Name : ______________________

Teacher/ Class : ______________________

SYDNEY TECHNICAL HIGH SCHOOL

HSC ASSESSMENT <u>TASK 1</u>

DECEMBER 2009

MATHEMATICS - EXTENSION 1

Time Allowed : **70 minutes**

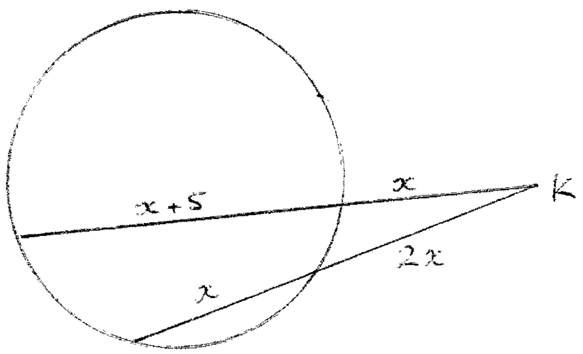
Instructions:

- Write your name and class at the top of each page.
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- Start **each** question on a new **page.**
- Diagrams, unless otherwise, stated are not to scale.

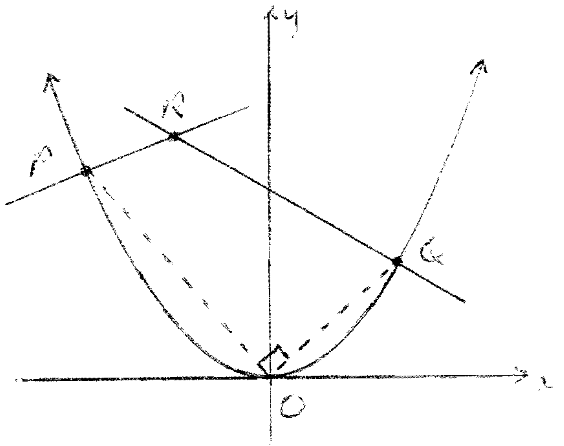
Q1	Q2	Q3	Q4	Q5	TOTAL
/10	/10	/10	/10	/10	/50

Question 2

a) Secants to the circle are drawn to meet at K. Find the value of x. 2



b)



For the parabola $x = 2at, y = at^2$, points P and Q subtend a right angle at the vertex.

The normals at P and Q intersect at R.

i) Prove that $pq = -4$, where p and q are the parameters for P and Q. 1

ii) Derive the gradient of the normal at P and show that its equation is $yp - ap^3 = -x + 2ap$. 2

iii) R has a y coordinate of $ap^2 - 2a + aq^2$. Find its x coordinate. 2

iv) Show that the cartesian locus for R is the parabola $x^2 = 16a(y - 6a)$. 2

v) Find the coordinates of the focus of the parabola in iv). 1

Question 3

a) Accurately describe how to find the centre of the circle passing through points A, B, C. 1

B.

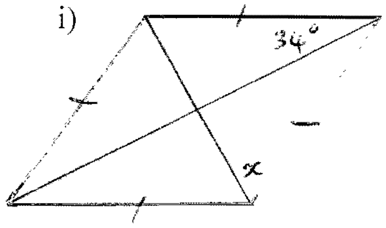
C.

A.

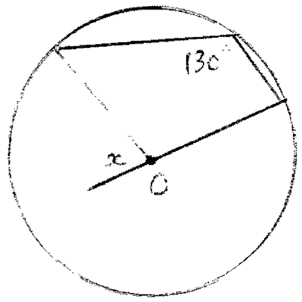
Question 1

a) Find the value of each pronumeral. O is the centre of each circle. Reasons are <u>not</u> necessary. 5

i)



ii)



