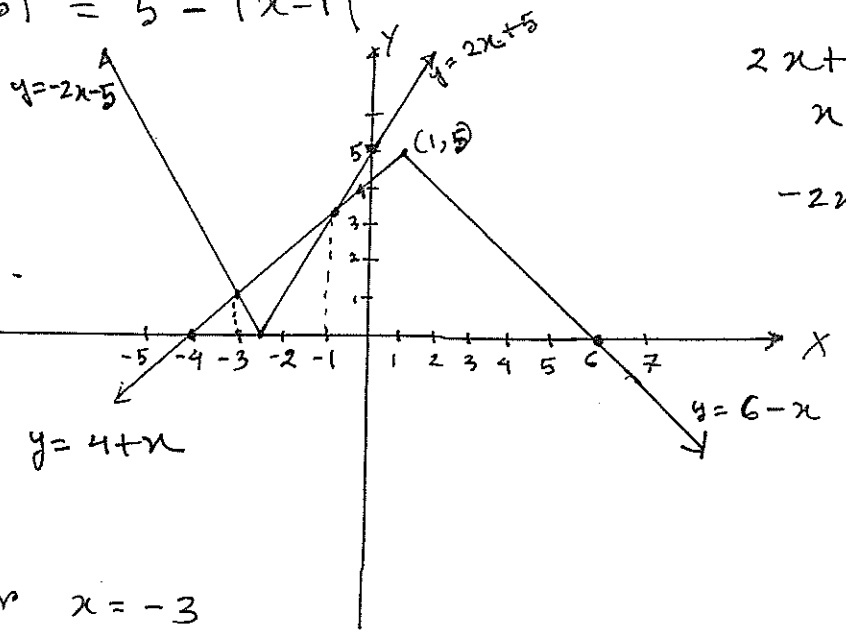
ii) $-3 \le x \le -1$

10. a) domain: All real x

range: $\{y \in \mathbb{R}, y \leq 5\}$

b) $|2x+5| + |x-1| \geq 5$

$|2x+5| = 5 - |x-1|$



$2x+5 = 4+x$

$x = -1$

$-2x-5 = 4+x$

$x = -3$

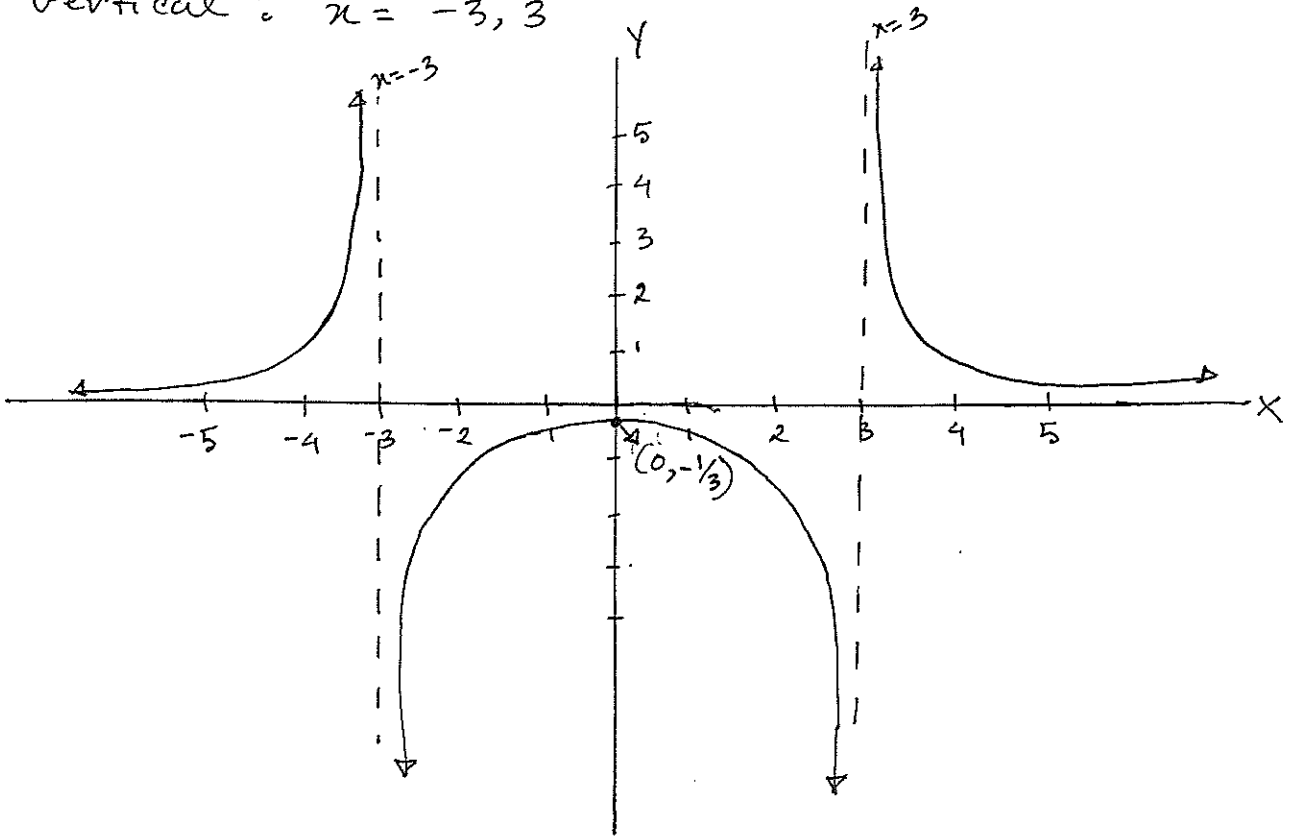
$x = -1$ or $x = -3$

c) i) $f(-x) = \frac{3}{(-x)^2 - 9} = \frac{3}{x^2-9} = f(x)$ $\therefore$ even

ii) horizontal: $y = 0$

vertical: $x = -3, 3$

iii)



Extension 1 year 11 Assessment 1-2017

Markers' Comments

Question 6

Indices: 16 does not equal 8^2

Difference of 2 squares twice – fully factorise

Complete the square for a circle

Asymptotes are written with equals sign. Read question: don't find domain and range

Question 7

a) When proving geometry questions, its best to draw a diagram. Label all relevant information and give reasoning. When listing either co-interior or alternate angles, you must list which lines are parallel. When listing angle sum of a triangle, name the triangle you are working with. Do not construct extra lines without a diagram to show what you have doing. Marker will give zero marks in that situation.

b) Poor algebra skills meant that students did not acquire all marks.
Needed to find both points of intersections, not just the x coordinates.

c) i) Students MUST learn how to write out formal proofs for similarity. Do not use congruency reasoning for similarity proofs.

ii) Poor reasoning given in this question. Students need to learn the properties of a parallelogram. The question did NOT state that RQ//MP (you needed to prove that)

Question 8

a) Use a substitution $m = \sqrt{x}$ or $m^2 = x$:
Students need to realise that they cannot simply square each individual term as $x + y = c$ is not the same as $x^2 + y^2 \neq c^2$ ALSO answers need to be tested if solving using substitution to ensure no false answers have been introduced. Any answer that is discarded should have an explanation as to why – not just vanish.

b) In a composite graph the x values need to be substituted in at the start by checking the domain in which they lie and choosing the correct 'rule' to then sub them into.

