Name: ..... Maths Class: .....

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 10 YEARLY EXAMINATION

Mathematics
PART A

OCTOBER 2009

TIME ALLOWED: 70 minutes

Instructions:

- Write your name and class at the top of this page,
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Calculators may be used
- ALL questions are worth 12 marks, and part marks are shown.

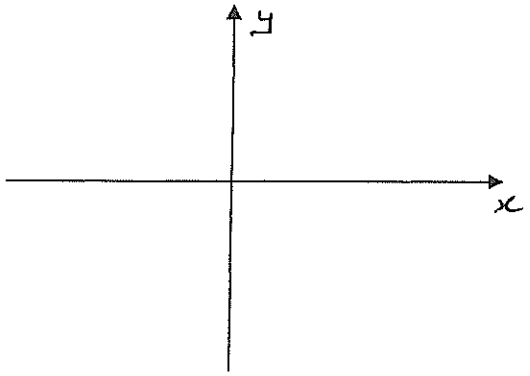
(FOR MARKERS USE ONLY)

PART A					
Q1	Q2	Q3	Q4	Q5	TOTAL
/12	/12	/12	/12	/12	/60

QUESTION 1:

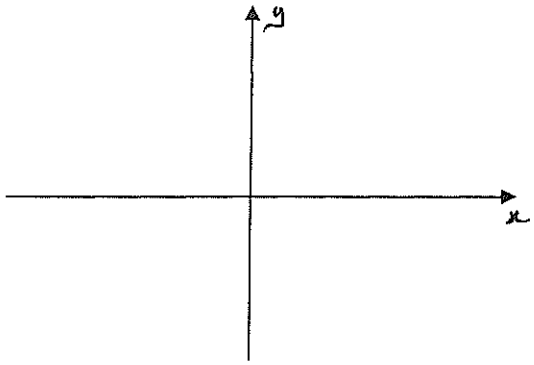
Marks

(a) (i) Using the axes provided, sketch the curve $y = \frac{2}{x}$ (showing all major features): **2**

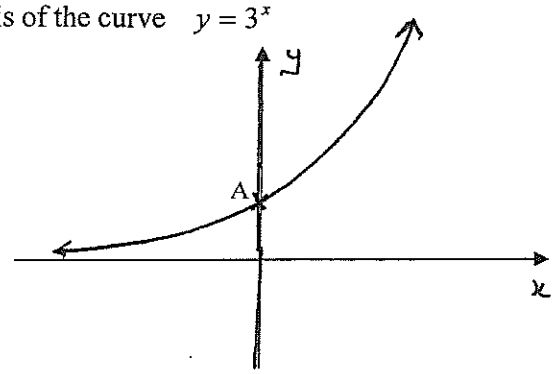


(ii) What are the asymptotes to this curve? **1**

(b) Using the axes provided, sketch the curve $x^2 + y^2 = 9$ (showing the points where it cuts the axes): **1**



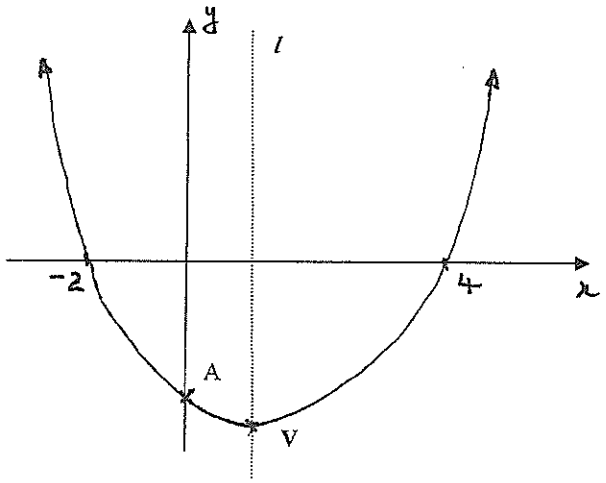
(c) The diagram below is of the curve $y = 3^x$



(i) What are the coordinates of the point A? **1**

(ii) On the diagram, draw in the curve $y = 3^{-x}$ **1**

(d) You are given this sketch of the parabola $y = x^2 + bx + c$



(i) Find the values of b and c **2**

(ii) Find the y-value of the point A marked on the y-axis. **1**

(iii) Find the equation of the line l drawn on the graph. **1**

(iv) Find the co-ordinates of the vertex, V **1**

(v) On the diagram, draw the curve $y = 2x - x^2 + 8$ **1**

QUESTION 2:

