

Name: Maths Class:

SYDNEY TECHNICAL HIGH SCHOOL



YEAR 11 MATHEMATICS

HSC ASSESSMENT 1

December 2007

TIME ALLOWED: 70 minutes

Instructions:

- Write your name and class at the top of this page, and at the top of each answer sheet.
- At the end of the examination this examination paper must be attached to the front of your answers.
- All necessary working must be shown. Marks may not be awarded for careless or badly arranged work.
- Marks indicated are a guide only and may be varied if necessary.
- **START EACH QUESTION ON A NEW PAGE**

(FOR MARKERS USE ONLY)

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	TOTAL
/7	/6	/7	/6	/7	/7	/7	/7	/54

QUESTION 1: (7 MARKS)

(a) Evaluate $\sum_{r=1}^4 \frac{1}{r(r+1)}$ 1

(b) By considering $0.\dot{8}\dot{3}$ as an infinite series (and by no other means),
express this recurring decimal as a simplified fraction 2
(SHOW ALL WORKING)

(c) The n^{th} term of a sequence is given by $T_n = \frac{1}{3}(2^{n-1})$

(i) Find the values of the first, second and third terms 4

(ii) Prove that the series is geometric and find the common ratio

(iii) Find the sum of the first 10 terms

QUESTION 2: (6 MARKS) START A NEW PAGE

Find derivatives of the following:

(a) $y = \frac{x^4}{4} + 2x + 3$ 2

(b) $y = \sqrt{4 - x^2}$ 2

(c) $y = \frac{3}{x-1}$ 2

QUESTION 3: (7 MARKS) (START ON A NEW PAGE)

(a) For the sequence given by

4

4, 7, 10,,421

- (i) Find an expression for the n^{th} term
- (ii) Find the 45^{th} term
- (iii) How many terms are there in the sequence?
- (iv) Find the sum of all of the terms of this sequence

(b) Find the equation of the normal to the curve $y = x(x - 2)(x - 1)$ at the point where $x = 2$

3

QUESTION 4: (6 MARKS) (START ON A NEW PAGE)

(a) Find the value(s) of x for which the curve $y = x^2 - 3x + 1$ is increasing

2

(b) Consider the series given by

4

$1 + 2x + 4x^2 + \dots$

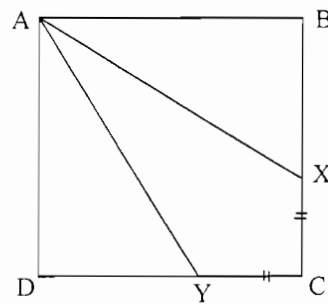
- (i) For what value(s) of x does this series have a limiting sum?
- (ii) Assuming that the conditions on x in part (i) are satisfied, find the value of x for which the limiting sum is $\frac{4}{3}$

QUESTION 5: (7 MARKS) (START ON A NEW PAGE)

- (a) A woman invests \$20 000 into a fund earning compound interest at 6% p.a.
- (i) Find the value of her investment after the first 6 years. 2
- (ii) After 6 years the compound interest rate rises to 8% p.a. and she leaves her money in the fund for a further 4 years. 1
Find the value of her investment to the nearest dollar at the end of the 10 years.

- (b) In the diagram at right,
ABCD is a square

$$CX = CY$$



- (i) Prove, giving all reasons, that $\triangle ABX \equiv \triangle ADY$ 3
- (ii) Hence prove that $\triangle AXY$ is isosceles 1

QUESTION 6: (7 MARKS) (START ON A NEW PAGE)

- (a) Frank invests \$250 per month into a superannuation fund, which earns interest of 12% per annum compounded monthly.
- (i) What is the value of his investment at the end of 1 month? 1
- (ii) He continues to invest \$250 each month into the fund for 10 full years, the last payment being in the fund for a whole month. 3

What is the total value of his investment after the 10 years?

