SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS

Year 11

2 Unit

Task 1
2013

Common Test

Time Allowed: 70 mins

Name :_____

Teachers Name :_____

Instructions:
- Begin each question on a new page
- Marks shown are approximate and may be varied
- Show necessary working
- Full marks may not be awarded if working is poorly set out or difficult to read
- Write all answers in simplest form

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
/8	/8	/8	/8	/8	/8	/8	/8	/64

Question 1   **8 marks**

a) Arrange in ascending order:

$$3^{\frac{1}{2}} \quad , \qquad 2^{\frac{5}{6}} \quad , \qquad \left(\frac{3}{11}\right)^{-\frac{1}{3}}$$ 1

b) Calculate $\dfrac{9.62 \times 10^{-4}}{3.67 \times 10^{-3} \times 2.67 \times 10^{2}}$ giving your answer correct to 2 significant figures   1

c) Write $\dfrac{x}{y} - 2$ as a single fraction   1

d) Given $s = \dfrac{n}{2}[2a + (n-1)d]$ find s when n=16, a=5, d=4   1

e) Gold, a very soft metal, can be hammered into sheets of thickness 1.02×10^{-4} mm.
    How many such sheets are needed to make a pile 1 cm thick?   1

f) Express $0.29\dot{8}$ as a fraction in lowest terms.   2

g) Rewrite $2\left(x^{2} - 1\right)^{-\frac{1}{2}}$ as an expression with no negative or fractional indices   1

Question 2   **8 marks**

a) Simplify $\dfrac{1}{\sqrt{3}+\sqrt{x}}+\dfrac{1}{\sqrt{3}-\sqrt{x}}$ 2
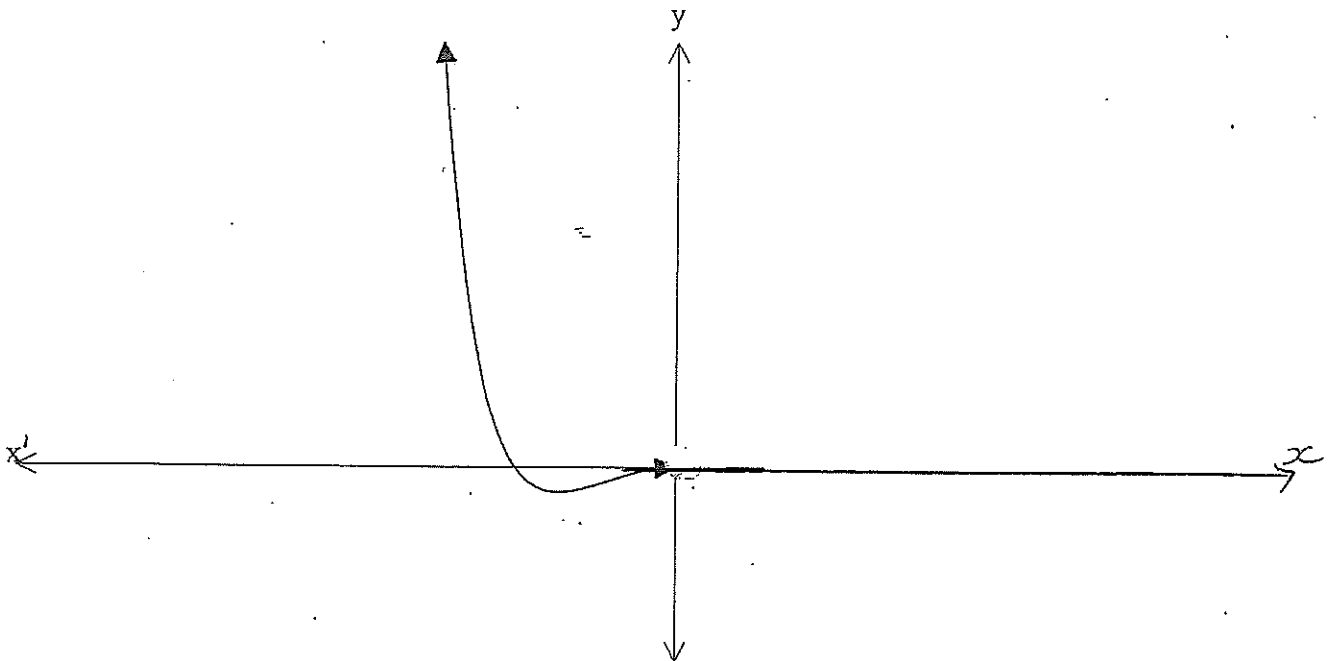
b) Subtract $3x^2-1$ from x^3-2x^2+3 1

c) The area of a circle is found using the formula $A=\pi r^2$. If the area of a particular circle is 45cm^2, find the radius correct to 1 decimal place   1

d) The diagram shows part of a function $y=f(x)$

Copy this diagram onto your answer sheet.

