Probability & Counting Technique

- Counting Technique
- $\dfrac{n!}{x!\,y!\ldots}$
- Arrangements in a line
- Arrangements in a circle
- Unordered selections
- Probability

Counting Technique

Counting Technique reduces confusion.

Example 1

Using the number 0 – 9, how many combinations of

a) 2 numbers	i) Repetition	9	10		= 90	0 can't be the starting number.
	ii) Non Repeating	9	9		= 81	
b) 3 numbers	i) Repetition	9	10	10	= 900	
	ii) Non Repeating	9	9	8	= 648	

Example 2

In the Melbourne Cup there are 24 horses. How many ways can 1st, 2nd and 3rd take place?

24	23	22	= 12144

$^{24}P_3$ = 12144

Example 3

From the word "PROBLEMS" how many 5 letter words are possible?

a) No Restrictions	8	7	6	5	4	= 6720
b) Must begin with P and end with an S	1	6	5	4	1	= 120
c) P must be in the word, but not at the beginning. No M	6	6	5	4	3	= 2160

$$\boxed{\frac{n!}{x!\,y!\ldots}}$$

Total number of letters

Number of repeated letters

Example 1 – **AUSTRALIA**

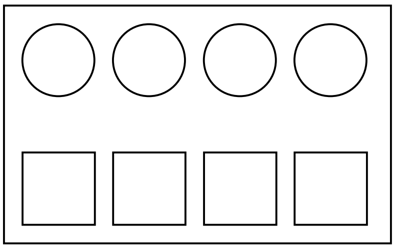
$$\frac{9!}{3!} = 60480$$

Example 2 - **RECOMMENDED**

$$\frac{11!}{3!.2!.3!} = 1663200$$

Arrangements

Example 1



Arrangements of people in a train carriage.

8 seats

6 people

a)	No Restrictions	$= 8\times7\times6\times5\times4\times3$	$= 20160$
b)	X must face the front Y must face the back	$= 4\times4\times6\times5\times4\times3$	$= 5760$
c)	X & Y must be in the corners	$= 4\times3\times6\times5\times4\times3$	$= 4320$
d)	X & Y must sit together.	$= 2\times6\times6\times5\times4\times3$	$= 4320$

Arrangements in a line

Example 1

5 girls lining up in a line. How many different ways are there?

5	4	3	2	1	$= 5!$	$= 120$

Example 2

5 boys, 4 girls standing in a line.

a)	No restrictions	$9!$	$= 362880$
b)	Alternative	$5! \times 4!$	$= 2880$
c)	Separate groups	$5! \times 4! \times 2$	$= 5760$
d)	2 of the girls must sit together	$8! \times 2$	$= 80640$

Arrangements in a circle

Example 1

Compare 4 people in a line, and in a circle

Line	$4\times3\times2\times1 = 4!$	$= 24$
Circle	$1\times3\times2\times1 = 3!$	$= 6$

Example 2

Find the number of ways 5 boys and 5 girls sit around a table.

a)	No restrictions	$1\times9\times8\times7\times6\times5\times4\times3\times2\times1 = 9!$	$= 362880$
b)	A particular girl must sit between 2 boys	$1\times2\times7\times6\times5\times4\times3\times2\times1 = 2\times7!$	$= 10080$
c)	2 particular girls wish to sit directly opposite	$1\times2\times8\times7\times6\times5\times4\times3\times2\times1 = 2\times8!$	$= 80640$
d)	2 particular people do not want to sit together	$1\times7\times8\times7\times6\times5\times4\times3\times2\times1 = 7\times8!$	$= 282240$

Unordered selections

What we want to do now is to 'choose' a sub-group from a totally of things.

Example 1

We select 3 students (any order) from a group of 24 students.