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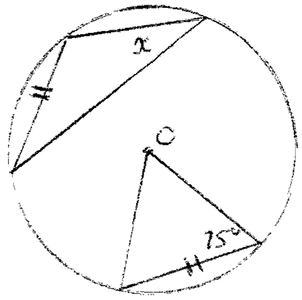
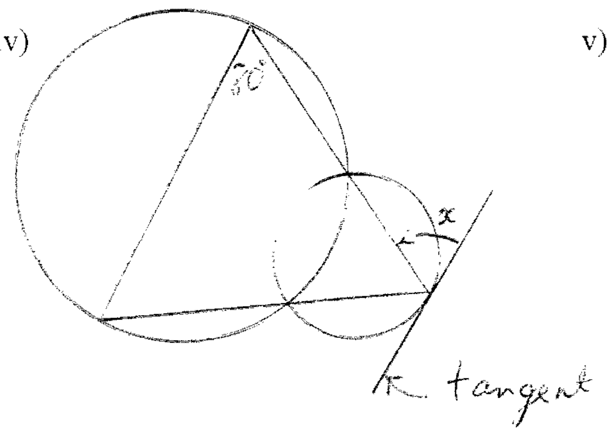
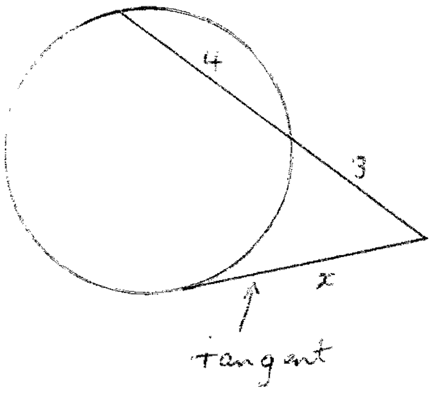


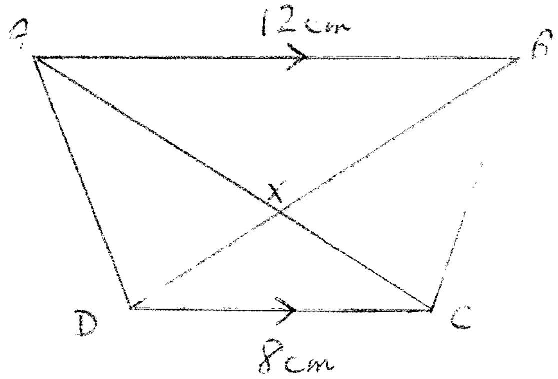
iv)



v)



b) $ABCD$ is a trapezium with $AB \| DC$. $AB = 12cm, DC = 8cm, AC = 9cm$. Diagonals intersect at X.



i) State which two triangles are similar (do not prove similarity). 1

ii) Hence, find the length of AX 2

c) Find the first negative term of the sequence 496,489,482, ……… 2

Question 5

a) The sum of the first two terms of a geometric series is 6, and the sum of the second and third terms is -5. 1

Find the common ratio for this series.

b) Kenny begins his retirement with \$500,000 and invests it to earn 6% p.a.

Interest is calculated on the balance at the end of each month and added to the remaining balance.

Immediately after the interest calculation, Kenny withdraws an amount, M, for living expenses.

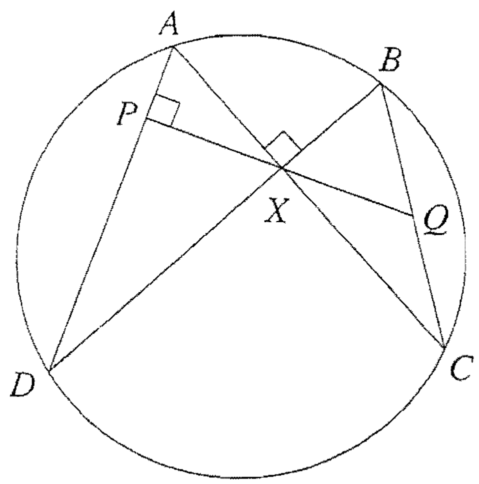
i) Let A_n be the balance remaining after the nth withdrawal.

Show that $A_2 = 500000 \times 1.005^2 - 1.005M - M$ 1

ii) How much can he afford to withdraw each month is he plans to live for another 20 years? 2

iii) If, instead, he withdraws \$5000 per month, show that the number of months can be found using $1.005^n = 2$. (do not solve this). 1

c)



NOT TO SCALE

The diagram shows points, A, B, C and D on a circle. The lines AC and BD are perpendicular and intersect at X. The perpendicular to AD through X meets AD at P and BC at Q.

