SYDNEY TECHNICAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 3
Term 2 2016

Mathematics

Name ....

Teacher ....

General Instructions

- Reading Time - 5 minutes.
- Working Time - 90 min.
- Write using a blue or black pen.
- Board approved calculators may be used.
- The reference sheet is provided at the back of this paper.
- In Questions 6-9, show relevant mathematical reasoning and /or calculations.
- Begin each question on a fresh sheet of paper.

Total marks (58)

- Attempt Questions 1-9.
- All questions are of equal value.

Multiple Choice		5
Question 6		13
Question 7		13
Question 8		13
Question 9		14
TOTAL		**58**

Section 1
Multiple Choice (5 marks)
Use the multiple choice answer sheet for Question 1-5

1. Evaluate

$$\lim_{h \to 4} \frac{4-h}{16-h^2}$$

(A) 0

(B) $\frac{1}{8}$

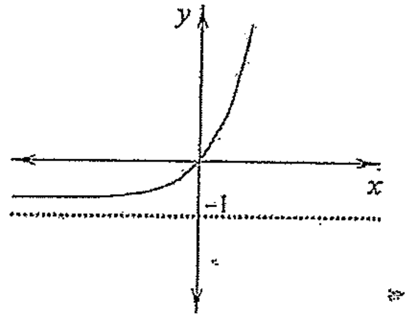
(C) $\frac{1}{4}$

(D) 4

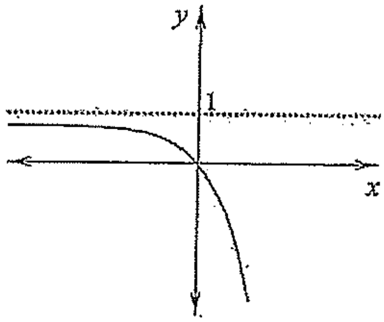
2.

Which of the following graphs could have the equation $y = 1 - e^x$?

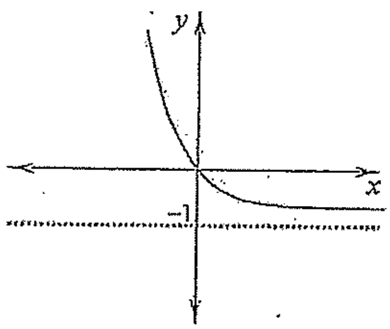
(A)



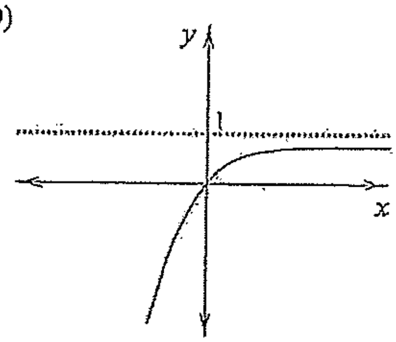
(B)



(C)



(D)



3. What is the value of $8e^{-2}$ correct to 3 significant figures

(A) 1.08

(B) 1.082

(C) 1.083

(D) 1.10

4. What is the radius of the circle $x^2 + y^2 - 4x + 8y + 11 = 0$

(A) 2

(B) 4

(C) 9

(D) 3

5. Solve for x: $\tan x + \sqrt{3} = 0, \quad -\pi \leq x \leq \pi$

(A) $\frac{2\pi}{3}, \frac{5\pi}{3}$

(B) $\frac{\pi}{3}, \frac{2\pi}{3}$

(C) $\frac{5\pi}{6}, \frac{-\pi}{6}$

(D) $\frac{2\pi}{3}, \frac{-\pi}{3}$

Section II

Total Marks (64)
Attempt Questions 6 – 9.

