

# SYDNEY TECHNICAL HIGH SCHOOL



## PRELIMINARY HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK 3

SEPTEMBER 2013

2 UNIT

# Mathematics

### General Instructions

- Working time - 120 minutes
- Write using black or blue pen
- Board-approved calculators may be used
- All necessary working should be shown in questions 11 to 18
- Start each question on a new page

Total marks - 82

### Section 1 - 10 marks

Attempt Questions 1 – 10.  
Allow about 15 minutes for this section.

### Section 2 - 72 marks

Attempt Questions 11 – 18.  
Allow about 105 minutes for this section.

Name : \_\_\_\_\_

Teacher : \_\_\_\_\_

## Section 1

10 marks

Attempt Questions 1 – 10

Allow about 15 minutes for this section

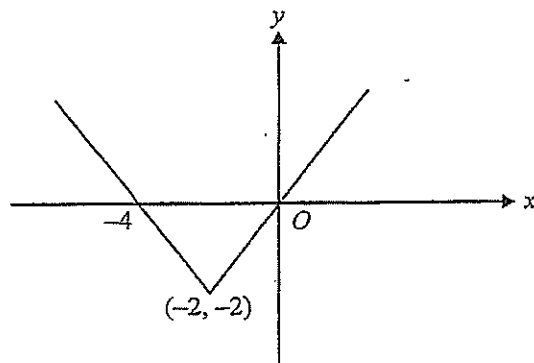
Use the multiple-choice answer sheet in your answer booklet for Questions 1 – 10.  
Do not remove the multiple-choice answer sheet from your answer booklet.

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1. If  $x = -4$  is a root of the equation  $2x^2 + kx + 4 = 0$ , what is the value of  $k$ ?

- (A) 7
- (B) 8
- (C) 9
- (D) 10

2.



The rule of the function whose graph is shown is

- (A)  $y = |x| - 4$
- (B)  $y = |x - 2| + 2$
- (C)  $y = |x + 2| - 2$
- (D)  $y = |2 - x| - 2$

3. If  $\sqrt{12} + \sqrt{3} = \sqrt{b}$  then

(A)  $b = \sqrt{15}$

(B)  $b = 3\sqrt{3}$

(C)  $b = 15$

(D)  $b = 27$

4. The  $x$  coordinates of the points of intersection of  $y = x^2$  and  $x + y = 6$  are the solutions of

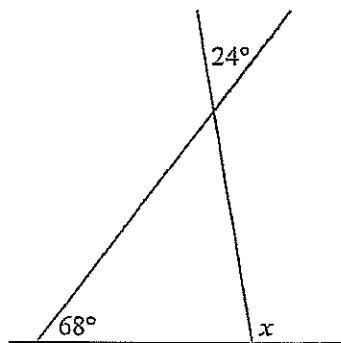
(A)  $x^2 - x - 6 = 0$

(B)  $x^2 + x - 6 = 0$

(C)  $x^2 - x + 6 = 0$

(D)  $x^2 + x + 6 = 0$

5.



The size of the angle  $x$  is

(A)  $68^\circ$

(B)  $88^\circ$

(C)  $92^\circ$

(D)  $112^\circ$

6. Given  $y = a x^n$  then  $\frac{dy}{dx} = ?$

(A)  $a \times n \times x^{n-1}$

(B)  $a \times n \times x^{n+1}$

(C)  $n \times x^{n-1}$

(D)  $a \times x^{n-1}$

7. Find the values of  $m$  for which  $24 + 2m - m^2 \leq 0$

(A)  $m \leq -4$  or  $m \geq 6$

(B)  $m \leq -6$  or  $m \geq 4$

(C)  $-4 \leq m \leq 6$

(D)  $-6 \leq m \leq 4$

8. For  $y = (4x + 1)(x + 2)^3$ ,  $\frac{dy}{dx}$  is equal to

(A)  $12(x + 2)^2$

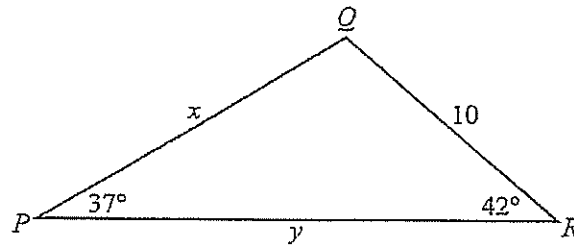
(B)  $(x + 2)^2(16x + 11)$

(C)  $3(x + 2)^2(4x + 1)$

(D)  $(x + 2)^2(12x + 7)$

9. PQR is a triangle with side lengths  $x$ , 10 and  $y$ , as shown below.

In this triangle, angle RPQ =  $37^\circ$  and angle QRP =  $42^\circ$ .



