SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

YEAR 11 COMMON TEST

MAY 2006

Time allowed: 70 minutes

Instructions:

- Show all necessary working in every question

- Start each question on a new page

- Attempt all questions

- All questions are not of equal value

- Marks shown are approximate and may be changed

- Full marks may not be awarded for careless or badly arranged work

- Your sketches must be neat. Use a ruler to draw axes.

- Approved calculators may be used

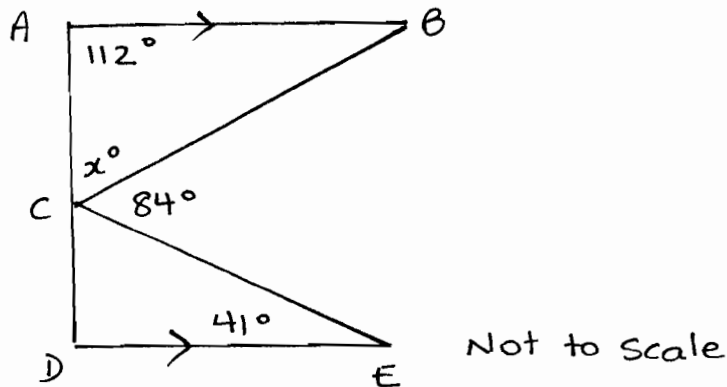- These questions are to be handed in with your answers.

Name:_____

Class:_____

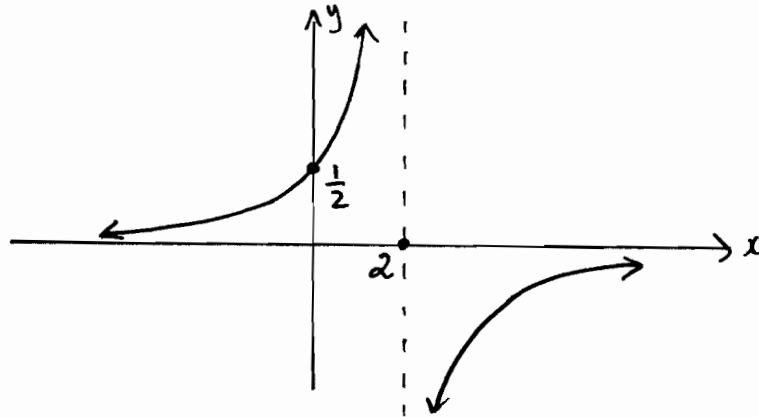| Question 1	Question 2	Question 3	Question 4	Question 5	Question 6	Total
/10	/10	/11	/9	/10	/10	/60

Question 1 (10 marks)

a) Find the value of x (1)

b) Find the exact value of cot(120°) (2)

c) Write down the equation of the hyperbola shown on the Cartesian plane below (2)

d) If $\sin\alpha = \dfrac{-p}{q}$ and $\cos\alpha > 0$, find an expression for $\tan\alpha$ (2)

e) Solve $|2x - 4| = x + 2$ (3)

Question 2 (10 Marks) **Start a new page**

a) Find the largest possible domain for $f(x) = \sqrt{2 + x} + \sqrt{2 - x}$ (2)

b) Simplify $a + \dfrac{1}{a}$ if $a = \sqrt{5} + 1$. Express your answer in its simplest form (2)

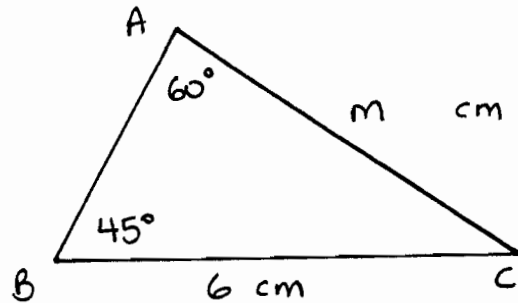c) Solve $\dfrac{1}{|x - 3|} \geq \dfrac{1}{2}$ (3)

d) Simplify $\tan A.\sin A + \cos A$ (2)

e) Simplify $\dfrac{\sin(90 - \theta)}{\sin\theta}$ (1)

Question 3   (11 marks)   **Start a new page**

a)   Solve $\dfrac{2x+3}{x-4} > 1$ (3)

b)   Find the exact value of m in triangle ABC   (2)

c)   Find the range for the function $f(x) = 2x^2 + 4x + 5$ (2)

d)   Consider the curve $y = \dfrac{2-x}{2x+3}$ (4)

    i.   State the equation of the vertical asymptote

    ii.   Find the equation of the horizontal asymptote

       (hint: examine y as $x \to \pm\infty$)

    iii.   Find the x and y intercepts.

