SYDNEY TECHNICAL HIGH SCHOOL

(Est. 1911)

Year 11 Assessment Task 1

May 2007

Extension 1 Mathematics

Time allowed: 70 mins.

Instructions:

- Write neatly and do not cramp your work. Use a ruler to draw straight lines in diagrams.
- Marks may not be awarded for careless or badly arranged work.
- **Staple these questions to the front of your answers.**
- All necessary working must be shown.
- Indicated marks are for guidance and may be changed slightly during the marking process.

Q1	Q2	Q3	Q4	Q5	TOTAL
/11	/11	/12	/12	/12	/58

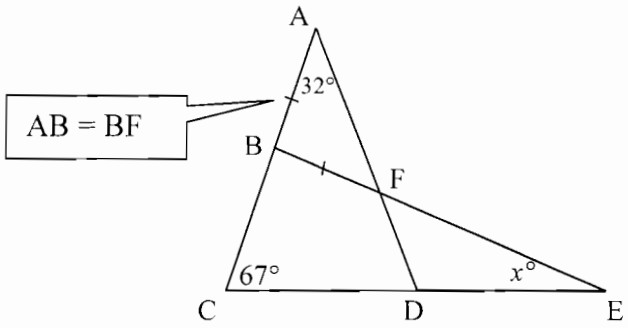
Question 1.

a) Factorise fully: $4a^2 - 9b^2 - 4a - 6b$ 2

b) Simplify $\frac{|x|}{x}$ $(x \neq 0)$, stating the values of x for which your answer applies. 2

c) Test the function $f(x) = \frac{x}{2+x^2}$ and state whether it is *odd*, *even* or *neither*. 1

d)



From the following solution, copy the lines i), ii) and iii) onto your page and fill in the missing parts:

To find x:

$\angle BAF = 32°$

i) $\therefore$ _____ $= 32°$ (_____) 1

ii) $\therefore$ _____ $= 64°$ (_____) 1

iii) Now $64° +$ ___ + ___ $= 180°$ (_____) 1

$\therefore \quad x = 49$

e) Solve $\frac{4}{x-3} - \frac{6}{x+1} = 1$ 3

Question 2. (Begin a new page)

a) Simplify $\dfrac{\dfrac{1}{a-1}+\dfrac{1}{a}}{\dfrac{1}{a-1}-\dfrac{1}{a}}$ 1

b) i) Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)^2$ 1

ii) Hence or otherwise find values for x and y such that 2

$$\sqrt{7+2\sqrt{10}}=\sqrt{x}+\sqrt{y}$$

c) i) Sketch the function $y=\sqrt{4-x}$ showing clearly any x intercepts. 1
(Use a ruler to draw the axes)

ii) Shade in the region $y \geq \sqrt{4-x}$ 2

d) Solve for x: $\dfrac{3x+1}{x-3} \geq 2$ 4

Question 3. (Begin a new page)

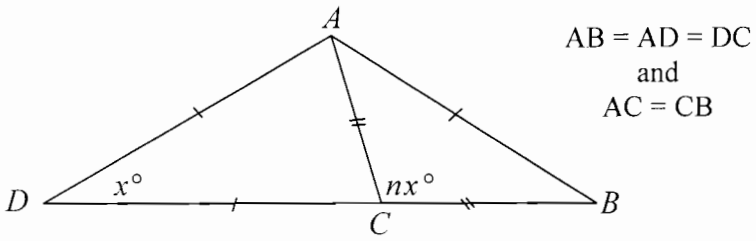
a) Solve $|x+2| = |3x-6|$ 2

b) Simplify fully:

i) $\dfrac{\tan A}{\sec A}$ 2

ii) $\cot A(\sec^2 A - 1)$ 2

c) Copy the following diagram onto your writing page.



