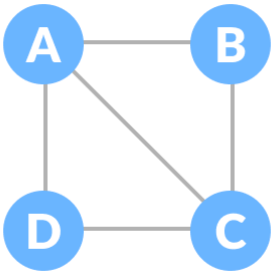
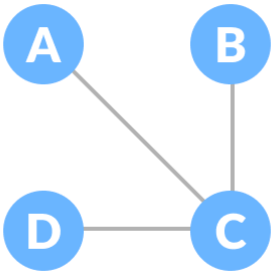
Spanning Tree and Minimum Spanning Tree

Before we learn about spanning trees, we need to understand two graphs: **undirected graphs and connected graphs.**

An **undirected graph** is a graph in which the **edges do not point in any direction** (ie. the edges are bidirectional).



A **connected graph** is a graph in which **there is always a path from a vertex to any other vertex.**



Spanning Tree

A spanning tree is a **sub-graph of an undirected connected graph**, which **includes all the vertices of the graph with a minimum possible number of edges**. If a vertex is missed, then it is not a spanning tree.

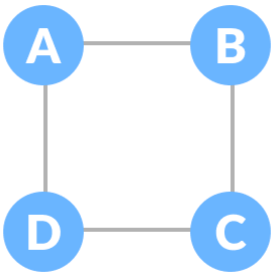
> The total number of spanning trees with n vertices that can be created from a complete graph is equal to $n^{(n-2)}$

If we have $n = 4$, the maximum number of possible spanning trees is equal to $4^{4-2} = 16$. Thus, 16 spanning trees can be formed from a complete graph with 4 vertices.

Example of a Spanning Tree

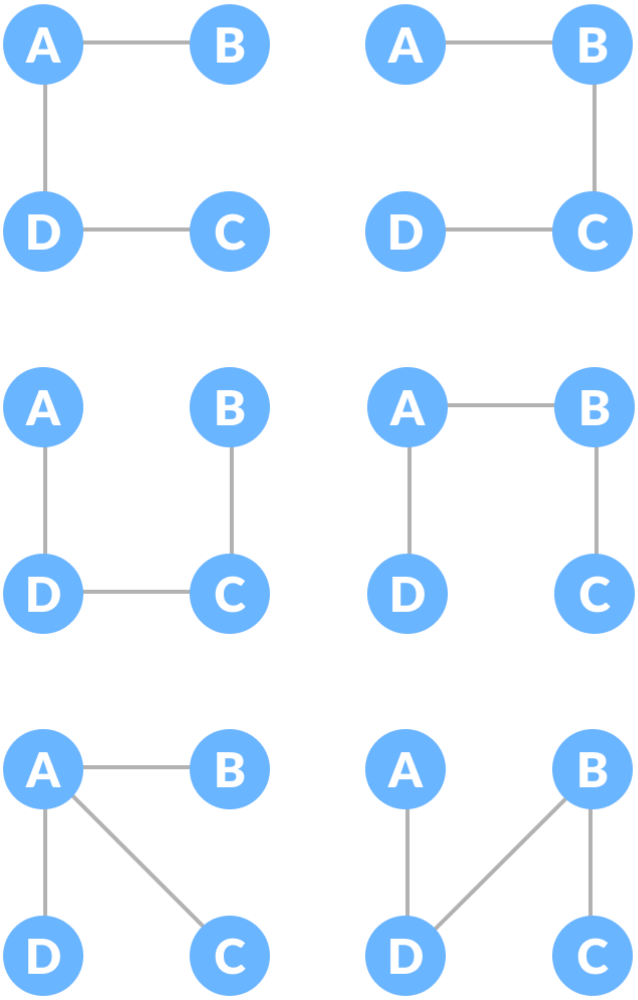
Let's understand the spanning tree with examples below:

Let the original graph be:



Normal graph

Some of the possible spanning trees that can be created from the above graph are:



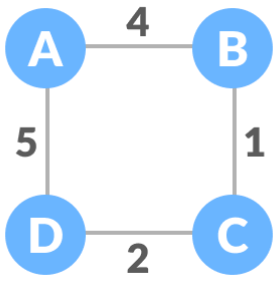
Minimum Spanning Tree

A minimum spanning tree is a spanning tree **in which the sum of the weight of the edges is as minimum as possible.**

Example of a Spanning Tree

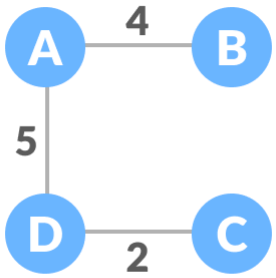
Let's understand the above definition with the help of the example below.

The initial graph is:



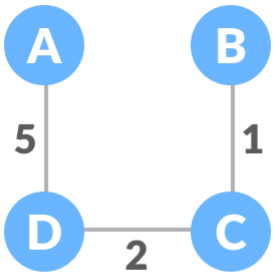
Weighted graph

The possible spanning trees from the above graph are:



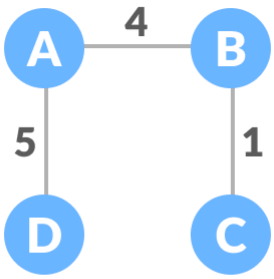
sum = 11

Minimum spanning tree - 1



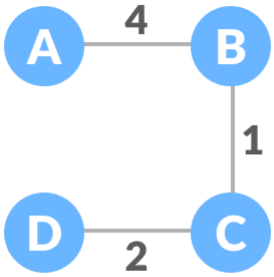
sum = 8

Minimum spanning tree - 2



sum = 10

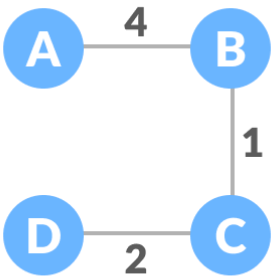
Minimum spanning tree - 3



sum = 7

Minimum spanning tree - 4

The minimum spanning tree from the above spanning trees is:



sum = 7

The minimum spanning tree from a graph is found using the following algorithms:

1. Prim's Algorithm
2. Kruskal's Algorithm

Spanning Tree Applications

- Computer Network Routing Protocol

Minimum Spanning tree Applications

- **To find paths in the map**