SYDNEY TECHNICAL HIGH SCHOOL

HIGHER SCHOOL CERTIFICATE
PRELIMINARY ASSESSMENT TASK 2
July 2016

Extension 1 Mathematics

Name .....

Teacher .....

General Instructions

- Reading Time - 5 minutes.
- Working Time - 90 min.
- Write using a blue or black pen.
- Board approved calculators may be used.
- The BOSTES reference sheet is provided at the back of this paper.
- In Questions 1-11, show relevant mathematical reasoning and /or calculations.
- Begin each question on a fresh sheet of paper.

Total marks (65)

Section I- Pages 2-3

5 Marks

- Attempt Questions 1-11.

Section II Pages 4-8

- All questions are of equal value.

Section 1
Multiple Choice (5 marks)
Use the multiple choice answer sheet for Question 1-5

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1. Which of the following is an expression for $\frac{9^n - 6^n}{9^n - 4^n}$?

(A) $\frac{6^n}{4^n}$

(B) $\frac{3^n}{5^n}$

(C) $\frac{3^n}{3^n - 2^n}$

(D) $\frac{3^n}{3^n + 2^n}$

2. Which of the following is an expression for $\cos(A + B) - \cos(A - B)$

(A) $-2\sin A \sin B$

(B) $-2\cos A \cos B$

(C) $2\cos A \cos B$

(D) $2\sin A \sin B$

3. Find the domain and range of $y = \sqrt{2x - 6}$

(A) Domain: $x \leq 3$, Range: $y \geq 0$

(B) Domain: all real x, Range: $y \geq 0$

(C) Domain: $x \geq 3$, Range: all real y

(D) Domain: $x \geq 3$, Range: $y \geq 0$

4. If $x = 2at$ and $y = 3at^2$, which of the following is an expression for $\frac{dy}{dx}$?

(A) t
(B) 2t
(C) 3t
(D) 6t

5. If $R\cos(x + \alpha) \equiv \cos x - \sin x$, what are the values of $R\cos\alpha$ and $R\sin\alpha$?

(A) $R\cos\alpha = -1$, $R\sin\alpha = -1$
(B) $R\cos\alpha = -1$, $R\sin\alpha = 1$
(C) $R\cos\alpha = 1$, $R\sin\alpha = -1$
(D) $R\cos\alpha = 1$, $R\sin\alpha = 1$

Section II

Total Marks (60)
Attempt Questions 6 – 11.

Answer each question in your writing booklet.
In Questions 6-11, your responses should include relevant mathematical reasoning and/or calculations.

Question 6 (10 Marks) Use a Separate Sheet of paper

(a) Find $\lim_{x \to 4} \dfrac{x^2 + x - 20}{x - 4}$ **1**

(b) Find the acute angle between the lines $x - 3y + 2 = 0$ and $2x + y - 3 = 0$ (to the nearest degree). **2**

(c) Find the exact value of $\sin 105°$ **3**

(d) Find the coordinates of the point that divides the interval joining A(7, 1) and B(0, -6) internally in the ratio 4 : 3. **2**

(e) Differentiate $f(x) = 3x - 5$ using first principles. **2**

End of Question 6

Question 7 (10 Marks) Use a Separate Sheet of paper

(a) i) On the same axis sketch the graph $y = |x|$ and $y = (x - 2)^2$ **2**

ii) Hence solve $|x| < (x - 2)^2$. **3**

(b) Solve $\dfrac{4}{|x + 1|} < 3$ **2**

(c) Solve $\sin(\theta + 10) = \cos(\theta - 20)$ if θ is acute. **1**

(d) Find the equation of the tangent to $y = x^2 + 3x + 4$ at the point where $x = 0$. **2**

End of Question 7

Question 8 (10 Marks) Use a Separate Sheet of paper

(a) Differentiate the following expressions, leaving your answer with no negative or fractional indices. There is no need to simplify to a single fraction.

