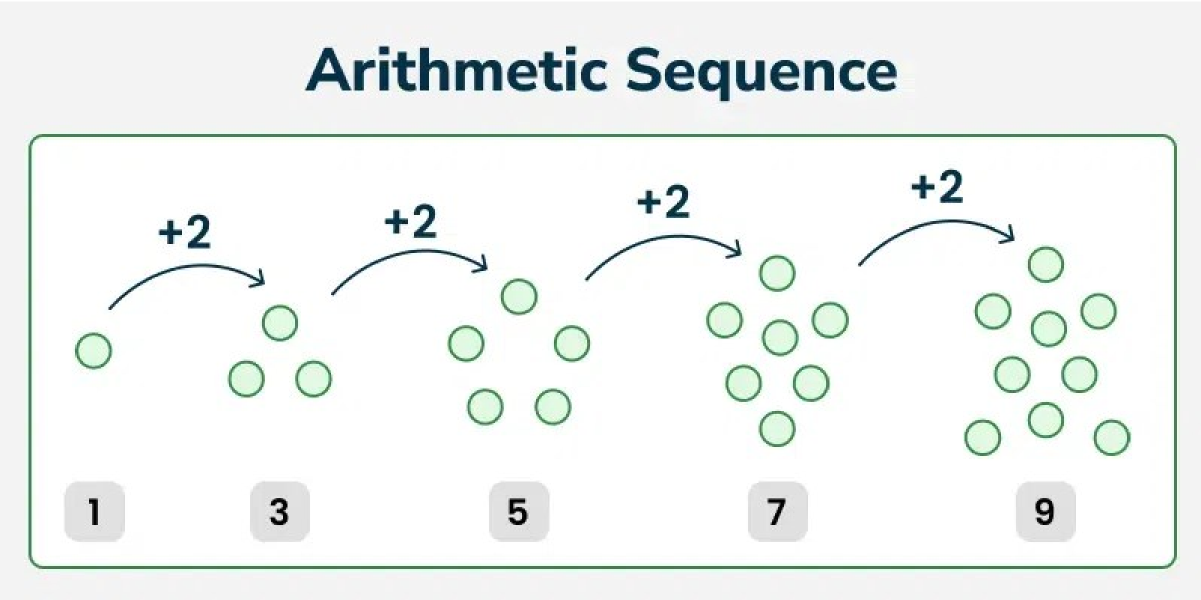
Sequence And Series

Sequence

- A **sequence** is an **ordered list of numbers following a specific pattern** or rule.
- **Example**: 2,4,6,8
- Each number in a sequence is called a **term**.



An **arithmetic sequence** (or arithmetic progression) is a sequence of numbers **in which the difference between consecutive terms is constant**. This difference is called the [common difference] (denoted as d).

For example:

👉 2, 5, 8, 11, 14, . . . (first term = 2 and common difference = 3)

A **geometric sequence** (or geometric progression) is a sequence of numbers in which the ratio between consecutive terms is constant. This ratio is known as the **common ratio** (denoted as r)

For example:

👉 3, 6, 12, 24, 48, . . . (first term = 3 and common ratio = 2)

Series:

- A **series** is the sum of the terms of a sequence.
- Series as Terms of a Sequence connected by **positive (+)** or **negative (-)** signs.
- **Example**: 2+4+6+8

Infinite vs. Finite Sequences and Series

Finite Sequence:

- A sequence with a **limited** number of terms.
- **Example:** 1,2,3,4,5

Infinite Sequence:

- A sequence with an **unlimited** number of terms.
- Example: 1,2,3,4,5, …

General Term or nth Term

- The **general term** of a sequence represents any term **(Tn)** in the sequence using a formula.
- It is **used to find the value of a term at a specific position**.

