Name: ..... **Maths Teacher:** .....

SYDNEY TECHNICAL HIGH SCHOOL

Year 11
Mathematics

Preliminary HSC Course

Assessment 2

July, 2016

Time allowed: 90 minutes

General Instructions:

- Marks for each question are indicated on the question.
- Approved calculators may be used
- All necessary working should be shown
- Full marks may not be awarded for careless work or illegible writing
- ***Begin each question on a new page***
- Write using black or blue pen
- All answers are to be in the writing booklet provided
- BOSTES reference sheet attached and may be removed if required.

Section 1	Multiple Choice Questions 1-6 6 Marks
Section II	Questions 7-14 64 Marks

SECTION I

6 marks
Attempt Questions 1-6
Allow about 8 minutes for this section

Use the multiple choice answer sheet for Questions 1-6.

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1. What is the exact value of $\tan 30°$?

   A. $\sqrt{3}$

   B. $-\sqrt{3}$

   C. $\frac{1}{\sqrt{3}}$

   D. $-\frac{1}{\sqrt{3}}$

2. What is the gradient of the line perpendicular to the line $2x - y + 3 = 0$?

   A. -2

   B. $-\frac{1}{2}$

   C. $\frac{1}{2}$

   D. 2

3. If $x = \frac{3}{5}$ and $y = \frac{1}{4}$, then $\frac{x + y}{3x - 5y}$ equals:

   A. $\frac{17}{61}$

   B. $\frac{17}{11}$

   C. $\frac{61}{17}$

   D. $\frac{11}{17}$

4. $8p^3 + 1$ factorises to:

   A. $(8p + 1)(p^2 - 2p + 1)$

   B. $(2p - 1)(p^2 - 4p + 1)$

   C. $(2p + 1)(p^2 + 4p + 1)$

   D. $(2p + 1)(p^2 - 2p + 1)$

5. What is the value of $f(-1)$ if $f(x) = x^3 - 4x$

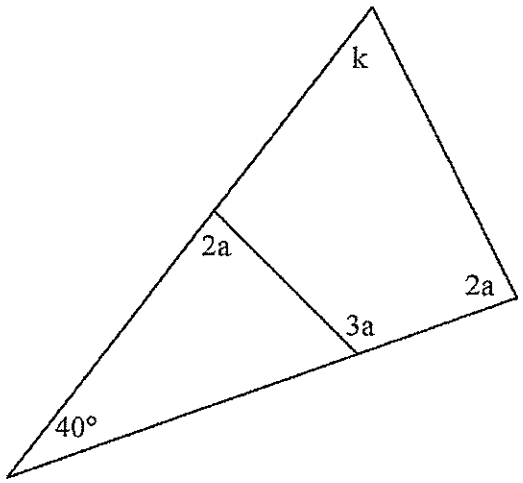
   A. -3

   B. -5

   C. 3

   D. 5

6. What is the value of k?



   A. 40°

   B. 35°

   C. 60°

   D. 105°

END OF SECTION 1

SECTION II

64 marks
Attempt Questions 7-14
Allow about 82 minutes for this section

Answer each question on a new page in the answer booklet.

In Questions 7-14, your responses should include relevant mathematical reasoning and/or calculations.

Question 7 (8 marks) Start a NEW page. **Marks**

a) Solve $(x-2)(x+3) > 0$ and show the solution on a number line. **2**

b) If $\frac{3}{5+\sqrt{2}} = a + b\sqrt{2}$, find the values of a and b. **2**

c) Rewrite the following as a single fraction **2**

$$\frac{1}{x-1} + \frac{x^2+1}{x+1}$$

d) Evaluate $\frac{a^2}{ab-4b}$ if $a = -\frac{1}{2}$ and $b = \pi$. Give your answer to 2 significant figures. **2**