- (b) (i) Find the value of the derivative of the curve $y = x^3 + 3x^2 - 24x + 2$ at the point (2, -26). 2
- (ii) What is the geometric significance of this result with respect to the tangent to the curve at this point? 1

QUESTION 7: (7 MARKS) (START ON A NEW PAGE)

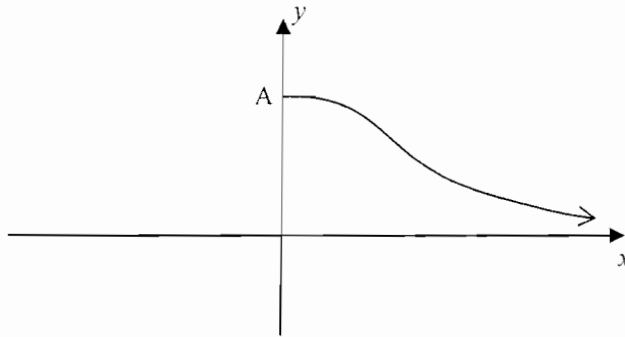
(a) Consider the graph of $y = f(x)$ where $f(x) = \frac{1}{x^2 + 4}$

(i) Prove algebraically that this is an even function

1

(ii) One half of the graph of $y = \frac{1}{x^2 + 4}$ is shown below:

2



Find the coordinates of the point A in the diagram and draw the complete sketch of the curve on your answer sheet.

(b) (i) On a set of axes, draw the curve $y = |x - 5|$ showing all important features

2

(ii) By drawing (and labelling) another line on this same set of axes, or otherwise, find all values of x for which

2

$$|x - 5| \leq x + 5$$

QUESTION 8: (7 MARKS) (START ON A NEW PAGE)

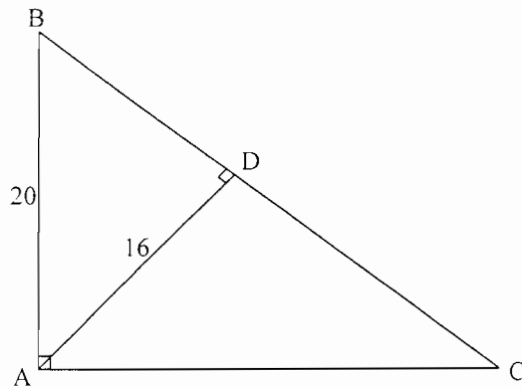
- (a) Show that the curve $y = \frac{1-x}{2-x}$ is decreasing for all values of x

2

- (b) From the vertex A of the right angled triangle ABC shown at right, AD is drawn perpendicular to BC

AB=20 cm

AD= 16 cm



- (i) Prove that $\triangle ABC$ is similar to $\triangle DAC$ giving all reasons
- (ii) Find the length of BD
- (iii) It can also be shown that $\triangle ABC$ is similar to $\triangle DBA$.
(YOU DO NOT HAVE TO PROVE THIS)

2

1

2

Using this result and parts (i) and (ii) above, find the length of DC

(END OF EXAMINATION)

2 UNIT SOLUTIONS

① (a) $\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \frac{1}{4 \times 5} = \frac{48}{60} = \frac{4}{5}$ ← ONLY THIS REQUIRED for 1 MARK.

(b) $0.83 = 0.838383 \dots$
G.P. $a = 0.83$
 $r = 0.01$

$S_{\infty} = \frac{0.83}{1 - 0.01} \leftarrow \text{① MARK}$
 $= \frac{83}{99} \leftarrow \text{① MARK}$

(c) (i) $T_1 = \frac{1}{3}$ $T_2 = \frac{2}{3}$ $T_3 = \frac{4}{3}$ ← ① MARK for all.