Example 1 (Arithmetic Sequence):

- Sequence: 2,4,6,8
- General term: **an=2n**

Sequence and Series

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Arithmetic Sequence →	10, 15, 20, 25, 30, 35, ... , u_n
Arithmetic Series →	10 + 15 + 20 + 25 + 30 + 35 + ... + u_n

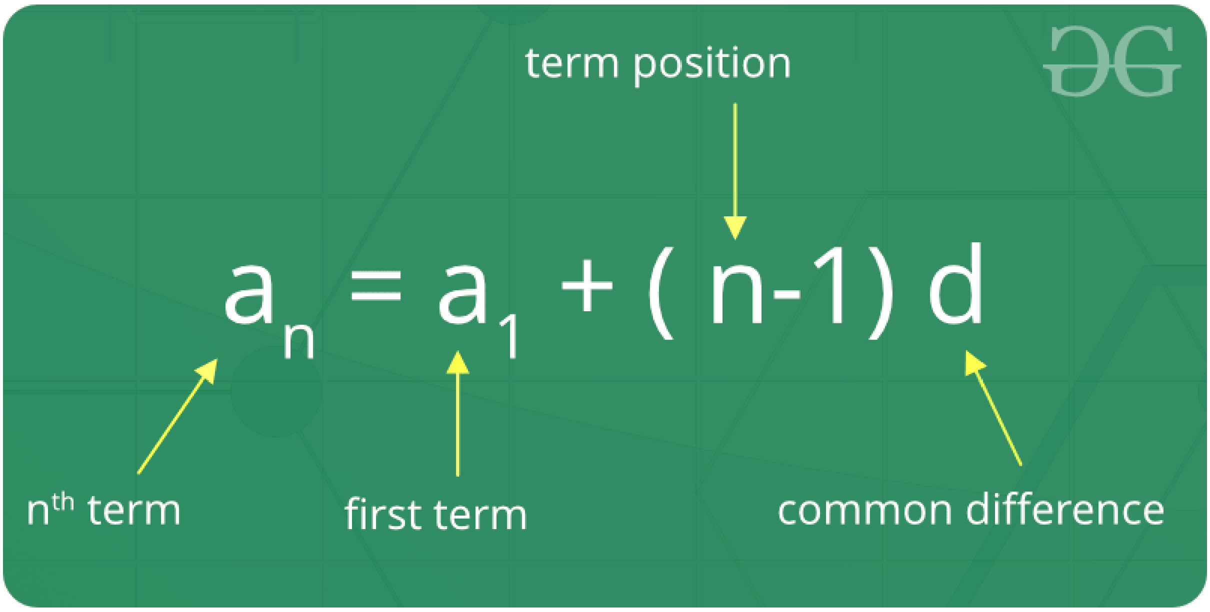
Geometric Sequence →	100, 50, 25, 12.5, 6.25, 3.125, ... , u_n
Geometric Series →	100 + 50 + 25 + 12.5 + 6.25 + 3.125 + . . . + u_n

👉 General term or nth term of Arithmetic Series

$$an = a + (n-1)d$$

Where,

- **a** is the first term
- **d** is the common difference
- **n** is the number of terms
- **an** is the nth term



Arithmetic Series for 3 Numbers:

$$a-d, a, a+d$$

Arithmetic Series for 4 Numbers:

$$a-3d, a-d, a+d, a+3d$$

Arithmetic Series for 5 Numbers:

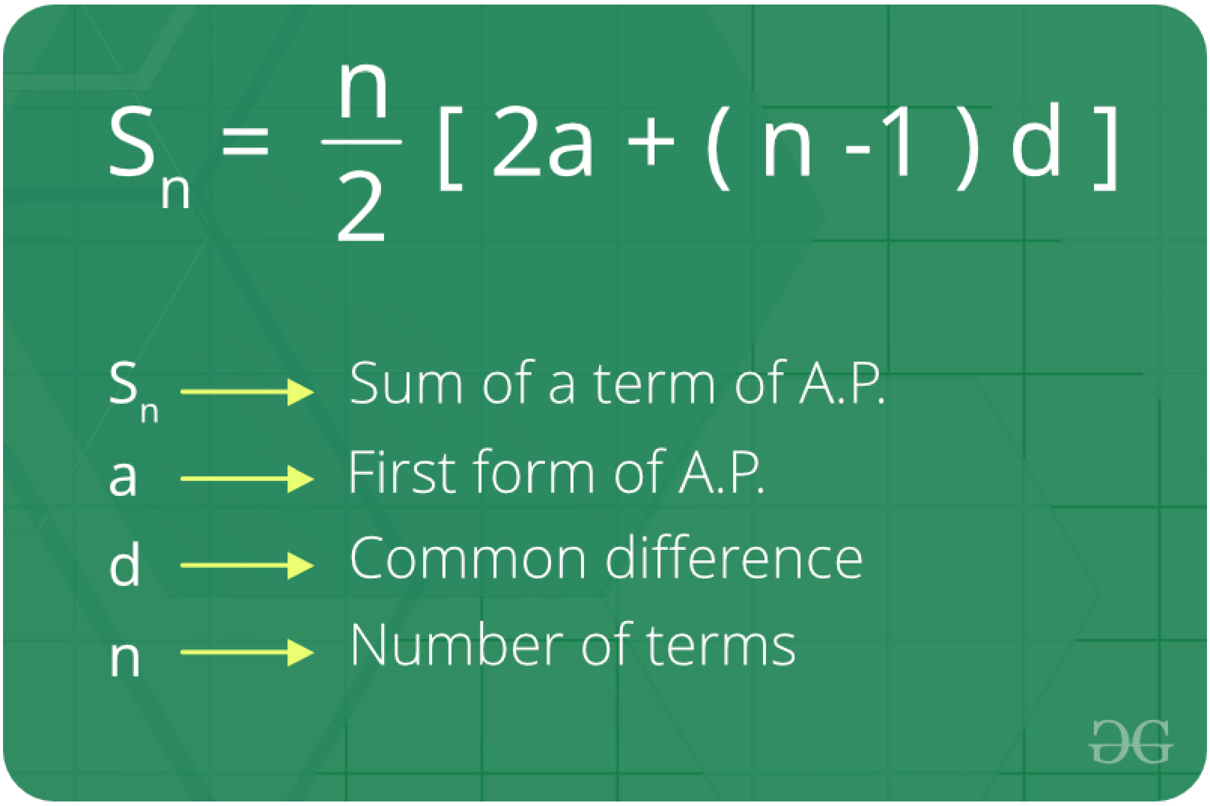
$$a-2d, a-d, a, a+d, a+2d$$

Sum of First n Terms

$$S_n = n/2\ [2a + (n-1)d]$$

Where,

- **a** is the first term
- **d** is the common difference
- **n** is the number of terms.



If we know the **last term** of an **arithmetic sequence**, the **formula** to find the **sum** of the first **n** terms **(Sn)** is:

Sn= n / 2 ·(First Term+Last Term)

Number of terms

nth term

$$S_n = \frac{n}{2}(a + l)$$

Sum of n terms

First term

Quadratic Equation

Standard Form of a Quadratic Equation

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Coefficient of x

$$ax^2 + bx + c = 0$$

Coefficient of x^2

Constant

Quadratic Formula

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$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Arithmetic Mean

The **Arithmetic Mean (AM)** is the average of two terms in an Arithmetic Progression (AP).