END OF QUESTION 7

Question 8 (8 marks) Start a NEW page. **Marks**

a) Factorise fully $\frac{3x^2}{4} - 12$ **2**

b) Find the exact value of $\cos x$ if $\tan x = -\frac{2}{3}$ and $\sin x$ is negative. **2**

c) Solve $|2x-3| = 12$. **2**

d) Given that: **2**

$$f(x) = \begin{cases} 2, & \text{for } x > 1 \\ 2x, & \text{for } -1 \leq x \leq 1 \\ -2x, & \text{for } x < -1 \end{cases}$$

find the value of $f(2) - f(-2) + f(1)$

END OF QUESTION 8

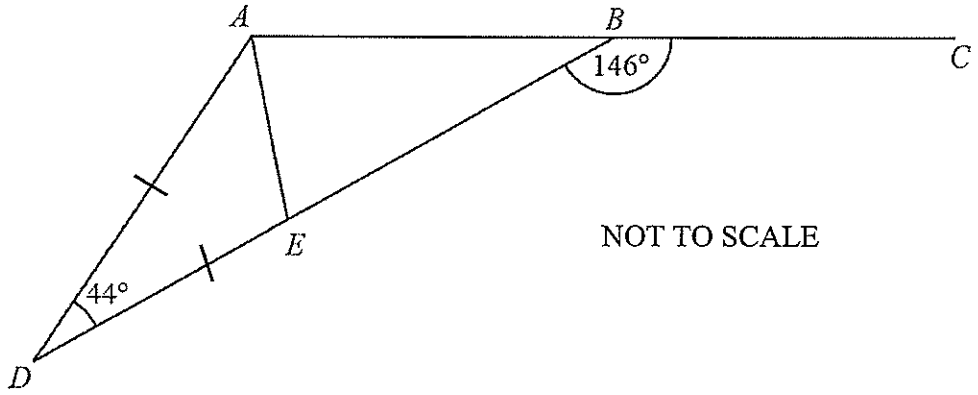
Question 9 (8 marks) Start a NEW page. **Marks**

a) Show that $\sec^2\theta - \tan^2\theta = 1$. **2**

b) Simplify: **3**

$$\tan(-A)\sin(90-A)$$

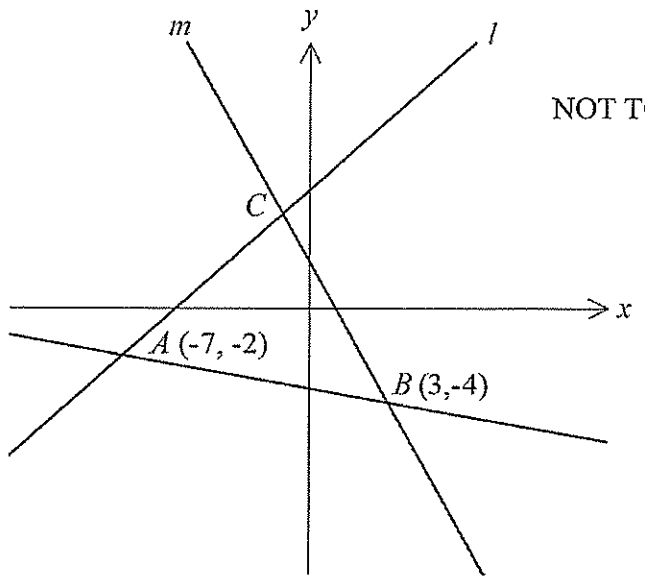
c) In the diagram below, ABC is a straight line and E lies on BD. **3**
$AD = DE, \angle ADE = 44°, \angle CBE = 146°$.



Copy the diagram below into your answer booklet and find $\angle BAE$, giving reasons.

END OF QUESTION 9

Question 10 (8 marks) Start a NEW page. **Marks**



NOT TO SCALE

The point A (-7,-2) lies on the line l given by the equation $y = x + 5$ and the point B (3,-4) lies on the line m given by $2x + y - 2 = 0$.

a) Show that C, which is the point of intersection of the lines l and m, has the coordinates (-1,4). **1**

b) Find the midpoint of the interval AB. **1**

c) Show the equation of line AB is $x + 5y + 17 = 0$. **2**

d) Show the perpendicular distance of the point A from line m is $\frac{18}{\sqrt{5}}$ units. **2**

