Sydney Technical High School

HSC Assessment Task 1

December 2011

Mathematics

Time allowed — 70 minutes

Instructions

- Use a blue or black pen.
- Approved calculators may be used.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Total marks — 55
- Attempt all questions, using the booklets provided.
- Marks awarded are shown on each question.
- Start each question on a new page.

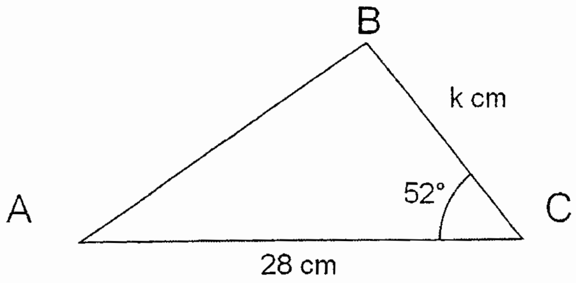
Question 1 **8 Marks**

a) Solve $cosx = -\frac{1}{\sqrt{2}}\ for\ 0° \leq x \leq 360°$ **2**

b) Given $\tan\theta = \frac{-5}{12}$, find $\sin\theta$, if θ is obtuse. **1**

c) The area of the triangle is 253.74 cm^2 and $\angle ACB = 52°$. **2**

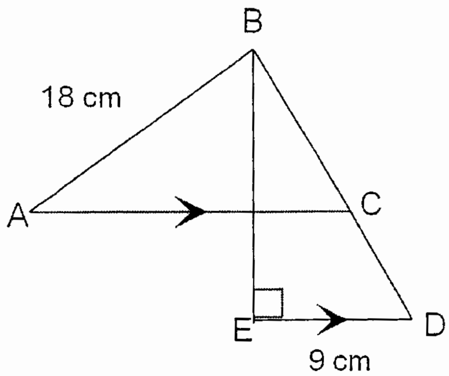
Calculate the value of k to the nearest centimetre.



d) In the diagram, $AB = 16\ cm$, $AC = 20\ cm$, $BE = 15\ cm$ and $ED = 9\ cm$.
$BE \perp ED$ and $AC \parallel ED$.

(i) Find the size of $\angle BDE$, to the nearest degree. **1**

(ii) Using the Sine Rule, find the size of $\angle ABC$, to the nearest degree. **2**

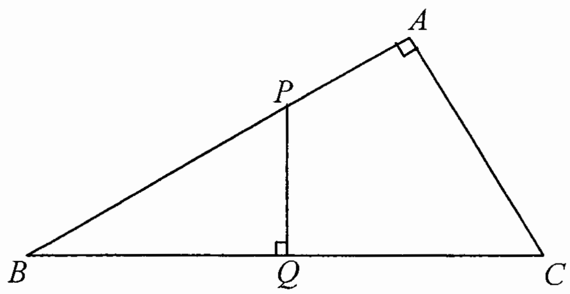


Question 2 **8 Marks**

a) A point P(x,y) moves so that the sum of the squares of its distance from each of the points A(0,0) and B(4,0) is equal to 40. **3**

Show that the locus of P(x,y) is a circle, and state its radius and centre.

b)



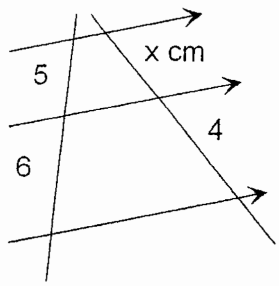
In the diagram, ABC is a right angled triangle with $AB = 8$ cm and $AC = 6$ cm.

If the triangle is folded along the line PQ, vertex B coincides with vertex C.

(i) Show that triangles ABC and QBP are similar. **2**

(ii) Find the length of PQ. **1**

c) Find the value of x in the diagram. (Give a reason) **2**



Question 3 **8 Marks**

a) The first three terms of a sequence are *7, 13* and *19.*

(i) Is this sequence arithmetic, geometric or neither? (Show reasoning) **1**

(ii) Find a simplified expression for the *nth* term, T_n. **1**

(iii) Find T_{32}. **1**

(iv) Is *243* a term of the sequence? (Show appropriate working.) **2**

b) Find the values of *A*, *B* and *C* if $3x^2 + 5x \equiv A(x+1)^2 + B(x+1) + C$ **3**

Question 4 **7 Marks**

a) (i) The fourth term of a geometric sequence is $\frac{2}{9}$ and the seventh term is $\frac{2}{243}$. **3**
Find the first three terms.

