On Solving MIS with Rydberg Atom Array

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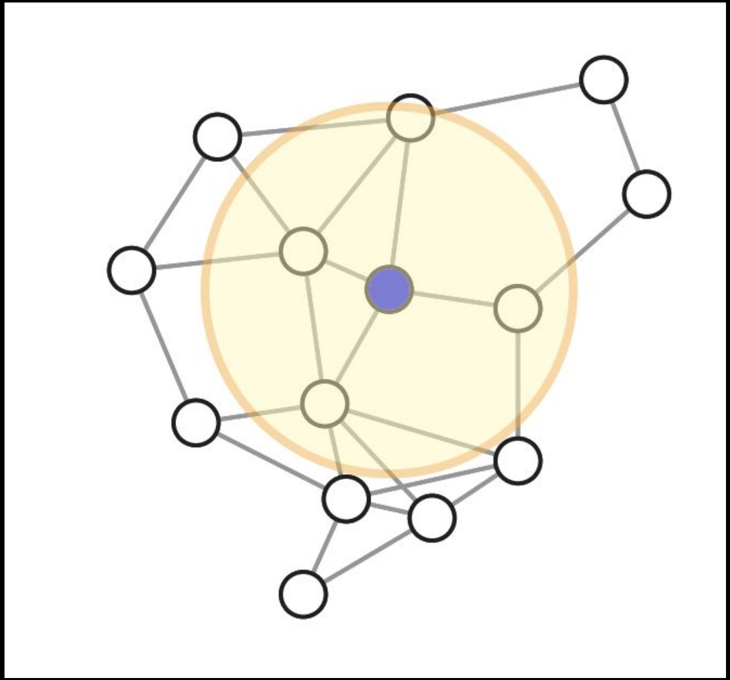
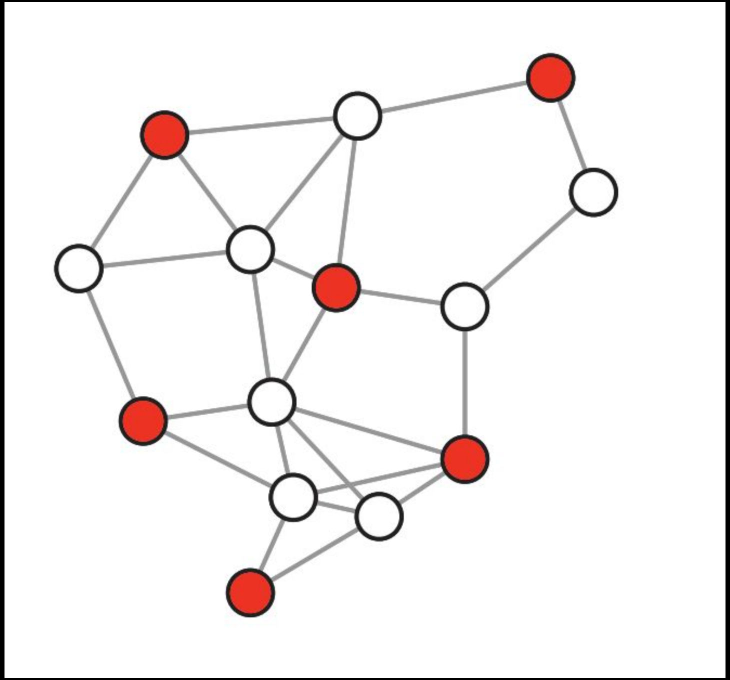
iQuHACK 2024

Problem Introduction

The Maximum Independent Set Problem (MIS)