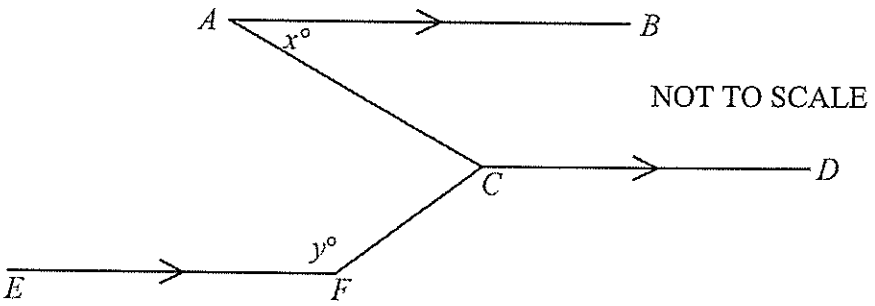
e) Find the length of BC. **1**

f) Hence, or otherwise, find the area of triangle ABC. **1**

END OF QUESTION 10

Question 11 (8 marks) Start a NEW page. **Marks**

a)



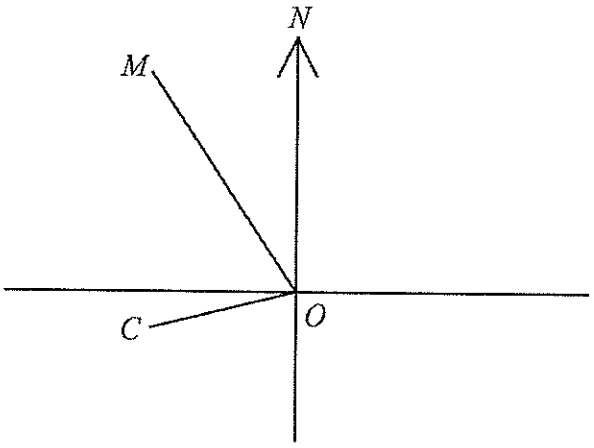
In the diagram, $AB \parallel CD \parallel EF$, $\angle BAC = x°$ and $\angle CFE = y°$.

i) Copy the diagram into your answer booklet and place all the information onto the diagram.

ii) Find the value of $\angle ACF$ in terms of x and y, giving complete reasons. **3**

b) What is the size of an exterior angle of a regular pentagon? **1**

c) Martin (M), a surveyor, walks 3.2km away from a wharf (O) at a bearing of 295° and his colleague (C) walks to their car parked S25°W from the wharf. The colleague is now due south of the surveyor.



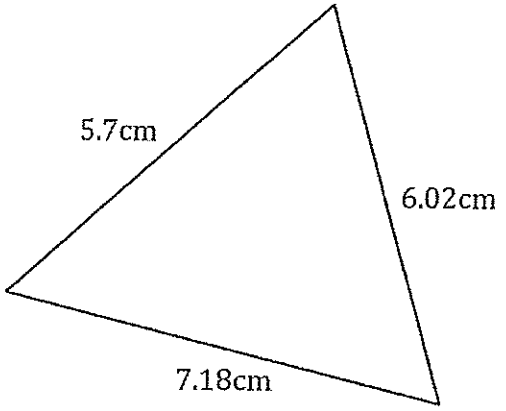
i. Copy the diagram in your answer booklet, making sure it takes up **half a page.** Add all information given and find the size of $\angle MOC$, adding it to your diagram. **1**

ii. Find, correct to the nearest 100m, the distance that the colleague walked from the wharf to the car. **3**

END OF QUESTION 11

Question 12 (9 marks) Start a NEW page. **Marks**

a) Consider the triangle below.



NOT TO SCALE

i) Show the size of the smallest angle to the nearest degree is 50°. **2**

ii) Hence, or otherwise, find the area of the triangle giving your answer with 3 significant figures. **2**

b) Solve θ for the domain $0 \leq \theta \leq 360°$: **3**

$$sin\theta - \sqrt{3}cos\theta = 0$$

c) Sketch and shade the region that represents the inequality **2**

$$(x-2)^2 + (y+3)^2 < 16.$$

END OF QUESTION 12

Question 13 (7 marks) Start a NEW page. **Marks**

a) A, B and C are the points (2,-1), (-2,11) and (8, 1) respectively.