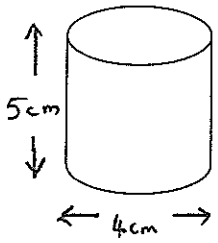
(a) Calculate the volume of a sphere with radius 3 cm.
Give your answer correct to 2 dec. places. **1**

WORKING:

SOLUTION:

(b) (i) Calculate the Surface Area of the solid right cylinder in figure A shown at right, correct to 2 dec. places **2**

FIGURE A



WORKING:

SOLUTION:

(ii) The surface area of the solid figure B shown at right is approximately **1**

$314.28\ cm^2$

FIGURE B

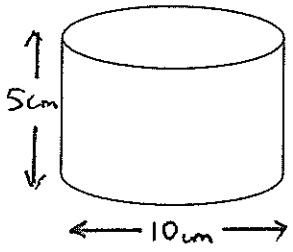
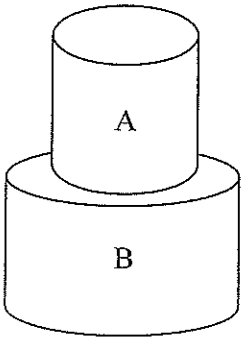


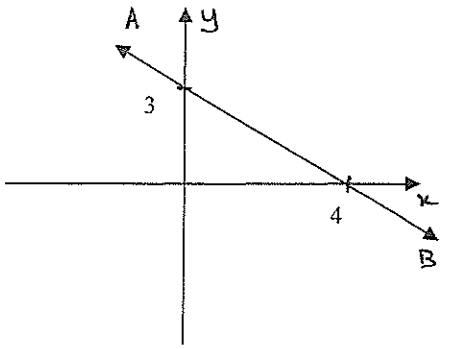
Figure A is pasted on top of figure B to form a new solid as shown. What is the surface area of the new figure?

WORKING:



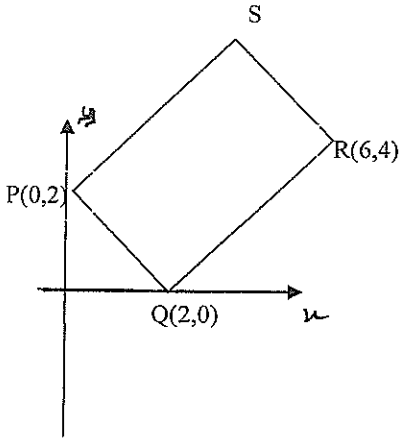
SOLUTION:

(c) Find the equation of the line AB in the diagram below (give your answer in general form): **3**



GENERAL FORM:

(d) In the diagram at below, PQRS is a rectangle



(i) Find the co-ordinates of the point S **1**

(ii) Find the midpoint of QR **1**

(iii) Find the length of QR **1**

(iv) Find the area of the rectangle PQRS **1**

(v) Find the equation of QR **1**

QUESTION 3:

(a) Complete the following frequency table, and use it to find the mode and the mean of the distribution.

Score (x)	*Frequency (f)*	*fx*
12.6	2	
12.8	3	
13.0		91.0
13.2		52.8
13.4	4	
	$\sum f =$	$\sum fx =$

1

MODE: _____ MEAN: _____ 2

(b) The following stem and leaf plot lists the scores for a batsman in a Club cricket team..

Stem	leaf
3	2 4
4	1 1 3 5
5	2 4 5 5 6 7 7
6	1 3 9
7	1
8	3

Use the information provided to find:

ANSWER:

(i) the number of times the player batted 1

(ii) the mean score for the batsman 1

(iii) the range of scores 1

(iv) the median score 1

(v) the interquartile range 1

(c) The statistical results for tests given to all year 10 in Maths and Science were recorded as follows

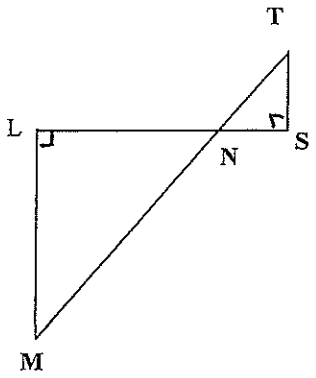
	Mean	Standard Deviation
Maths	65	12.5
Science	80	6.3

