SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11
PRELIMINARY HIGHER SCHOOL CERTIFICATE
ASSESSMENT TASK 3

SEPTEMBER 2015

Mathematics Extension 1

General Instuctions

- Working time - 90 minutes
- Write using black or blue pen
- Approved calculators may be used
- In questions 6 to 11, show relevant mathematical reasoning and/or calculations
- Start each question in section 2 on a new page
- Full marks may not be awarded for careless or badly arranged work

Total marks - 66

Section 1 - 5 marks

Attempt Questions 1 – 5.
Allow about 8 minutes for this section.

Section 2 - 61 marks

Attempt Questions 6 – 11.
Allow about 82 minutes for this section.

Name : ____________________________

Teacher : ____________________________

Section 1

5 marks
Attempt Questions 1 – 5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet in your answer booklet for Questions 1 – 5.
Do not remove the multiple-choice answer sheet from your answer booklet.

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1. What is the remainder when $P(x) = 5x^3 - 17x^2 - x + 11$ is divided by $x - 2$?

   A) -147

   B) -95

   C) -19

   D) 11

2. How many asymptotes does the graph of $y = \frac{x^2}{3x(x+1)}$ have ?

   A) 3

   B) 2

   C) 1

   D) 0

3. A function is represented by the parametric equations

   $$x = 2t + 1 \text{ and } y = t - 2 .$$

   Which of the following is the Cartesian equation of this function ?

   A) $x - 2y + 3 = 0$

   B) $x - 2y - 3 = 0$

   C) $x + 2y + 5 = 0$

   D) $x - 2y - 5 = 0$

4. What is the focus of the parabola $(x-3)^2 = -8y$?

A) $(3, -2)$

B) $(3, 2)$

C) $(0, -2)$

D) $(-2, 3)$

5. $\sin 2x$ equals

A) $\frac{1-\tan^2 x}{1+\tan^2 x}$

B) $\frac{2\tan x}{1+\tan^2 x}$

C) $\frac{2\tan x}{1-\tan^2 x}$

D) $\frac{1+\tan^2 x}{1-\tan^2 x}$

SECTION 2 BEGINS ON THE NEXT PAGE

Section 2

61 marks
Attempt Questions 6 – 11
Allow about 82 minutes for this section

Answer each question in your answer booklet. Start each question on a new page.

In Questions 6 – 11, your response should include relevant mathematical reasoning and/or calculations.

Question 6 (11 marks)

a) Find the coordinates of the point that divides the interval from $A(1,6)$ to $B(-8,2)$ internally in the ratio 2:1. **2**

b) Draw a neat sketch of the polynomial $y = (2-x)^2(6-x)$ clearly labelling all intercepts. **2**

c) Express $\cos A \sin 2A + \cos 2A \sin A$ in terms of $3A$ **1**

d) Differentiate $(1+2\sqrt{x})^5$ **2**

e) Evaluate $\lim_{x\to\infty} \frac{2x^3-x^2+x+3}{x^3-1}$ **1**

f) Find the exact value or values of m if the acute angle between the lines $y = 2x$ and $y = mx + 5$ is 60 degrees. **3**

Question 7 (10 marks) Start a new page.

a) Factorise $x^3 + 125$ 1

b) Find the vertex and focus of the parabola $y = x^2 + 4x + 3$. 2

c) Given $P(x) = 2x^3 + 5x^2 - 11x + 4$

i) Evaluate $P(1)$ 1

ii) Hence, or otherwise, fully factorise $P(x)$. 2

d) Solve $4\cos\theta = \sec\theta$ for $0 \le \theta \le 2\pi$, giving your answer in radians. 2

e) Find the possible values of $\sin\theta$ if $\cos 2\theta = \frac{3}{25}$. 2

Question 8 (10 marks) Start a new page.

