

SYDNEY TECHNICAL HIGH SCHOOL

MATHEMATICS EXTENSION 1

COMMON TEST

MAY 2003

Time Allowed: 70 minutes

Instructions:

- Show all necessary working
- Start each question on a new page
- All questions are of equal value
- Approximate marks are shown alongside each question

Q1	Q2	Q3	Q4	Q5	Q6	Total



Question 1

(a) If $\sqrt[3]{p} = 32$ and $\sqrt{q} = 243$ find $\sqrt[3]{pq}$ (2)

(b) Factorise $x^4 + 8x^2 - 9$ (2)

© Solve $\cos 2x = -\frac{\sqrt{3}}{2}$ for $0 \leq x \leq 360$ (2)

(d) Simplify $\frac{\cos(360 - \theta)^\circ}{\sin(-\theta)^\circ}$ (2)

(e) If $x - \frac{1}{x} = 2\sqrt{2}$ find the value of $x^2 + \frac{1}{x^2}$ (2)

Question 2

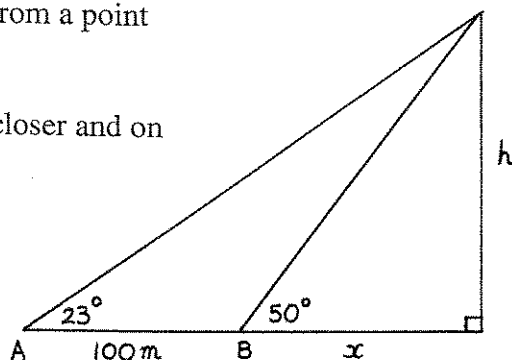
(a) (i) Factorise $2^{n+1} + 2^n$ (3)

(ii) Hence, write $\frac{2^{1001} + 2^{1000}}{3}$ as a power of 2

(b) Given that $\tan A = p$ and $180^\circ < A < 270^\circ$ (2)
find an expression for $\cos A$ in terms of p .

- © The angle of elevation of a tower top from a point A is 23° (5)

The angle of elevation from B, 100m closer and on the same horizontal plane as A is 50° .



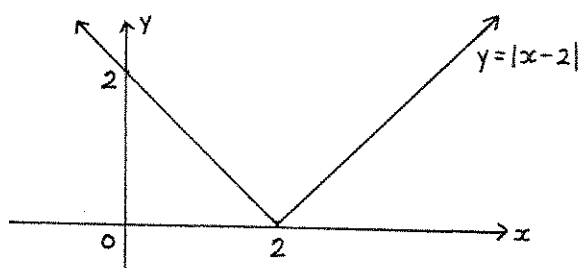
- Find an expression for $\tan 67^\circ$ and $\tan 40^\circ$ in terms of x and h .
- Hence show that $h = \frac{100}{\tan 67^\circ - \tan 40^\circ}$
- Hence find the height of the tower (correct to 2 decimal places.).

Question 3

(a) Solve $\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 3$ (2)

(b) Solve $2\sin^2 x + \cos x - 1 = 0$ for $-180^\circ \leq x \leq 180^\circ$ (4)

- © The graph of $y = |x - 2|$ is given below (4)

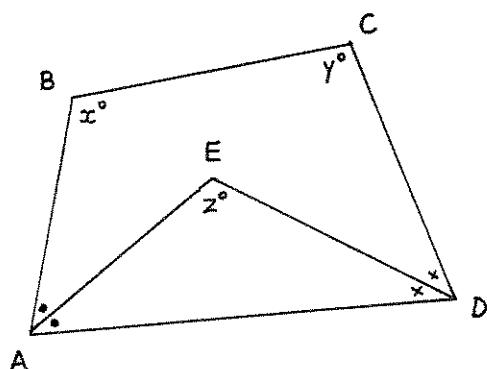


- Sketch $y = |x - 2| - 1$
- Hence, sketch $y = \left| |x - 2| - 1 \right|$ on a separate number plane
- If $\left| |x - 2| - 1 \right| = a$, where a is an integer, has 3 solutions, find the value of a .

Question 4

(a) Simplify $\frac{15^n \times 3^{n+1}}{5^{n-1}}$ (2)

(b) $\angle BAE = \angle EAD$ and $\angle CDE = \angle EDA$. (5)



i) Copy and complete: $\angle EAD + \angle EDA = \underline{\hspace{2cm}}$ ()

ii) Hence find an expression for $x + y$.

Give a reason.

(c) Prove $(1 + \tan A + \sec A)(1 + \tan A - \sec A) = 2 \tan A$ (3)

Question 5

(a) State the natural domain of $y = \frac{\sqrt{x+4}}{x}$ (2)

(b) Factorise $x^2 - y^2 + 6y - 9$ (2)

© Solve $\frac{x^2 - 4}{x} \leq 0$ (3)

(d) Solve simultaneously $\begin{cases} 2^x + 3^y = 5 \\ 2^{x+3} - 3^{y+2} = 23 \end{cases}$ (3)

Question 6

(a) Solve $\frac{1}{|x-3|} > \frac{1}{2}$ (3)

(b) Consider the function $f(x) = \frac{x}{x^2 - 1}$ (7)

i) For what values of x is $f(x)$ undefined?

ii) Show that $y = f(x)$ is an odd function

iii) What is the graphical significance of part ii.

iv) Hence, sketch the function showing important features.