If three numbers **a,A,b** are in AP, the middle term A is the **Arithmetic Mean** of a and b, and it is given by:

$$A = \frac{a+b}{2}$$

a2=a+d

a3=a+2d

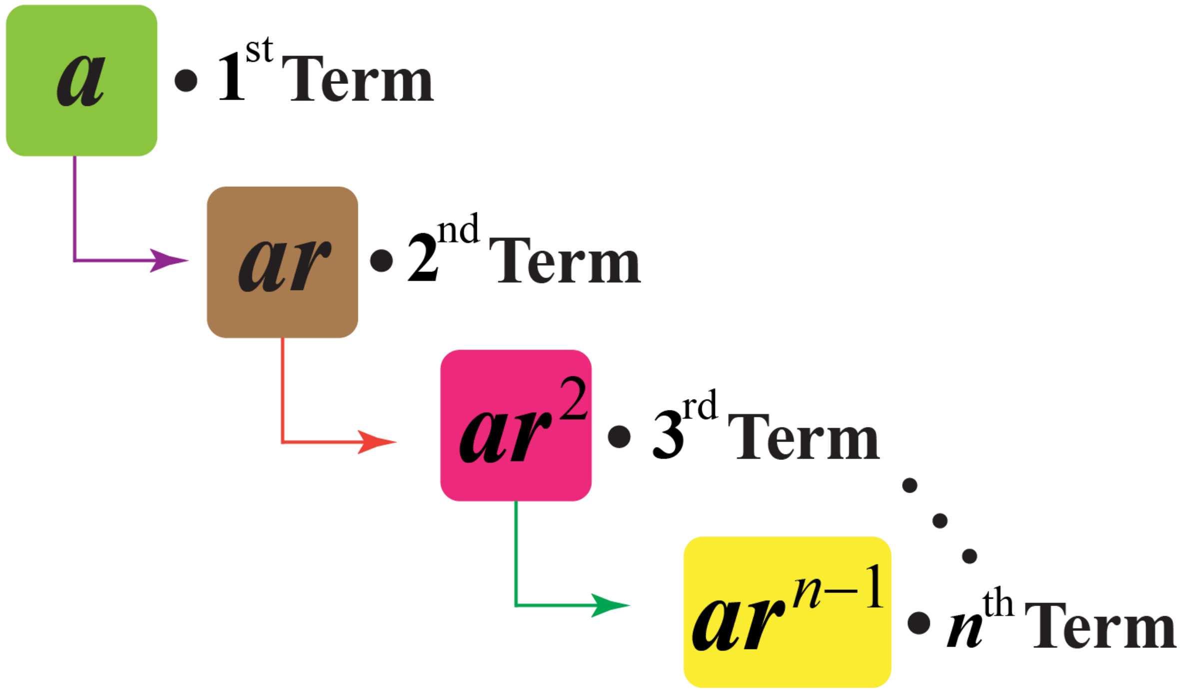
a4=a+3d

Example:

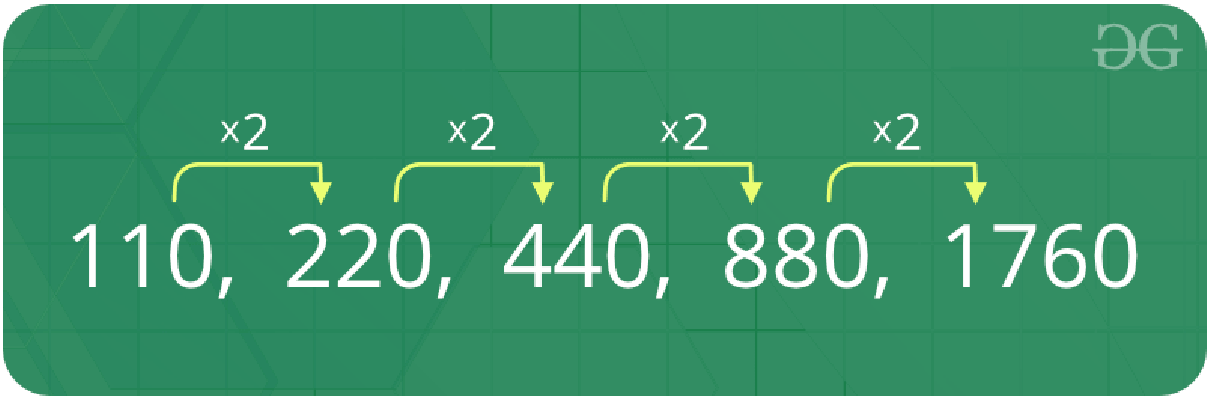
1. Given $a = 2$ and $d = 3$, find a_2, a_3, a_4, a_5:

   - $a_2 = a + d = 2 + 3 = 5$
   - $a_3 = a + 2d = 2 + 2(3) = 8$
   - $a_4 = a + 3d = 2 + 3(3) = 11$
   - $a_5 = a + 4d = 2 + 4(3) = 14$

So, the sequence is: $2, 5, 8, 11, 14$.



Geometric Progression (GP)



General Form of Geometric Progression

The nth term of the Geometric series is denoted by **an**

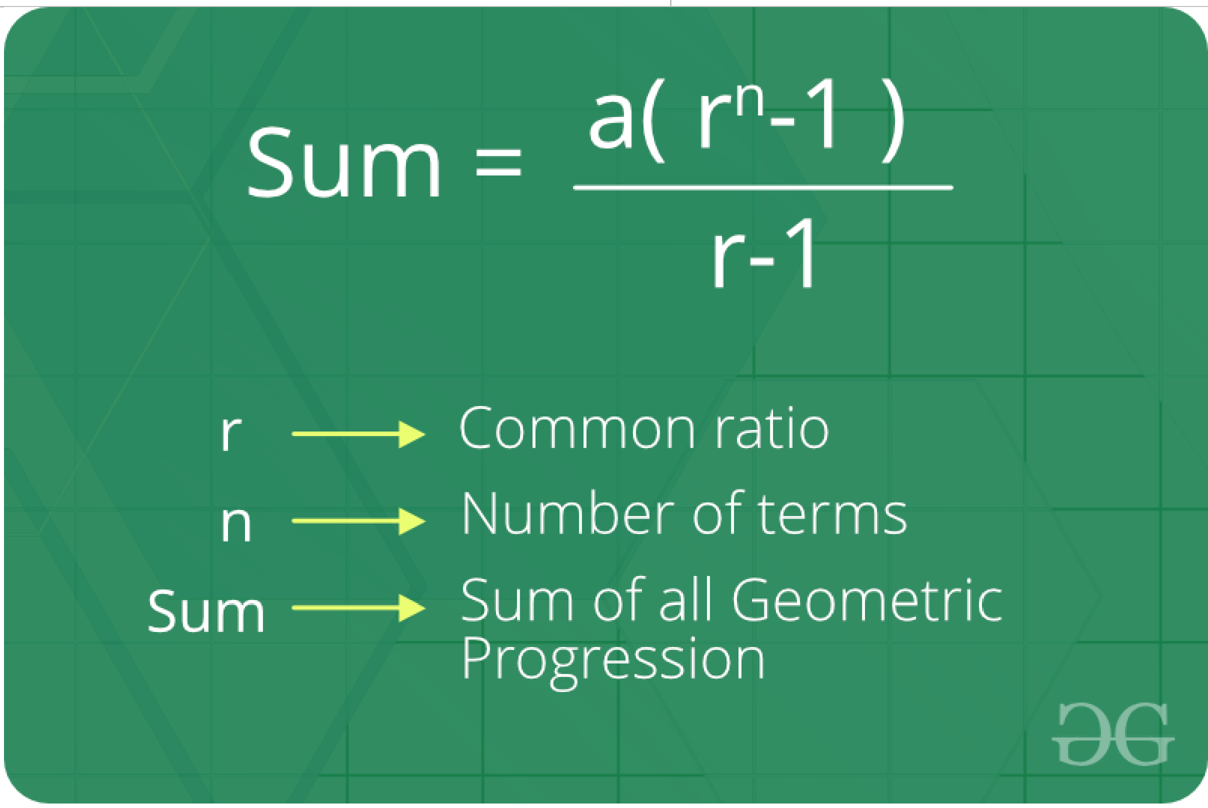
$$a1 = a$$
$$a2 = a*r$$
$$a3 = a*r^2$$
$$a4 = a*r^3$$
$$an = a*r^{n-1}$$