Ezra scored 80 in Maths and 86 in Science, which caused his father to be very upset. Ezra wants to convince his father that he actually did BETTER in Maths than he did in Science compared to the rest of year 10. What argument should he use? **2**

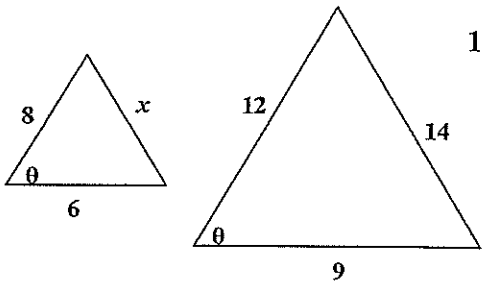
(d) A set of scores has a mean of 52 and a standard deviation of 6.5. A score of 54 is added. Will the standard deviation go UP or DOWN? **2**
(You must justify your answer.)

QUESTION 4:

(a) For the diagram at right, state why ΔLMN is similar to ΔSTN (you must give a reason for each statement): **1**

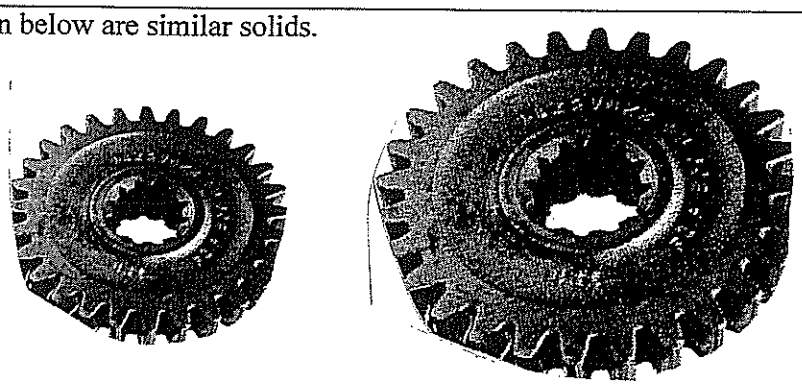


(b) (i) State the reason why these two triangles are similar **1**



(ii) Find the value of x in the triangle at left. **1**

(c) The two cogs shown below are similar solids. **3**

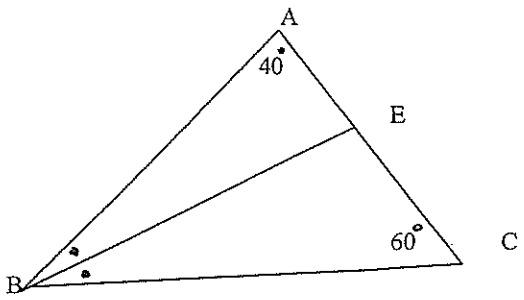


The area of the front of the cog at left is $60cm^2$ while the front area of the shape on the right is $135cm^2$

What is the volume of the larger cog if the volume of the smaller cog is 150 cm^3

WORKING:

(d) For the diagram below, you are given that angle BAC = 40° the angle ACB = 60°, and that EB bisects the angle ABC.



(i) Find the size of the angle EBC (give reasons):

1

(ii) Setting out a formal proof, prove that ΔABC is similar to ΔBEC.

2

(iii) If $\frac{AB}{BC} = \frac{x}{y}$ and $EC = z$, prove that $BE = \frac{xz}{y}$

2

QUESTION 5:

(a) Solve the following quadratic equation, leaving your answers as surds in simplest form **3**

$$2x^2 - 4x - 5 = 0$$

WORKING:

SOLUTION:

(b) Solve the following quadratic equation, giving your answers correct to 2 decimal places: **2**

$$x^2 + 6x - 2 = 0$$

WORKING:

SOLUTION:

(c) Solve the equation $x^2 = 2x - 1$, giving your answer in exact form **1**

WORKING:

SOLUTION:

(d) (i) Write an equation which solves the following problem:

"3 times a positive whole number plus twice its reciprocal is 7. What is the number?" **1**

(ii) Solve the equation you have just formed: **2**

(iii) In your answer to Part (ii) above, one answer is not a solution to the problem. Why not? **1**

(e) Solve the following for x: **2**

$$(\frac{1}{x})^2 + (\frac{1}{x}) - 6 = 0$$

WORKING:

SOLUTION:

Name: SOLUTIONS Maths Class: .....