Use a ruler to draw the axes.

Use about $\frac{1}{3}$ of a page.

Question 1

$$(a) p = 32^3 \quad q = 243^2$$

$$\begin{aligned} \sqrt[5]{pq} &= \sqrt[5]{32^3 \times 243^2} \\ &= \underline{\underline{72}} \end{aligned}$$

$$\begin{aligned} (b) \quad x^4 + 8x^2 - 9 \\ &= (x^2 + 9)(x^2 - 1) \\ &= \underline{\underline{(x^2 + 9)(x + 1)(x - 1)}} \end{aligned}$$

$$\begin{aligned} (c) \quad \cos 2x &= -\frac{\sqrt{3}}{2} \\ 2x &= 150, 210, 510, 570 \\ x &= \underline{\underline{75, 105, 255, 285}} \end{aligned}$$

$$\begin{aligned} (d) \quad \frac{\cos(360 - \theta)}{\sin(-\theta)} &= \frac{\cos \theta}{-\sin \theta} \\ &= \underline{\underline{-\cot \theta}} \end{aligned}$$

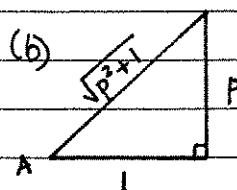
$$\begin{aligned} (e) \quad x - \frac{1}{x} &= 2\sqrt{2} \\ \left(x - \frac{1}{x}\right)^2 &= x^2 - 2 + \frac{1}{x^2} \end{aligned}$$

$$\begin{aligned} x^2 + \frac{1}{x^2} &= (2\sqrt{2})^2 + 2 \\ &= 8 + 2 \\ &= \underline{\underline{10}} \end{aligned}$$

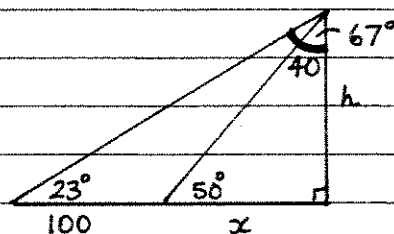
Question 2

$$(a) \text{ i. } 2^n(2+1) = \underline{\underline{3 \times 2^n}}$$

$$\begin{aligned} \text{ii. } \frac{2^{1001} + 2^{1000}}{3} &= \frac{3 \times 2^{1000}}{3} \\ &= \underline{\underline{2^{1000}}} \end{aligned}$$

$$\begin{aligned} (b) \quad \cos A &= \frac{-1}{\sqrt{p^2 + 1}} \end{aligned}$$


(c)



$$\text{i. } \tan 67^\circ = \frac{100 + x}{h}$$

$$\tan 40^\circ = \frac{x}{h}$$

$$\begin{aligned} \text{ii. } h \tan 67^\circ - 100 &= x \\ h \tan 40^\circ &= x \end{aligned}$$

$$\begin{aligned} h \tan 40^\circ &= h \tan 67^\circ - 100 \\ 100 &= h \tan 67^\circ - h \tan 40^\circ \\ 100 &= h(\tan 67^\circ - \tan 40^\circ) \end{aligned}$$

$$\therefore h = \frac{100}{\tan 67^\circ - \tan 40^\circ}$$

$$\text{iii. } h = \underline{\underline{65.93}}$$

Question 3

$$(a) \frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 3$$

$$\sqrt{x+1} + \sqrt{x-1} = 3\sqrt{x+1} - 3\sqrt{x-1}$$

$$4\sqrt{x-1} = 2\sqrt{x+1}$$

$$2\sqrt{x-1} = \sqrt{x+1}$$

$$4(x-1) = (x+1)$$

$$4x - 4 = x + 1$$

$$3x = 5$$

$$\therefore x = \underline{\underline{\frac{5}{3}}}$$

$$(b) \quad 2 \sin^2 x + \cos x - 1 = 0$$

$$2(1 - \cos^2 x) + \cos x - 1 = 0$$

$$2 - 2\cos^2 x + \cos x - 1 = 0$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$(2\cos x + 1)(\cos x - 1) = 0$$