Answer each question in your writing booklet.
In Questions 6-9, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 **(13 Marks)** Use a Separate Sheet of paper

(2 marks)

a) Differentiate $\frac{2x^4}{\cos x}$ with respect to x

(1 mark)

b) Find $\int \sec^2\left(\frac{x}{2}\right) dx$

(2 marks)

c) Find the primitive function of

i) $(3x-4)^6$

ii) $\frac{6}{x^2}$

(4 marks)

d)

i) Show that $x = \frac{2\pi}{3}$ is a solution of $\cos x = \cos 2x$

ii) Sketch on the same set of axes the functions $y = \cos x$ and $y = \cos 2x$ for $0 \leq x \leq 2\pi$

iii) How many solutions does $\cos x = \cos 2x$ have for the domain $0 \leq x \leq 2\pi$?

(2 marks)

e) Evaluate

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\frac{x}{2}\, dx$$

(2 marks)

f) Solve $\sin x = \frac{-\sqrt{3}}{2}$ for $0 \leq x \leq 2\pi$

End of Question 6

Question 7 **(13 Marks)** Use a Separate Sheet of paper

(3 marks)

a) Evaluate $\int_{-1}^{3}\left(x^2+\frac{x}{2}\right)dx$

(3 marks)

b) The region bounded by the curve $y = 2-\sqrt{x}$ and the y axis between $y = 0$ and $y = 2$ is rotated about the y axis to form a solid. Find the volume of the solid in simplest exact form.

(3 marks)

c)

i) Find the first and second derivatives of $f(x) = xe^{-x}$

ii) Find the set of values of x for which the function $f(x) = xe^{-x}$ is both decreasing and concave down.

(4 marks)

d) If $\frac{d^2y}{dx^2} = 8\cos(2x)$

i) Find y, given that there is a stationary point at $\left(\frac{\pi}{2}, 1\right)$

ii) Show that $\frac{d^2y}{dx^2} + 4y + 4 = 0$

End of Question 7

Question 8 **(13 Marks)** Use a Separate Sheet of paper

(7 marks)

a)

i) Solve the equation $1 - 2\cos x = 0$ for $0 \leq x \leq 2\pi$

ii) Sketch the graph of the curve $y = 1 - 2\cos x$ for $0 \leq x \leq 2\pi$ showing clearly the coordinates of the endpoints and the maximum turning point.

iii) Find in simplest exact form the area of the region bounded by the curve $y = 1 - 2\cos x$ and the x axis between $x = 0$ and $x = \pi$

(3 marks)

b) Evaluate and give the exact value of

$$\int_0^1 (e^{3x} + 1)\ dx$$

(3 marks)

c) Using a substitution, or otherwise, solve

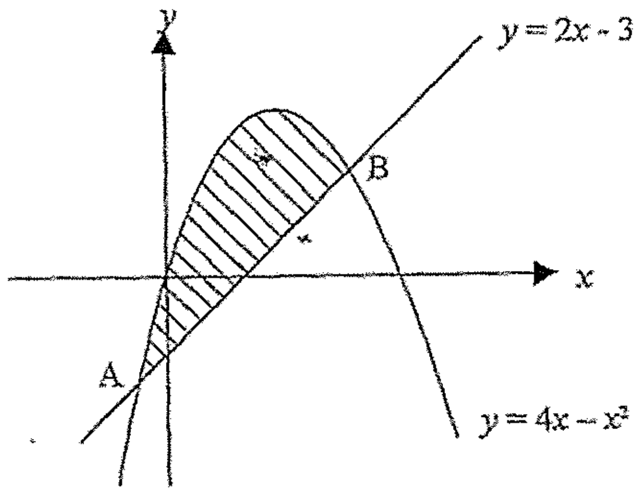
$$2^{2x} - 8(2^x) = 0$$

End of Question 8

Question 9 **(14 Marks)** Use a Separate Sheet of paper

(5 marks)

a) The following diagram shows parts of the graphs of $y = 2x - 3$ and $y = 4x - x^2$