(ii) Explain why a limiting sum exists. (Show reasoning) **1**

(iii) Find the limiting sum (sum to infinity). **1**

b) Find the geometric series whose first term is 3 and whose limiting sum is 9. **2**

Question 5 **9 Marks**

a) The equation of a parabola is given as $(x - 2)^2 = 4y$.

(i) Find the coordinates of the focus. **1**

(ii) What is the equation of the directrix? **1**

(iii) Sketch the parabola, showing all important features. **2**

b) Find the equation of the parabola with vertex $(-1, 2)$, axis parallel to the x axis and passing through the point $(7, 10)$. **2**

c) Find the equation of the tangent to the parabola $x^2 = -16y$ at the point $(-2, -\frac{1}{4})$. **3**

Question 6 **8 Marks**

a) For the parabola with equation $y = 10 + 3x - x^2$, find: **4**

(i) the equation of the axis of symmetry.

(ii) the coordinates of the vertex.

(iii) the x intercepts.

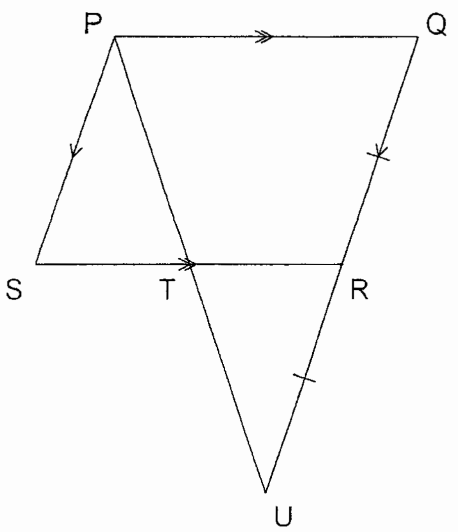
(iv) the values of x for which $10 + 3x - x^2 > 0$.

b) Write a quadratic equation with roots α and β if $\alpha + \beta = -2$ and $\alpha\beta = 6$. **1**

c) Make a suitable substitution and solve $(x^2 - 2)^2 - 7(x^2 - 2) = 0$. **3**

Question 7 **7 Marks**

a) In the diagram, $PQRS$ is a parallelogram. QR is produced to U so that $QR = RU$.



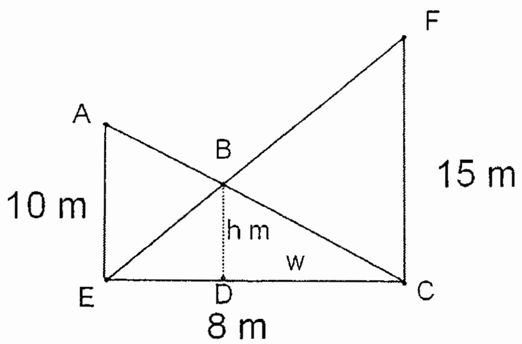
(i) Giving appropriate reasons, prove that ΔPST and ΔURT are congruent. **3**

(ii) Hence or otherwise, show that T is the midpoint of SR. **1**

b) Two vertical poles of height $10m$ and $15m$ are $8m$ apart. Wire stretches from the top of each pole to the foot of the other. See diagram below.

(i) Use the properties of similar figures to find an expression for w in terms of h. [Hint: Let $DC = w$] **1**

(ii) Find the height, h m above the ground where the wires cross. **2**



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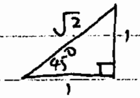
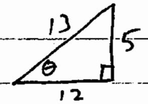
HSC ASSESSMENT 1 SOLUTIONS MATHEMATICS 2011

Q1 a) $\cos x = -\frac{1}{\sqrt{2}}$



$\therefore x = 135^\circ, 225^\circ$

b) $\tan\theta = \frac{-5}{12}$



$\therefore \sin\theta = \frac{5}{13}$

c) $A = \frac{1}{2}ab\sin C$

$\therefore 253.74 = \frac{1}{2} \times 28 \times k \times \sin 52^\circ$

$= 14k\sin 52^\circ$

$\therefore k = \frac{253.74}{14\sin 52^\circ}$

$\doteqdot 23$ cm

d) (i) $\tan B\hat{D}E = \frac{15}{9}$

$= 1\frac{2}{3}$

$\therefore B\hat{D}E \doteqdot 59^\circ$

(ii) $\therefore A\hat{C}B = 59^\circ$ (corresponding $\angle$s)