c) CHOOSE A **METHOD** FOR THESE QUESTIONS AND LEARN IT !!!!!! Very poorly done, as students need to commit to a method and set it out – important to write $x \neq 0$ at the start and check their solutions.

d) Ratio of intercepts needs to be revised so that the **correct ratio** is written – **test answers** and explain why any answers are discarded. Students who had the correct value but from incorrect ratios (usually making way easier algebra) were not awarded any marks.

Question 9

(a) You were not solving an equation, so there were no "answers". You needed to factorise both $(x^3 + 1)$ and $(x^3 - 27)$

(b) You cannot assume that ED was parallel to AB (and so, also CP). It is true, but do you <u>really</u> know this, or is it a gut feeling. Additionally, some people extended PC to F. The question is, do you know that CP extended goes through F or is it, again, because it makes sense? Similarly for CP "bisecting the angle BCD." – It isn't given, and you do not know it.

(c) The graph $y = |2x + 5|$ meets the x-axis at $x = -\frac{5}{2}$

Question 10

(a) Sketch the graph if you are not sure about the range, the y value cannot be more than 5.

(b) If you solve this question algebraically, there will be four cases: ++,+ -, - +, - - you need to show all cases and always check your answer!!
If you are solving graphically you cannot just draw $|2x + 5|$ and $|x - 1|$ separately. You need to sketch $|2x + 5| + |x - 1|$ by adding coordinates and then check where the graph intersects with the line y=5.

(c) i) You need to give reasons, read questions!
ii) Asymptotes are written with equal sign "=". Eg. Vertical asymptote: x = -3, 3
iii) Label the graph properly with all asymptotes and x, y intercepts.

Question 11

(a) Proving the two pairs of triangles similar did not lead to proving the ratios of INTERCEPTS not sides of triangles were equal. It is OK to quote "Ratio of intercepts theorem" or ("Intercepts on parallel lines").

(b) (ii) Most forgot the x=3 which is where y=0.

(c) It will always be easier to simplify original fractions first, then substitute the rationalise denominator ONCE, not for 3 separate fractions.