Complete the graph of $y=f(x)$ given that it is an even function.   1

e) Simplify $\sqrt{\dfrac{a^3b^7}{ab^3}}$ 2

f) Simplify $a^7 \times 5a^{-3} \div 15a^{-4}$ 1

Question 3   8 marks

Factorise fully

a) $x^4 - x^2$ 2

b) $3a^2 + 2a - 8$ 2

c) $x^2 - 12xy + 20y^2$ 2

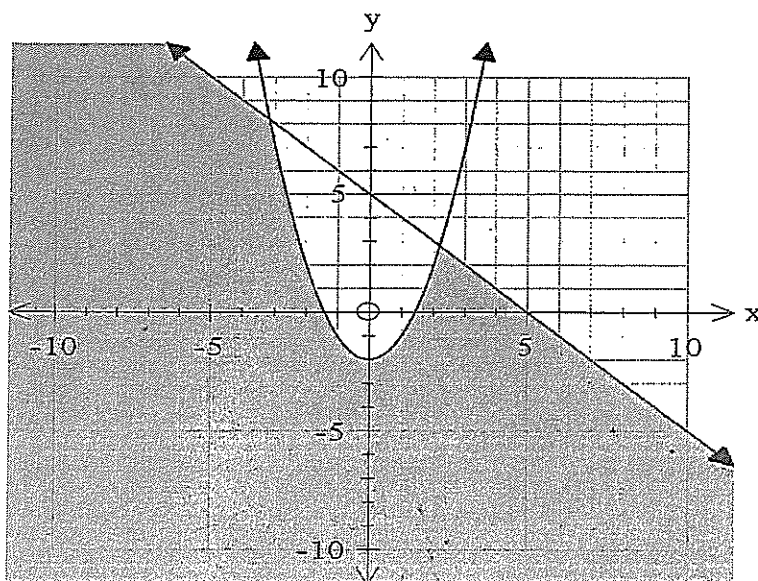d) $16x^4 - 2x$ 2

Question 4   8 marks

a) (i) Solve the following equations simultaneously:

$$\left\{ \begin{array}{c} x^2 + y^2 = 21 \\ x + y = 3 \end{array} \right\}$$ 2

   (ii) What does this solution represent in relation to the graphs of:

$$x^2 + y^2 = 21 \text{ and } x + y = 3 ?$$ 1

b) Using the process of completing the square, solve the following leaving your

answer in surd form: $x^2 + 4x = 1$ 2

c) State the two inequalities which represent the shaded region below: 3

Question 5 8 marks

Solve the following

a) $\dfrac{x-5}{4}+3=\dfrac{5x}{3}$ 2

b) $4x^2-4x+1=0$ 2

c) $|x+2|\le 2$ 2

d) $|2x+6|=3x-1$ 2

Question 6 8 marks

a) Sketch the following graphs on separate number planes. Use a ruler to draw all straight lines. Label any important points.

    i. $y=|x-2|$ 2

    ii. $x^2+y^2=4$ 2

    iii. $y=(x+1)^2+2$ 2

b) From a visual perspective, which of the previous questions are functions?
(Do not provide a formal proof). 2

Question 7   **8 marks**

a) Simplify fully $\sqrt{98} \times \sqrt{48}$ 2

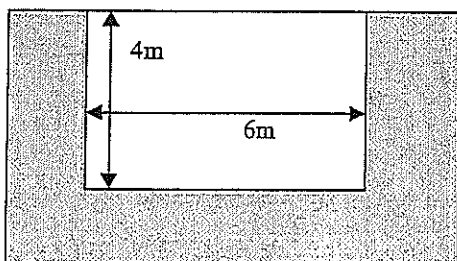b) Simplify $\dfrac{a^3 + 1}{a^2 - a + 1}$ 1

c) The function $f(x)$ is defined as

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x > 3 \\ 3x & \text{if } -2 \le x \le 3 \\ 2 & \text{if } x < -2 \end{cases}$$

   Find $f(-3) + f(4) - f(3)$ 3

d) A rectangular garden bed; 6m long and 4m wide has a path of uniform width around three sides as shown

If the area of the path is 25.5m^2, use a quadratic equation to find the width of the path.

                                                                                            2

Question 8 8 marks

a) For the function $y = \dfrac{x^2 + 3x}{x + 3}$

    i. State any discontinuities 1

    ii. State the domain and range of the function 2

    iii. Sketch the graph of the function, showing all important points. 3

b) Find $f(x)$ for all x given that $f(x-1) = x^2 - 1$ 2

End of Paper

Q2

(1) $\sqrt{3} = 1.732\ldots$

$2^{5/6} = 1.78179$

$3^{1/11} = 1.54202\ldots$

$2^{-1/13}$

$\dfrac{1}{\sqrt{3}+\sqrt{x}} + \dfrac{1}{\sqrt{3}-\sqrt{x}}$

$= \dfrac{\sqrt{3}-\sqrt{x} + \sqrt{3}+\sqrt{x}}{(\sqrt{3}+\sqrt{x})(\sqrt{3}-\sqrt{x})}$