(i) $(x^2+3)\sqrt{2x^3-5}$ **2**

(ii) $\dfrac{2x+3}{(x-1)^2}$ **3**

(b) Factorise fully $2a^4-3a^3+16a-24$ **2**

(c) The lines $mx-y=0$ and $mx-6y=0$ where $(m>0)$ are inclined at an angle of $45°$ to each other.

i) Show that $m^2-5m+6=0$. **2**

ii) Hence find the values of m. **1**

End of Question 8

Question 9 (10 Marks) Use a Separate Sheet of paper

(a) i) Use the substitution $t=\tan\dfrac{x}{2}$ to show that $\sin x+\cos x+1=\dfrac{2(t+1)}{1+t^2}$ **2**

ii) Hence solve the equation $\sin x+\cos x=-1$ for $0° \leq x \leq 360°$ **2**

(b) i) Find the value of $f'(8)$ if $f(x)=3\sqrt{x+1}$ **2**

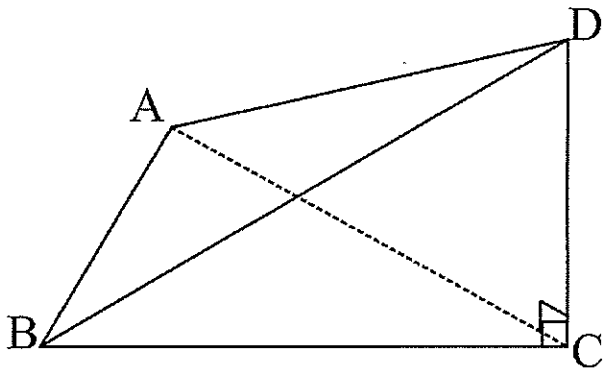
ii) Hence find the equation of the normal to the curve $y=f(x)$ at the point on it where $x=8$. **2**

(c) Find the exact distance between the parallel lines $3x+2y-5=0$ and $3x+2y=1$ **2**

End of Question 9

Question 10 (10 **Marks**) Use a Separate Sheet of paper

(a) In the diagram a vertical tower CD of height h metres stands with its bottom C on horizontal ground. A and B are two points on the ground such that AB = 28 metres. From B the bearing of the tower is 050° and the angle of elevation of the top D of The tower is 30°. From A the bearing of the tower is 110° and the angle of elevation of the top of the tower is 60°.



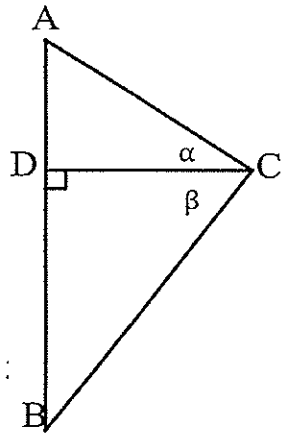
i) Draw a top view diagram and on it show why $\angle ACB = 60°$ **1**

ii) Hence find the exact height of the tower. **3**

(b) Given x and y are rational, solve the following for x and y. **3**

$$x + y + \sqrt{x^2 + y} = 10 + 2\sqrt{13}$$

(c)



Given triangle ABC where D is a point on AB such that CD⊥AB, $\angle ACD = \alpha$ and $\angle DCB = \beta$.

i) Write down the expressions for tand α and tan β **1**

ii) Show that $DB = \dfrac{AD \cos\alpha \sin\beta}{\cos\beta \sin\alpha}$ **2**

End of Question 10

Question 11 (10 Marks) Use a Separate Sheet of paper

(a) i) Express $\sqrt{3}\cos x + \sin x$ in the form $R\sin(x + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$ **2**

ii) Hence find the range of the function $f(x) = \sqrt{3}\cos x + \sin x$ **1**

(b) If $\cos x = \frac{7}{9}$ and $\cos y = \frac{1}{3}$ where x and y are acute.

i) Show that $\sin x = \sin 2y$ **2**

ii) Hence show that the exact value of $\cos 3y = -\frac{23}{27}$ **2**

(c) Prove the identity $\frac{\cot A - \tan A}{\cot 2A} = 2$ **3**

End of Question 11

End of Examination

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Student Name: _____ Teacher Name: _____

