# iQuHack 2024: Qubit Noise Reduction Challenge

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## 1 Background

Noise mitigation has been a big issue in quantum computing since the beginning, with many of the current hardware unable to achieve significant breakthrough due to noise constraints (including only being able to factor 35 with Shor's Algorithm, a far cry from its potential). Although noise mitigation techniques have been proposed with logical qubits, such as Shor Code, these schemes often require large physical qubit volume, a struggle during the NISQ era. However, for certain algorithms and simulations, a qubit remap could resolve the problem of high fidelity in two-qubit gates. Specifically, by exploring the topology and fidelity of the system, we can often remap qubits to another system such that the most important gates and qubits are mapped to ones that have the highest fidelity.

## 2 Graph Theory

Graph Theory is a branch of mathematics that studies the relation between mathematical objects. We define a graph as a set of ordered pairs G = (V,E,$\phi$) where V is a node, or point, E is an edge, and $\phi$ is the incidence function that maps an edge to a node. For our purposes, we define it as

$$\phi : E \rightarrow \{(x, y)|x, y \in V\}$$

(this is a definition of the incidence function that allows for loops and directed graphs)

## 3 Noise and the Kraus formalism

Ideally, we would be able to produce the a pure state via a quantum operation. However, this is unrealistic, and error will always seep into our results. The general solution to quantum states in the prescence of noise is as the outer product of pure states —0¿ and —1¿ which we describe by the density matrix:

$$\sum_i \rho_i |\psi_r\rangle \langle\psi_r| \tag{1}$$

Where $\rho$ is the probability that $\psi$ is in a certain state. Time evolutions can be described by these density matrices, which have to be completely positive and trace preserving (CPTP) so that the density matrix still describes an actual quantum state. According to Kraus's theorem, any CPTP matrix can be expressed as a linear map:

$$\epsilon(\rho) = \sum_i K_i \rho K_i^* \tag{2}$$

where

$$\sum_i K_i^* K_i = 1 \tag{3}$$

As such, since noise describes what happens to a quantum state, any noise in a quantum system can be represented by the Kraus formalism. An example of this is a bit flip error which can be described as:

$$(1 - \rho)\rho + \rho X \rho X \tag{4}$$

# 4 Our system

For this project, we were not working with a physical chip, rather, we were working in a purely simulational context. To this degree, we were able to run an ideal circuit with all qubits maximally connected and with the option of introducing simulated noise into a circuit.

# 5 Our Solution

## 5.1 Noise Simulation

The first step we did was take our circuit and run some global noise through it. We used the data from the noisy circuit to output every possible ordered pair and the fidelity associated with each path.

## 5.2 Graph Theory Implementation

### 5.2.1 Abstracting the Problem

Fundamentally, what we have is an issue of how to optimize qubit connections given pairwise relationships that constrain how we may best connect qubits. As such, it is almost natural to map a quantum circuit onto a graph theory problem, which we can do through the Networkx python package. We let Q be the graph that models the quantum circuit, defined by

$$\phi : E \to \{(x, y) | x, y \in V\}$$

Such that Q is a multiple, directed graph allowing loops. What this means is that each node can have multiple edges, edges are ordered pairs such that

$$\{D : x \to y | x, y \in Q\}, \{S : y \to x | x, y \in Q\}$$

But D may not necessarily equal S, and there exists:

$$\{E : x \to x | x \in Q\}$$

Q in our simulations is a maximally connected, or complete graph, such that $Q = K_{\mathrm{n}}$. We are then able to create Q as a graph using the Networkx package for Python. For the purposes of the simulation, we are not assuming a physical chip, and so the distance:

$$\forall u, v, w \in Q, d(u, v) = d(v, w) = d(u, w)$$

#### 5.2.2 The Degree Algorithm

Between each qubit pairing, we have a certain fidelity in an ordered pair such that we model it in our graph by making weighted, directed edge. Then for all $x$ we can find the degree $deg(x)$, which sums over all edges that connect to that node (including the self-loop). We then sort over those values and return $\Delta Q$. Finally, we find all paths that connect to $\Delta Q$, and sort them by their individual weights, which will allow us to find the best possible connections around a specific node.

Finally, we can implement self-loops with weights for each node that represents the total simulated noise that we inserted into a circuit so we can take the inherent noise of a circuit into account independant of other qubits. Finally, we made a visual interpreter that plots the graph with all of the connections.

### 5.3 Mapping functions

To map the qubit abstractions to the actual qubits in our circuit, we use a combination of certain algorithms to further optimize and then map the best possible connections.

#### 5.3.1 Qubit Frequency compatibility

This algorithm will analyze the makeup of the specified quantum circuit such that depending on the amount of gates that require ordered pairs (like CNOT gates) the algorithm will weigh that target qubit more in favor of being mapped to $\Delta Q$ and the target of the CNOT gate as the optimal qubit path from $\Delta Q$.

The second half of the algorithm will analyze the fidelity between both ordered pairs that the graph algorithm to determine if an optimal connection is backwards compatible. If not, it will rank a new optimal edge that is compatible in both directions.

The last part of the algorithm will then finalize the mapping such that $\Delta Q$ is mapped to the qubit that has the most ordered pair gates, with other qubits mapped assigned by $\Delta Q$'s optimal connections.

## 6 Limitations and Future

By performing experiments on the 3-qubit GHZ state, we discovered that there is an 11% reduction in the middle-noisy states. This represents a significant improvement over naive optimization. However, our results are limited in scope due to the limited qubit counts as well as a simulated noisy model, which does not take into account things such as the decomposition of SWAP gates or Hadamard gates. However, our system should be able to scale well due to our functional calls. More investigation upon the topology of the computer will be needed to further optimize in a system that is not fully connected.