Pulse Optimization

We tried to use Sels and Polkovnikov's counterdiabatic protocol. The driving Hamiltonian

$$H = H_0 + A = -\Delta(t)\sum_i n_i + \sum_{<i,j>} V_{ij} n_i n_j + \sum_i \frac{\Omega(t)}{2}\left(e^{i\phi(t)}\left|g_i\right\rangle\left|e_i\right\rangle + e^{-i\phi(t)}\left|e_i\right\rangle\left|g_i\right\rangle\right)$$

where \i$, j >$ represents nearest neighbor in our graph. It can be viewed as the sum of a base

$$H_0 = -\Delta(t)\sum_i n_i + \sum_{<i,j>} V_{ij} n_i n_j + \sum_i \frac{\Omega(t)}{2}\cos\left(\phi(t)\right)\left(\left|g_i\right\rangle\left\langle e_i\right| + \left|e_i\right\rangle\left\langle g_i\right|\right)$$

and a counterdiabatic component

$$A = \sum_i \frac{\Omega(t)}{2} i \sin\left(\phi(t)\right)\left(\left|g_i\right\rangle\left\langle e_i\right| - \left|e_i\right\rangle\left\langle g_i\right|\right)$$

in which way H_1 is a purely imaginary part of H_0, respecting the constraint according to [1].

It is worth noting that the two terms can be rewritten using Pauli operators defined on the $\left|e_i\right\rangle, \left|g_i\right\rangle$ bases as

$$\begin{aligned} H_0 &= -\Delta(t)\sum_i \frac{\sigma_i^z + 1}{2} + \sum_{<i,j>} \frac{V_{ij}}{4}(\sigma_i^z + 1)(\sigma_j^z + 1) + \sum_i \frac{\Omega(t)}{2}\cos(\phi)\sigma_i^x \\ &= \sum_{<i,j>} \frac{V_{ij}}{4}\sigma_i^z\sigma_j^z + \sum_i \sigma_i^z\left(\frac{1}{4}\bar{V}_i - \frac{\Delta(t)}{2}\right) + \sum_i \frac{\Omega(t)}{2}\cos(\phi)\sigma_i^x - \Delta(t)\frac{N}{2} + \sum_{<i,j>}\frac{V_{ij}}{4} \end{aligned}$$

and

$$A = \sum_i \frac{\Omega(t)}{2}\sin(\phi)\sigma_i^y$$

given the total interaction potential felt by atom $\bar{V}_i = \sum_{j\in\mathrm{neighbor}(i)} V_{ij}$. Now evaluate the G operator defined in Sels' paper

$$\begin{aligned} G &= \partial_t H_0 + \frac{i}{\hbar}[A, H] \\ &= -\sum_i \sigma_i^z\frac{\Delta'(t)}{2} + \sum_i \frac{\Omega'(t)}{2}\cos(\phi)\sigma_i^x + \frac{\Omega}{2}\sin(\phi)\left(\Omega\cos(\phi)\sum_i \sigma_i^z - \sum_{<ij>} V_{ij}\sigma_i^x\sigma_j^z - \sum_i\left(\frac{1}{2}\bar{V}_i - \Delta(t)\right)\sigma_i^x\right) \\ &= \left(\frac{\Omega^2}{4}\sin(2\phi) - \frac{\Delta'}{2}\right)\sum_i \sigma_i^z + \sum_i\left(\frac{\Omega'}{2}\cos(\phi) + \frac{\Omega}{2}\sin(\phi)\left(\Delta - \frac{\bar{V}_i}{2}\right)\right)\sigma_i^x - \frac{\Omega}{2}\sin(\phi)\sum_{<ij>} V_{ij}\sigma_i^x\sigma_j^z \end{aligned}$$

The Schmidt norm is

$$S = 2^{-N}\operatorname{Tr}\left(G^2\right) = N\left(\frac{\Omega^2}{4}\sin(2\phi) - \frac{\Delta'}{2}\right)^2 + \sum_i\left(\frac{\Omega'}{2}\cos(\phi) + \frac{\Omega}{2}\sin(\phi)\left(\Delta - \frac{\bar{V}_i}{2}\right)\right)^2 + \left(\frac{\Omega}{2}\sin(\phi)\right)^2\sum_{<ij>} V_{ij}^2$$

Although this quantity above is graph-dependent and an analytical optimization generally hard to obtain, we can still get insights into how to optimizing the pulse parameters $\Omega, \phi, \Delta, \Delta$ by examining the form of S. For example, it is beneficial to set $\frac{\Omega^2}{4}\sin(2\phi) \approx \frac{\Delta'}{2}$ and $\Omega' \approx \Omega\tan(\phi)\left(\Delta - \frac{\bar{V}_i}{2}\right)$.

In the experiment, we left Δ as default to be a piecewise linear function, constrained $\phi = \pi/4$ to minimize the absolute value of Ω, and set $\Omega_{max} = \sqrt{2\Delta'}$, fulfilling the first condition above.

Building upon this setting, we used cubic spline functions to interpolate the points into continuous smooth evolutions, as shown below.