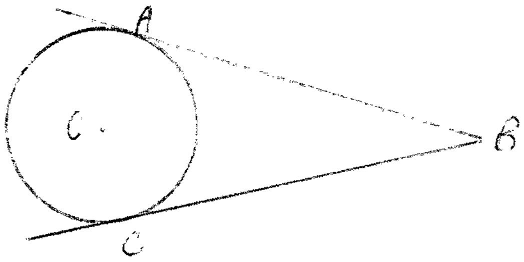
i) Neatly redraw the diagram onto your answer page.

ii) Prove that $\angle QXB = \angle QBX$. 3

iii) Prove that Q bisects BC. 2

END OF PAPER

b) Tangents BA and BC are drawn to the circle, with centre O.



i) Neatly redraw the above diagram onto your answer page.

ii) Prove that $AB = CB$ 2

iii) Join AC and OB. Prove that $AC \perp OB$ 3

c) Express $0.6\dot{5}$ as the sum of an infinite geometric series and find its limiting sum in fraction form. 2

d) For a particular series, the sum of the first n terms is given by $S_n = 3n^2 - 2n$.
Find the simplified expression for the $(n+1)th$ term, T_{n+1} 2

Question 4

a) The first three terms of a certain geometric series are: $x + 2 + 1\frac{1}{2} + \ldots\ldots$

i) Find the value of x 1

ii) Find an expression for the tenth term in the form $\frac{3^a}{2^b}$ 1

b) Evaluate $\sum_{n=10}^{30}(2^n + 2n + 2)$ 3

c) John invests \$1000 into a savings account.

i) If the account earns 6% p.a. compounded annually, find the account's value at the end of 15 years. 1

ii) If the account is to have a value of \$5000 after 15 years, find the annual compound interest rate needed to achieve this. Give your answer correct to 1 decimal place. 2

d) Mary invests \$1000 into an account, earning 6% p.a. interest, at the beginning of 2010. She continues to deposit \$1000 at the beginning of each subsequent year into the account. Find the total value of her savings at the end of 2024. 2

SOLUTIONS

1) a) i) $56°$ ii) $80°$ iii) $15°$

iv) $80°$ v) $x = \sqrt{21}$

b) i) $\triangle ABX$ & $\triangle CDX$

ii) $\frac{AX}{XC} = \frac{3}{2}$

$2AX = 3XC$

$2AX = 3(9 - AX)$

$5AX = 27$

$\therefore \underline{AX = 5.4 \text{ cm}}$

c) Need $T_n = 496 + (n-1)(-7) < 0$

$\therefore 496 - 7n + 7 < 0$

$-7n < -503$

$n > \frac{503}{7}$

$\therefore$ $\underline{T_{72}(-1) \text{ is first neg. term}}$

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2) a) $x(2x+5) = 2x(3x)$

$2x^2 + 5x = 6x^2$

$4x^2 - 5x = 0$

$x(4x-5) = 0$

$x = 0$ or $5/4$ $(x > 0)$

$\therefore \underline{x = 5/4}$.

b) i) $M_{OP} \times M_{OQ} = -1$

$\therefore \frac{ap^2}{2ap} \times \frac{aq^2}{2aq} = -1$

$\therefore \frac{p}{2} \times \frac{q}{2} = -1$

$\therefore pq = -4$

ii) $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= 2at \times \frac{1}{2a}$

$= t$

at P, $M_T = p$

$\therefore m_N = \underline{\frac{-1}{p}}$

eqn. of normal at P is

$y - ap^2 = -\frac{1}{p}(x - 2ap)$

$\therefore \underline{yp - ap^3 = -x + 2ap}$ ⊛

(Simly for Q: $yq - aq^3 = -x + 2aq$)

iii) Sub $y = ap^2 - 2a + aq^2$ into ⊛ above

$\therefore (ap^2 - 2a + aq^2)p - ap^3 = -x + 2ap$

$\therefore x = \cancel{ap^3} - \cancel{ap^3} + 2ap - aq^2p + 2ap$

$= 4ap - aq(-4)$

$= \underline{4ap + 4aq}$

iv) $x = 4ap + 4aq = 4a(p+q)$

$\therefore p + q = \frac{x}{4a}$

and $y = a(p^2 + q^2 - 2)$

$= a\left[(p+q)^2 - 2pq - 2\right]$

$= a\left[\left(\frac{x}{4a}\right)^2 - 2(-4) - 2\right]$

$= a\left(\frac{x^2}{16a^2} + 8 - 2\right)$

$= \frac{x^2}{16a} + 6a$

$\therefore y - 6a = \frac{x^2}{16a}$

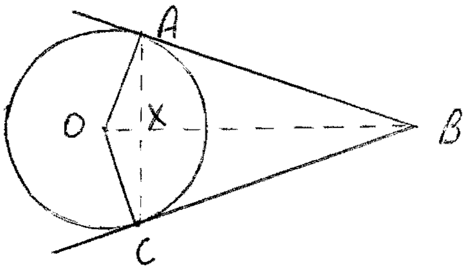
$\therefore x^2 = 16a(y - 6a)$ as reqd.

