SYDNEY TECHNICAL HIGH SCHOOL

Mathematics Extension 1

Preliminary Assessment 1
May 2010

Time allowed — 70 minutes

Instructions

- Use a blue or black pen.

- Approved calculators may be used.

- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.

- Marks awarded are shown on each question.

- Total marks — 55

- Attempt all questions.

- Start each question on a new page.

- Hand in your examination paper and solutions in one bundle.

Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Total

Question 1   **Marks 8**

a)   Solve: $-12 \leq 1 - 2x \leq -3$.   (2)

b)   Sketch the function $f(x) = \begin{cases} 1 - x, for\ x < 0 \\ -1, for\ x = 0 \\ 1 - x^2\ for\ x > 0 \end{cases}$ (2)

c)   Solve $\dfrac{3}{x-3} - \dfrac{5}{x+1} = 1$.   (2)

d)   Solve by completing the square $x^2 - 4x + 1 = 0$ (2)

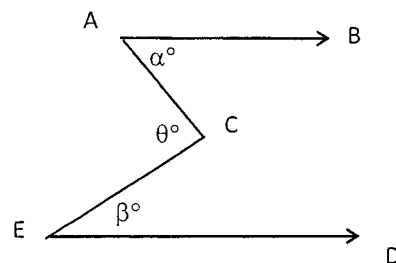**Question 2**   (Start a new page)   **Marks 10**

a)   Find the values x and y such that: $6 + \sqrt{x - y} = x + y + 3\sqrt{2}$.   (3)

b)   For the diagram shown, prove giving reasons that $p + q + r = 360$.   (2)
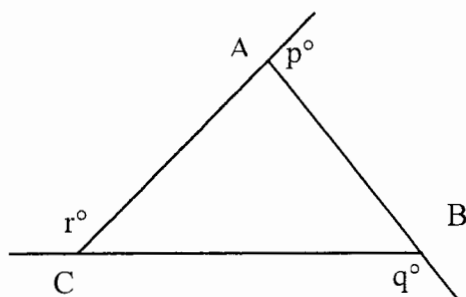
c)   Solve for x and show on a number line: (3)

$\dfrac{4x+1}{x-4} \geq 2$

d)   Write down the exact value of *cosec 240°*, leaving denominator irrational.   (2)

Question 3   (Start a new page)   **Marks 9**

a)   For the function $y = \frac{1}{x^2 - 1}$:

    (i)   State the domain.   *(1)*

    (ii)   Find all asymptotes.   *(2)*

    (iii)   Hence, sketch the curve.   *(2)*

b)   Solve $|2x + 1| = |x - 5|$.   *(2)*

c)   Express θ in terms of α and β, giving reasons.   *(2)*

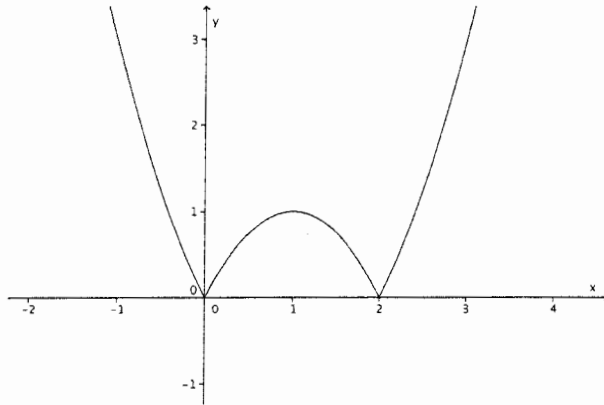**Question 4**   (Start a new page)   **Marks 10**

a)   Solve $2\cos 2x = \sqrt{3}$, for $0° \leq x° \leq 360°$.   *(3)*

b)   Solve for $0° \leq \theta \leq 360°$ *(2)*

    $\sin \theta = \cos \theta$

c)   Show that $(1 + \tan A + \sec A)(1 + \tan A - \sec A) = 2 \tan A$ *(3)*

d)   If $x - \frac{1}{x} = 3\sqrt{5}$, find the value of $x^2 + \frac{1}{x^2}$. *(2)*