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PART A					
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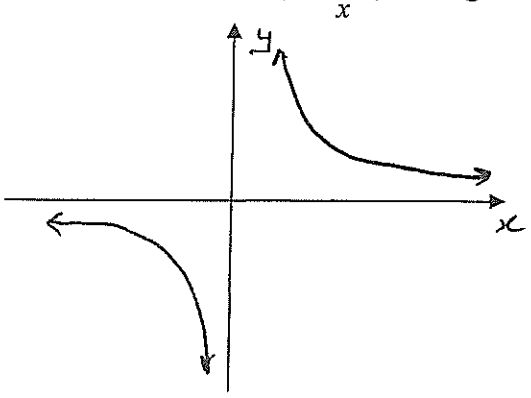
QUESTION 1:

Marks

(a) (i) Using the axes provided, sketch the curve $y = \frac{2}{x}$ (showing all major features): **2**

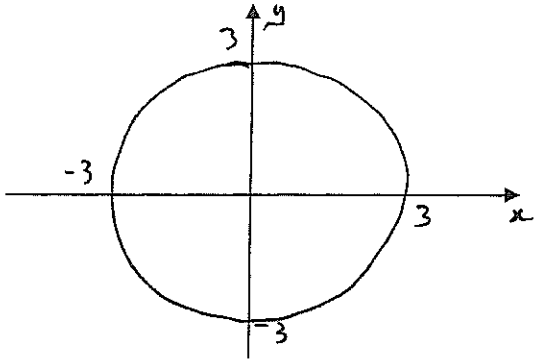


(1) only if only 1 part is shown (no points required)

(ii) What are the asymptotes to this curve? **1**

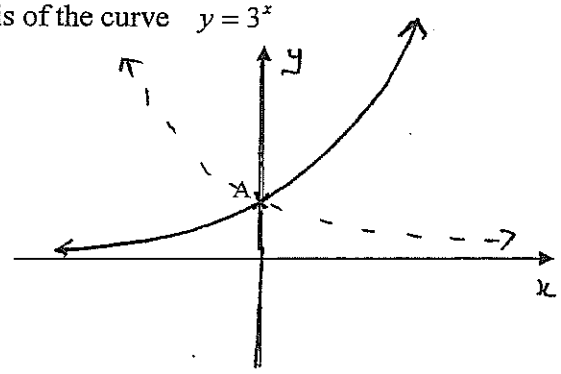
OR { x and y - axes / x = 0, y = 0

(b) Using the axes provided, sketch the curve $x^2 + y^2 = 9$ (showing the points where it cuts the axes): **1**



No marks if intercepts are not shown.

(c) The diagram below is of the curve $y = 3^x$

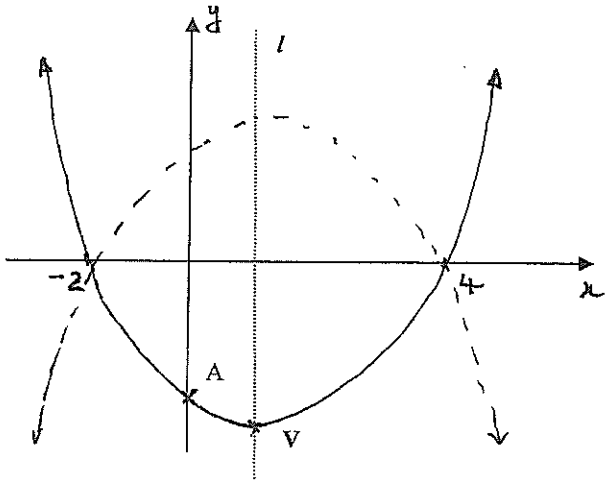


(i) What are the coordinates of the point A? **1**

(0,1).

(ii) On the diagram, draw in the curve $y = 3^{-x}$ **1**

(d) You are given this sketch of the parabola $y = x^2 + bx + c$



(i) Find the values of b and c **2**

$y = (x-4)(x+2)$

$y = x^2 - 2x - 8$ ← accept this or,

$b = -2$, $c = -8$ 1 each

(ii) Find the y-value of the point A marked on the y-axis. **1**

A is $(0, -8)$

ok $y = -8$

(accept just -8)

(iii) Find the equation of the line l drawn on the graph. **1**

$x = \frac{2}{2}$

$x = 1$

DO NOT ACCEPT $(1, 0)$ OR just 1.

