Sydney Technical High School

Mathematics

HSC Assessment Task 3 - 2 unit

June 2014

General Instructions

- Working Time – 75 minutes.
- Approved calculators may be used.
- Write using blue or black pen.
- A table of Standard Integrals is provided at the back of this paper.
- In Question 6 - 9, show relevant mathematical reasoning and/or calculations.
- Begin each question on a new side of the answer booklet.
- Marks shown are a guide and may need to be adjusted.
- Full marks may not be awarded for careless work or illegible writing.

NAME________________________

TEACHER ____________________

Total Marks – 57

MULTIPLE CHOICE **Pages 2 – 3**
5 marks

FREE RESPONSE **Pages 3 – 5**
52 marks

MULTIPLE CHOICE Q1-5

Question 1

What is the derivative of sin^2x?

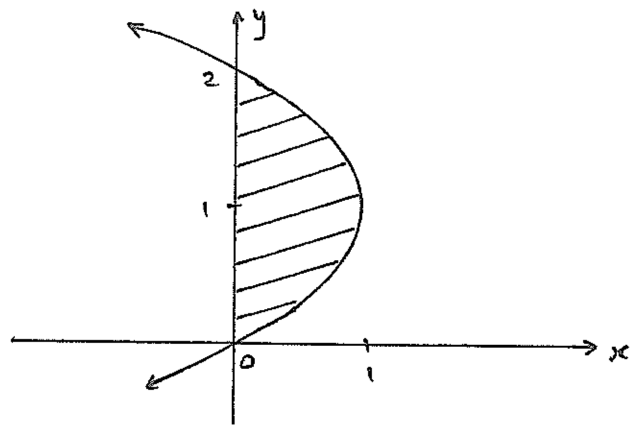
A. cos^2x B. $2\sin x$ C. $2\cos x \sin x$ D. $\frac{sin^3x}{3}$

Question 2

Which of the following curves has period π units and amplitude 2 units?

A. $y = \pi \tan \frac{x}{2}$ B. $y = \pi \sin 2x$ C. $y = 2\cos 2x$ D. $y = 2\sin \frac{x}{2}$

Question 3



The shaded area can be found using:

A. Area = $\int_0^1 f(y)\,dy$

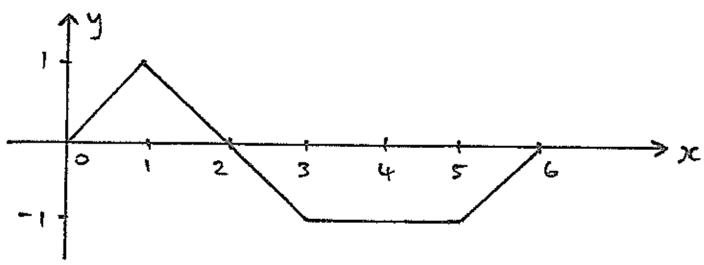
B. Area = $\int_0^1 f(x)\,dx$

C. Area = $\int_0^2 f(y)\,dy$

D. Area = $\int_0^2 f(y)\,dx$

Question 4

Use the graph of $y = f(x)$ below to evaluate $\int_0^6 f(x)dx$



A. 2 B. -2 C. -3 D. 4

Question 5

$\frac{d}{dx}(\cot x) = ?$

A. $-cosec^2 x$ B. $-cos^2 x$ C. $\sec x \tan x$ D. $-\cot x \; cosec\, x$

FREE RESPONSE Q6-9

Question 6 (13 marks)

a) Express 10° in radians. 1

b) Write the exact value of $\cos\frac{\pi}{6}$ 1

c) A circle, with radius 10 cm, has a sector subtending $\frac{\pi}{5}$ radians at the centre. Find the area of the sector in exact form. 1

d) Solve for $0 \leq x \leq 2\pi$: (answers in exact form)

i) $\tan x = \sqrt{3}$ 2

ii) $2sin^2 x + \sin x = 0$ 3

e) Differentiate:

i) $y = \sin 3x$ 1

ii) $y = x cos\, x$ 2

iii) $y = \sqrt{\tan x}$ 2

Question 7 (13 marks) Start a new page.

a) For the curve $y = \cos 3x$:

i) What is its period? 1

ii) Sketch the curve over the domain $0 \leq x \leq \pi$. Use a ruler and show intercepts on the axes. 2

b) A curve has gradient function $\frac{dy}{dx} = 3x^2 + x - 1$. If the curve passes through (-1,1), find the equation of the curve. 2

c) Find the gradient of the curve $y = 5x + 2\sin x$ when $x = 0$. 2

d) If $\int_0^a (2x+2)^3\, dx = 30$, find the value of a. 2

e) Consider the curve given by $y = \sin x - \cos x$. Find the coordinates of all stationary points in the domain $0 \leq x \leq 2\pi$ and determine their nature. Do not sketch the curve. 4

Question 8 (13 marks) Start a new page.