Question 5   (Start a new page)   **Marks 8**

a)   Show algebraically whether the function $f(x) = 3x(3 - x^2)$ is *odd*, *even* or neither.   (2)

b)   Write a possible equation for the curve shown: (2)

c)   Simplify $\dfrac{3^{n+2} \times 12^n}{4^{n-1}}$ (2)

d)   Eliminate θ from the pair of equations. (Leave in factorised form).   (2)

$x = 1 + \sec \theta$

$y = 2 + \tan \theta$

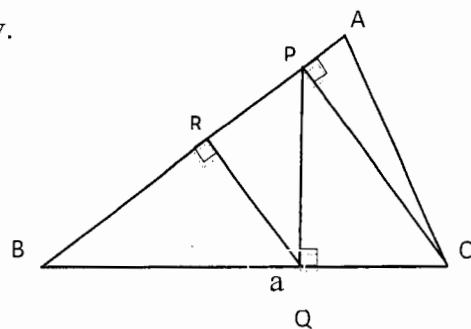**Question 6**   (Start a new page)   **Marks 10**

a)   By treating the expression as a difference of two squares,   (2)
     express it as the product of four factors: $x^6 - y^6$.

b)   In triangle ABC, lines CP, PQ and QR are drawn
     perpendicular to AB, BC and AB respectively.

  (i)   Explain why $\angle RBQ = \angle RQP = \angle QPC$.   (3)

  (ii)   Show that $QR = a \sin B \cos^2 B$.   (2)

c)   Simplify $\dfrac{1}{(a-b)(a-c)} + \dfrac{1}{(b-c)(b-a)} + \dfrac{1}{(c-a)(c-b)}$ (3)
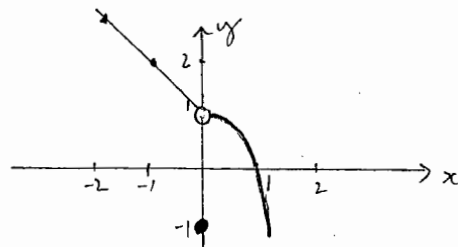
Q1 a) Solve: $-12 \leq 1-2x \leq -3$

$$-12 \leq 1-2x \leq -3$$
$$\therefore -13 \leq -2x \leq -4$$
$$\therefore 4 \leq 2x \leq 13$$
$$\therefore 2 \leq x \leq 6.5$$

b)

c) $\dfrac{3}{x-3} - \dfrac{5}{x+1} = 1$

$$\therefore 3(x+1) - 5(x-3) = (x-3)(x+1)$$
$$\therefore 3x+3 - 5x+15 = x^2 - 2x - 3$$
$$\therefore x^2 = 21$$
$$\therefore x = \pm\sqrt{21}$$

d) $x^2 - 4x + 1 = 0$

$$\therefore (x-2)^2 - 4 + 1 = 0$$
$$\therefore (x-2)^2 = 3$$
$$\therefore x - 2 = \pm\sqrt{3}$$
$$\therefore x = 2 \pm \sqrt{3}$$

Q2

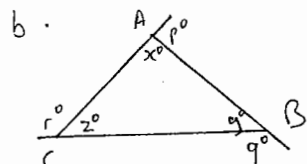**a)** $6 + \sqrt{x-y} = x + y + 3\sqrt{2}$
$$= x + y + \sqrt{18}$$

$$\therefore x + y = 6$$
$$x - y = 18$$
$$\therefore 2x = 24$$
$$\therefore x = 12$$
$$\therefore y = -6$$

b.

