[illegible]

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The Function $f(n) = O(g(n))$ if there exist positive constant c and n_0 such that,

$f(n) \le c * g(n)$ for all $n \ge n_0$

$\le$ = less than equal

$\ge$ = greater than

Eg: $f(n) = 2n+3$

$2n+3 \le 10n \longrightarrow n \ge 1$

$f(n)$ c $g(n)$

The Function $f(n) = \Theta(g(n))$ if there exist positive constant c_1, c_2 and n_0 such that

$c_1 * g(n) \le f(n) \le c_2 * g(n)$

$1*n \le 2n+3 \le 5*n$ → Average

$c_1\, g(n)$ $f(n)$ $c_2\, g(n)$

Logarithms

$a^b = c$ — b: Power, a: Base — It is called exponent.

$\log_a c = b$ — It is called logarithm.

$\log a$ — base a

(1) Base same = Power same

Property

(1) $\log_a mn = \log_a m + \log_a n$ Ex: $\log(5\times3) = \log 5 + \log 3$

(2) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

(3) $\log a^b = b \log a$ Ex: $\log 2^3 = 3 \log 2$

(4) $\log_{10} 990 = \frac{\log_e 990}{\log_e 10}$; $\log_b n = \frac{\log_a n}{\log_a b}$

(5) $\log_x x = 1$ ($\log 10 = 1$)

Mantissa and characteristics

$\log x = 3.120$

So characteristics of $x = 3$

and mantissa of $x = .120$

Sequence and Series

(i) Sequence ⟹ 2, 4, 6, 8, 10
All difference is 2 (same)
All should be order

Ex: 3 6 9 12 15

3 9 27 81 ⟶ $\frac{9}{3}=3, \frac{27}{9}=3, \frac{81}{27}=3$

(3 multiplied case arrived)

⟹ Sequence is arrangement of Number according to certain fixed Rules.

finite sequence ⟹ 2, 4, 6, 8 (2: 1st term, 4: 2nd term)

Infinite sequence ⟹ 3, 6, 12, 24 ....

General term

(1) 2, 4, 6, 8, 10 .... n term
$2*1$ $2*2$ $2*3$ $2*4$ $2*5$ ...... $2*n$

⟹ $t_n = 2n$

Series

⟹ 2+4+6+8+10 Up a series

The term of sequence connected by positive or negative sign form a series.

(1) Find the 17 and 24 term of the sequence whose nth term is $t = 4n-3$

$t_{17} ⟹$ $t_{24} ⟹$

$t_n = 4n-3$

$t_{17} = 4*17-3$

⟹ 65

AP

Arithematic Progression ⇒ A sequence in which the difference between every term and preceding is always constant, is called AP.