Which one of the following expressions is correct for triangle PQR ?

- (A)  $x = \frac{10}{\sin 37^\circ}$
- (B)  $x = 10 \times \frac{\sin 42^\circ}{\sin 37^\circ}$
- (C)  $y = 10 \times \frac{\sin 37^\circ}{\sin 101^\circ}$
- (D)  $10^2 = x^2 + y^2 - 2xy \cos 42^\circ$

10. For  $y = \sqrt{1 - f(x)}$ ,  $\frac{dy}{dx}$  is equal to

- (A)  $\frac{2f'(x)}{\sqrt{1-f(x)}}$
- (B)  $\frac{-1}{2\sqrt{1-f(x)}}$
- (C)  $\frac{1}{2} \sqrt{1-f'(x)}$
- (D)  $\frac{-f'(x)}{2\sqrt{1-f(x)}}$

## Section 2

72 marks

Attempt Questions 11 – 18

Allow about 105 minutes for this section

Start each question on a new page

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### Question 11 (9 marks)

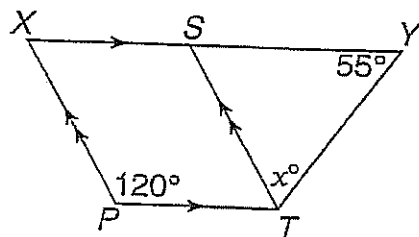
- a) Simplify  $\frac{3p^2-3q^2}{6p-6q}$  2
- b) If  $\frac{\sqrt{2}}{9} = 2^m \times 3^n$  find the values of  $m$  and  $n$ . 2
- c) Solve  $\frac{x+1}{x+3} = 5$  2
- d) Differentiate  $y = 4x^3 - 3x^2 - x + 2$  2
- e) Find the exact value of  $\tan 330^\circ$  1

### Question 12 (9 marks) Start a new page

- a) The points  $A(1, 7)$ ,  $B(-3, 5)$  and  $C(4, -1)$  lie on a number plane.
- i) Find the length of AC. 1
- ii) Find the gradient of AC. 1
- iii) Find the equation of the line AC. 1
- iv) Find the perpendicular distance of B from the line AC. 1
- v) If ABCD is a parallelogram, find the coordinates of D. 1
- b) Find the exact solution of  $2x^2 + 4x - 5 = 0$  2
- c) Simplify  $\sqrt{60} + (\sqrt{5} - \sqrt{3})^2$  2

**Question 13** (9 marks) [Start a new page](#)

- a) Find the area bounded by the line  $4x - y = 8$ , the  $x$  axis and the  $y$  axis. 2
- b) If  $f(x) = x^2 + 2x$  find  $\frac{f(x+h)-f(x)}{h}$  in simplest form. 2
- c) Find the equation of the tangent to  $y = (x - 3)^3$  at the point  $(1, -8)$  3
- d) The diagram shows  $XY$  parallel to  $PT$ ,  $XP$  parallel to  $ST$ ,  
angle  $XPT = 120^\circ$  and angle  $SYT = 55^\circ$ .



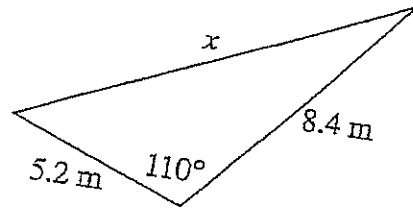
Find , with reasons, the value of  $x$ .

2

**Question 14** (9 marks) [Start a new page](#)

- a) Find the value of  $x$ , correct to 1 decimal place.