    iv.   Sketch the curve

Question 4   (9 marks)   **Start a new page**

a)   Solve $2\cos\theta = \sqrt{2}$ for $0 \le \theta \le 360°$ (3)

b)   Solve $\sin 2x = \dfrac{-1}{2}$ for $0 \le x \le 360°$ (3)

c)   Sketch the circle $x^2 + y^2 = 4x$. Showing any intercepts with

    the coordinate axes.   (3)

Question 5 (10 marks)   **Start a new page**

a)   Simplify $\dfrac{18^n \times 6^{n+1}}{3^n}$ (2)

b)   i.   Sketch on separate diagrams $(\alpha)\, y = |x+2|$ (2)

$(\beta)\, y = |x+2| + |x-2|$ (2)

ii.   Hence solve $|x+2| = 4 - |x-2|$ (2)

c)   Sketch the region indicated by the intersection of $y \le \sqrt{4-x^2}$ (2)

and $x + y + 2 \ge 0$

Question 6 (10 marks)   **Start a new page**

a) $y = f(x)$ represents a continuous <u>odd</u> function. Write down an expression for A. (1)

$$f(x) = \quad x(2-x) \qquad \text{where } x \ge 0$$
$$ = \quad A \qquad\quad \text{where } x < 0$$

b)   Find x and y if

$$x - 2y + 2\sqrt{x-y} = 7 + \sqrt{20}$$ (3)

c)   In the diagram below, triangle QRP has a right angle at R,
$\angle RQS = 45°, \angle SQP = 15°$ and QR = RS = 2 cm.
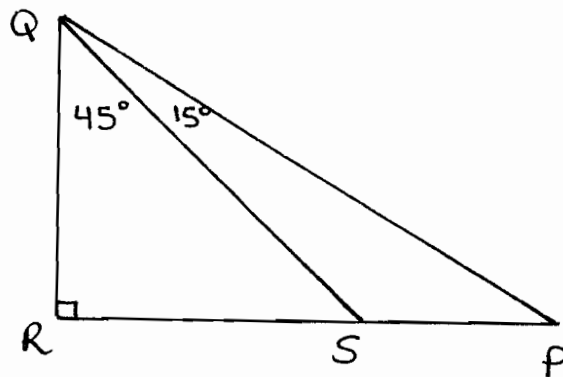
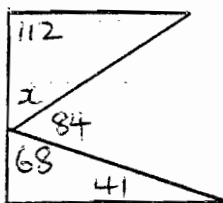i.   Using triangle QRS find the exact length of QS. (1)

ii.   Using triangle QRP find the exact length of PR and hence the exact
length of PS. (2)

iii.   Use the sine rule in an appropriate triangle to prove that

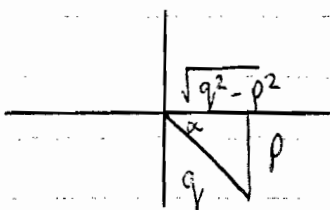$$\sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$$ (3)

Question 1

a)

112
x
84
68
41

$x = \cancel{25}$ ° ✓

$a + \dfrac{1}{a} = \sqrt{5}+1 + \dfrac{\sqrt{5}-1}{4}$

$= \dfrac{4\sqrt{5}+4 + \sqrt{5}-1}{4}$

$= \dfrac{5\sqrt{5}+3}{4}$ ✓

b) $\dfrac{1}{\tan 120°} = -\dfrac{1}{\sqrt{3}}$ ✓ ✓

c) $y = \dfrac{-1}{x-2}$

① = neg ✓
② = equat ✓

c) $\dfrac{1}{|x-3|} \geqslant \dfrac{1}{2}$ $x \neq 3$

$2 \geqslant |x-3|$

$|x-3| \leq 2$

$x-3 \leq 2$ $-x+3 \leq 2$

$x \leq 5$ ✓ $-x \leq -1$

$x \geqslant 1$ ✓

∴ $1 \leq x \leq 5$, $x \neq 3$. ✓

d)

$\sqrt{q^2-\rho^2}$
x
ρ
q

$\tan x = \dfrac{-\rho}{\sqrt{q^2-\rho^2}}$ ✓

d) $\dfrac{1}{\tan A} \sin A + \cos A$

$= \dfrac{\sin A}{\cos A} \cdot \sin A + \cos A$

$= \dfrac{\sin^2 A + \cos^2 A}{\cos A}$ ✓ ①

$= \dfrac{1}{\cos A}$

$= \sec A.$ ✓ ①

e)

$2x-4 = x+2$ $-(2x-4) = x+2$

$x-4 = 2$ $-2x+4 = x+2$

$x = 6$ $-3x = -2$

test ✓ $x = \dfrac{2}{3}$

test ✓

∴ $x = 6$, $x = 2/3$ ✓✓✓

e)