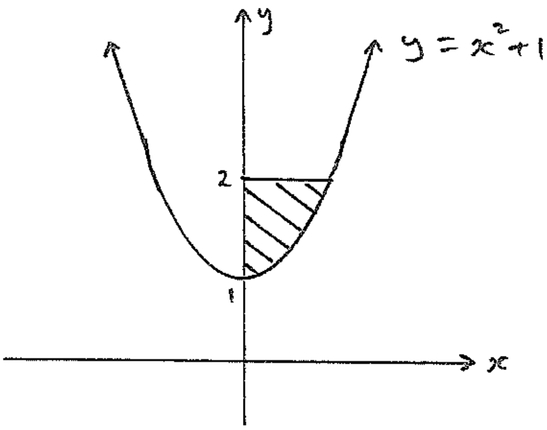
a) Differentiate $y = \dfrac{\sin 2x}{2x}$. Simplify your answer. 2

b) i) Use two (2) applications of the Trapezoidal rule to approximate the value of $\int_1^3 \frac{1}{x}\,dx$ 2

ii) Will your answer in part i) be an over OR under-estimation of the true integral value? Accurately explain why. 1

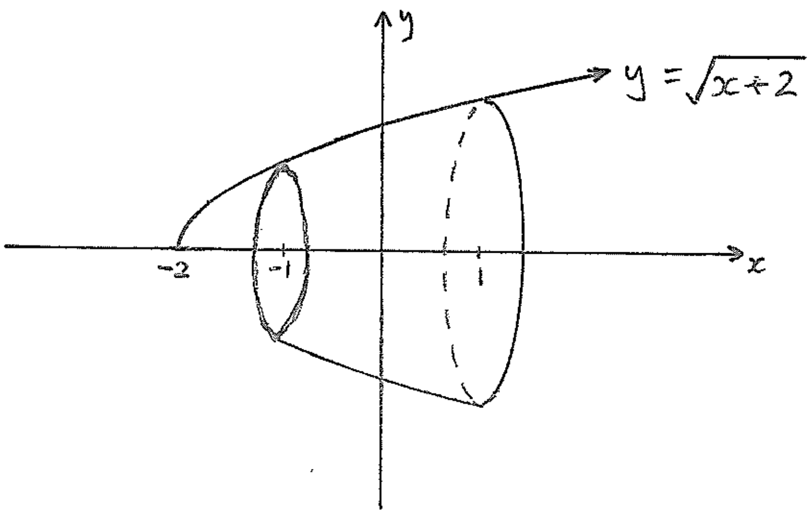
c) i) Given $y = x^2 + 1$, express x in terms of y. 1

ii) Using your answer above, find the shaded area below. 2



iii) Derive the same shaded area above using the x axis. 2

d) A hollow bowl is made by rotating part of the curve $y = \sqrt{x+2}$ between $x = -1$ and $x = 1$ around the x axis, as shown below. Find the exact volume occupied by the bowl. 3



Question 9 (13 marks) Start a new page.

a) Solve $\cos x = 0.7256$ for $0 \leq x \leq 2\pi$. Give your answer in radians correct to two decimal places. 2

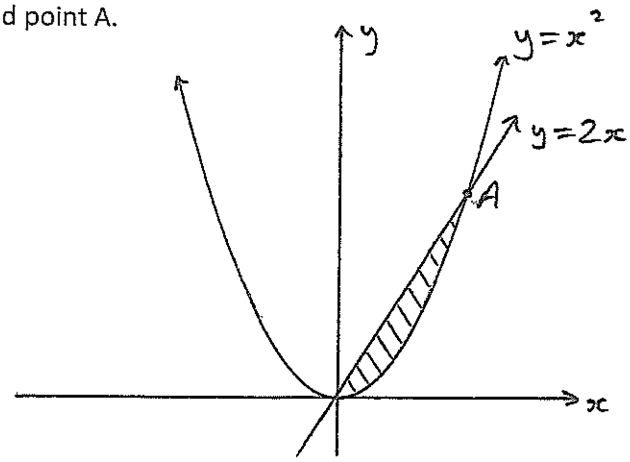
b) i) Differentiate $sin^4 x$ 1

ii) Hence, find $\int \cos x \, sin^3 x \, dx$ 1

c) Use Simpson's Rule and the five (5) function values in the table below to approximate $\int_3^7 f(x)dx$ 2