(ii) $T_2/T_1 = 2$ $T_3/T_2 = 2$ ← ① for showing this
 $r = 2 \leftarrow \text{① MARK for stating}$

(iii) $S_n = \frac{\frac{1}{3}(1 - 2^{n+1})}{1 - 2}$ or $\frac{\frac{1}{3}(2^{n+1} - 1)}{1}$ } ① for any of these.
 $= 1023/3$
 $= 341$

(2) (a) $dy/dx = x^3 + 2$ ← ① for each part = ②

(b) $dy/dx = \frac{1}{2}(-2x)(4 - x^2)^{-1/2}$ } ← ② but only ① if no negative
 $= \frac{-x}{\sqrt{4 - x^2}}$

(c) $dy/dx = -3(x-1)^{-2}$ } EITHER for ② but only ① for no negative.
 $= -\frac{3}{(x-1)^2}$

(3) (a) (i) A.P. $a = 4$, $d = 3$

$T_n = 4 + (n-1)3$
 $= 3n + 1 \leftarrow \text{① MARK}$

(ii) $T_{15} = 136 \leftarrow \text{① MARK}$

(iii) $3n + 1 = 421$

$n = 140 \leftarrow \text{① MARK}$

(iv) $S_{140} = \frac{140}{2}(4 + 421) = 29750$ } ① for any of these.
 $= \frac{140}{2}(8 + 139 \times 3)$

(b) $\frac{dy}{dx} = 3x^2 - 6x + 2 \leftarrow \text{① MARK}$

At $x = 2$ $m_T = 2$
 $m_N = -1/2 \leftarrow \text{① MARK}$

\therefore tangent is $y = -\frac{1}{2}(x-2)$ } ①
ie. $x + 2y - 2 = 0$

Student's Name/No:

Teacher's Name:

(4) (a) $\frac{dy}{dx} = 2x - 3$

For increasing $\frac{dy}{dx} > 0 \leftarrow \textcircled{1}$ for showing this angle.

$\therefore x > \frac{3}{2} \leftarrow \textcircled{1}$ mark

(b) G.P. $a = 1, r = 2x$

(i) $-1 < 2x < 1$
 $\therefore -\frac{1}{2} < x < \frac{1}{2}$ } $\leftarrow \textcircled{2}$ MARKS. Only $\textcircled{1}$ if only the solution $x < \frac{1}{2}$ is given
NIL for $-1 < x < 1$

(ii) $S_{\infty} = \frac{4}{3}$
 $\therefore \frac{1}{1-2x} = \frac{4}{3} \leftarrow \textcircled{1}$ MARK
 $\therefore 4 - 8x = 3$
 $x = \frac{1}{8} \leftarrow \textcircled{1}$ MARK.

(5) (a) (i) $A_1 = 20000 (1.06)^6 \leftarrow \textcircled{1}$
 $= 28,370.38 \leftarrow \textcircled{1}$
 $A_{10} = 20,000 (1.06)^6 (1.08)^4$
 or
 $28,370.38 (1.08)^4$
 $= \$38,598.00$ } $\textcircled{1}$ only for answer.

(b) (i) In $\triangle ABX$ and $\triangle ADY$
 $AB = AD$ (sides of a square)
 $BX = DY$ (since $BC = DC$, sides of square and $CX = CY$, given)
 $\angle ABX = \angle ADY$ ($= 90^\circ$, angles in a square)
 $\therefore \triangle ABX \equiv \triangle ADY$ (SAS) \leftarrow subtract $\textcircled{1}$ if RHS

(ii) $AX = AY$ (corresponding sides in congruent triangles)
 $\therefore \triangle AXY$ is isosceles (2 sides equal) $\leftarrow \textcircled{1}$

⑥ (a) (i) $A_1 = 250(1.01) \leftarrow \textcircled{1} \text{ mark for } 0.01$
 $= 252.50$

$$(iv) A_p = 250(1.01)^{120} + 250(1.01)^{119} + \dots + 250(1.01)$$

$$= 250 \left[\frac{1.01 (1.01^{120} - 1)}{1.01 - 1} \right]$$

① MARK for ↑

① for 120.

