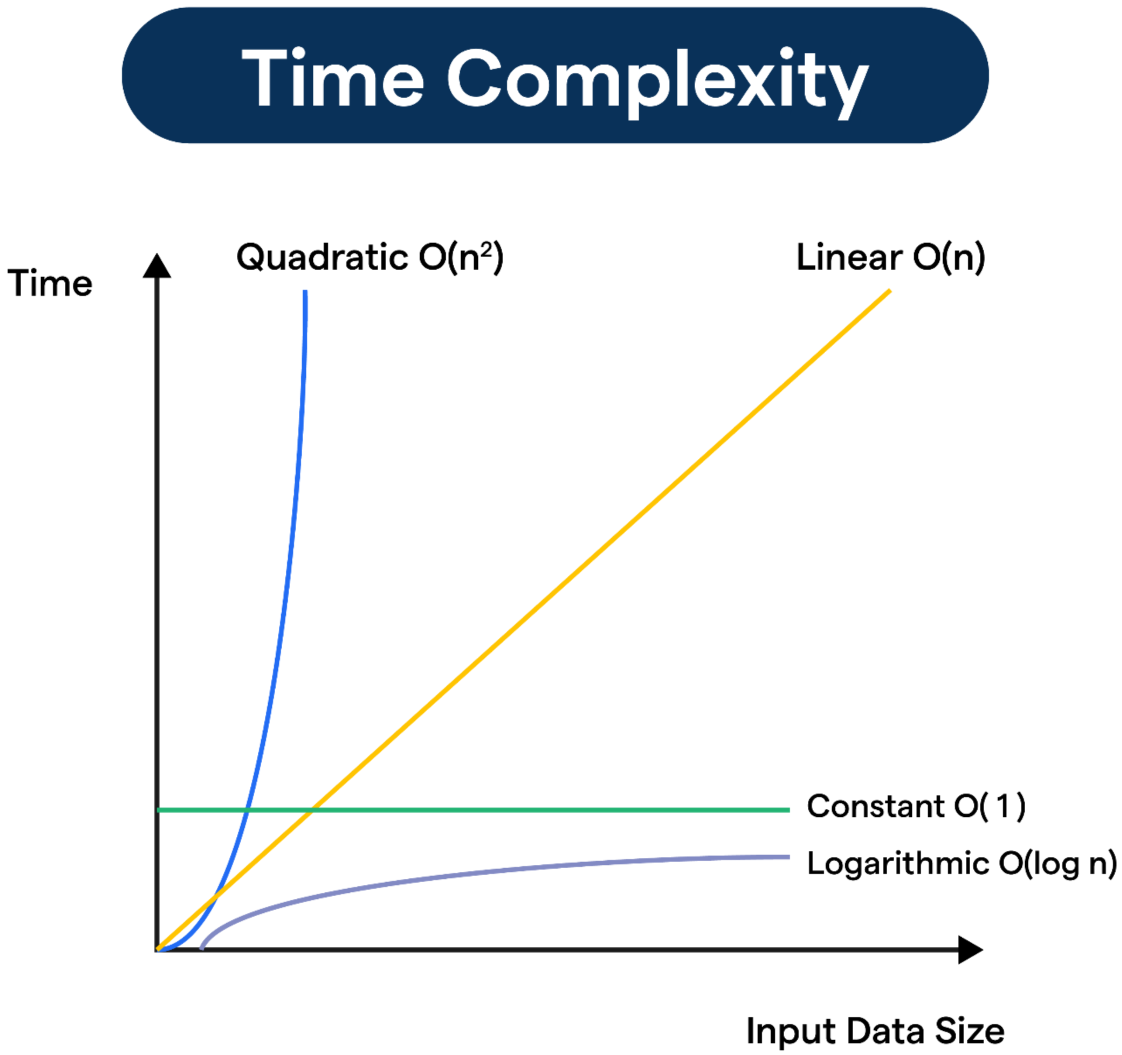
Time Complexity



💻 **When a single problem has multiple solutions, we need to analyze the algorithms**

- Algorithm analysis helps us to determine the **efficiency** of different solutions **in terms of time** and space required.

- The goal is to identify the most **optimal solution** for a given problem.

Types of Algorithm Analysis:

1. **Aposteriori Analysis:**
    - This is the **practical analysis** of an algorithm.
    - Dependent on factors like **compiler optimization**, **processor speed**, and **data characteristics**.

Example:

- **Measuring runtime using tools like a stopwatch**.

2. **Apriori Analysis:**
    - This is the **theoretical analysis** of an algorithm. Focuses on **mathematical estimations**. Independent of system hardware and software.
    - Performed **before implementing** the algorithm.
    - **Evaluates the algorithm based on input size and computational steps without executing it**.