Year 11 Extension One Mathematics
July 2016 Task #2

Multiple Choice

1. D
2. A
3. D
4. C
5. D

1) $\frac{(3\times3)^n-(3\times2)^n}{(3\times3)^n-(2\times2)^n}$

$= \frac{3^{2n}-3^n\times2^n}{3^{2n}-2^{2n}}$

$= \frac{3^n(3^n-2^n)}{(3^n-2^n)(3^n+2^n)}$

$= \frac{3^n}{3^n+2^n}$

2) $\cos A\cos B-\sin A\sin B-(\cos A\cos B+\sin A\sin B)$

$= -2\sin A\sin B$

3. $y=\sqrt{2x-6}$

$2x-6\geq 0.$

Domain $x\geq 3$

Range $R: y\geq 0$

4. $\frac{dy}{dx}=\frac{dy}{dt}\div\frac{dx}{dt}$

$= 6at\div 2a$

$= 3t$

5 $\cos x-\sin x = R\cos x\cos\alpha - R\sin x\sin\alpha$

$R\cos\alpha = 1 \qquad R\sin\alpha = 1$

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Question 6

a) $= \lim_{x\to 4} \frac{x^2+x-20}{x-4}$

$= \lim_{x\to 4} \frac{(x+5)(x-4)}{(x-4)}$

$= \lim_{x\to 4} x+5$

$= 9$

b) $x-3y+2=0 \qquad 2x+y-3=0$

$3y = x+2 \qquad y = 3-2x$

$y = \frac{x}{3}+\frac{2}{3}$

$m_1 = \frac{1}{3} \qquad m_2 = -2$

$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\tan\theta = \left|\frac{\frac{1}{3}-(-2)}{1+\frac{1}{3}\times(-2)}\right|$

$\tan\theta = \left|\frac{7/3}{1/3}\right|$

$\tan\theta = 7$

$\theta = 82^\circ$

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c) $\sin 105^\circ = \sin(60^\circ + 45^\circ)$

$= \sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$

$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$ or $\frac{\sqrt{6}+\sqrt{2}}{4}$

d) A(7,1), B(0,-6), 4:3

$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$

$= \left(\frac{4\times 0 + 3\times 7}{7}, \frac{4\times -6 + 3\times 1}{7}\right)$

$= \left(\frac{21}{7}, \frac{-21}{7}\right)$

$= (3, -3)$

e) $f(x) = 3x - 5$

$f(x+h) = 3(x+h) - 5$

$= 3x + 3h - 5$

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{3x + 3h - 5 - (3x - 5)}{h}$

$= \lim_{h \to 0} \frac{3h}{h}$

$= \lim_{h \to 0} 3$

$f'(x) = 3.$

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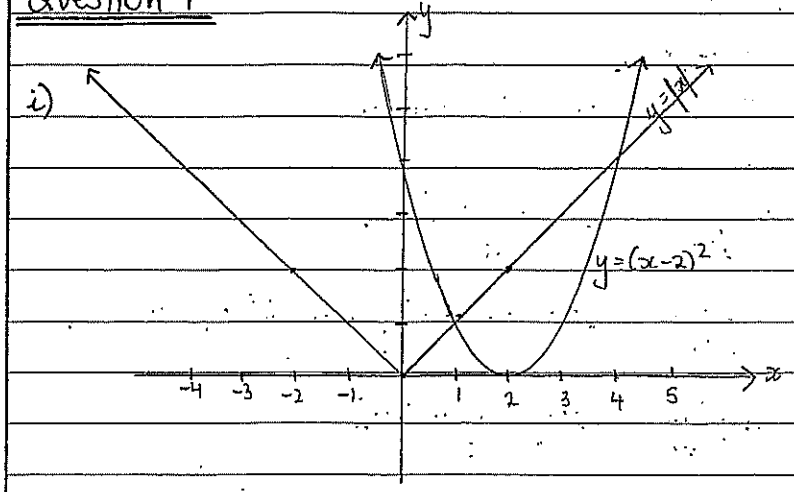
Question 7

i)



ii) $x = (x-2)^2$

$x = x^2 - 4x + 4$

$0 = x^2 - 5x + 4$

$0 = (x-4)(x-1)$

$x = 1$ $\quad$ $x = 4$

$x < 1$ $\quad$ $x > 4$

b) $\frac{4}{|x+1|} < 3$

$4 < 3(x+1)$ or $4 < -3(x+1)$

$4 < 3x + 3$ $\quad$ $4 < -3x - 3$

$1 < 3x$ $\quad$ $7 < -3x$

$x > \frac{1}{3}$ $\quad$ $x < -\frac{7}{3}$

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c) $\cos(4\alpha) = \cos 2(2\alpha)$

$= 2\cos^2 2\alpha - 1$

$= 2(2\cos^2\alpha - 1)^2 - 1$

$= 2(4\cos^4\alpha - 4\cos^2\alpha + 1) - 1$

$= 8\cos^4\alpha - 8\cos^2\alpha + 1$

d) $y = x^2 + 3x + 4$

$\frac{dy}{dx} = 2x + 3$

When $x = 0$ $y = 4$

$\frac{dy}{dx} = 2 \times 0 + 3$

$\frac{dy}{dx} = 3$

$y - y_1 = m(x - x_1)$

$y - 4 = 3(x - 0)$

$y - 4 = 3x$

$y = 3x + 4$

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Question 8

a-i) $y = (x^2+3)(\sqrt{2x^3-5})$

$u = x^2 + 3$ $\quad v = (2x^3-5)^{\frac{1}{2}}$

$\frac{du}{dx} = 2x$ $\quad \frac{dv}{dx} = \frac{1}{2}(2x^3-5)^{-\frac{1}{2}} \times 6x^2$