Ex: 2, 4, 6, 8, 10

$d \begin{cases} a_2 - a_1 = 2 \\ a_3 - a_2 = 2 \end{cases}$

General term of AP :-

$$a_n = a + (n-1)d$$

Where a = 1st term
and d = common difference
n = term

Q. Find the general term of an AP 12, 5, -2, -9

$\Rightarrow T_n = 12 + (n-1) \times -7$

$\Rightarrow 12 + (-7n + 7)$

$\Rightarrow 19 - 7n$

Q. Which term of series 12 + 9 + 6 + ... is equal to -30.

$\Rightarrow a = 12 \quad -30 = 12 + (n-1) \times (-3)$

$d = -3 \quad \Rightarrow n = 15$

$T_n = -30$

Q. Sum of three number is 9 in AP and sum of their square is 35. Find the number.

$\Rightarrow \boxed{a-d,\ a,\ a+d} \longrightarrow$ Formula

$\boxed{a-3d,\ a-d,\ a+d,\ a+3d} \longrightarrow$ Four term

→ Let three Number in AP = $a-d, a, a+d$

$a-d+a+a+d = 9$

$3a = 9$

$a = 3$

$(a-d)^2 + a^2 + (a+d)^2 = 35$

$\Rightarrow a^2+d^2-2ad + a^2 + a^2+d^2+2ad$

$\Rightarrow a^2+d^2+a^2+a^2+d^2 = 35$

$\Rightarrow 3a^2 + 2d^2 = 35$

$\Rightarrow 3 \cdot 3^2 + 2d^2 = 35$ $\Rightarrow 1, 3, 5$

$\Rightarrow 2d^2 = 35 - 27$ $\Rightarrow 5, 3, 1$

$\Rightarrow d^2 = 4$

$\Rightarrow d = \pm 2$

Sum of nth term $\Rightarrow$ $\boxed{S_n = \frac{n}{2}[2a+(n-1)d]}$

Q Find the sum of 17th term of the AP 5, 9, 13, 17, ...

$\Rightarrow n=17, a=5, d=4$

$\Rightarrow S_{17} = \frac{17}{2}(2*5+(17-1)*4]$ BODMAS

$\Rightarrow S_{17} = \frac{17}{2}(2*5+16*4]$

$\Rightarrow \frac{17}{2}(10+64]$

$\Rightarrow \frac{17}{2} * 74$

$\Rightarrow 17 * 37$

$\Rightarrow S_{17} \Rightarrow 629$

$\boxed{S_n = \frac{n}{2}(a+a_n)} \longrightarrow$ Formula

$\hookrightarrow S_n = \frac{n}{2}(a+l)$ l - last term

Find the sum of odd integer from 1 to 2001.

$\Rightarrow 1+3+5+7 \ldots\ldots +2001$

$\Rightarrow a=1 \quad n=?$
$l=2001$

$d=2$

$\boxed{a_n = a+(n-1)d}$

$\Rightarrow 2001 = 1+(n-1)\times 2$

$\Rightarrow 2001 = 1+2n-2$

$\Rightarrow 2001 = -1+2n$

$\Rightarrow 2n = 2002$

$n \Rightarrow 1001$

$\Rightarrow S_{1001} \Rightarrow \frac{1001}{2}(1+2001)$

$\Rightarrow 1002001$

Arithmetic Mean

Insert four AM between 4 and 29.

$\Rightarrow$ 4 A_1 A_2 A_3 A_4 29

a a_2 a_3 a_4 a_5 a_6

$a_2 = t_2 \Rightarrow a+d$

$a_3 \Rightarrow a+2d$

$a_4 \Rightarrow a+3d$

$a_5 \Rightarrow a+4d$

$a_6 \Rightarrow a+5d$

$a+5d = 29$

$4+5d = 29$

$5d = 29-4$

$5d = 25$

$d = 5$

$4+5 \rightarrow 9$

$4*5*2 \rightarrow 14$

$4*5*3 \rightarrow 19$

$4*5*4 \rightarrow 24$

$\rightarrow 29$

Find the sum of 100 AM between 11 and 15.

$\Rightarrow \left(n * \frac{a+b}{2} \right) \Longrightarrow$ Formula

$\Rightarrow 100 * \frac{11+15}{2}$

$\Rightarrow 100 * \frac{26}{2}$ 13 $\Rightarrow \underline{\underline{1300}}$

GP

Geometric Progression :-

$r = 2$

2, 4, 8, 16, ...

$\boxed{\frac{a_2}{a_1} \text{ equal}} \Rightarrow \frac{4}{2} = 2, \frac{8}{4} = 2, \frac{16}{8} = 2$

A sequence of non zero number in which every term except the first term, bears a constant ratio with its proceeding term, is called geometric progression.

General term of GP ⇒

Common ratio $= r$

$\Rightarrow a_n \Rightarrow a \cdot r^{n-1}$

Find the 6th term of the sequence 1, -1/2, 1/4, -1/8

$\Rightarrow \frac{-\frac{1}{2}}{\frac{1}{1}} = \frac{-1}{2} * \frac{1}{1} = \frac{-1}{2}$ its a GP series

$\Rightarrow \frac{\frac{1}{4}}{\frac{-1}{2}} = \frac{1}{4} * \frac{2}{-1} = \frac{-1}{2}$