Example:

- Count how many times a loop runs.
- Count the number of comparisons or assignments.

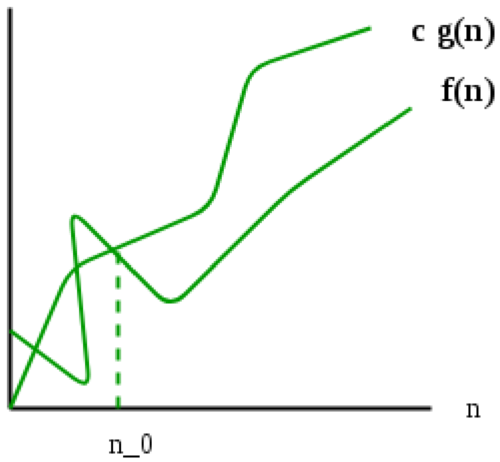
Asymptotic Notations:

Asymptotic Notations **are mathematical tools used to analyze the performance of algorithms** by understanding **how their efficiency changes as the input size grows**.

1. **Big-O Notation (O):**
   - Describes the **upper bound** of the time complexity.
   - Represents the **worst-case scenario**.
   - Example: $O(n)$, $O(n^2)$, $O(\log n)$.

Represents the **maximum time** required for an algorithm.

Example: A loop that runs **n** times has time complexity **O(n)**.



f(n) = O(g(n))

Mathematical Definition:

f(n) = O(g(n))

If there **exist positive constants c** and $\mathbf{n_0}$

such that $0 \le f(n) \le cg(n)$ for all $n \ge n_0$

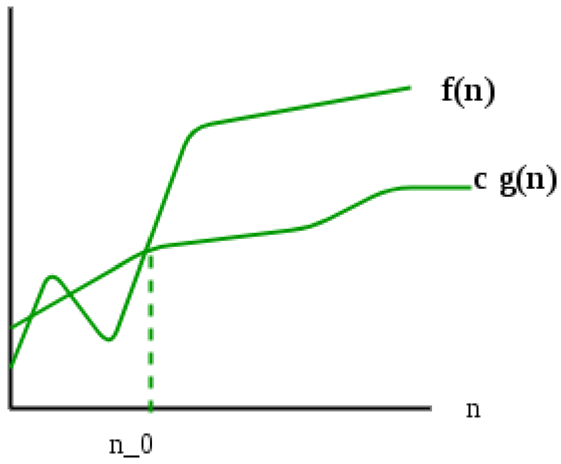
Example:

Let $f(n) = 3n + 2$ and $g(n) = n$.

- $f(n) = 3n + 2 \le 4n$ for $n \ge 2$.
- Here, $c = 4$ and $n_0 = 2$.
- Thus, $f(n) \in O(n)$.

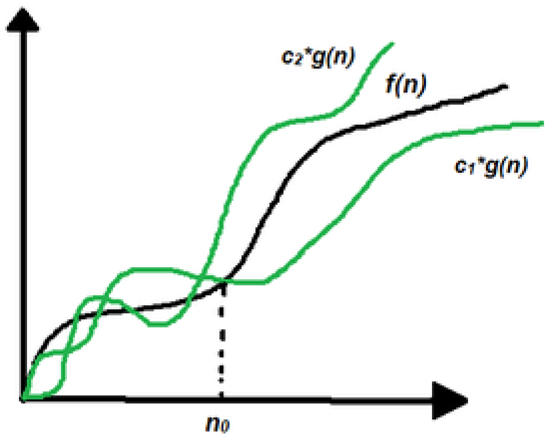
2. **Omega (Ω):**
   - Describes the **lower bound** of the time complexity.
   - Represents the **best-case scenario**.
   - **Example:** Ω(n), Ω(log n).

Represents the **minimum time** required for an algorithm.



f(n) = Omega(g(n))

3. **Theta (Θ):**
   - Describes the **tight bound** of the time complexity.
   - Represents the **average-case scenario**.
   - Example: Θ(n), Θ(log n).



Represents the **average-case time** of an algorithm.