58 084.77

① for answer

(b) (i) $\frac{dy}{dx} = 3x^2 + 6x - 24 \leftarrow 1 \text{ MARK}$

$$A + (3, -26) \quad m_1 = 0 \quad \leftarrow \textcircled{1} \text{ mark}$$

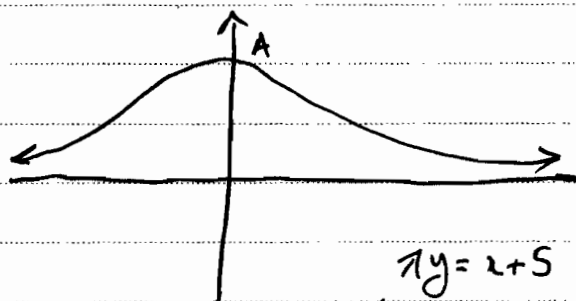
(ii) The tangent is parallel to the x-axis (1 mark)

⑦ (a) (i). $f(a) = \frac{1}{a^2+4}$

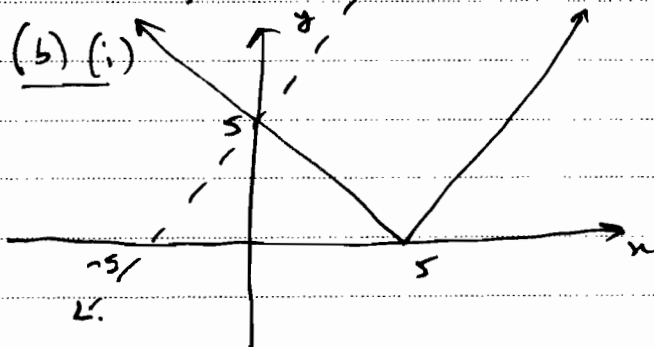
$$\begin{aligned} f(-a) &= \frac{1}{(-a)^2+4} \\ &= \frac{1}{a^2+4} \\ &= f(a) \end{aligned}$$

① mark
must show that $(-a)$ has
been substituted.

(i) A is $(0, 1/4)$ ① mark.



① for shape.



① for shape

① for the two intercepts.

(ii) ① mark for the line $y = x + 5$
① mark for $x \geq 0$ (Don't penalise for $x > 0$)

OR

$$\begin{aligned} x - 5 &\leq x + 5 \quad \text{or} \quad x - 5 \geq -x - 5 & (1) \\ -5 &\leq 5 & 2x \geq 0 \\ \text{NO SOLN} & & x \geq 0 & (1) \end{aligned}$$

(8)

(a) $y = \frac{1-x}{2-x}$

$$\frac{dy}{dx} = \frac{(2-x)(-1) - (1-x)(-1)}{(2-x)^2}$$

$$= \frac{-2+x+1-x}{(2-x)^2}$$

$$= -\frac{1}{(2-x)^2}$$

← ① mark

This is always negative for all x } ① for a statement about
∴ Always decreasing. } the negativity of y'

(b) In $\triangle ABC$ and $\triangle DAC$

$$\angle BAC = \angle ADC = 90^\circ \text{ (given)}$$

$\angle ACB$ is common.

∴ $\triangle ABC \parallel \triangle DAC$ (equiangular)

} ① each reason
= 2.

① mark

(ii)

$$BD = \sqrt{20^2 - 16^2}$$

$$= 12 \text{ cm}$$

① mark.

(iii) Since $\triangle ABC \parallel \triangle DBA$ (given)

and $\triangle ABC \parallel \triangle DAC$ (above)

∴ $\triangle DBA \parallel \triangle DAC$.

$$\therefore \frac{AD}{BD} = \frac{DC}{AD}$$

$$\therefore DC = \frac{AD^2}{BD}$$

$$= \frac{256}{12}$$

$$= 21\frac{1}{3} \text{ cm}$$

① mark