$\dfrac{\sin(90-\theta)}{\sin\theta}$

$= \dfrac{\cos\theta}{\sin\theta}$

$= \cot\theta$ ①

Question 2

a) $\dfrac{\sqrt{x+2}}{\sqrt{2-x}}$ $x \geqslant -2$
                    $x \leq 2$ ✓

∴ $-2 \leq x \leq 2$ ✓

b) $\dfrac{1}{\sqrt{5}+1} = \dfrac{\sqrt{5}-1}{5-1}$

a) $\dfrac{2x+3}{x-4} > 1$ $x \neq 4$ ✓

$(2x+3)(x-4) > (x-4)^2$ ✓

$(2x+3)(x-4) - (x-4)^2 > 0$

$(x-4)\left[2x+3-(x-4)\right] > 0$

$(x-4)(x+7) > 0$
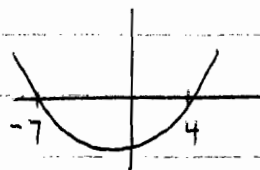
$x < -7$, $x > 4$ ✓

b) $\dfrac{6}{\sin 60°} = \dfrac{m}{\sin 45°}$

$6\sin 45° = m\sin 60°$ ✓

$6 \times \dfrac{1}{\sqrt{2}} = m \times \dfrac{\sqrt{3}}{2}$

$m = \dfrac{6}{\sqrt{2}} \times \dfrac{2}{\sqrt{3}}$

$= \dfrac{12\sqrt{6}}{6}$ ✓

$m = 2\sqrt{6}$

c) $x = \dfrac{-b}{2a}$ $x = \dfrac{-4}{4}$

$x = -1$ ✓

$y = 2(-1)^2 + 4(-1) + 5$

$= 3$ ✓

∴ range: $y \geqslant 3$ ✓

$y = \dfrac{x - ..}{2x+3}$

I. $x = -3/2$ ①

II. $y = -1/2$ ①

III. when $x = 0$ $y = 2/3$ y int
                                   ①
     when $y = 0$ $x = 2$ x int

IV.

$x = \dfrac{-3}{2}$ ①

Question 4

a) $2\cos\theta = \sqrt{2}$

$\cos\theta = \dfrac{\sqrt{2}}{2}$ ✓

$\cos\theta = 1/\sqrt{2}$ $\begin{array}{c|c} S & A \\ \hline T & C \end{array}$
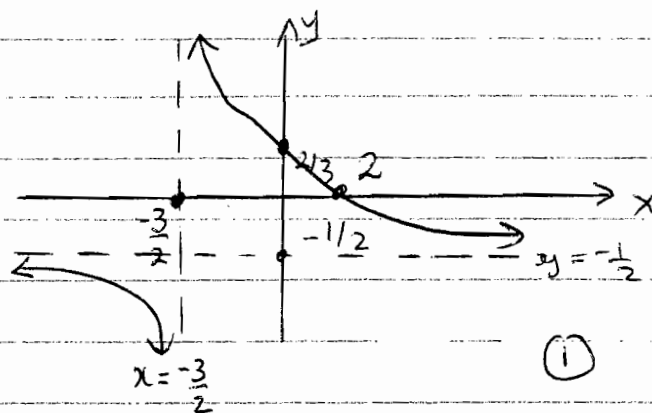
∴ $\theta = 45°, 315°$ ✓✓

b) $\sin 2x = \dfrac{-1}{2}$ $0 \leqslant 2x \leqslant 720$

$\begin{array}{c|c} S & A \\ \hline T & C \end{array}$

$2x = 210°, 330°, 570°$

$690°$

∴ $x = 105°, 165°, 285°, 345°$

c) $x^2 - 4x + 4 + y^2 = 4$ ✓

$(x-2)^2 + y^2 = 4$

$C = (2,0)$, $r = 2$ ✓
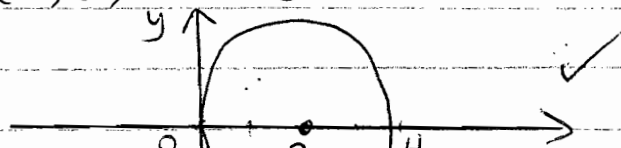
Question 5

a) $\dfrac{18^n \times 6^{n+1}}{3^n} = \dfrac{6^n \times 3^n \times 6^{n+1}}{3^n}$ ✓

$$= 6^{2n+1}$$ ✓

b) i.

$y = |x+2|$

ii.