Let $C\hat{A}B = x°$
$A\hat{B}C = y°$
$B\hat{C}A = z°$

$p + x = 180$ (straight angle)
$q + y = 180$ ("   "   )
$r + z = 180$ ("   "   )

$$\therefore p + x + q + y + r + z = 3 \times 180$$
$$= 540$$
$$\therefore p + q + r + x + y + z = 540$$

But $x + y + z = 180$ (angle sum of $\triangle$)

$$\therefore p + q + r + 180 = 540$$
$$\therefore p + q + r = 360 \quad \text{QED}$$

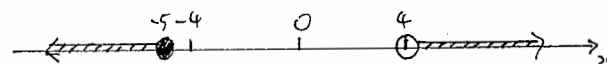**c)** $\dfrac{4x+1}{x-4} \geq 2$ NB $x \neq 4$

$$\therefore (4x+1)(x-4) \geq 2(x-4)^2$$
$$\therefore 4x^2 - 15x - 4 \geq 2x^2 - 16x + 32$$
$$\therefore 2x^2 + x - 36 \geq 0$$
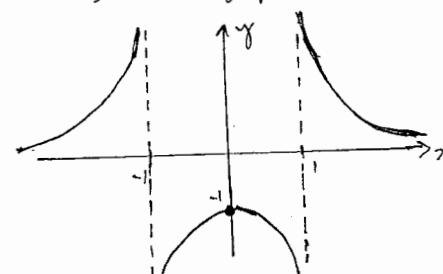$$\therefore (2x+9)(x-4) = 0 \quad \left]\text{provides boundaries}\right.$$
$$\therefore x = -4.5, 4$$

For $x < -4.5$, $\dfrac{4x+1}{x-4} \geq$

For $-4.5 < x < 4$, $\dfrac{4x+1}{x-4}$

For $x > 4$, $\dfrac{4x+1}{x-4} \geq 2$

For $x = 4$, $\dfrac{4x+1}{x-4}$ is undef

d) $\operatorname{cosec} 240° = \dfrac{1}{\sin 240°}$

$$= \dfrac{-1}{\sin 60°}$$
$$= \dfrac{-2}{\sqrt{3}}$$
$$\left(= \dfrac{-2\sqrt{3}}{3} \right)$$

Q3

a) $y = \dfrac{1}{x^2 - 1}$

(i) domain $\Rightarrow$ all real x except $x = \pm 1$

(ii) vertical asymptotes when $x = \pm 1$
horizontal asymptotes when $y = 0$

(iii)

As $x \to 1^-$, $y \to -\infty$
As $x \to -1^+$, $y \to -\infty$
As $x \to 1^+$, $y \to \infty$
As $x \to -1^-$, $y \to \infty$
As $x \to \infty$, $y \to 0^+$
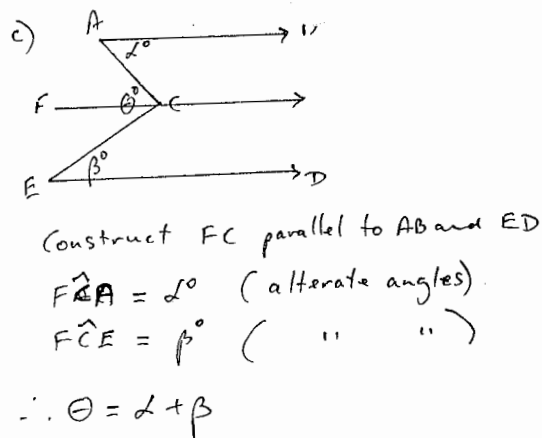As $x \to -\infty$, $y \to 0^+$

23 cont

b) $|2x+1| = |x-5|$

$(2x+1)^2 = (x-5)^2$

$\therefore 4x^2 + 4x + 1 = x^2 - 10x + 25$

$\therefore 3x^2 + 14x - 24 = 0$

$\therefore (3x-4)(x+6) = 0$

$\therefore x = \frac{4}{3}, -6$

c)

Construct FC parallel to AB and ED

$F\hat{A}A = \alpha°$ (alternate angles)

$F\hat{C}E = \beta°$ (" ")

$\therefore \theta = \alpha + \beta$

Q4

a) $2\cos 2x = \sqrt{3}$, for $0° \le x \le 360°$

$\therefore \cos 2x = \frac{\sqrt{3}}{2}$

$\therefore 2x = 30°, 330°, 390°, 690°$

$\therefore x = 15°, 165°, 195°, 345°$

b) $\sin\theta = \cos\theta$ $\therefore$ first quadrant.