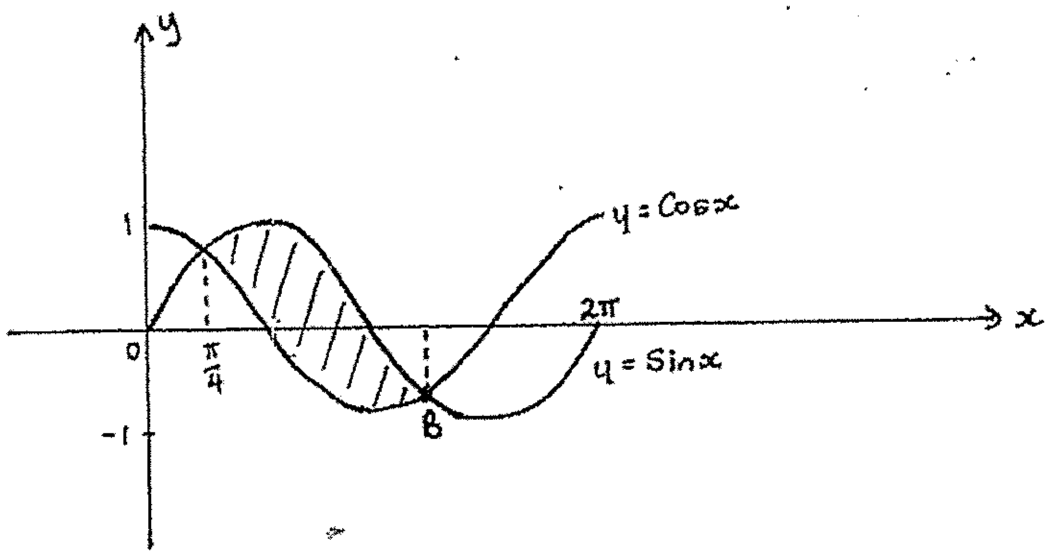


i) A is the point $(-1, -10)$. Find the x value at the point B.

ii) Find the exact area between the curves (shaded)

(4 marks)

b)

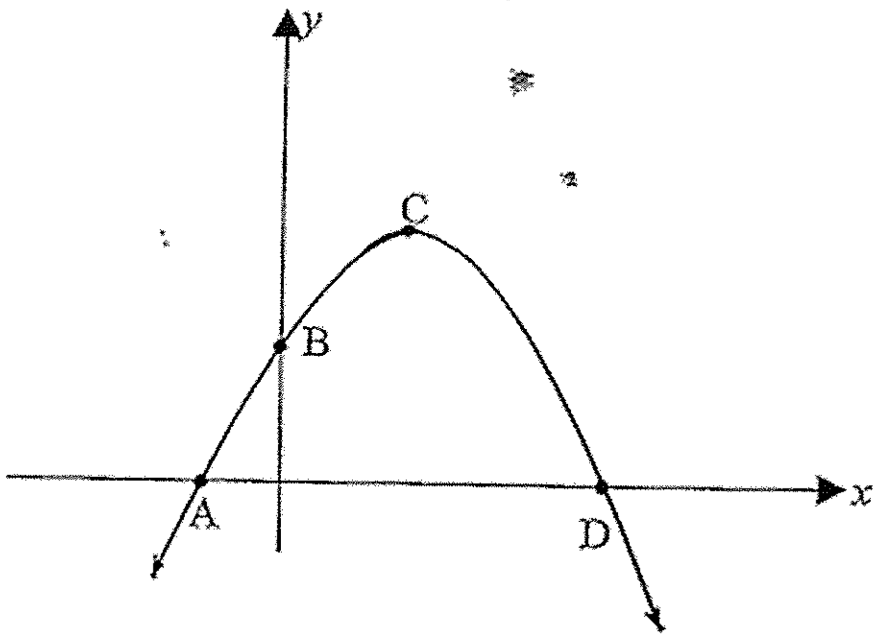


i) State the x coordinate of B

ii) Find the exact value of the shaded area

(5 marks)

c) A portion of the curve $y = 3\sin(x + \frac{\pi}{6})$ is shown below.