$= 3x^2(2x^3-5)^{-\frac{1}{2}}$

$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$

$= (x^2+3)\left[3x^2(2x^3-5)^{-\frac{1}{2}}\right] + (2x^3-5)^{\frac{1}{2}} \times 2x$

$= \frac{3x^2(x^2+3)}{\sqrt{2x^3-5}} + 2x\sqrt{2x^3-5}$

aii $y = \frac{2x+3}{(x-1)^2}$

$u = 2x + 3$ $\quad v = (x-1)^2$

$du = 2$ $\quad dv = 2(x-1)$

$\frac{dy}{dx} = \frac{(x-1)^2 \times 2 - (2x+3) \times 2(x-1)}{(x-1)^4}$

$\frac{dy}{dx} = \frac{2x^2 - 4x + 2 - 4x^2 + 2x - 3}{(x-1)^4}$

$\frac{dy}{dx} = \frac{-2x^2 - 2x + 1}{(x-1)^4}$

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b) $2a^4 - 3a^3 + 16a - 24$

$= a^3(2a-3) + 8(2a-3)$

$= (2a-3)(a^3+8)$

$= (2a-3)(a+2)(a^2-2a+4)$

c) i) $mx - y = 0$ $\quad mx - by = 0$

Gradient $= m$ $\quad$ Gradient $= \frac{1}{6}m$

where $m > \frac{1}{6}m > 0$

$\therefore \tan 45 = \left| \frac{m - \frac{1}{6}m}{1 + \frac{1}{6}m^2} \right|$

$1 = \frac{5m}{6 + m^2}$

$6 + m^2 = 5m$

$m^2 - 5m + 6 = 0$

ii) $m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$m = 2 \quad m = 3$

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Question 9

a i) $t = \tan\frac{x}{2}$ $\qquad \sin x + \cos x + 1 = \frac{2(t+1)}{1+t^2}$

$L.H.S = \frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2} + 1$

$= \frac{2t + (1-t^2) + (1+t^2)}{1+t^2}$

$= \frac{2t+2}{1+t^2}$

$= \frac{2(t+1)}{1+t^2}$

$= R.H.S$

ii) for $x \neq 180°$

Sub $t = \tan\frac{x}{2}$ $\qquad \frac{2(t+1)}{1+t^2} = 0$

$t = -1$

$\therefore \tan\frac{x}{2} = -1$

$0° < \frac{x}{2} < 180°$

$\therefore \frac{x}{2} \neq 135°$

$x = 270°$

$\sin x + \cos x = 1$ for $0° \leq x \leq 360°$

For $x = 180°$ $\qquad \sin x + \cos x = 0 + (-1) = -1$

$x = 180°$

$\sin x + \cos x = -1$ for $0° \leq x \leq 36°$

has solutions $x = 180°, 270°$

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b i) $y = 3\sqrt{x+1}$ $\quad y = 3(x+1)^{1/2}$

$\frac{dy}{dx} = \frac{3}{2}(x+1)^{-1/2}$

When $x = 8$ $\quad y = 9$

$\frac{dy}{dx} = \frac{1}{2}$ $\quad m_1 = \frac{1}{2}$ $\quad m_2 = -2$

Equation of the normal $y - y_1 = m(x - x_1)$

$y - 9 = -2(x - 8)$

$y - 9 = -2x + 16$

$y = -2x + 25$

c) $3x + 2y - 5 = 0$ $\quad 3x + 2y = 1$

when $x = 0$ $\quad y = \frac{1}{2}$

$d = \frac{|3 \times 0 + 2 \times \frac{1}{2} + (-5)|}{\sqrt{3^2 + 2^2}}$

$d = \frac{|-4|}{\sqrt{13}}$

$d = \frac{4}{\sqrt{13}}$ or $\frac{4\sqrt{13}}{13}$ units

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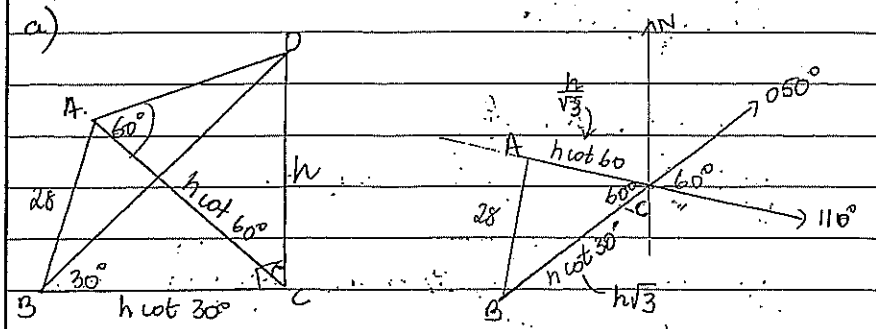
Question 10

a)