AB = AD = DC
and
AC = CB

i) $\angle ACB$ is a multiple of x. Find the value of n. 1

ii) Determine any values of x which make this configuration of triangles possible. 2

d) Solve: $2\sin^2 2x = 1$ for $0° \le x \le \cancel{360°}$ 180 3

Question 4. (Begin a new page)

a) If $\sin x = \frac{3}{5}$ and $90° \le x \le 270°$, find the value of $\tan x$. 2

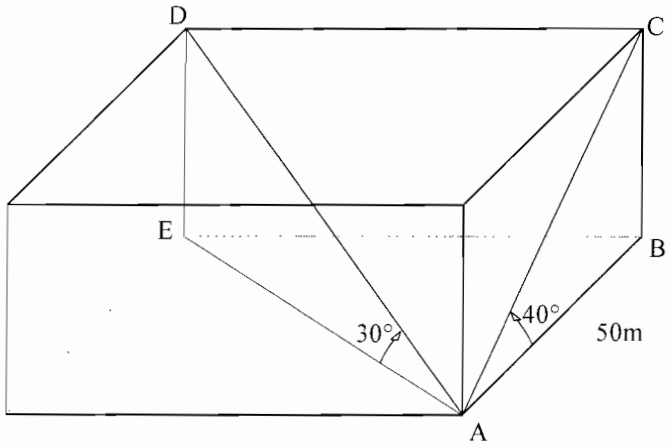
b) Find the sum of the interior angles of a regular polygon if each interior angle is 150°. 2

d) i) Sketch $y = |2x - 4|$ showing clearly the x and y intercepts. 1

(*Use a ruler to draw the axes*).

ii) Use your sketch to help solve $|2x - 4| < x + 4$. 3

d) In a rectangular prism the face diagonal AC is inclined 40° to the base while the internal diagonal AD is inclined 30° to the base. The length of AB is 50m.



i) Use trigonometry to find an *expression* for the height CB. 1

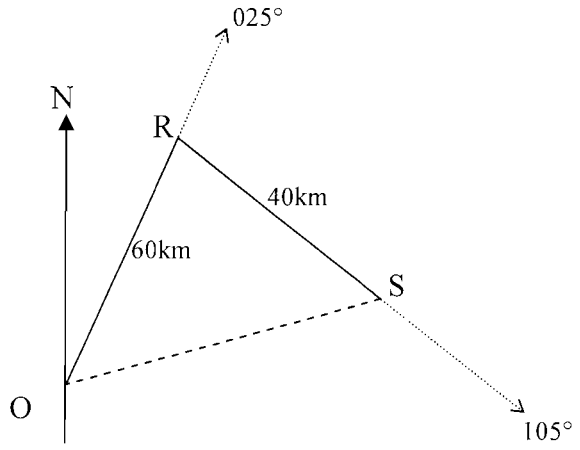
ii) Use the result of part i) to find an *expression* for the length of EA. 1

iii) Hence find the length of EB to the nearest metre. 2

Test continues over page. . .

Question 5. (Begin a new page)

a) A ship sails for 60 kilometres from O to R on a bearing of 025°. It then changes course and sails for 40 kilometres to S on a bearing of 105°.



i) Calculate the ship's distance from O to the nearest kilometre. 3

ii) Find the size of $\angle ROS$ to the nearest degree. 2

iii) Hence find the bearing of the ship from O (to the nearest degree). 1

b) i) Sketch the graph of $y = x^2 - 1$ *(Use a ruler to draw the axes)* 1

ii) On the same diagram sketch the graph of $y = \dfrac{1}{x^2 - 1}$ showing clearly any intercepts and asymptotes. 2

iii) The two graphs intersect at (0, -1) and at two other points. Find the other two points of intersection. 3

End of Test

Year 11 Assessment Task No 1 May 2007 Suggested solutions and Marking Scheme

Question 1

a) $4a^2 - 9b^2 - 4a - 6b = (2a - 3b)(2a + 3b) - 2(2a + 3b)$
$= (2a + 3b)[(2a - 3b) - 2]$
$= (2a + 3b)(2a - 3b - 2)$

2 marks
allow 1 for diff. of 2 squares

b) $\frac{|x|}{x} = 1$ for $x > 0$,
$= -1$ for $x < 0$

1 mark for each

c) $f(-x) = \frac{-x}{2 + (-x)^2}$
$= -\frac{x}{2 + x^2}$
$= -f(x)$
$\therefore$ function is odd