i. Find the distance of the interval joining BC. **1**

ii. Find the equation of the circle which has BC as the diameter. **2**

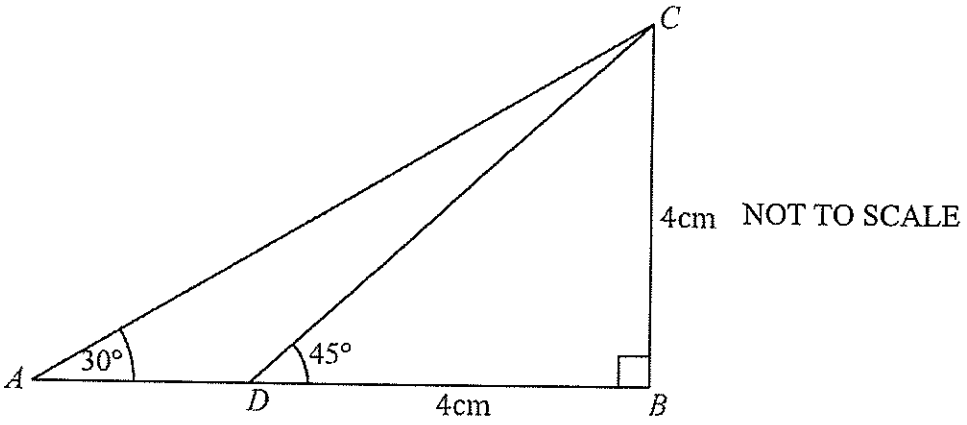
iii. Does this circle pass through A? Justify your answer. **1**

b) Prove that $\dfrac{(1+\tan^2 x)\cot x}{\operatorname{cosec}^2 x} = \tan x$ **3**

END OF QUESTION 13

Question 14 (8 marks) Start a NEW page. **Marks**

a)



In the above diagram $DB = CB = 4$cm, $\angle CDB = 45°$ and $\angle CAD = 30°$.

i) Copy the diagram into your answer booklet and show that $AD = 4(\sqrt{3}-1)$ cm. **2**

ii) Explain why $\angle ACD$ is $15°$. **1**

iii) Use the sine rule to show that $\sin 15° = \dfrac{\sqrt{3}-1}{2\sqrt{2}}$ **2**

b) Solve $2\sin^2 x - 3\sin x = -1$ for $0 \leq x \leq 360°$ **3**

END OF PAPER

2010 11 11 Assessment 2 Solutions

Section I

1. C
2. B
3. B
4. D
5. C
6. C

Section II

7a.

$x < -3$ or $x > 2$

b. $\frac{3}{5+\sqrt{2}} \times \frac{5-\sqrt{2}}{5-\sqrt{2}}$

$= \frac{3(5-\sqrt{2})}{5^2-2}$

$= \frac{15}{23} - \frac{3\sqrt{2}}{23}$

$\therefore a = \frac{15}{23}, \quad b = -\frac{3}{23}$

c. $\frac{1}{x-1} + \frac{x^2+1}{x+1}$

$= \frac{1}{x-1} \times \frac{x+1}{x+1} + \frac{x^2+1}{x+1} \times \frac{x-1}{x-1}$

$= \frac{(x+1) + (x^2+1)(x-1)}{(x-1)(x+1)}$ OR $\frac{x^3-x^2+2x}{(x-1)(x+1)}$

d. $\frac{a^2}{ab-4b} = \frac{(-\frac{1}{2})^2}{(-\frac{1}{2})(\pi)-4(\pi)}$

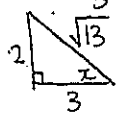
$= -0.0176\ldots$

$\doteq -0.018$ (2 sig. figs.)

8a. $\frac{3x^2}{4} - 12 = 3(\frac{x^2}{4} - 4)$

$= 3(\frac{x}{2}-2)(\frac{x}{2}+2)$

b. $\tan x = -\frac{2}{3}$



$\therefore \cos x = \frac{3}{\sqrt{13}}$

c. $|2x-3| = 12$

so: $2x-3 = 12$ and $-(2x-3) = 12$

$2x = 15$ $\quad 2x-3 = -12$

$x = \frac{15}{2}$ $\quad 2x = -9$

$x = -\frac{9}{2}$

d. $f(2) - f(-2) + f(1) = 2 - 4 + 2 = 0$

9. a. $\sec^2\theta - \tan^2\theta = 1$

$LHS = \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} = \frac{1-\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta}$

$= 1 = RHS$

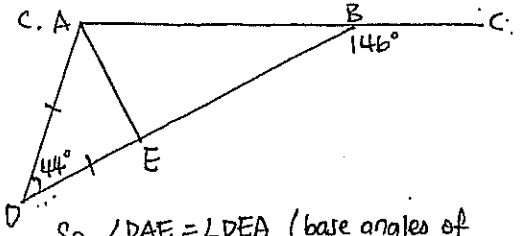
b. $\tan(-A)\sin(90-A)$

$= -\tan A \cos A$

$= -\frac{\sin A}{\cos A}\cos A$

$= -\sin A$

c.