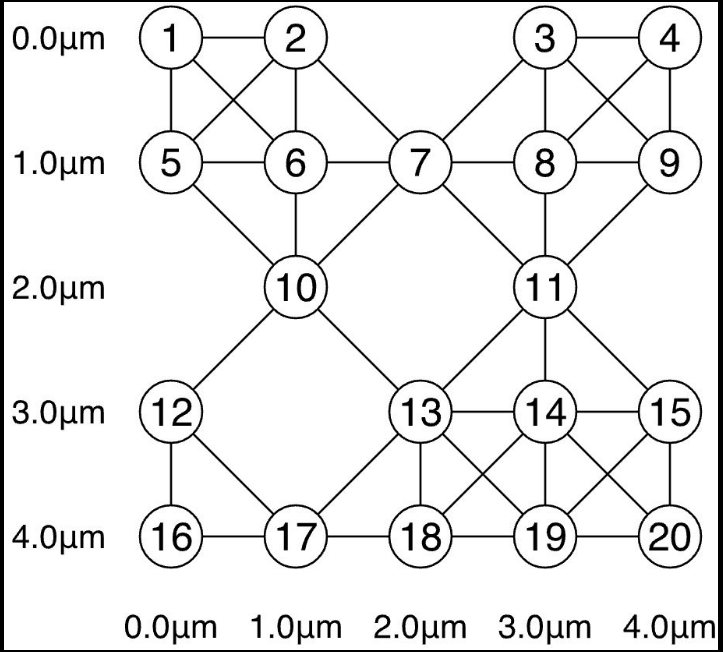


Unit-Disk Graph

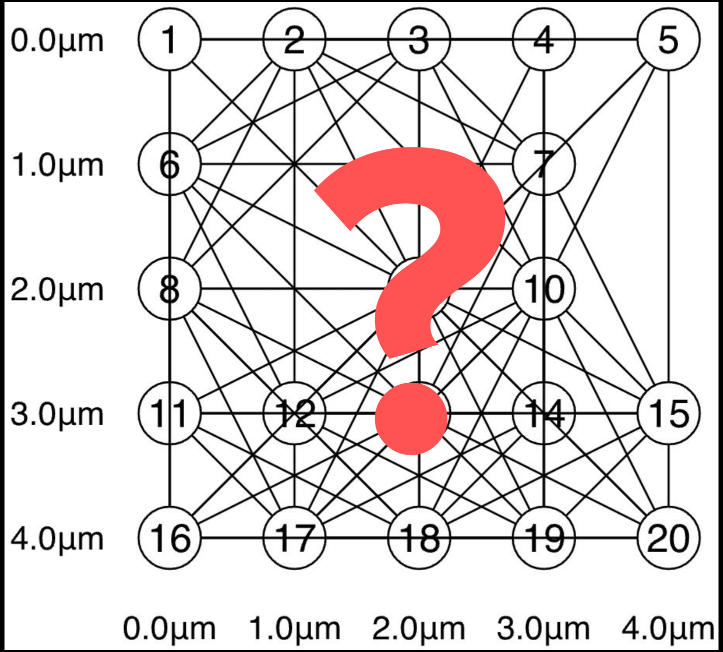


MIS Solution

Beyond the King's Graph



Adjacent and Diagonal Connections



Three Lattice Constant Radius

Theory

Set-up

Ebadi et. al.: grid of Rydberg atoms as in QuEra to simulate interactions

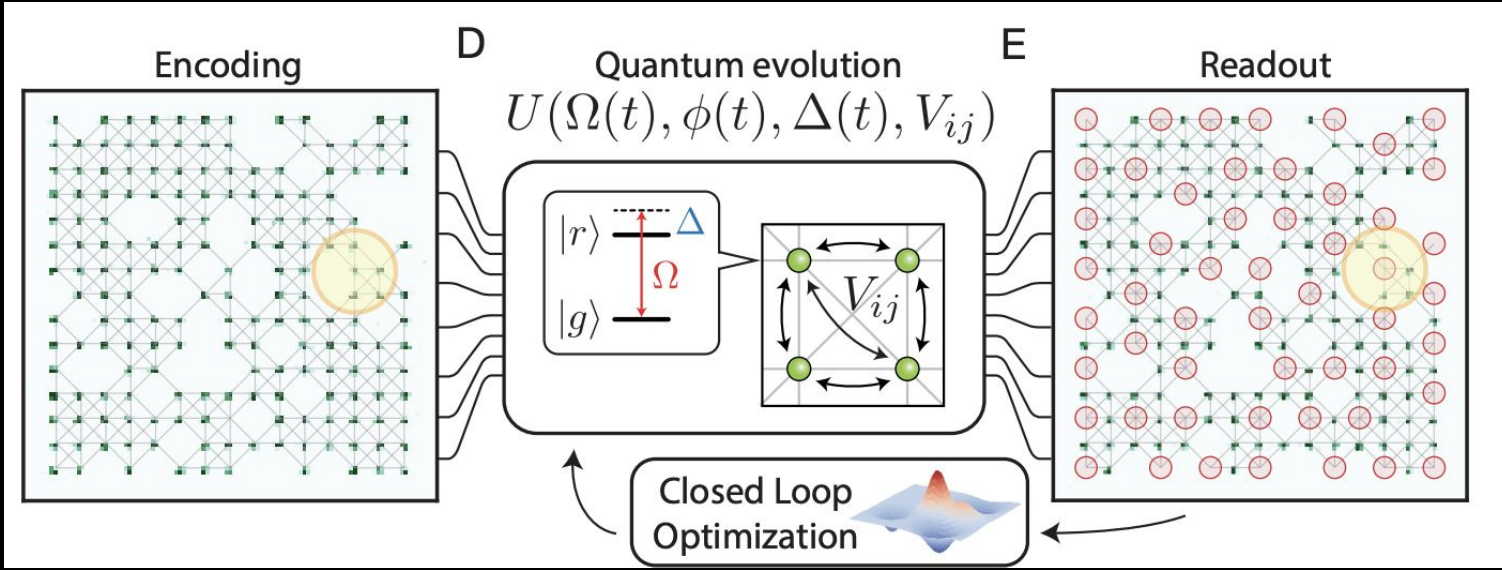