1 if odd (even if test not shown)

d) i) $\therefore$ $\angle$AFB = 32° (angles opposite equal sides *or similar*) 1

ii) $\therefore$ $\angle$CBE (*or equivalent*) = 64° (exterior angle of triangle) 1

iii) Now 64° + 67° + x° = 180° (angle sum of triangle) 1

e) $\frac{4}{x-3} - \frac{6}{x+1} = 1$

3 marks

$4(x+1) - 6(x-3) = (x-3)(x+1)$ 1

$-2x + 22 = x^2 - 2x - 3$ 1

$x^2 - 25 = 0$

Therefore $x = \pm 5$ 1

Year 11 Assessment Task No 1 May 2007 Suggested solutions and Marking Scheme

a)
$$\frac{\frac{1}{a-1}+\frac{1}{a}}{\frac{1}{a-1}-\frac{1}{a}} = \frac{\left(\frac{1}{a-1}+\frac{1}{a}\right)\times a(a-1)}{\left(\frac{1}{a-1}-\frac{1}{a}\right)\times a(a-1)}$$
$$= \frac{a+a-1}{a-(a-1)}$$
$$= \frac{2a-1}{1}$$
$$= 2a-1$$

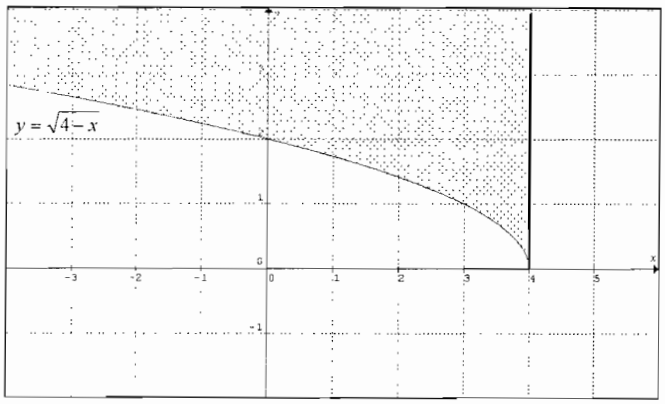
1 mark (*working need not be shown*)

b) i) $\left(\sqrt{x}+\sqrt{y}\right)^2 = x+y+2\sqrt{xy}$ 1 mark

ii)
$$\sqrt{7+2\sqrt{10}} = \sqrt{x}+\sqrt{y}$$
$$\therefore 7+2\sqrt{10} = x+y+2\sqrt{xy}$$
$$\therefore x+y=7 \text{ and } xy=10$$
$$\therefore x = 2,\ 5$$
$$y = 5,\ 2$$

2 marks,
only 1 set of values need be shown

c)



1 for correct graph

1 for shading above

1 for correct right boundary

d) $\frac{3x+1}{x-3} \geq 2$ Critical values at $x = 3$ ($x \neq 3$) and when $3x + 1 = 2x - 6$ i.e. $x = -7$ 2 marks

When $x < -7$, $\frac{3x+1}{x-3} > 2$ ✓

When $-7 < x < 3$, $\frac{3x+1}{x-3} < 2$ ×

When $x > 3$, $\frac{3x+1}{x-3} > 2$ ✓

1 mark

Correct answer – 4 marks total - suggestions shown.

Allow 1 for excluding 3 from the solution

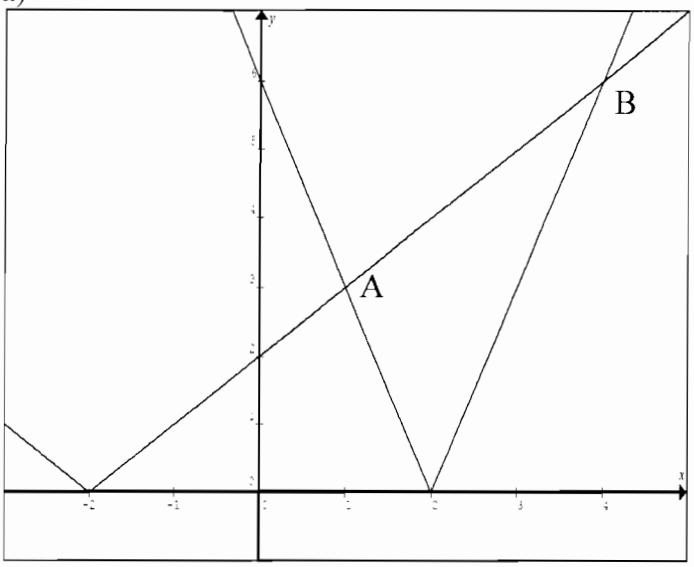
Suggest in general, 1 mark off for each error.