2



- b) Solve  $|2x - 4| < 2$

2

- c) Evaluate  $\lim_{x \rightarrow 2} \frac{3x^2 - 5x - 2}{x - 2}$

2

- d) Indicate, by shading, the region where points simultaneously satisfy the inequalities  $y \leq x^2 - 1$  and  $x^2 + y^2 \leq 4$

3



**Question 15** (9 marks) Start a new page

- a) If  $\sin \theta = \frac{2}{3}$  and  $\cos \theta < 0$  find the exact value of  $\tan \theta$ . 2
- b) Differentiate the following with respect to  $x$
- i)  $y = (5x - 3)^4$  1
- ii)  $y = \frac{6}{x^2}$  1
- iii)  $y = 12\sqrt{x^3}$  1
- c) If the quadratic equation  $ax^2 + bx + c = 0$  has a discriminant equal to 4, what does this tell us about the nature of the roots of the equation? 2
- d) If the lines  $2x - 5y + 3 = 0$  and  $ax + 4y + 12 = 0$  are perpendicular, find the value of  $a$ . 2

**Question 16** (9 marks) Start a new page

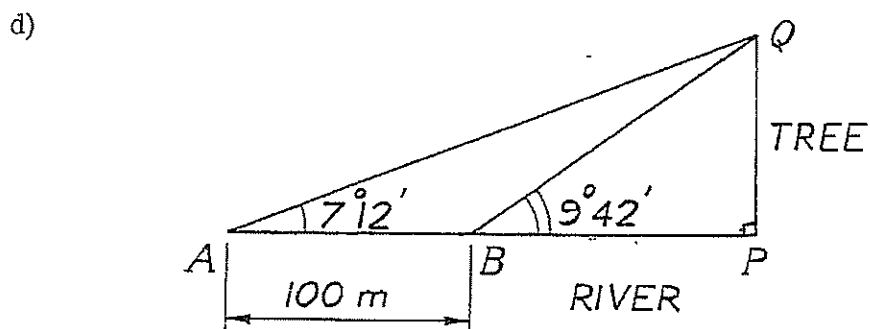
- a) Solve  $\sin^2 \theta = \frac{3}{4}$  for  $0 \leq \theta \leq 360^\circ$ . 2
- b) If  $f(x) = x\sqrt{2x+1}$  evaluate  $f'(4)$ . 2
- c) Find, correct to the nearest degree, the acute angle the line  $3x - y - 3 = 0$  makes with the  $x$  axis. 2
- d) Evaluate  $\lim_{x \rightarrow \infty} \frac{x^2-4}{2x^2+x}$  1
- e) Simplify  $\sin^3 A \sec A + \sin A \cos A$  2

**Question 17** (9 marks) Start a new page

a) State the domain of  $y = \sqrt{x+4}$  1

b) If  $y = \frac{2x-1}{x+4}$  find  $\frac{dy}{dx}$ . 2

c) Simplify  $\sin(90^\circ - \theta) \operatorname{cosec} \theta$  2



The diagram above was sketched by a surveyor, who measured the angle of elevation of a tree top on the other side of a river to be  $7^\circ 12'$  at the point A. 4

At the point B, 100 metres directly towards the tree from A, the angle of elevation was  $9^\circ 42'$ .

Calculate the height of the tree, correct to 3 significant figures.

**Please turn over**

**Question 18** (9 marks) Start a new page

- a) Find the gradient of the normal to the curve

$$y = x^2 + 6x + 3 \text{ at the point } (1, -2) \quad 2$$

- b) Find all values of  $k$  for which the quadratic equation 3

$$kx^2 - 8x + k = 0 \text{ has real roots.}$$

- c) The curve  $y = ax + \frac{b}{x^2}$  cuts the  $x$  axis at the point  $(2, 0)$  4

and the gradient of the tangent to this curve at the point  $(2, 0)$  equals 1.

Find the values of  $a$  and  $b$ .