a) If α , β and γ are the roots of the equation $2x^3 - 6x^2 + 4x - 1 = 0$

find the value of

i) $\alpha + \beta + \gamma$ 1

ii) $\alpha^2 + \beta^2 + \gamma^2$ 2

b) Find the equation of an odd polynomial of degree 3 which passes through the points $(2, 0)$ and $(3, 30)$. 2

c) Simplify $\cot\theta - 2\cot 2\theta$ 2

d) For what values of x is the gradient of $y = x - \frac{4}{x}$ greater than 5 ? 3

Question 9 (10 marks) Start a new page.

a) The point $P(x, y)$ moves so that its distance from the point $A(1,4)$ is always double its distance from the point $B(1,1)$.

i) Show that the locus of $P(x, y)$ is a circle. 2

ii) Find the centre and radius of this circle. 2

b) $P(4p, 2p^2)$ and $Q(4q, 2q^2)$ are two points on the parabola $x^2 = 8y$.

i) Show that the equation of the chord PQ is given by 2

$$y - \frac{1}{2}(p + q)x + 2pq = 0.$$

ii) The chord PQ passes through the point $(0, -2)$. 1

Show that $pq = 1$

iii) If S is the focus of the parabola, 3

and SP and SQ are the distances from S to P and Q respectively,

show that $\frac{1}{\text{SP}} + \frac{1}{\text{SQ}} = \frac{1}{2}$

Question 10 (10 marks) Start a new page.

a) For what value, or values of x is the function $y = |x+3|$ not differentiable ? 1

b) Find the gradient of the curve $y = \frac{(2x-1)(x-3)}{x-5}$ at the point $(6, 33)$. 2

c) Solve $3\sin 2x = 5\sin x$ for $0° \leq x \leq 360°$. (nearest degree) 4

d) Solve the equation $5x^3 - 63x^2 + 136x - 60 = 0$ 3

given that the product of two of its roots is 6 .

Question 11 (10 marks) Start a new page.

a) Use the t results, where $t = \tan\frac{\theta}{2}$, to solve 3

$8\cos\theta - \sin\theta = 4$ for $0° \leq \theta \leq 360°$ correct to the nearest degree.

b) $P(2ap, ap^2)$ and $Q(2aq, aq^2)$ are points on the parabola $x^2 = 4ay$.

i) Show that the equation of the tangent to the parabola at P 2

is given by $y = px - ap^2$.

ii) Find the point of intersection of the tangent at P and the tangent at Q. 2

iii) If O is the vertex of the parabola and OP is perpendicular to OQ, 1

show that $pq = -4$.

iv) If $OPRQ$ is a rectangle then R has coordinates $(2a(p+q),\ a(p^2+q^2))$. 2

<u>Do not prove this</u>.

Find the locus of R.

End of Paper.

SOLUTIONS EXT 1 SEPT 2015

1. C

2. B

3. D

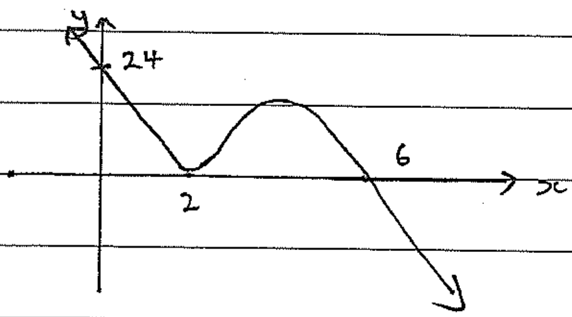
4. A

5. B

6 a) $\left(\frac{1 \times 1 + 2 \times -8}{3}, \frac{1 \times 6 + 2 \times 2}{3} \right)$