$\therefore \frac{\sin A\hat{B}C}{20} = \frac{\sin 59^\circ}{18}$

$\therefore \sin A\hat{B}C = \frac{10\sin 59^\circ}{9}$

$= 72^\circ 15'$

$\doteqdot 72^\circ$

Q2 a) $(x-0)^2 + y^2 + (x-4)^2 + y^2 = 40$

$\therefore x^2 + y^2 + x^2 - 8x + 16 + y^2 = 40$

$\therefore 2x^2 + 2y^2 - 8x = 24$

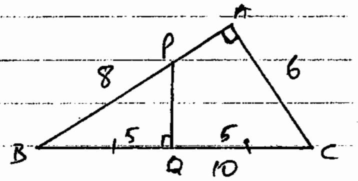
$\therefore x^2 + y^2 - 4x = 12$

$\therefore (x-2)^2 + y^2 = 16$

circle: centre $(2, 0)$

radius 4

b)



(i) In Δs QBP and ABC,

$A\hat{B}C$ is common

$B\hat{Q}P = B\hat{A}C = 90^\circ$

$\therefore \Delta QBP \,|||\, \Delta ABC$ (AAA)

(ii) $\frac{PQ}{BQ} = \frac{6}{8}$

$\therefore PQ = 5 \times \frac{6}{8}$

$= 3.75$ cm

c) $\frac{x}{4} = \frac{5}{6}$

$\therefore 6x = 20$

$\therefore 3x = 10$

$\therefore x = 3\frac{1}{3}$ cm

Intercepts in proportion across parallel lines.

Q3 a) (i) For AP

$13 - 7 = 6$

$19 - 13 = 6$

$\therefore$ AP

For GP

$\frac{13}{7} = 1\frac{6}{7}$

$\frac{19}{13} = 1\frac{6}{13}$

$\therefore$ Not GP

(ii) $T_n = a + (n-1)d$

$= 7 + 6(n-1)$

$= 6n + 1$

(iii) $\therefore T_{32} = 6 \times 32 + 1$

$= 193$

(iv) Let $T_n = 243$

$\therefore 6n + 1 = 243$

$\therefore 6n = 242$

$\therefore n = \frac{242}{6}$

$= 40\frac{1}{3}$

243 is not a term as $40\frac{1}{3} \notin \mathbb{N}$

b) $3x^2 + 5x \equiv A(x+1)^2 + B(x+1) + C$

$RHS = A(x+1)^2 + B(x+1) + C$

$= A(x^2 + 2x + 1) + Bx + B + C$

$= Ax^2 + 2Ax + A + Bx + B + C$

$= Ax^2 + (2A+B)x + (A+B+C)$

$\therefore A = 3$

$2A + B = 5$

$\therefore B = -1$

$A + B + C = 0$

$\therefore 3 - 1 + C = 0$

$\therefore C = -2$

$$\left.\begin{aligned} A &= 3 \\ B &= -1 \\ C &= -2 \end{aligned}\right\}$$

Q4 a) (i) $T_n = ar^{n-1}$

$\therefore T_4 = ar^3 = \frac{2}{9}$

$T_7 = ar^6 = \frac{2}{243}$

$\therefore \frac{ar^6}{ar^3} = \frac{2}{243} \times \frac{9}{2}$

$\therefore r^3 = \frac{1}{27}$

$\therefore r = \frac{1}{3}$

$\frac{a}{27} = \frac{2}{9}$

$\therefore a = \frac{2 \times 27}{9}$

$= 6$

Seq $\Rightarrow$ $6, 2, \frac{2}{3}$

(ii) Limiting sum exists because $-1 < r < 1$ [or $|r| < 1$]

(iii) $S_\infty = \frac{a}{1-r}$

$= \frac{6}{1-\frac{1}{3}}$

$= 9$

4b) $a = 3$

$\frac{a}{1-r} = 9$

$\therefore \frac{3}{1-r} = 9$

$\therefore \frac{1}{1-r} = 3$

$\therefore 1 - r = \frac{1}{3}$

$\therefore r = 1 - \frac{1}{3}$

$= \frac{2}{3}$

Seq $\Rightarrow$ $3, 2, \frac{4}{3}, \ldots$

Q5 a) $(x-2)^2 = 4y$

General form $\Rightarrow (x-x_1)^2 = 4a(y-y_1)$

$\therefore a = 1,\ x_1 = 2,\ y_1 = 0$

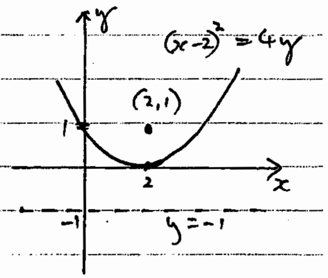
vertex $(2,0)$

(i) Parabola upright $\therefore$ focus $(2,1)$

(ii) Directrix $y = -a$

$\therefore y = -1$

(iii)