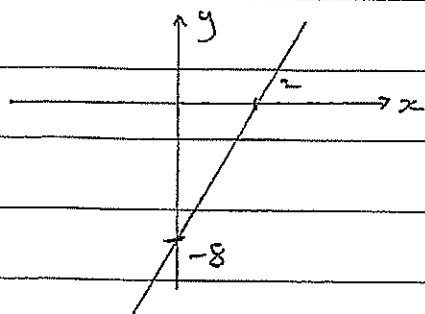
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CANDIDATE NO: \_\_\_\_\_

## SOLUTIONS (JUNE 2013 YEARLY)

1. C	12. a. i. $AC = \sqrt{(4-1)^2 + (-1-7)^2}$
2. C	$= \sqrt{73}$ units
3. D	
4. B	ii. $m = \frac{7-1}{1-4}$
5. C	$= \frac{8}{-3}$
6. A	
7. A	iii. $y-7 = -\frac{8}{3}(x-1)$
8. B	$3y-21 = -8x+8$
9. B	$8x+3y-29=0$
10. D	
	iv. $d = \frac{ -3 \times 8 + 5 \times 3 - 29 }{\sqrt{8^2 + 3^2}}$
11. a. $\frac{3(p-q)(p+q)}{6(p-q)}$	$= \frac{38}{\sqrt{73}}$ units
$= \frac{p+q}{2}$	
	v. D (8,1)
b. $m = \frac{1}{2}, n = -2$	
	b. $x = \frac{-4 \pm \sqrt{4^2 - 4 \times 2 \times -5}}{4}$
c. $x+1 = 5x+15$	$= \frac{-4 \pm \sqrt{56}}{4}$
$4x = -14$	
$x = \frac{-7}{2}$	$= \frac{-2 \pm \sqrt{14}}{2}$
d. $y' = 12x^2 - 6x - 1$	c. $2\sqrt{15} + 5 - 2\sqrt{15} + 3$
	$= 8$
e. $-\frac{1}{\sqrt{3}}$	

13. a.



$$\begin{aligned} \text{Area} &= \frac{1}{2} \times 8 \times 2 \\ &= 8 \text{ sq units} \end{aligned}$$

$$\text{b. } \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 + 2(x+h) - (x^2 + 2x)}{h}$$

$$= \frac{x^2 + 2xh + h^2 + 2x + 2h - x^2 - 2x}{h}$$

$$= 2x + h + 2$$

$$\text{c. } y' = 3(x-3)^2$$

$$\text{sub } x=1$$

$$\begin{aligned} m_T &= 3(1-3)^2 \\ &= 12 \end{aligned}$$

$$\therefore y + 8 = 12(x-1)$$

$$y + 8 = 12x - 12$$

$$y = 12x - 20$$

$$\text{d. } \angle XST = 120^\circ \text{ (opposite angles of a parallelogram)}$$

$$\therefore x = 65 \text{ (exterior angle of a triangle equals the sum of the opposite interior angles)}$$

$$14. \text{ a. } x^2 = 5 \cdot 2^2 + 8 \cdot 4^2 - 2 \times 5 \cdot 2 \times 8 \cdot 4 \times \cos 110^\circ$$

$$x = 11.3 \text{ m}$$

$$\text{b. } -2 < 2x - 4 < 2$$

$$2 < 2x < 6$$

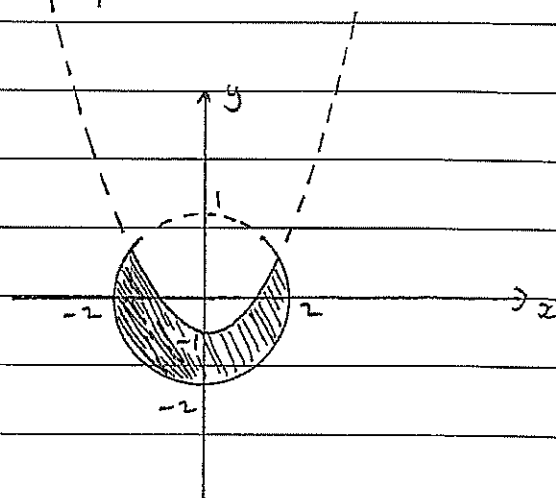
$$1 < x < 3$$

$$\text{c. } \lim_{x \rightarrow 2} \frac{(3x+1)(x-2)}{x-2}$$

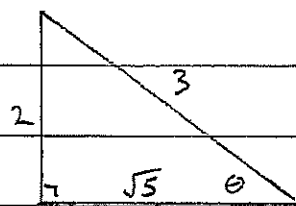
$$= \lim_{x \rightarrow 2} (3x+1)$$

$$= 7$$

d.