$y = |x+2| + |x-2|$
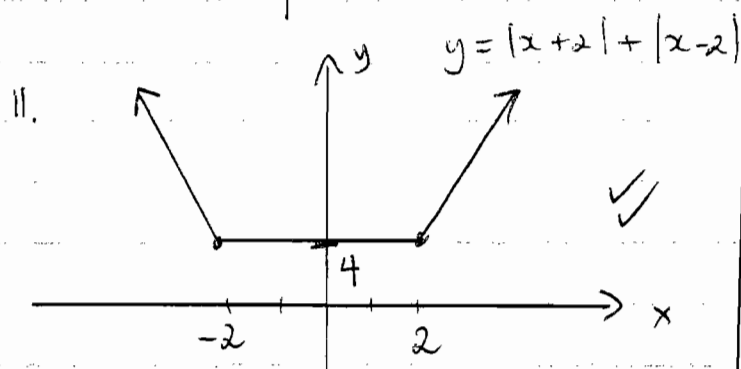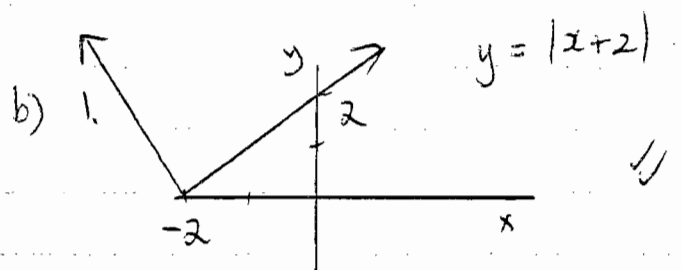
iii. $|x+2| = 4 - |x-2|$

$|x+2| + |x-2| = 4$ ✓

∴ $-2 \le x \le 2$ ✓

c)

need

Question 6

a)

odd fnc ✓
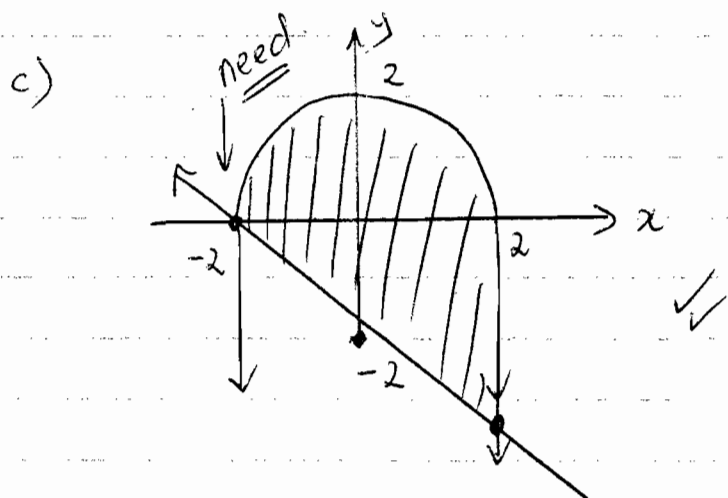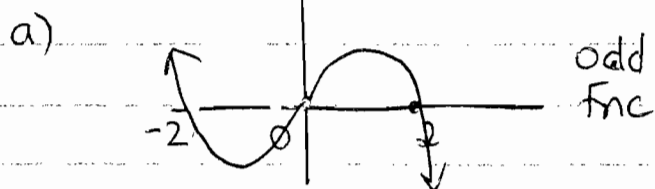
$A = x(x+2)$ or $x^2 + 2x$

b)

$x - 2y + \sqrt{4x - 4y} = 7 + \sqrt{20}$

$\to x - 2y = 7 \to 2x - 4y = 14.$

$\to 4x - 4y = 20 \to 4x - 4y = 20$

$2x = 6$

$x = 3$

∴ $3 - 2y = 7$

$-2y = 4$

$y = -2$ ✓

∴ $x = 3$ ✓ $y = -2$ ✓

c)
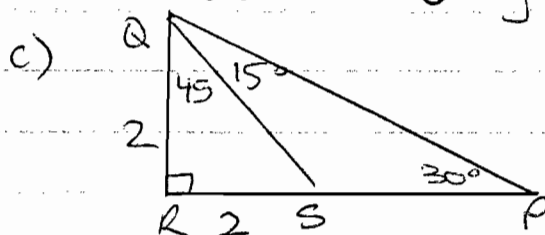
i. $QS^2 = 2^2 + 2^2$

∴ $QS = 2\sqrt{2}$ ✓

ii. $\tan 30° = 2/RP$

$1/\sqrt{3} = 2/RP$

$RP = 2\sqrt{3}$ ✓

∴ $PS = 2\sqrt{3} - 2$ ✓

iii. $\triangle QSP$

$\dfrac{\sin 15°}{PS} = \dfrac{\sin 30°}{QS}$ ①

$\sin 15° = \dfrac{PS}{QS} \cdot \sin 30°$ ①

$= \dfrac{2\sqrt{3} - 2}{2\sqrt{2}} \cdot \dfrac{1}{2}$

$= \sqrt{3} - 1$ ①