$\therefore \frac{\sin\theta}{\cos\theta} = 1$

$\therefore \tan\theta = 1$

$\therefore \theta = 45$ only

c) RTS $(1 + \tan A + \sec A)(1 + \tan A - \sec A) = 2\tan A$

LHS $= (1 + \tan A + \sec A)(1 + \tan A - \sec A)$

$= (1 + \tan A)^2 - \sec^2 A$

$= 1 + 2\tan A + \tan^2 A - \sec^2 A$

$= 2\tan A + \sec^2 A - \sec^2 A$

$= 2\tan A$

$=$ RHS   QED

d) $x - \frac{1}{x} = 3\sqrt{5}$

$\therefore \left(x - \frac{1}{x}\right)^2 = 45$

$\therefore x^2 - 2 + \frac{1}{x^2} = 45$

$\therefore x^2 + \frac{1}{x^2} = 47$

Q5

a) $f(x) = 3x(3 - x^2)$

$f(-x) = -3x(3 - (-x)^2)$

$= -3x(3 - x^2)$

$= -f(x)$

$\therefore$ the function is odd.

d) $x = 1 + \sec\theta$

$y = 2 + \tan\theta$

$\therefore \sec\theta = x - 1$ —(i)

$\tan\theta = y - 2$ —(ii)

Now $\sec^2\theta = 1 + \tan^2\theta$

$\therefore (x-1)^2 = 1 + (y-2)^2$

b)(i) $R\hat{Q}B = 90 - R\hat{B}Q$ (right-angled $\triangle$)

$\therefore R\hat{A}P = 90 - (90 - R\hat{B}Q)$ (complementary angles)

$= R\hat{B}Q$

Now $R\hat{P}Q = 90 - R\hat{A}P$

So $C\hat{P}Q = 90 - (90 - R\hat{A}P)$ (complementary angles)

$= R\hat{A}P$

$= R\hat{B}Q$

$\therefore R\hat{B}Q = R\hat{A}P = Q\hat{P}C$ QED

(ii) $PC = a\sin B$ in $\triangle PBC$

$\therefore \frac{PQ}{a\sin B} = \cos B$ in $\triangle PQC$

$\therefore PQ = a\sin B \cos B$

$\therefore \frac{RQ}{a\sin B \cos B} = \cos B$ in $\triangle RQP$

$\therefore RQ = a\sin B \cos^2 B$ QED

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b) $y = |x(x-2)|$

$= |x^2 - 2x|$

c) $\dfrac{3^{n+2} \times 12^n}{4^{n-1}} = \dfrac{3^{n+2} \times 3^n \times 4^n}{4^{n-1}}$

$= 3^{2n+2} \times 4$

$= 4 \times 3^{2(n+1)}$

Q6

a) $x^6 - y^6 = (x^3 - y^3)(x^3 + y^3)$

$= (x-y)(x^2 + xy + y^2)(x+y)(x^2 - xy - $

c) $\dfrac{1}{(a-b)(a-c)} + \dfrac{1}{(b-c)(b-a)} + \dfrac{1}{(c-a)(c-b)}$

$= \dfrac{(b-c) - (a-c) + (a-b)}{(a-b)(b-c)(a-c)}$

$= \dfrac{b - c - a + c + a - b}{(a-b)(b-c)(a-c)}$

$= 0$