15. a.



2nd quadrant

$$\therefore \tan \theta = -\frac{2}{\sqrt{5}}$$

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b. i.  $y' = 20(5x-3)^3$

ii.  $y = 6x^{-2}$

$$y' = -12x^{-3}$$

$$= \frac{-12}{x^3}$$

iii.  $y = 12x^{\frac{3}{2}}$

$$y' = 12 \times \frac{3}{2} x^{\frac{1}{2}}$$

$$= 18\sqrt{x}$$

c. roots are real, different and rational

d.  $m_1 = \frac{2}{5}$      $m_2 = -\frac{9}{4}$

but  $m_1 \times m_2 = -1$

$$\frac{2}{5} \times -\frac{9}{4} = -1$$

$$2a = 20$$

$$a = 10$$

16. a.  $\sin \theta = \pm \frac{\sqrt{3}}{2}$

$$\theta = 60^\circ, 120^\circ, 240^\circ, 300^\circ$$

b.  $f'(x) = \sqrt{2x+1} + \frac{x}{\sqrt{2x+1}}$

$$f'(4) = \sqrt{9} + \frac{4}{\sqrt{9}}$$

$$= 4\frac{1}{3}$$

c.  $m = 3$

$$\therefore \tan \theta = 3$$

$$\therefore \theta = 72^\circ$$

d.  $\frac{1}{2}$

e.  $\sin A \left( \frac{\sin^2 A}{\cos A} + \cos A \right)$

$$= \sin A \left( \frac{\sin^2 A + \cos^2 A}{\cos A} \right)$$

$$= \frac{\sin A}{\cos A}$$

$$= \tan A$$

17. a.  $x \geq -4$

b.  $\frac{dy}{dx} = \frac{(x+4)(2) - (2x-1)(1)}{(x+4)^2}$

$$= \frac{2x+8-2x+1}{(x+4)^2}$$

$$= \frac{9}{(x+4)^2}$$

c.  $\sin(90^\circ - \theta) \operatorname{cosec} \theta$

$$= \frac{\cos \theta}{\sin \theta}$$

$$= \cot \theta$$

$$d. \angle AQB = 2^\circ 30'$$

$$\frac{BQ}{\sin 7^\circ 12'} = \frac{100}{\sin 2^\circ 30'}$$

$$BQ = \frac{100 \sin 7^\circ 12'}{\sin 2^\circ 30'}$$

$$\text{and } \sin 9^\circ 42' = \frac{h}{BQ}$$

$$\therefore h = \frac{100 \sin 7^\circ 12' \sin 9^\circ 42'}{\sin 2^\circ 30'}$$

$$= 48.4 \text{ m}$$

$$18. a. y' = 2x + 6$$

$$\text{when } x = 1$$

$$m_T = 8$$

$$\therefore m_N = -\frac{1}{8}$$

$$b. \text{ real roots } \Rightarrow \Delta \geq 0$$

$$\therefore (-8)^2 - 4 \times k \times k \geq 0$$

$$64 - 4k^2 \geq 0$$

$$k^2 \leq 16$$

$$-4 \leq k \leq 4$$

$$c. (2, 0) \text{ satisfies } y = ax + \frac{b}{x^2}$$

$$\therefore 0 = 2a + \frac{b}{4}$$

$$\text{or } 8a + b = 0$$

$$y = ax + bx^{-2}$$

$$y' = a - 2bx^{-3}$$

$$\text{when } x = 2, y' = 1$$

$$\therefore 1 = a - \frac{2b}{8}$$

$$8 = 8a - 2b$$

$$4 = 4a - b$$

$$\therefore \text{Solve simultaneously}$$

$$8a + b = 0$$

$$4a - b = 4 \quad \text{add}$$

$$12a = 4$$

$$a = \frac{1}{3}$$

$$\therefore b = -\frac{8}{3}$$