~~28~~ Using the cosine rule in $\triangle ABC$

$28^2 = (h\sqrt{3})^2 + (\frac{h}{\sqrt{3}})^2 - 2(h\sqrt{3})(\frac{h}{\sqrt{3}})\cos 60°$

$28^2 = 3h^2 + \frac{1}{3}h^2 - 2h^2 \cos 60°$

$784 = 3h^2 + \frac{h^2}{3} - h^2$

$784 = \frac{7h^2}{3}$

$h^2 = 336$

$h = \sqrt{336}$ or $4\sqrt{21}$ metres tall

b) $x + y + \sqrt{x^2 + y} = 10 + 2\sqrt{13}$

$x + y = 10$ — ①

$x^2 + y = 52$ — ②

$y = 10 - x$ — ③

Sub ③ into 2

$x^2 + y = 52$

$x^2 + (10 - x) = 52$

$x^2 + 10 - x = 52$

$x^2 - x - 42 = 0$

$(x - 7)(x + 6) = 0$

$x = 7$ $\quad x = -6$

$y = 3$ $\quad y = 16$.

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c) $\angle ACD = \alpha$ $\angle DCB = \beta$ $CD \perp AB$

In $\triangle ADC$ $\tan\alpha = \frac{AD}{DC}$

$$DC = \frac{AD}{\tan\alpha}$$

$$= AD \times \frac{\cos\alpha}{\sin\alpha}$$

In $\triangle BDC$ $\tan\beta = \frac{DB}{DC}$

$$DC = \frac{DB}{\tan\beta}$$

$$DC = DB \cdot \frac{\cos B}{\sin\beta}$$

ii) $\frac{DB \cdot \cos B}{\sin\beta} = \frac{AD \cdot \cos\alpha}{\sin\alpha}$

$$DB = \frac{AD \cdot \cos\alpha \sin\beta}{\cos\beta \sin\alpha}$$

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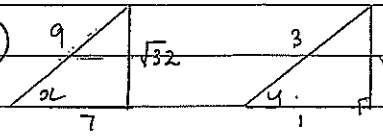
Question 11

a) i) $\sqrt{3}\cos x + \sin x = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right)$

$$= 2\sin(x + 60°)$$

ii) The range of $f(x)$ is $y: -2 \le y \le 2$

b)



$\sin x = \frac{\sqrt{32}}{9}$ $\sin 2y = 2\sin y \cos y$

$$= 2 \cdot \frac{\sqrt{8}}{3} \cdot \frac{1}{3}$$

$$= \frac{2 \times 2\sqrt{2}}{9}$$

$$= \frac{4\sqrt{2}}{9} \rightarrow \frac{\sqrt{32}}{9}$$

$\sin x = \sin 2y$

$x = 2y$ (x, y acute)

b iii) $\cos 3y = \cos(2y + y)$

$$= \cos 2y \cos y - \sin 2y \sin y$$

$$= -\frac{7}{9} \times \frac{1}{3} - \frac{\sqrt{32}}{9} \times \frac{\sqrt{8}}{3}$$

$$= -\frac{7}{27} - \frac{16}{27}$$

$\cos 2y = 2\cos^2 y - 1$

$= 2\left(\frac{1}{3}\right)^2 - 1$

$= -\frac{7}{9}$

$$= -\frac{23}{27}$$

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$$\frac{\cot A - \tan A}{\cot 2A} = 2$$

$$L.H.S = \frac{\cot A - \tan A}{\cot 2A}$$

$$= \frac{\frac{1}{\tan A} - \tan A}{\frac{1-\tan^2 A}{2\tan A}}$$

$$= \frac{1-\tan^2 A}{\tan A} \div \frac{1-\tan^2 A}{2\tan A}$$

$$= \frac{1-\tan^2 A}{\tan A} \times \frac{2\tan A}{1-\tan^2 A}$$

$$= 2$$

$$= R.H.S$$