Find

i) the x value at A (in terms of π)

ii) the y value at B

iii) the coordinates of C (give x value in terms of π)

iv) the x value at D (in terms of π)

End of Examination

☺☺☺

1. B 2. B 3. A 4. D

5. D

6. a)

$\frac{d}{dx}\frac{2x^4}{\cos x} = \cancel{\cos(8x^3)+}$

$= \frac{\cos x(8x^3)+2x^4\sin x}{\cos^2 x}$

$= \frac{2x^3(4\cos x+x\sin x)}{\cos^2 x}$

b) $\int \sec^2\frac{x}{2}\,dx$

$= 2\tan\frac{x}{2}+C$

c) i) $\int(3x-4)^6\,dx$

$= \frac{(3x-4)^7}{21}+C$

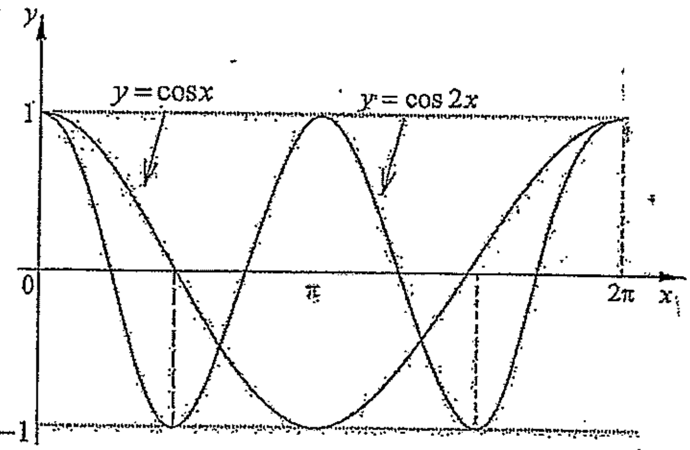
ii) $y' = 6x^{-2}$

$y = \frac{-6}{x}+C$

d) i) $\cos\frac{2\pi}{3} = -\frac{1}{2}$

$\cos\frac{4\pi}{3} = -\frac{1}{2}$

ii)



iii) 4 solutions

e) $\int_{\frac{\pi}{2}}^{\pi} \sin\frac{x}{2}\,dx$

$= \left[-2\cos\frac{x}{2}\right]_{\frac{\pi}{2}}^{\pi}$

$= -2\cos\frac{\pi}{2} + 2\cos\frac{\pi}{4}$

$= 0 + \frac{2}{\sqrt{2}}$

$= \frac{2}{\sqrt{2}}$ or $\sqrt{2}$

g) $\sin x = \frac{-\sqrt{3}}{2}$

S A
T C

$x = \frac{4\pi}{3}, \frac{5\pi}{3}$

7.

a) $\int_{1}^{3} x^2 + \frac{x}{2}\,dx$

$= \left[\frac{1}{3}x^3 + \frac{1}{4}x^2\right]_{-1}^{3}$

$= \frac{1}{3}\left(3^2 - (-1)^3\right) + \frac{1}{4}\left(3^2 - (-1)^2\right)$

$= \frac{28}{3} + \frac{8}{4}$

$= 11\frac{1}{3}$

b) $V = \int_0^2 \pi x^2\,dy$

$= \int_0^2 \pi(2-y)^4\,dy$

$= \frac{-\pi}{5}\left[(2-y)^5\right]_0^2$

$= \frac{-\pi}{5}\left(0 - 2^5\right)$

$= \frac{32\pi}{5}$ units^3

c) $f(x) = xe^{-x}$

$f'(x) = 1.e^{-x} + x.(-e^{-x})$

$= (1-x)e^{-x}$

$\therefore f'(x) < 0$ for $x > 1$

$f''(x) = -e^{-x} - (1-x)e^{-x}$

$= (x-2)e^{-x}$

$\therefore f''(x) < 0$ for $x < 2$

d) ii)

$y'' + 4y + 4 = 0$

LHS $= 8\cos 2x + 4(-2\cos 2x - 1) + 4$

$= 8\cos 2x - 8\cos 2x - 4 + 4$

$= 0$

$=$ RHS

d) i)

$y' = \int 8\cos 2x \, dx$

$= 4\sin 2x + C_1$

$y' = 0$ when $x = \frac{\pi}{2}$

$\therefore C_1 = 0$

$y = \int 4\sin 2x \, dx$

$= -2\cos 2x + C_2$

$y = 1 \quad x = \frac{\pi}{2}$

$1 = -2\cos \pi + C_2$

$= -2(-1) + C_2$

$C_2 = -1$

$\therefore y = -2\cos 2x - 1$

8.