$\therefore x < -7, \quad x > 3$ 1 mark

Marking rubric here should be

Year 11 Assessment Task No 1 May 2007 Suggested solutions and Marking Scheme

a)



Solve $|x+2| = |3x-6|$

Solutions occur when

$x+2 = 6-3x$ A

and

$x+2 = 3x-6$ B

Which give $x = 1$

and $x = 4$

1 for each correct solution

b)

i) $\dfrac{\tan A}{\sec A} = \dfrac{\sin A}{\cos A} \times \cos A$

$= \sin A$

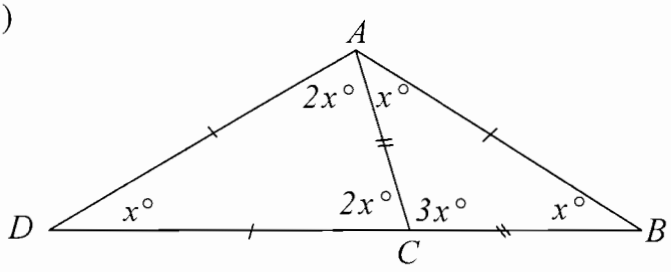
2 marks
Allow 1 for substitution

ii) $\cot A(\sec^2 A - 1) = \dfrac{1}{\tan A} \times \tan^2 A$

$= \tan A$

2 marks
Allow 1 for substitution

c)



i) $n = 3$ 1 mark

ii) $5x = 180$

$\therefore x = 36$ 2 marks
Allow 1 for correct equation

d) Solve: $2\sin^2 2x = 1$ for $0° \le x \le 180°$

$\sin^2 2x = \frac{1}{2}$

$\sin 2x = \pm\dfrac{1}{\sqrt{2}}$

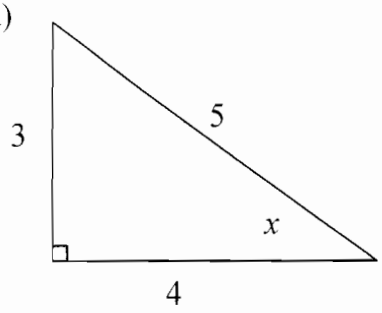
$2x = 45°, 135°, 225°, 315°$

$x = 22\frac{1}{2}°, 67\frac{1}{2}°, 112\frac{1}{2}°, 157\frac{1}{2}°$

3 marks
(1 mark each step)

Year 11 Assessment Task No 1 May 2007 Suggested solutions and Marking Scheme

a)



$\sin x = \frac{3}{5}$ and $90° \le x \le 270°$,

x must be in the second quadrant

$\therefore \tan x = -\frac{3}{4}$

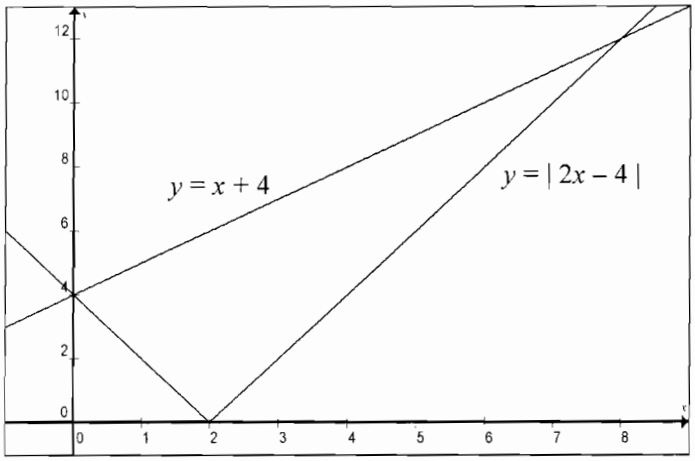
1 for ¾
1 for correct sign

b) Exterior angle = 30° $\therefore$ polygon has 12 sides — 1 mark

Interior sum = 12 x 150°

= 1800° — 1 mark

c)