$\Rightarrow a_n = a r^{n-1}$

$a = 1$

$r = \frac{-1}{2}$

$n = 6$

$a_6 = 1 * \left(\frac{-1}{2}\right)^{(6-1)}$

$\Rightarrow \frac{-1}{2} * \frac{-1}{2} * \frac{-1}{2} * \frac{-1}{2} * \frac{-1}{2}$

$\Rightarrow \frac{-1}{32}$

Q. Find the 12 term of GP whose 8th term is 192 and common ratio 2.

$$\Rightarrow a_{12} = ?$$

$$a_8 = 192 \Rightarrow a_8 = a \cdot r^{8-1}$$

$$r = 2$$

$$\Rightarrow a \cdot r^7 \Rightarrow 192$$

$$\Rightarrow a_{12} \Rightarrow ar^{11}$$

$$\Rightarrow ar^7 r^4$$

$$\Rightarrow 192 \cdot r^4$$

$$\Rightarrow 192(2)^4$$

$$\Rightarrow \underline{\underline{3072}}$$

$a^m \cdot a^n \Rightarrow a^{m+n}$

If base same then add not multiply

* Sum of nth term in GP ⇒

$\Rightarrow \frac{a(1-r^n)}{1-r}$ if $r<1$ where, a is 1st term, r is common ratio, n is nth term

$\Rightarrow \frac{a(r^n-1)}{r-1}$ if $r>1$

* Find the sum of $1, 1/2, 1/4, 1/8 \dots$ to 10 th term.

$\Rightarrow a=1$, $r=\frac{1}{2}$, $n=10$

$S_n = \frac{a(1-r^n)}{1-r}$

$= \frac{1\left(1-\left(\frac{1}{2}\right)^{10}\right)}{1-\frac{1}{2}}$

$\Rightarrow \frac{1-\left(\frac{1}{2}\right)^{10}}{\frac{1}{2}} \Rightarrow \frac{1-\frac{1}{2^{10}}}{\frac{1}{2}}$

$\Rightarrow \frac{\frac{2^{10}-1}{2^{10}}}{\frac{1}{2}} = \frac{2^{10}-1}{2^{10}} \times \frac{2}{1}$

$\Rightarrow \frac{2^{10}-1}{2^{9}}$

⇒

* Sum of GP to infinity

$\Rightarrow S_\infty = \frac{a}{1-r}$

* Sum the series to infinity

$1, -1/3, 1/9, -1/27 \dots \infty$

$\Rightarrow a=1$, $r=-\frac{1}{3}$ $\Rightarrow \frac{1}{1-\left(-\frac{1}{3}\right)}$

$\Rightarrow \frac{1}{1+\frac{1}{3}} \Rightarrow \frac{1}{\frac{3+1}{3}}$

$\Rightarrow \frac{1}{\frac{4}{3}}$

$\Rightarrow \frac{3}{4}$

Geometric Mean ⇒

$a \quad b \Rightarrow GM = \sqrt{ab}$

Geometric mean of a and b

$\underline{M = \sqrt{ab}}$

Insert 3 GM between 3 and 432:

$\Rightarrow 3 \quad G_1 \quad G_2 \quad G_3 \quad 432$

$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$

$a = 3$

$a_5 = 432$ (a.r^{n-1} ⇒ nth term formula)

$\Rightarrow a \cdot r^4 = 432$

$\Rightarrow 3 \cdot r^4 = 432$

$r^4 = \frac{432}{3}$

$\Rightarrow r^4 \Rightarrow 144$

$r^4 = 2^4 * \sqrt{3}^2$

$\Rightarrow 2*2*2*2*\sqrt{3}*\sqrt{3}*\sqrt{3}*\sqrt{3}$

$\Rightarrow 2^4 * (\sqrt{3})^4$

$\Rightarrow (2*\sqrt{3})^4$

$\Rightarrow \underline{2*\sqrt{3}}$

$\Rightarrow a_2 = ar^1 = 3 * 2\sqrt{3}$

$\Rightarrow 6\sqrt{3}$

$\Rightarrow 4\sqrt{3}^{2+1} = 36\sqrt{3}$

$\Rightarrow \underline{\cancel{\ }} = \underline{72\sqrt{3}}$