(iv) Find the co-ordinates of the vertex, V **1**

$y = 1 - 2 - 8$

$= -9.$

$\therefore$ V is $(1, -9)$ ← ONLY correct answer.

(v) On the diagram, draw the curve $y = 2x - x^2 + 8$ **1**

see graph.

QUESTION 2:

(a) Calculate the volume of a sphere with radius 3 cm.
Give your answer correct to 2 dec. places. **1**

WORKING:

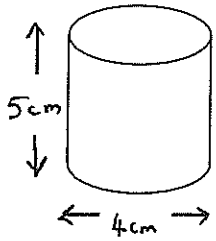
$V = \frac{4}{3}\pi (3)^3$

$= 36\pi$

SOLUTION: $Vol \approx$ { $113.10\ cm^3$ / $113.09\ cm^3$ / $113.04\ cm^3$ } accept any of (no penalty if not to 2 dec places)

(b) (i) Calculate the Surface Area of the solid right cylinder in figure A shown at right, correct to 2 dec. places **2**

FIGURE A



WORKING:

$SA = 2\pi rh + 2\pi r^2$

$= 2\pi(2)5 + 2\pi(2)^2$

$= 28\pi$

≈ 87.9645

SOLUTION: { $87.97\ cm^2$ / $87.96\ cm^2$ all OK. / $87.964\ cm^2$ } (1) mark for $62.83\ cm^2$

(ii) The surface area of the solid figure B shown at right is approximately **1**

$314.28\ cm^2$

FIGURE B

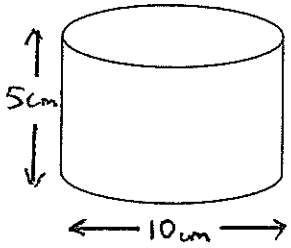


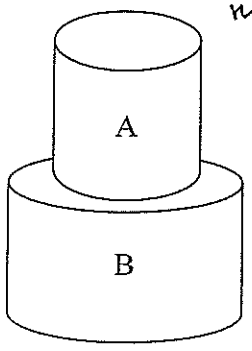
Figure A is pasted on top of figure B to form a new solid as shown. What is the surface area of the new figure?

WORKING:

$SA = 314.28 + 87.97 - 2\pi r^2$ ← this part is what the mark is for.

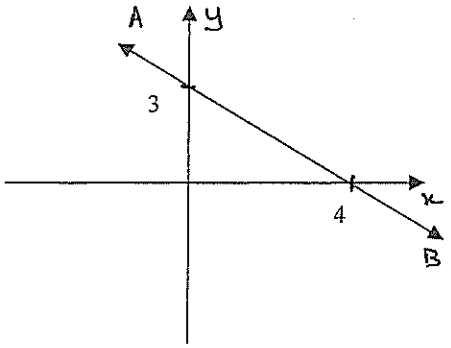
$= 314.28 + 87.97 - 25.13$

$= 377.12$



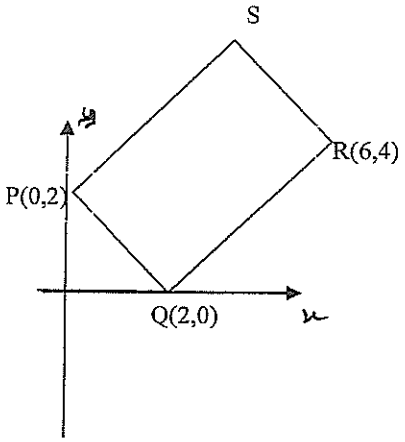
SOLUTION: $377.12\ cm^2$

(c) Find the equation of the line AB in the diagram below (give your answer in general form): **3**



GENERAL FORM:

(d) In the diagram at below, PQRS is a rectangle



(i) Find the co-ordinates of the point S **1**

(ii) Find the midpoint of QR **1**

(iii) Find the length of QR **1**

(iv) Find the area of the rectangle PQRS **1**

(v) Find the equation of QR **1**

QUESTION 3:

(a) Complete the following frequency table, and use it to find the mode and the mean of the distribution.

Score (x)	*Frequency (f)*	*fx*
12.6	2	25.2
12.8	3	38.4
13.0	7	91.0
13.2	4	52.8
13.4	4	53.6
	$\Sigma f = 20$	$\Sigma fx = 261$

1 for table correct

1

MODE: 7 MEAN: 13.05 } 1 each. be careful of "subsequent errors"

2

(b) The following stem and leaf plot lists the scores for a batsman in a Club cricket team..