v) Focal length $4a$, vertex $(0, 6a)$

$\therefore$ $\underline{\text{focus at } (0, 10a)}$

(3) a) Centre is at point of intersection of two perpendicular bisectors of say, AB and AC.

b) i)



ii) $OA = OB$ (equal radii)

OB is common

$\angle OAB = \angle OCB = 90°$ (radius $\perp$ tangent)

$\therefore \triangle OAB \equiv \triangle OCB$ (RHS)

$\therefore AB = BC$ (corresponding sides in congruent triangles)

iii) $\angle AOX = \angle COX$ (corresponding angles in congruent triangles)

OX is common

$OA = OC$ (above)

$\therefore \triangle AOX \equiv \triangle COX$ (SAS)

$\therefore \angle AXO = \angle CXO$ (corresponding angles in congruent triangles)

$\therefore \angle AXO$ & $\angle CXO = 90°$ each.

(AXC is a straight line)

$\therefore AC \perp OB$ as reqd.

c) $0.6\dot{5} = \frac{6}{10} + \frac{5}{100} + \frac{5}{1000} + \dots$

$= \frac{6}{10} + \frac{a}{1-r}$

$= \frac{6}{10} + \frac{\frac{5}{100}}{1-\frac{1}{10}}$

$= \frac{6}{10} + \frac{5}{100} \times \frac{10}{9}$

$= \frac{6}{10} + \frac{5}{90}$

$= \frac{59}{90}$

d) $T_{n+1} = S_{n+1} - S_n$

$= 3(n+1)^2 - 2(n+1) - 3n^2 + 2n$

$= \cancel{3n^2} + 6n + 3 - \cancel{2n} - 2 - \cancel{3n^2} \cancel{+2n}$

$= 6n + 1$

(4) a) i) $\frac{2}{x} = \frac{1\frac{1}{2}}{2} = \frac{3}{4}$

$3x = 8$

$x = \frac{8}{3}$

ii) $T_{10} = ar^9$

$= \frac{8}{3} \times \left(\frac{3}{4}\right)^9$

$= \frac{2^3}{3} \times \frac{3^9}{2^{18}}$

$= \frac{3^8}{2^{15}}$

b) $\sum_{n=10}^{30}(2^n) + \sum_{n=10}^{30}(2n+2)$

$= \frac{2^{10}(2^{21}-1)}{2-1} + \frac{21}{2}(22+62)$

$= \underline{\underline{2,147,483,506}}$

c) i) $A = 1000(1.06)^{15}$

$= \underline{\underline{\$2396.56}}$

ii) $5000 = 1000(1+r)^{15}$

$(1+r)^{15} = 5$

$r = \sqrt[15]{5} - 1$

$= 0.1132\ldots$

$\doteqdot \underline{\underline{11.3\%\ p.a.}}$

d) First \$1000 $\Rightarrow 1000(1.06)^{15}$

Second \$1000 $\Rightarrow 1000(1.06)^{14}$

$\vdots$

last \$1000 $\Rightarrow 1000(1.06)^{1}$

$\therefore$ total $= \frac{1000(1.06)(1.06^{15}-1)}{1.06-1}$

$= \underline{\underline{\$24672.53}}$.

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(5) a) $a + ar = a(1+r) = 6$

$ar + ar^2 = ar(1+r) = -5$

$\therefore r = \underline{\underline{-5/6}}$

b) i) $A_1 = 500000 \times 1.005 - M$

$A_2 = A_1 \times 1.005 - M$

$= (500000 \times 1.005 - M) \times 1.005 - M$

$= 500000 \times 1.005^2 - 1.005M - M$

ii) $A_{240} = 500000 \times 1.005^{240} - 1.005^{239}M - \ldots - M$

$= 500000 \times 1.005^{240} - (1.005^{239}M + \ldots + M)$

$= 500000 \times 1.005^{240} - \frac{M(1.005^{240}-1)}{1.005-1}$

and $A_{240} = 0$,

$\therefore M = \frac{(500000 \times 1.005^{240}) \times 0.005}{1.005^{240}-1}$

$= \underline{\underline{\$3582.16 \text{ per month.}}}$

iii)

$5000 = \frac{(500000 \times 1.005^n) \times 0.005}{1.005^n - 1}$

$\therefore 5000 \times 1.005^n - 5000 = 2500 \times 1.005^n$

$\therefore 2500 \times 1.005^n = 5000$

$\therefore 1.005^n = 2$

as req'd.