Geometric Progression Formula

General Form	$a, ar, ar^2, ar^3, \ldots$	***a*** **is the first term**, and ***r*** **is the common ratio.**

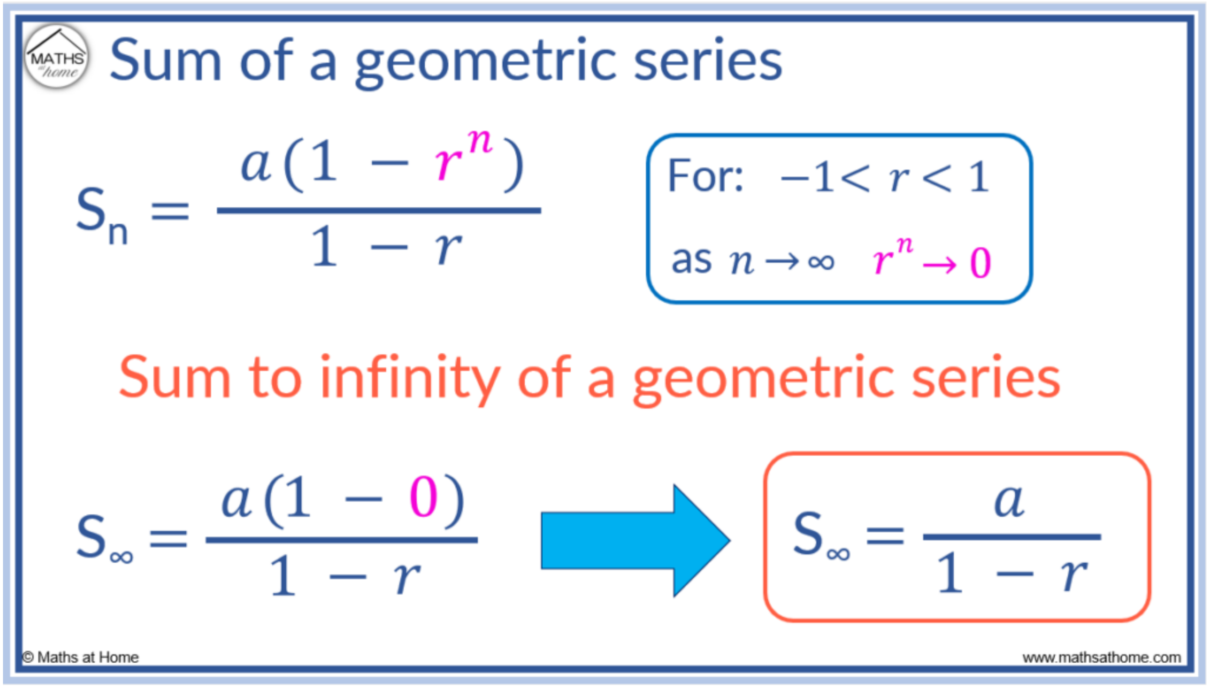
👉 **nth Term of a *GP***	**$Tn = ar^{n-1}$**	**Tn is the nth term**, *a* is the first term, and r is the common ratio.

Sum of First n Terms ($r > 1$)	**$Sn = a[(r^n - 1)/(r - 1)]$**
Sum of First n Terms ($r < 1$)	**$Sn = a[(1 - r^n)/(1 - r)]$**



👉 Infinite Terms of a G.P

The formula **$a/1-r = S_n$** works when n is equal to infinity.



Geometric Mean

Geometric Mean of A and B

$$G = \sqrt{ab}$$