Stem	leaf
3	2 4
4	1 1 3 5
5	2 4 5 5 6 7 7
6	1 3 9
7	1
8	3

Use the information provided to find:

	ANSWER:	
(i) the number of times the player batted	18	1
(ii) the mean score for the batsman	53.83	1
(iii) the range of scores	51	1
(iv) the median score	55	1
(v) the interquartile range	18	1

Do not accept 83−32

(c) The statistical results for tests given to all year 10 in Maths and Science were recorded as follows

	Mean	Standard Deviation
Maths	65	12.5
Science	80	6.3

Ezra scored 80 in Maths and 86 in Science, which caused his father to be very upset. Ezra wants to convince his father that he actually did BETTER in Maths than he did in Science compared to the rest of year 10. What argument should he use? 2

They have to convince you that he was in the middle range of Scieme (ie less than 1 S.D. in Science) but more than 1 S.D. in Maths

If they use mid 67% for Science and in top 16% for Maths pay it (it may not be a normal distribution)

(d) A set of scores has a mean of 52 and a standard deviation of 6.5. A score of 54 is added. Will the standard deviation go UP or DOWN? 2
(You must justify your answer.)

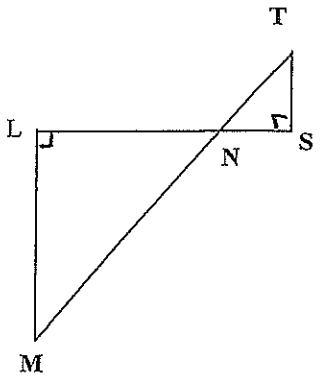
DOWN (1)

Because the score added is less than 1 standard deviation above the mean and will "contract" the scores. (1)

(have to convince you)

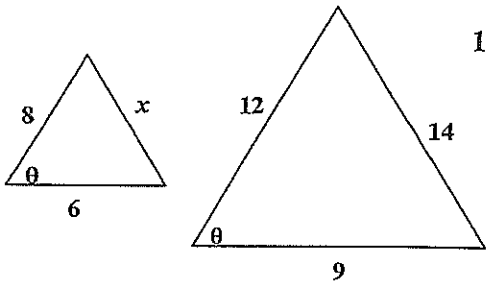
QUESTION 4:

(a) For the diagram at right, state why ΔLMN is similar to ΔSTN (you must give a reason for each statement): 1



(1) Both triangles have 90°
$\angle LNM = \angle SNT$ (vertically opposite angles)
$\therefore$ triangles are equiangular.

(b) (i) State the reason why these two triangles are similar 1



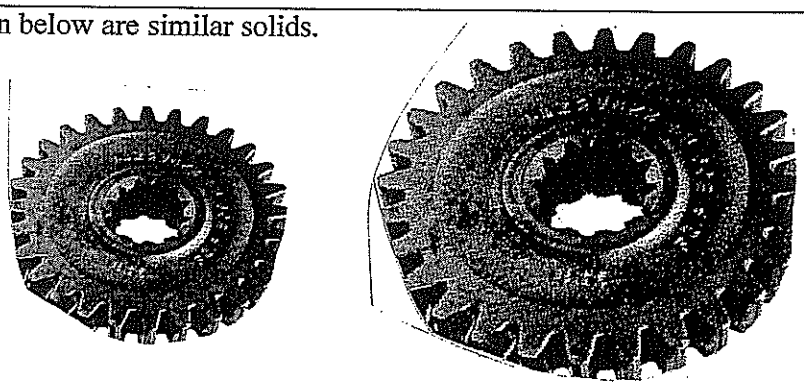
Sides in proportion about the same angle.