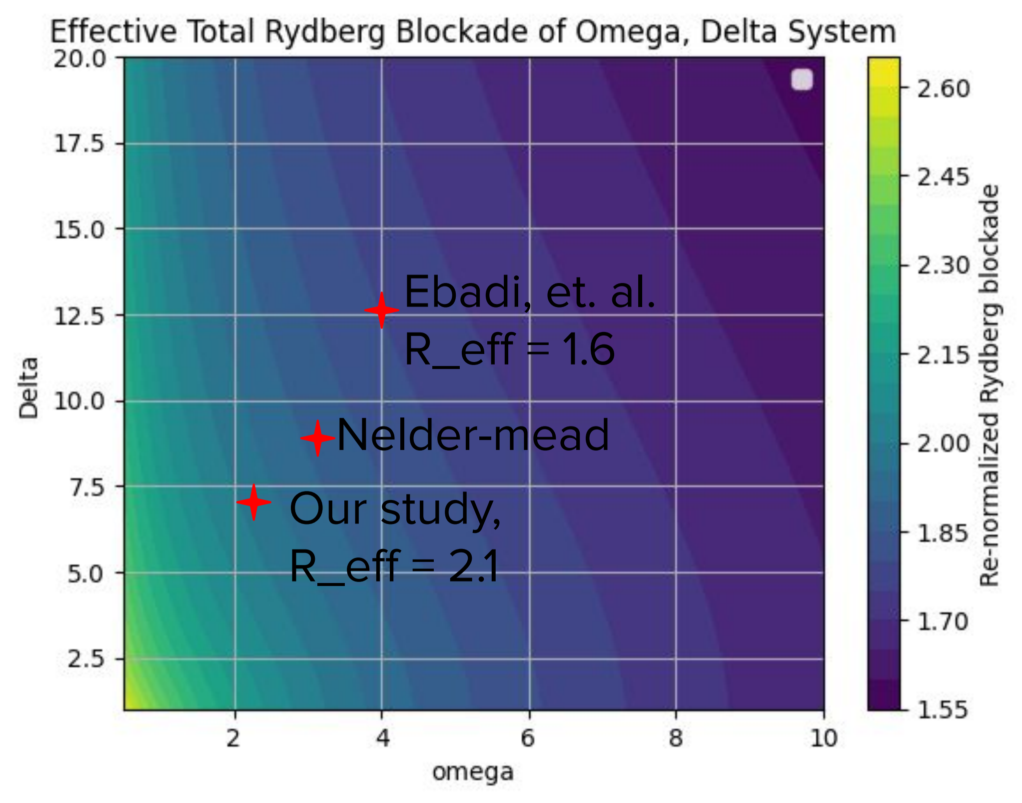


Fig. 1 Ebadi et. al.

Effective Rydberg Radius Function of Detuning, Rabi freq.



From first principles:

$$\frac{\int_0^{\Delta_f} (S_R(\Delta(t),\Omega) R_B(\Delta(t),\Omega) dt)}{\int_0^{\Delta_f} S_R(\Delta(t),\Omega) dt}$$

What range is feasible on a quantum computer?