$= \dfrac{2\sqrt{3}}{3-x}$

(2) $\dfrac{x-2y}{y}$

(b) $x^3 - 2x^2 + 3$

$- \dfrac{(3x^2-1)}{x^3-5x^2+4}$

(c) $\pi r^2 = 45$

$r = \sqrt{45/\pi}$

$= 3.784\ldots$

$= 3.8$

(d)

(e) $= wb^2$

(f) $= \dfrac{a}{3}$

(1) $\sqrt{3} = 1.732\ldots$

$2^{5/6} = 1.78179$

$3^{1/11} = 1.54202\ldots$

(2) 1.02×10^{-4})$\overline{10.00}$

$= 98039.21$

$= 98040$ sheets

(3) $1000n = 298.989898\ldots$

$10n = 2.9898\ldots$

$990n = 296$

$n = \dfrac{148}{495}$

(4) $\dfrac{-2}{\sqrt{(x^2-1)}}$

Q3

(a) $x^2(x-1)(x+1)$

(b) $(a+2)(3a-4)$

(c) $(x-2y)(x-10y)$

(d) $\left(2x \cdot \dfrac{3}{2x}-1\right)$

$= (2x-1)(2x-1)(4x^2+2x+1)$

Q4

(a) (i) $x^2 + y^2 = 21$ —①

$x + y = 3$ —②

$y = 3-x$ sub into ①

$x^2 + (3-x)^2 = 21$

$x^2 + 9 - 6x + x^2 = 21$

$2x^2 - 6x + 12 = 0$

$2(x^2 - 3x - 6) = 0$

$x = \dfrac{3 \pm \sqrt{33}}{2}$ $4.37, -1.37$

$x = \dfrac{3+\sqrt{33}}{2}$ $y = 6 - \dfrac{(3+\sqrt{33})}{2}$

$= \dfrac{3-\sqrt{33}}{2}$

$x = \dfrac{3-\sqrt{33}}{2}$ $y = \dfrac{3+\sqrt{33}}{2}$

(ii) Where graphs intersect.

Q2 (b)

$x^2 + 4x = 1$

$(x+2)^2 = 1+4$

$x+2 = \pm\sqrt{5}$

$x = \sqrt{5}-2,\ -\sqrt{5}-2$

(c) $y \le -x + 5$

$y \le x^2 - 2$

Q5

(a) $\dfrac{x-5}{4} + 3 = \dfrac{5x}{3}$

$\dfrac{x-5+12}{4} = \dfrac{5x}{3}$

$\dfrac{x+7}{4} = \dfrac{5x}{3}$

$3x+21 = 20x$

$21 = 17x$

$x = \dfrac{21}{17}$

(c) $|x+2| \le 2$

$x+2 \le 2$

$-x-2 \le 2$

$x \le 0$

$-x \le 4$

$x > -4$

$-4 \le x \le 0$

Q5

(a) $|2x+6| = 3x-1$

$2x+6 = 3x-1$

$7 = x$

check.

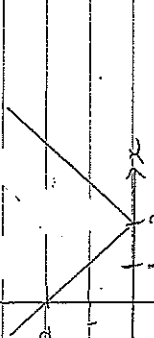LHS: 20

RHS: 20

$-2x-6 = 3x-1$

$-5 = 5x$

$x = -1$

check

LHS: 4

RHS: -4
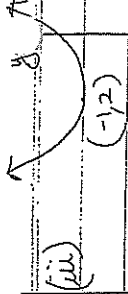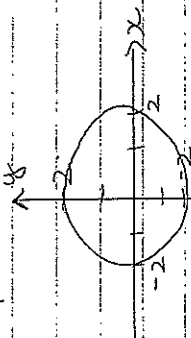
∴ only soln $x=7$

(iii)

Q6

(i) $y=|x-2|$

(ii) $x^2+y^2=4$

Q7

(a) (i) → (iii)

$\sqrt{98} \times \sqrt{48} = \sqrt{4704}$

$= \sqrt{16 \times 294}$

$= 4\sqrt{49 \times 6}$

$= 28\sqrt{6}$

(b) $\dfrac{a^3+1}{a^2-a+1} = \dfrac{(a+1)(a^2-a+1)}{a^2-a+1}$

$= a+1$

(c) $f(-3) = 2$

$f(4) = 17$

$f(9) = 9$

$2+17-9 = 10$

Q7

(a)

$A = 2(44x)x + 6x$

$25 \cdot 5 = 2x^2 + 14x$

$4x^2 + 28x - 51 = 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \dfrac{-28 \pm \sqrt{1600}}{8}$

$= \dfrac{-28 \pm 40}{8}$

$= \dfrac{3}{2}, \quad x \neq -68/8$

∴ width $= 3/2$ m

Q8

(a) $y = \dfrac{x^2+2x}{x+3}$

(ii) $x = -3$

(iii) domain $x: x \in \mathbb{R}\ x \neq$

range $y: y \in \mathbb{R}\ y \neq$

(b) $f(x-1) = x^2-1$

$f(x) = x^2+2x$