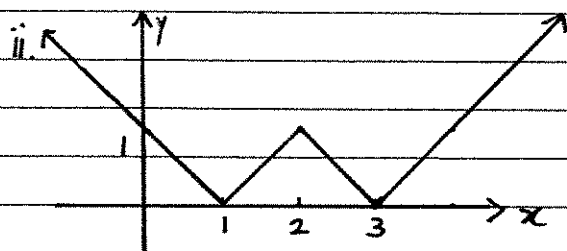
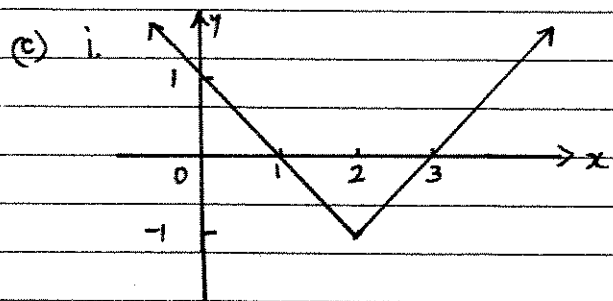
$$\cos x = -\frac{1}{2} \quad \cos x = 1$$

for $0 \leq x \leq 360$

$x = 120, 240$ and $0, 360$

\therefore for $-180 \leq x \leq 180$

$x = 120, -120, 0$



iii. $a = 1$

Question 4

(a) $\frac{15^n \times 3^{n+1}}{5^{n-1}} = \frac{3^n \times 5^n \times 3^{n+1}}{5^{n-1}}$
 $= \frac{3^{2n+1} \times 5}{5^{n-1}}$

(b) i. $180 - z$ (angle sum of Δ)

ii. $2(\angle EAD + \angle EDA) + x + y = 360^\circ$
 (angle sum of quad)

$2(180 - z) + x + y = 360$
 $360 - 2z + x + y = 360$
 $x + y = 2z$

(c) LHS = $(1 + \tan A + \sec A)(1 + \tan A - \sec A)$
 $= (1 + \tan A)^2 - \sec^2 A$
 $= 1 + 2 \tan A + \tan^2 A - \sec^2 A$
 $= 2 \tan A + \sec^2 A - \sec^2 A$
 $= 2 \tan A$
 $= \text{RHS}$

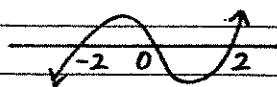
Question 5

(a) $x \geq -4, x \neq 0$

(b) $x^2 - (y^2 - 6y + 9)$
 $= x^2 - (y - 3)^2$
 $= [x + (y - 3)][x - (y - 3)]$
 $= (x + y - 3)(x - y + 3)$

(c) $x^2 \times \frac{x^2 - 4}{x} \leq 0 \times x^2$

$x(x + 2)(x - 2) \leq 0$



$\therefore x \leq -2, 0 < x \leq 2$

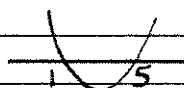
(d) $2^x + 3^y = 5$
 $2^x = 5 - 3^y$
 $2^x \cdot 2^3 - 3^y \cdot 3^2 = 23$
 $8(5 - 3^y) - 9 \cdot 3^y = 23$
 $40 - 8 \cdot 3^y - 9 \cdot 3^y = 23$
 $17 = 17 \cdot 3^y$
 $1 = 3^y$

$y = 0$
 $2^x + 3^0 = 5$
 $2^x = 4$
 $x = 2$

$\therefore x = 2$ and $y = 0$

Question 6

(a) $\frac{1}{|x-3|} > \frac{1}{2}$
 $(x-3)^2 \times \frac{1}{(x-3)^2} > \frac{1}{4} \times (x-3)^2$
 $4 > (x-3)^2$
 $(x-3)^2 - 4 < 0$
 $(x-3-2)(x-3+2) < 0$
 $(x-5)(x-1) < 0$



$\therefore 1 < x < 5, x \neq 3$

(b) $f(x) = \frac{x}{x^2-1}$

i. $x = \pm 1$

ii. $f(a) = \frac{a}{a^2-1}$

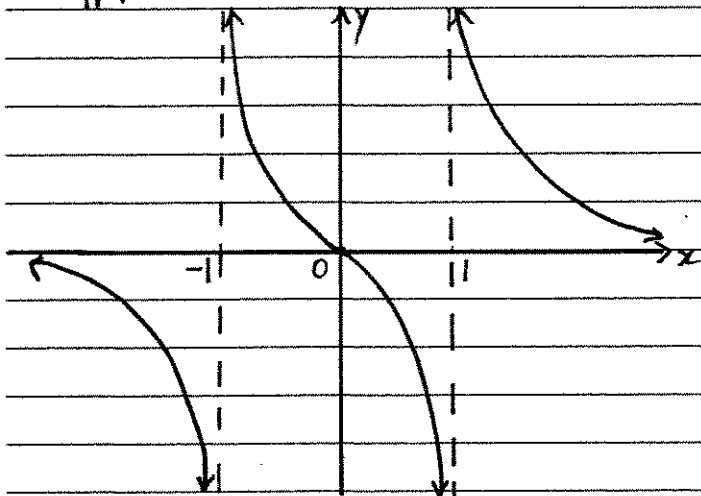
$$\begin{aligned} f(-a) &= \frac{-a}{(-a)^2-1} \\ &= \frac{-a}{a^2-1} \end{aligned}$$

\therefore function is odd since

$f(-a) = -f(a)$

iii. point symmetry about the origin

iv.



10/10/10