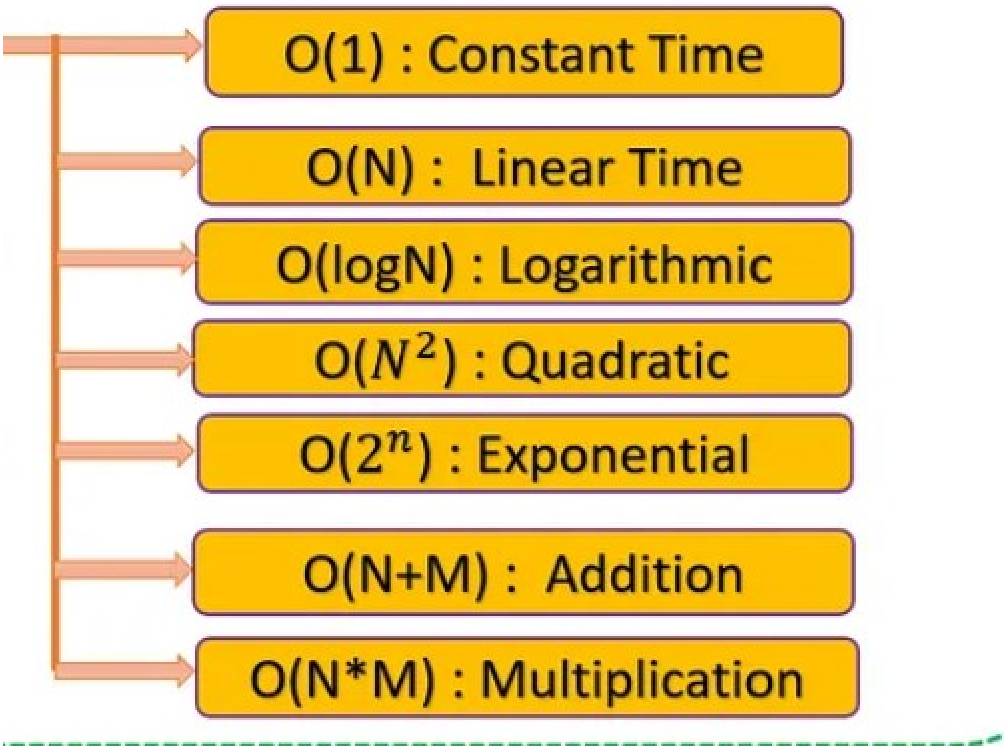
Best, Worst, and Average Cases:

1. **Best Case:**
   - The scenario where the algorithm performs the **minimum number of operations**.
   - Represented using **Ω notation**.

2. **Worst Case:**
   - The scenario where the algorithm performs the **maximum number of operations**.
   - Represented using **O notation**.

3. **Average Case:**

- Consider **all possible inputs** the algorithm can handle.
- Find the **time it takes for the algorithm to complete** for each input.
- Compute the **average time** across all inputs.
- Represented using **Θ notation**.



💻 O (1) – Constant Time

- The runtime doesn't depend on the input size.
- **Example:** Accessing an element in an array.

💻 O(log n) – Logarithmic Time

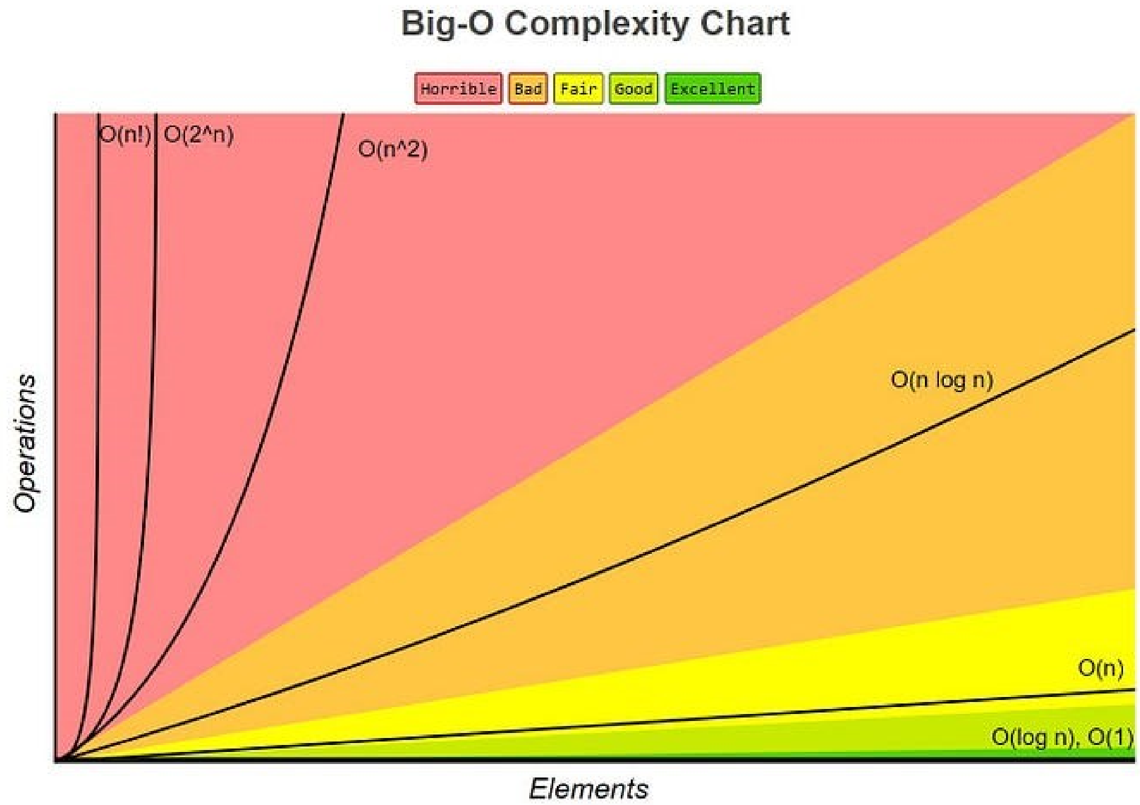
- The runtime grows slowly as input size increases.
- **Example**: Binary search.