b) $(y-y_1)^2 = 4a(x-x_1)$

$\therefore x_1 = -1$

$y_1 = 2$

$\therefore (y-2)^2 = 4a(x+1)$

But $(7,10)$ satisfies

$\therefore (10-2)^2 = 4a(7+1)$

$64 = 4a \times 8$

$\therefore 4a = 8$

$\therefore a = 2$

$\therefore (y-2)^2 = 8(x+1)$

c) $x^2 = -16y$ $\quad (-2, -\frac{1}{4})$

$y = -\frac{x^2}{16}$

$\therefore \frac{dy}{dx} = -\frac{2x}{16}$

$= -\frac{x}{8}$

$= \frac{1}{4}$ when $x = -2$

$\therefore$ equation of tangent $\Rightarrow$

$y - y_1 = m(x - x_1)$

$y + \frac{1}{4} = \frac{1}{4}(x+2)$

$\therefore 4y + 1 = x + 2$

$\therefore x - 4y + 1 = 0$

Q6 a) $y = -x^2 + 3x + 10$

(i) axis of symmetry when $x = \frac{-b}{2a}$

$\therefore x = \frac{-3}{2\times -1}$

$\therefore x = \frac{3}{2}$

(ii) $x = \frac{3}{2}$

$\therefore y = -(\frac{3}{2})^2 + 3\times\frac{3}{2} + 10$

$= -\frac{9}{4} + \frac{9}{2} + 10$

$= 12\frac{1}{4}$

$\therefore$ vertex $\Rightarrow (\frac{3}{2}, 12\frac{1}{4})$

(iii) $-x^2 + 3x + 10 = 0$

$\therefore x^2 - 3x - 10 = 0$

$\therefore (x-5)(x+2) = 0$

$\therefore x = -2, 5$

(iv) $a < 0$ $\therefore$ concave down

$\therefore$ positive when $-2 < x < 5$

Q6 b) $(x+\alpha)(x+\beta) = x^2 + (\alpha+\beta)x + \alpha\beta$

$\therefore x^2 + 2x + 6 = 0$ as $\alpha + \beta = -2$

$\alpha\beta = 6$

c) $(x^2-2)^2 - 7(x^2-2) = 0$

Let $w = x^2 - 2$

$\therefore w^2 - 7w = 0$

$\therefore w(w-7) = 0$

$\therefore w = 0, 7$

$\therefore x^2 - 2 = 0, 7$

$\therefore x^2 = 2, 9$

$\therefore x = \pm\sqrt{2}, \pm 3$

Q7 a) (i) In ΔPST and ΔURT,

$P\hat{T}S = U\hat{T}R$ (vertically opposite $\angle$s)

$P\hat{S}T = U\hat{R}T$ (alternate $\angle$s on parallel lines)

$QR = RU$ (data)

$SP = QR$ (opposite sides of parallelogram)

$\therefore RU = SP$

$\therefore \Delta PST \equiv \Delta URT$ (AAS)

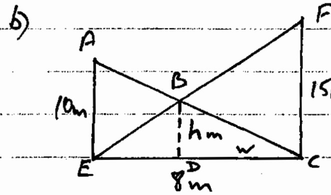
QED

(ii) $ST = RT$ (corresponding sides in congruent Δs)

$\therefore T$ is the midpoint of SR

QED

b)



It is interesting that if we change the distance between the poles, the value of h does not change. Try replacing the 8 with a variable (say d) to prove this.

Let $DC = w$

(i) Now $\frac{w}{h} = \frac{8}{10}$

$\therefore w = \frac{4h}{5}$

(ii) $\frac{8-w}{h} = \frac{8}{15}$

$\therefore \frac{8 - \frac{4h}{5}}{h} = \frac{8}{15}$

$\therefore 8h = 15(8 - \frac{4h}{5})$

$= 120 - 12h$

$\therefore 20h = 120$

$\therefore h = 6$ m