So $\angle DAE = \angle DEA$ (base angles of isosceles $\triangle ADE$)

$44° + \angle DAE + \angle DEA = 180°$ (angle sum of $\triangle ADE$)

$\angle DAE = 68°$

$\angle DEA = 68°$

$\angle DEA + \angle AEB = 180°$ (angle sum of straight line)

$\angle AEB = 180° - 68°$

$= 112°$

$\angle BAE + \angle AEB = 146°$ (exterior angle of $\triangle AEB$ equal to the sum of the opposite interior angles.

$\angle BAE = 146° - 112°$

$= 34°$

10. a. $y = x + 5$ ①

$2x + y - 2 = 0$ ②

sub ① into ②

$2x + x + 5 - 2 = 0$

$3x + 3 = 0$

$3x = -3$

$x = -1$

sub into ①

$y = -1 + 5$

$= 4$

$\therefore$ Intersection is $C(-1, 4)$

b. $M_{AB} = \left(\frac{-7+3}{2}; \frac{-2+-4}{2}\right)$

$= (-2, -3)$

c. $m_{AB} = \frac{-2--4}{-7-3}$

$= -\frac{1}{5}$

$y + 2 = -\frac{1}{5}(x+7)$

$y = -\frac{1}{5}x - \frac{17}{5}$

or $x + 5y + 17 = 0$

d. $d_{Am} = \frac{|2(-7)+1(-2)-2|}{\sqrt{2^2+1^2}}$

$\doteq \frac{18}{\sqrt{5}}$

$= \frac{18\sqrt{5}}{5}$

e. $d_{BC} = \sqrt{(3+1)^2 + (-4-4)^2}$

$= \sqrt{16+64}$

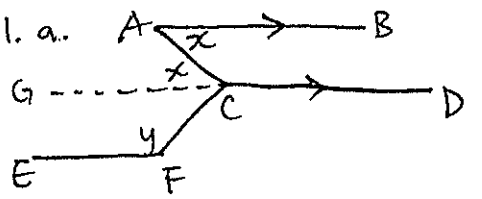
$= \sqrt{80}$

$= 4\sqrt{5}$

f. $A = \frac{1}{2} \times 4\sqrt{5} \times \frac{18\sqrt{5}}{5}$

$= 36$ units^2

11. a.



$\angle ACG = x°$ (alternate angles GD ∥ AB)

$\angle GCF + y = 180°$ (cointerior angles supplementary GD ∥ EF)

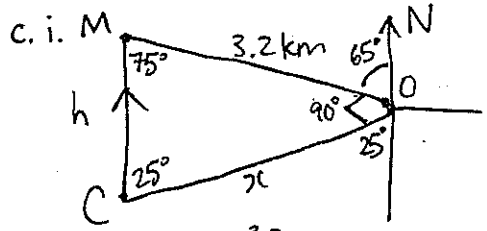
$\angle GCF = 180° - y°$

$\therefore \angle ACF = x° + 180° - y°$

b. Exterior angle sum of any polygon is 360°.

$360 \div 5 = 72°$

c. i.



ii. $\tan 25 = \frac{3.2}{x}$

$x = \frac{3.2}{\tan 25}$

$=$

$\doteq 6.9$ km (nearest 100m)

12. a. i. $\cos C = \frac{6.02^2 + 7.18^2 - 5.4^2}{2 \times 6.02 \times 7.18}$

$= 0.6397...$

$C = 50.228...$

$\doteq 50°$

ii. $A = \frac{1}{2}(6.02)(7.18)\sin 50$

$= 16.555...$

$\doteq 16.6\ cm^2$ (3 sig. figs.)