(ii) Find the value of x in the triangle at left. 1

$\frac{x}{14} = \frac{8}{12}$

$x = \frac{28}{3}$ or $9\frac{1}{3}$ or $9.\dot{3}$ or 9.3

(c) The two cogs shown below are similar solids. 3



The area of the front of the cog at left is $60cm^2$ while the front area of the shape on the right is $135cm^2$

What is the volume of the larger cog if the volume of the smaller cog is 150 cm^3

WORKING:

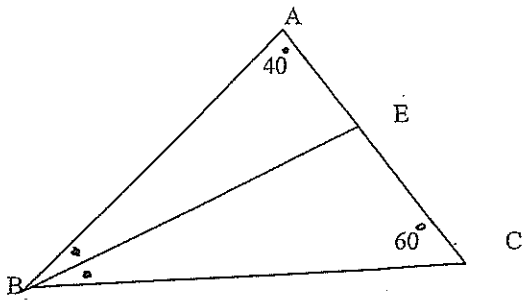
Ratio of areas $= \frac{135}{60}$ } (1)
$\therefore$ " " sides $= \frac{3}{2}$

$\therefore$ Ratio of volumes $= \frac{27}{8}$ (1)

$\therefore$ vol of large cog $= \frac{27}{8} \times \frac{150}{1}$

$= 506.25\ cm^3$ (1)

(d) For the diagram below, you are given that angle BAC = 40° the angle ACB = 60°, and that EB bisects the angle ABC.



(i) Find the size of the angle EBC (give reasons): 1

$\angle ABC = 80^\circ$ (angle sum of $\triangle ABC$)

$\therefore \angle EBC = 40^\circ$ (EB bisects $\angle ABC$)

(ii) Setting out a formal proof, prove that ΔABC is similar to ΔBEC. 2

In $\triangle ABC$ and $\triangle BEC$ (not necessary)

$\angle BAC = \angle EBC = 40^\circ$ (from above)

$\angle ACB = \angle ECB = 60^\circ$ (common)

$\therefore \triangle ABC \,|||\, \triangle BEC$ (equiangular)

must be "set out" for 2 marks

(iii) If $\frac{AB}{BC} = \frac{x}{y}$ and EC = z, prove that BE = $\frac{xz}{y}$ 2

$\frac{AB}{BC} = \frac{BE}{EC}$ (similar $\triangle$'s) (1)

$\therefore \frac{BE}{z} = \frac{x}{y}$

$\therefore BE = \frac{xz}{y}$ (1)

[OR similar]

QUESTION 5:

(a) Solve the following quadratic equation, leaving your answers as surds in simplest form **3**

$$2x^2 - 4x - 5 = 0$$

WORKING:

$$x = \frac{4 \pm \sqrt{16 + 40}}{4}$$

$$= \frac{4 \pm \sqrt{56}}{4} \leftarrow (2) \text{ for this}$$

$$= \frac{4 \pm 2\sqrt{14}}{4} \leftarrow (1) \text{ for this}$$

working is not necessary if correct

SOLUTION:

$$x = \frac{2 \pm \sqrt{14}}{2} \quad \text{or} \quad x = 1 \pm \tfrac{1}{2}\sqrt{14} \leftarrow (1)$$

(b) Solve the following quadratic equation, giving your answers correct to 2 decimal places: **2**

$$x^2 + 6x - 2 = 0$$

WORKING:

$$x = \frac{-6 \pm \sqrt{36 + 8}}{2}$$

$$= -3 \pm \sqrt{11}$$

} (1)

working is not necessary if correct

(1) accept either

SOLUTION: $x = 0.31\ (0.32)$ or $x = -6.32\ (-6.31)$ accept either

(c) Solve the equation $x^2 = 2x - 1$, giving your answer in exact form **1**

WORKING:

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

SOLUTION: $x = 1$

(d) (i) Write an equation which solves the following problem:

"3 times a positive whole number plus twice its reciprocal is 7. What is the number?" **1**

$3x + 2/x = 7$ ← (1)

$\therefore 3x^2 + 2 - 7x = 0$

$(3x - 1)(x - 2) = 0$

$\therefore x = 1/3$ or $x = 2$

(ii) Solve the equation you have just formed: **2**

see above $3x^2 + 2 - 7x = 0$

$\therefore (3x-1)(x-2) = 0$

$\therefore x = 1/3$ or $x = 2$ ← (1) each

(iii) In your answer to Part (ii) above, one answer is not a solution to the problem. Why not? **1**

$x = 2$ is the only solution because the question asked for a positive whole number.

(e) Solve the following for x: **2**

$$(\frac{1}{x})^2 + (\frac{1}{x}) - 6 = 0$$

WORKING:

Let $X = 1/x$

$X^2 + X - 6 = 0$

$(X + 3)(X - 2) = 0$

$X = -3$ or $X = 2$

$\therefore 1/x = -3$ $1/x = 2$

(1) for method

SOLUTION: $x = -1/3$ or $x = 1/2$ (1) for answers