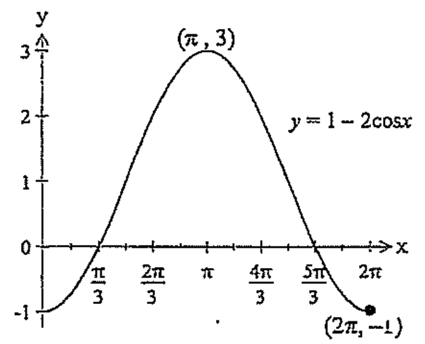
a) i)

$1-2\cos x = 0, \quad 0 \le x \le 2\pi$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3}, \frac{5\pi}{3}$

ii)



iii)

$A = -\int_0^{\frac{\pi}{3}} (1-2\cos x)\,dx + \int_{\frac{\pi}{3}}^{\pi} (1-2\cos x)\,dx$

$= -\left[x-2\sin x\right]_0^{\frac{\pi}{3}} + \left[x-2\sin x\right]_{\frac{\pi}{3}}^{\pi}$

$= -\left(\frac{\pi}{3} - 2\frac{\sqrt{3}}{2}\right) + \left[\left(\pi - \frac{\pi}{3}\right) - 2\left(0 - \frac{\sqrt{3}}{2}\right)\right]$

$= \frac{\pi}{3} + 2\sqrt{3}$

b)

$\int_0^1 (e^{3x}+1)\,dx$

$= \left[\frac{e^{3x}}{3} + x\right]_0^1$

$= \frac{e^3}{3} + 1 - \frac{1}{3}$

$= \frac{e^3}{3} + \frac{2}{3}$

c) $(2^x)^2 - 8(2^x) = 0$

$2^x(2^x - 8) = 0$

$2^x = 8$

$x = 3$

9.	b)
a) i)	i) B $x = \frac{5\pi}{4}$
$4x - x^2 = 2x - 3$	
$x^2 - 2x - 3 = 0$	ii)
$(x+1)(x-3) = 0$	$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin x - \cos x \, dx$
$\therefore x = -1, 3$	
At B, $x = 3$	$= \left(-\cos\frac{5\pi}{4} - \sin\frac{5\pi}{4}\right)$
	$- \left(-\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right)$
ii) Area =	
$\int_{-1}^{3} \left[(4x - x^2) - (2x - 3)\right] dx$	$= \left[-\left(\frac{-1}{\sqrt{2}}\right) - \left(\frac{-1}{\sqrt{2}}\right)\right] - \left(\frac{-1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right)$
$= \int_{-1}^{3} (2x - x^2 + 3) \, dx$	$= \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}}$
$= \left[x^2 - \frac{x^3}{3} + 3x\right]_{-1}^{3}$	$= \frac{4}{\sqrt{2}}$ or $2\sqrt{2}$ u^2
$= (9 - 9 + 9) - \left(1 + \frac{1}{3} - 3\right)$	
$= 10\frac{2}{3} u^2$	

c) i) $3\sin\left(x+\frac{\pi}{6}\right)=0$

$\therefore x+\frac{\pi}{6}=0, \pi$

$x=\frac{-\pi}{6}, \frac{5\pi}{6}$

At A $x=\frac{-\pi}{6}$

ii) $x=0$, $y=3\sin\frac{\pi}{6}$

$=1\frac{1}{2}$

iii) $y=3$

$\therefore 3\sin\left(x+\frac{\pi}{6}\right)=3$

$x+\frac{\pi}{6}=\frac{\pi}{2}$

$\therefore x=\frac{2\pi}{6}$

$=\frac{\pi}{3}$

iv) $x=\frac{5\pi}{6}$ (from i)