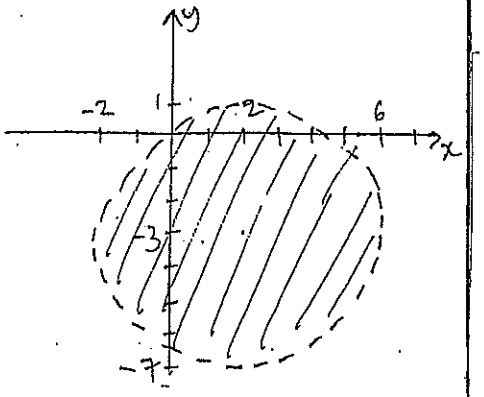
b. $\frac{\sin\theta}{\cos\theta} - \frac{\sqrt{3}\cos\theta}{\cos\theta} = \frac{0}{\cos\theta}$

$\tan\theta - \sqrt{3} = 0$

$\tan\theta = \sqrt{3}$

$\theta = 60°, 240°$

c.



13. a. i. $d_{BC} = \sqrt{(-2-8)^2 + (11-1)^2}$

$= \sqrt{200}$

$= 10\sqrt{2}$

ii. $M_{BC} = \left(\frac{-2+8}{2}, \frac{11+1}{2}\right)$

$= (3, 6)$

$(x-3)^2 + (y-6)^2 = 50$

iii) $A(2, -1)$

$(2-3)^2 + (-1-6)^2 = 50$

∴ Yes A lies on the circle as it satisfies the equation.

b. $\frac{(1+\tan^2 x)\cot x}{\operatorname{cosec}^2 x} = \tan x$

LHS $= \frac{(1+\tan^2 x)\cot x}{\operatorname{cosec}^2 x}$

$= \frac{\left(1 + \frac{\sin^2 x}{\cos^2 x}\right)\frac{\cos x}{\sin x}}{\frac{1}{\sin^2 x}}$

$= \frac{\left(\frac{\cos^2 x + \sin^2 x}{\cos^2 x}\right)\frac{\cos x}{\sin x}}{\frac{1}{\sin^2 x}}$

$= \frac{\left(\frac{1}{\cos^2 x}\right)\left(\frac{\cos x}{\sin x}\right)}{\frac{1}{\sin^2 x}}$

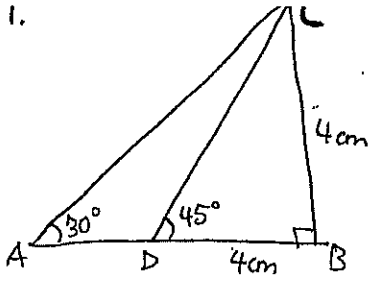
$= \frac{\frac{1}{\cos x \sin x}}{\frac{1}{\sin^2 x}}$

$= \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}}$

$= \frac{\sin x}{\cos x}$

$= \tan x$ = RHS

14 a. i.



$\tan 30° = \frac{4}{AD+4}$

$\frac{1}{\sqrt{3}} = \frac{4}{AD+4}$

$AD + 4 = 4\sqrt{3}$

$AD = 4\sqrt{3} - 4$

$= 4(\sqrt{3} - 1)$

ii. $\angle ADC + 45° = 180°$ (angles on a straight line)

$\angle ADC = 135°$

$135° + 30° + \angle ACD = 180°$ (angle sum of $\triangle ACD$)

$\angle ACD = 15°$

iii. $\frac{\sin 15}{AD} = \frac{\sin 30}{CD}$

$CD^2 = 4^2 + 4^2$ (Pythagoras')

$CD = 4\sqrt{2}$

$\frac{\sin 15}{4(\sqrt{3}-1)} = \frac{\frac{1}{2}}{4\sqrt{2}}$

$\sin 15 = \frac{\frac{1}{2}(4(\sqrt{3}-1))}{4\sqrt{2}}$

$= \frac{2(\sqrt{3}-1)}{4\sqrt{2}}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

b. Let $x = \sin x$

$2x^2 - 3x = -1$

$2x^2 - 3x + 1 = 0$

$(2x-1)(x-1) = 0$

$x = \frac{1}{2}, 1$

∴ $\sin x = \frac{1}{2}$, $\sin x = 1$

for $\sin x = \frac{1}{2}$

S ✓	A ✓
T	C

$x = 30°, 150°$

for $\sin x = 1$

boundary angle.

$x = 90°$

∴ $x = 30°, 90°, 150°$