$= \left(-5, \frac{10}{3}\right)$

b)



c) Sin 3A

d) $\frac{5}{\sqrt{x}}\left(1 + 2\sqrt{x}\right)^4$

e) 2

f) $\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$

$\therefore \quad \sqrt{3} = \left| \frac{m-2}{1+2m} \right|$

$\sqrt{3}\,|1 + 2m| = |m - 2|$

$\therefore \sqrt{3} + 2\sqrt{3}m = m - 2$

$(2\sqrt{3} - 1)m = -2 - \sqrt{3}$

$m = \frac{2 + \sqrt{3}}{1 - 2\sqrt{3}}$

or $\quad \sqrt{3} + 2\sqrt{3}m = -(m-2)$

$(2\sqrt{3} + 1)m = 2 - \sqrt{3}$

$m = \frac{2 - \sqrt{3}}{2\sqrt{3} + 1}$

$\therefore \quad m = \frac{2 + \sqrt{3}}{1 - 2\sqrt{3}}$ or $\frac{2 - \sqrt{3}}{2\sqrt{3} + 1}$

7 a) $(x + 5)(x^2 - 5x + 25)$

b) $y = x^2 + 4x + 3$

$y + 1 = (x + 2)^2$

$\therefore$ Vertex $(-2, -1)$

focus $\left(-2, -\frac{3}{4}\right)$

c) i) $P(1) = 0$

ii) $\therefore x - 1$ is a factor

$\therefore P(x) = (x-1)(2x^2 + 7x - 4)$

$= (x-1)(2x-1)(x+4)$

d) $4\cos\theta = \frac{1}{\cos\theta}$

$\cos^2\theta = \frac{1}{4}$

$\cos\theta = \pm\frac{1}{2}$

$\theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

e) $\cos 2\theta = \frac{3}{25}$

$1 - 2\sin^2\theta = \frac{3}{25}$

$\sin^2\theta = \frac{11}{25}$

$\sin\theta = \pm \frac{\sqrt{11}}{5}$

8 a) i) $\alpha + \beta + \gamma = 3$

ii) $\alpha^2 + \beta^2 + \gamma^2$

$= (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= 3^2 - 2(2)$

$= 5$

b) $P(x) = ax(x-2)(x+2)$

sub $(3, 30)$

$30 = 15a$

$a = 2$

$\therefore P(x) = 2x(x-2)(x+2)$

c) $\cot\theta - 2\cot 2\theta$

$= \frac{1}{\tan\theta} - \frac{2(1-\tan^2\theta)}{2\tan\theta}$

$= \frac{1 - (1-\tan^2\theta)}{\tan\theta}$

$= \tan\theta.$

d) $y = x - 4x^{-1}, \quad x \neq 0$

$y' = 1 + 4x^{-2}$

$\therefore 1 + 4x^{-2} > 5$

$\frac{4}{x^2} > 4$

$x^2 < 1$

$-1 < x < 1, \ x \neq 0$

9 a) i) $PA = 2 \times PB$

$\sqrt{(x-1)^2 + (y-4)^2} = 2\sqrt{(x-1)^2 + (y-1)^2}$

$(x-1)^2 + (y-4)^2 = 4\left[(x-1)^2 + (y-1)^2\right]$

$3x^2 - 6x + 3y^2 - 9 = 0$

$x^2 - 2x + y^2 - 3 = 0$

$(x-1)^2 + y^2 = 4$

which is a circle

ii) centre $(1, 0)$

radius 2 units

b) i) $m = \frac{2p^2 - 2q^2}{4p - 4q}$

$= \frac{p+q}{2}$

using $y - y_1 = m(x - x_1)$

$y - 2p^2 = \frac{p+q}{2}(x - 4p)$

$y - 2p^2 = \frac{1}{2}(p+q)x - 2p^2 - 2pq$

$y - \frac{1}{2}(p+q)x + 2pq = 0$

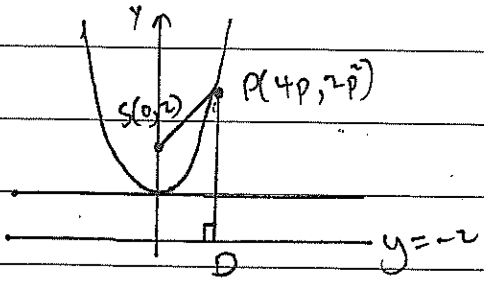
ii) sub $(0,-2)$

$-2-\frac{1}{2}(p+q)0+2pq=0$

$2pq=2$

$pq=1$

iii)