x	3	4	5	6	7
$f(x)$	2	5	1	3	4

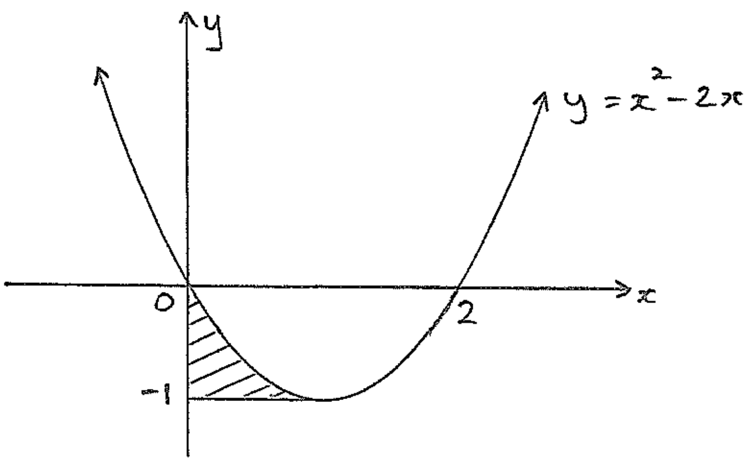
d) The area between the parabola $y = x^2$ and the line $y = 2x$ is shown. The graphs intersect at the origin and point A.



i) Find the coordinates of A. 1

ii) Find the exact value of the shaded area. 3

e) Find the shaded area below: 3



SOLUTIONS (2U Yr 12)

1) C 2) C 3) C 4) B 5) A

6) a) $10 \times \frac{\pi}{180} = \frac{\pi}{18}$

b) $\frac{\sqrt{3}}{2}$

c) $A = \frac{1}{2}r^2\theta$

$= \frac{1}{2} \times 100 \times \frac{\pi}{5}$

$= 10\pi \ u^2$

d) i) $x = \frac{\pi}{3}$ (1st, 3rd quads)

$= \frac{\pi}{3}, \frac{4\pi}{3}$

ii) $\sin x(2\sin x + 1) = 0$

$\sin x = 0$ or $-\frac{1}{2}$

$\therefore x = 0, \pi, 2\pi, \frac{7\pi}{6}, \frac{11\pi}{6}$

e) i) $y' = 3\cos 3x$

ii) $y' = 1 \times \cos x + (-\sin x)x$

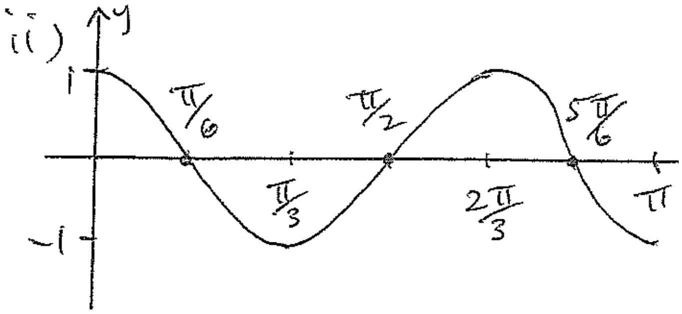
$= \cos x - x\sin x$

iii) $y' = \frac{1}{2}(\tan x)^{-\frac{1}{2}} \times \sec^2 x$

or $\frac{\sec^2 x}{2\sqrt{\tan x}}$

7) a) i) period is $\frac{2\pi}{3}$ units

ii)



b) $y = x^3 + \frac{x^2}{2} - x + c$

$(-1, 1) \rightarrow 1 = \cancel{-1} + \frac{1}{2} \cancel{+1} + c$

$(c = \frac{1}{2})$

$\therefore y = x^3 + \frac{x^2}{2} - x + \frac{1}{2}$

c) $y' = 5 + 2\cos x$

when $x = 0$, gradient $= 5 + 2$

$= 7$

d) $\left[\frac{(2x+2)^4}{8}\right]_0^a = 30$

$\frac{(2a+2)^4}{8} - \frac{2^4}{8} = 30$

$\frac{(2a+2)^4}{8} = 32$

$(2a+2)^4 = 256$

$2a + 2 = 4$

$\therefore a = 1$

(7) e) S.P.'s when $y' = \cos x + \sin x = 0$

$$\therefore \sin x = -\cos x$$
$$\tan x = -1$$
$$\therefore x = \pi/4 \text{ (2nd, 4th quads)}$$
$$= \frac{3\pi}{4} \text{ or } \frac{7\pi}{4}$$