1 mark for $y = |2x - 4|$

Points of intersection occur when
$x = 0$ and when $x + 4 = 2x\text{-}4$
i.e. $x = 0$ and $x = 8$ — 2 marks
$\therefore |2x - 4| \le x + 4$ when $0 \le x \le 8$ — 1 mark

d)

i) CB = 50 tan 40°. — 1 mark

ii)
$$\frac{EA}{DE} = \tan 60°$$
$$\therefore EA = DE \tan 60°$$
$$= CB \tan 60°$$
$$= 50 \tan 40° \tan 60°$$
1 mark

iii)
$$EB = \sqrt{EA^2 - AB^2}$$
$$= \sqrt{2500 \tan^2 40° \tan^2 60° - 2500}$$
2 marks
Allow 1 for correct

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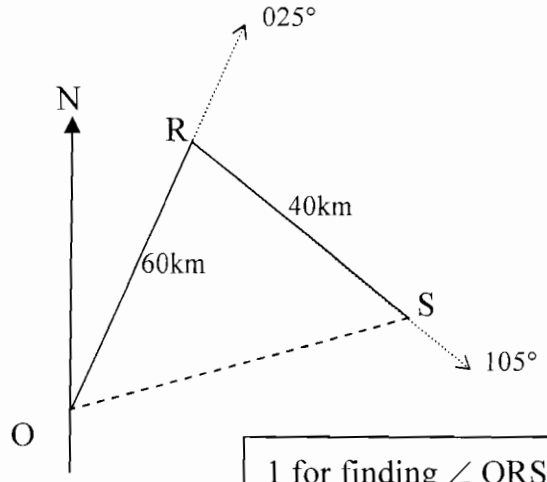
a)



1 for finding $\angle$ ORS and 1 for cosine formula

i) $OS^2 = 60^2 + 40^2 - 2.60.40.\cos 100°$ 2 marks

$\therefore OS = 78\,\text{km}$ 1 mark

ii) $\dfrac{\sin \angle ROS}{40} = \dfrac{\sin 100°}{78}$

$\angle ROS = \sin^{-1}\left(\dfrac{40\sin 100°}{78}\right)$

$\approx 30°$

OR

$\cos \angle ROS = \dfrac{78^2 + 60^2 - 40^2}{2.78.60}$

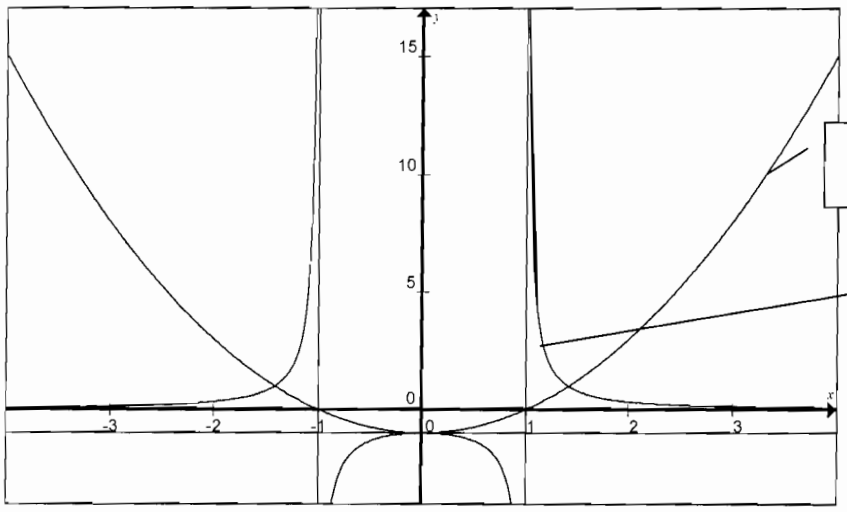
$\therefore \angle ROS = \cos^{-1}\left(\dfrac{78^2 + 60^2 - 40^2}{2.78.60}\right)$

$\therefore \angle ROS \approx 30°$

3 marks

iii) Bearing of the ship from O is 055°. 1 mark

b) i)



$y = x^2 - 1$ 1 mark

ii) $y = \dfrac{1}{x^2 - 1}$

1 for graph

1 for asymptotes

iii) $\dfrac{1}{x^2 - 1} = x^2 - 1$

$\therefore \left(x^2 - 1\right)^2 = 1$

i.e. $x^2 - 1 = \pm 1$

$\therefore \quad x^2 = 0 \text{ or } 2$