$SP = PD$ (by definition)

$= 2p^2+2$

$\therefore SQ = 2q^2+2$

$\therefore \frac{1}{SP}+\frac{1}{SQ}$

$= \frac{1}{2p^2+2}+\frac{1}{2q^2+2}$

$= \frac{1}{2}\left[\frac{q^2+1+p^2+1}{(p^2+1)(q^2+1)}\right]$

$= \frac{1}{2}\left[\frac{p^2+q^2+2}{p^2q^2+p^2+q^2+1}\right]$

but $pq=1$

$= \frac{1}{2}\left[\frac{p^2+q^2+2}{p^2+q^2+2}\right]$

$= \frac{1}{2}$

10 a) $x=-3$

b) $y' = \frac{(x-5)(4x-7)-(2x^2-7x+3)}{(x-5)^2}$

sub $x=6$

$m = \frac{(1)(17)-(33)}{1}$

$= -16$

c) $3\sin 2x = 5\sin x$

$6\sin x\cos x - 5\sin x = 0$

$\sin x(6\cos x-5)=0$

$\sin x = 0, \quad \cos x = \frac{5}{6}$

$x = 0°, 180°, 360°, 34°, 326°$

d) let α, β, γ be roots where $\beta\gamma=6$

$\therefore \alpha\beta\gamma = \frac{60}{5}$ but $\beta\gamma=6$

$\therefore \alpha = 2$

$\therefore x-2$ is a factor.

$5x^3-63x^2+136x-60$

$=(x-2)(5x^2-53x+30)$

$=(x-2)(5x-3)(x-10)$

$\therefore x = 2, 10, \frac{3}{5}$

alternatively may use the sum of the roots 1 and 2 at a time.

11 a) $8\cos\theta - \sin\theta = 4$

$8\left(\frac{1-t^2}{1+t^2}\right) - \frac{2t}{1+t^2} = 4$

$8 - 8t^2 - 2t = 4 + 4t^2$

$12t^2 + 2t - 4 = 0$

$6t^2 + t - 2 = 0$

$(3t+2)(2t-1) = 0$

$t = -\frac{2}{3}, \frac{1}{2}$

$\therefore \tan\frac{\theta}{2} = -\frac{2}{3}, \frac{1}{2}$

$\frac{\theta}{2} = 146^\circ 18', 26^\circ 34'$

$\theta = 53^\circ, 292^\circ$

b) i) $y = \frac{x^2}{4a}$

$y' = \frac{x}{2a}$

sub $x = 2ap$

$m_T = \frac{2ap}{2a}$

$= p$

using $y - y_1 = m(x - x_1)$

$y - ap^2 = p(x - 2ap)$

$y - ap^2 = px - 2ap^2$

$y = px - ap^2$

ii) tangent at Q: $y = qx - aq^2$

tangent at P: $y = px - ap^2$

Solve simultaneously

$px - ap^2 = qx - aq^2$

$(p-q)x = a(p-q)(p+q)$

$x = a(p+q)$

$\therefore y = pa(p+q) - ap^2$

$= apq$

$\therefore$ pt. of intersection $\left(a(p+q), apq\right)$

iii) $m_{OP} \times m_{OQ} = -1$

$\frac{ap^2}{2ap} \times \frac{aq^2}{2aq} = -1$

$\frac{pq}{4} = -1$

$pq = -4$

iv) locus: $x = 2a(p+q)$

$y = a(p^2 + q^2)$

$\therefore \frac{y}{a} = p^2 + q^2$

$\frac{y}{a} = (p+q)^2 - 2pq$

$\therefore \frac{y}{a} = \left(\frac{x}{2a}\right)^2 + 8$ $(pq = -4)$

$\therefore y = \frac{1}{4a}x^2 + 8a$