Testing :

x	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
y'	+	0	−

$\Rightarrow$ max. turning pt. at $\left(\frac{3\pi}{4}, \sqrt{2}\right)$

x	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
y'	−	0	+

$\Rightarrow$ min. turning pt. at $\left(\frac{7\pi}{4}, -\sqrt{}\right.$

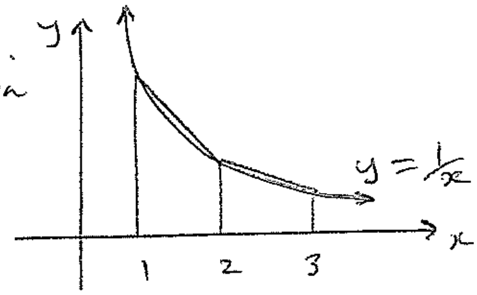
(8) a) $y = \dfrac{2\cos 2x \times 2x - 2\sin 2x}{4x^2}$

$$= \frac{2x\cos 2x - \sin 2x}{2x^2}$$

b) i) $\int_1^3 \frac{1}{x}\, dx \doteqdot \frac{1}{2}\left[f(1) + f(2)\right] + \frac{1}{2}\left[f(2) + f(3)\right]$

$$= \tfrac{1}{2}\left(1 + \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{1}{3}\right)$$
$$= \tfrac{1}{2}\left(2\tfrac{1}{3}\right)$$
$$= 1\tfrac{1}{6}$$

ii) answer is an <u>over</u> estimation as the trapeziums go <u>above</u> the curve



c) i) $x^2 = y - 1$

$$\therefore x = \pm\sqrt{y-1}$$

ii) Area $= \int_1^2 (y-1)^{1/2}\, dy$

$$= \left[\tfrac{2}{3}(y-1)^{3/2}\right]_1^2 = 2/3(1-0) = 2/3\ u^2$$

(8) c) iii) Area $= 2\times1\,(\text{rectangle}) - \int_0^1 (x^2+1)\,dx$

$$= 2 - \left[\frac{x^3}{3} + x\right]_0^1$$

$$= 2 - \left[\left(\tfrac{1}{3}+1\right) - (0+0)\right]$$

$$= 2 - 1\tfrac{1}{3}$$

$$= \tfrac{2}{3}\,u^2 \text{ as reqd.}$$

d) Vol. $= \pi \int_{-1}^{1} (x+2)\,dx$

$$= \pi \left[\frac{x^2}{2} + 2x\right]_{-1}^{1}$$

$$= \pi \left[\left(\tfrac{1}{2}+2\right) - \left(\tfrac{1}{2}-2\right)\right]$$

$$= \pi\left(2\tfrac{1}{2} - -1\tfrac{1}{2}\right)$$

$$= 4\pi\,u^3$$

(9) a) $x = 0{\cdot}76^c$ (1st, 4th quads)

$$= 0{\cdot}76^c,\ 5{\cdot}52^c$$

b) i) $y' = 4\sin^3 x \cos x$

ii) $\tfrac{1}{4}\sin^4 x + c$

c) $\int_3^7 f(x)\,dx \doteqdot \tfrac{1}{3}(2+4\times5+1) + \tfrac{1}{3}(1+4\times3+4)$

$$= \tfrac{1}{3}(2+20+2+12+4)$$

$$= \tfrac{1}{3}(40)$$

$$= 13\tfrac{1}{3}.$$

(9) d) i) graphs intersect when $x^2 = 2x$

$\therefore x^2 - 2x = 0$

$x(x-2) = 0$

$\therefore x = 0$ or 2

$\therefore A$ is $(2,4)$

ii) $\text{Area} = \int_0^2 (2x - x^2)\,dx$ OR $\left|\int_0^2 (x^2 - 2x)\,dx\right|$

$= \left[x^2 - \frac{x^3}{3}\right]_0^2$

$= (4 - 2\tfrac{2}{3}) - (0 - 0)$

$= 1\tfrac{1}{3}\ u^2$

e) (using x axis) $\text{Area} = 1\times 1$ square $-$ area to y axis

$= 1 - \left|\int_0^1 (x^2 - 2x)\,dx\right|$

$= 1 - \left|\left[\frac{x^3}{3} - x^2\right]_0^1\right|$

$= 1 - \left|\left(\tfrac{1}{3} - 1\right) - (0 - 0)\right|$

$= 1 - \left|-\tfrac{2}{3}\right|$

$= \tfrac{1}{3}\ u^2$

(some Ext 1 boys may use the y axis)

$y + 1 = x^2 - 2x + 1$

$(x-1)^2 = y + 1$

$x - 1 = \pm\sqrt{y+1}$

$\therefore x = 1 \pm \sqrt{y+1}$

(test, choose $-\sqrt{\ }$)

$\therefore x = 1 - \sqrt{y+1}$

$\therefore \text{area} = \int_{-1}^0 1 - (y+1)^{\frac{1}{2}}\,dy$

$= \left[y - \tfrac{2}{3}(y+1)^{\frac{3}{2}}\right]_{-1}^0$

$= (0 - \tfrac{2}{3}) - (-1 - 0)$

$= -\tfrac{2}{3} + 1$

$= \tfrac{1}{3}\ u^2$ as above.