💻 O(√n) – Square Root Time

- Grows faster than **O(log n)**, but slower than **O(n)**.
- **Example**:

💻 O(n) – Linear Time

- **Runtime grows directly with input size.**
- **Example:** Traversing an array.



💻 O (n log n) – Linearithmic Time

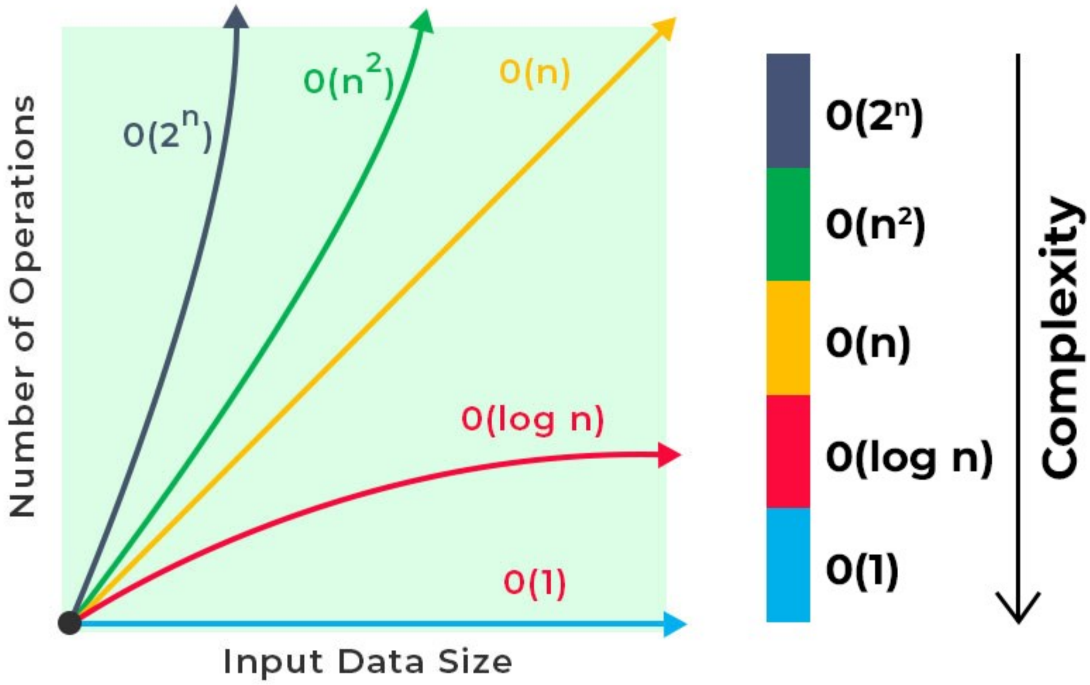
- **Common in divide-and-conquer algorithms like Merge Sort or Quick Sort.**

💻 O(n2) – Quadratic Time

- **Runtime grows proportional to the square of the input size (nested loops).**
- **Example: Bubble sort** or comparing all pairs in an array.

💻 O(n3) – Cubic Time

- Runtime grows **proportional to the cube of the input size** (triple nested loops).
- **Example: Matrix multiplication.**



💻 O(2n) – Exponential Time

- **Runtime doubles with each additional input.**
- **Example: Solving the Traveling Salesman Problem using brute force.**

🖥️ O(n!) – Factorial Time

- **Runtime grows extremely fast** (all possible permutations).
- **Example:** Solving the N-Queens problem using brute force.

🖥️ Growth Hierarchy (Smallest to Largest):

O(1) < O(logn) < O(n) < O(n) < O(nlogn) < O(n2) < O(n3) < O(2n) <O(n!)