${}^{24}C_3 \quad = 2024$

Example 2

Six students, 2 boys (A, B) included, are to be selected from 10 students.

a)	No restrictions	${}^{10}C_6$	$= 210$
b)	A and B inclusive	$1 \times {}^{8}C_4$	$= 70$
c)	A is excluded	${}^{9}C_6$	$= 84$
d)	A & B not in the same selected group.	${}^{8}C_5 + {}^{8}C_5 + {}^{8}C_6$	$= 140$

Example 3

Choose a committee of 5 from a group of 7 men and 4 women.

a)	No restrictions	${}^{11}C_5$	$= 462$
b)	3 men and 2 women	${}^{7}C_3 \times {}^{4}C_2$	$= 210$
c)	Male only	${}^{7}C_5$	$= 21$
d)	At leas 1 woman	${}^{4}C_1.{}^{7}C_4 + {}^{4}C_1.{}^{7}C_4 + {}^{4}C_1.{}^{7}C_4 + {}^{4}C_1.{}^{7}C_4$	
		$140 + 210 + 84 + 7$	$= 441$
	OR	${}^{11}C_5 - {}^{7}C_5$	$= 441$
e)	Majority women	${}^{4}C_4.{}^{7}C_1 + {}^{4}C_3.{}^{7}C_2$	
		$7 + 84$	$= 91$

Example 4

A dealer deals out 5 cards from a pack of 52 cards

a)	No restrictions	${}^{52}C_5$	$= 2\,598\,960$
b)	4 Aces	1×48	$= 48$
c)	3 diamonds, 2 hearts	${}^{13}C_3 \times {}^{13}C_2$	$= 22\,308$
d)	All clubs	${}^{13}C_5$	$= 1\,287$
e)	Picture cards	${}^{12}C_5$	$= 792$
f)	Same colour	$2 \times {}^{26}C_5$	$= 131\,560$

Probability

$$P(E) = \frac{n(E)}{n(S)}$$

So far we have been looking at arrangements and combinations without probability.

$n(S)$ = No restrictions

Example 1

The letters a, b, e, c, I, d, o, f are arranged in a circle. How many different orders are there if one of these arrangements is selected at random? Find the Probability.

a) At least 2 of the vowels are together

$$\frac{4!.3!}{7!} \qquad 1-\frac{4!.3!}{7!} \qquad = \frac{34}{35}$$

b) All the vowels are together

$$\frac{4!.4!}{7!} \qquad = \frac{4}{35}$$

Example 2

In a bag there is 6 Red, 4 White, 3 Black. 3 balls are selected simultaneously. Find the probability.

a) All red $= \frac{{}^6C_3}{{}^{13}C_3} = \frac{20}{286}$ $= \frac{10}{143}$

b) All white $= \frac{{}^4C_3}{{}^{13}C_3} = \frac{4}{286} = \frac{2}{143}$ $= \frac{2}{143}$

c) All same colour = P(all red) + P(all white) + P(all black)
$= \frac{10}{143} + \frac{2}{143} + \frac{{}^3C_3}{{}^{13}C_3}$ $= \frac{25}{143}$

d) Different colours $= \frac{{}^6C_1 \times {}^4C_1 \times {}^3C_1}{{}^{13}C_3}$ $= \frac{36}{143}$

e) 2 red, 1 white $= \frac{{}^6C_2 \times 4}{286}$ $= \frac{30}{143}$

f) Exactly 2 white $= \frac{{}^4C_2 \times 9}{286}$ $= \frac{27}{143}$

g) At least 1 white $= 1 - \frac{{}^9C_3}{{}^{13}C_3}$ $= \frac{101}{143}$

h) Including 1 particular ball $= \frac{{}^{12}C_2}{286}$ $= \frac{3}{13}$

i) At most 2 white $= \frac{{}^4C_3 + {}^4C_1.{}^4C_2 + {}^4C_2.{}^4C_1}{286}$ $= \frac{141}{143}$

j) Majority red $= \frac{{}^6C_3 + {}^6C_2 + {}^7C_1}{286}$ $= \frac{125}{286}$