For a small Rabi Frequency, small mixing times and decoherence times will drastically impact our qubits as we scale the problem

Given our effective Rydberg Blockade averaged estimate, we need a really small Rabi to reach r=3. Thus we claim this is likely unfeasible as our lattice scales beyond n=10 for current hardware limitations

However, r=(20^0.25)>2 is in the feasible range (Rabi ~ 2MHz, Detuning ~5 MHz), **so we expect r=2 is possible as we scale to a 15x15 lattice.**

Further optimizations

Discussed a method based on individual addressing (existing technology as in Mennssen et. al.) to choose a smart initial state

Choose initial state with full grid solution, then drop detunings of atoms which we want to drop out of lattice as our adiabatic approach

We expect the time of algorithm to be much smaller, thus errors will scale accordingly

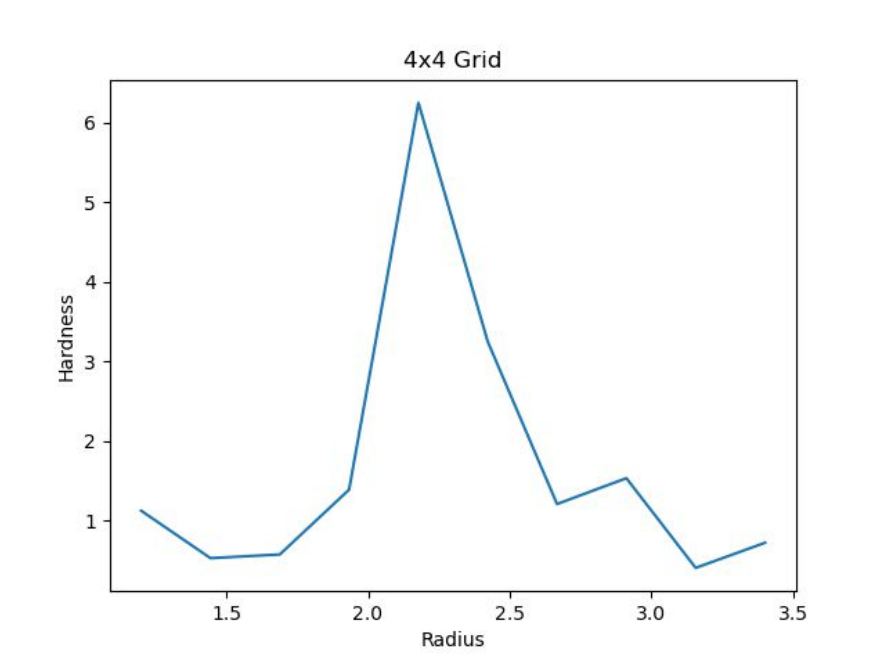
If qubits can be brought within 2.85 um we can then expect to robustly solve $r=3$ case for $n>20$.

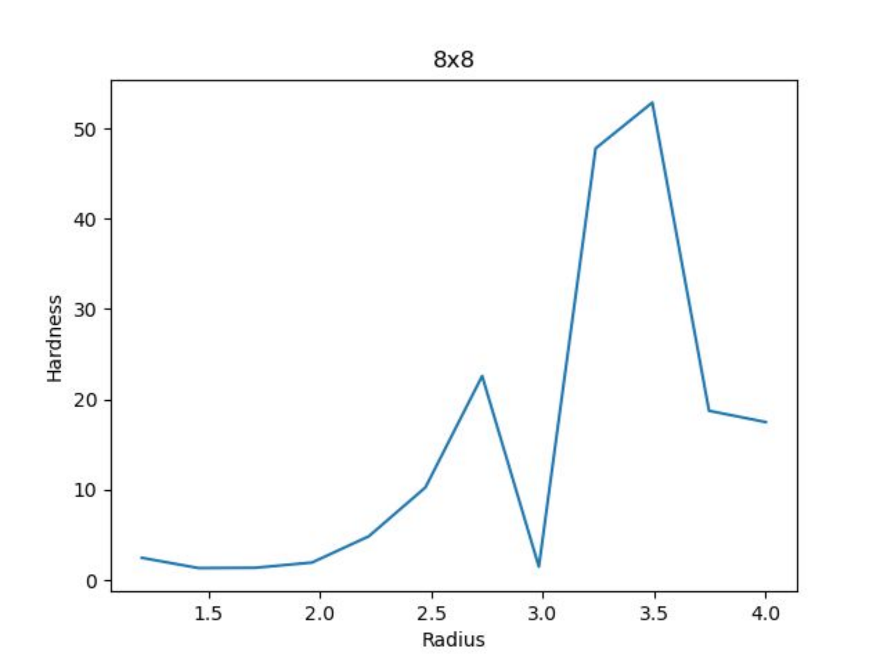
Programming and Experiments

The Hardest Problems (Classically)

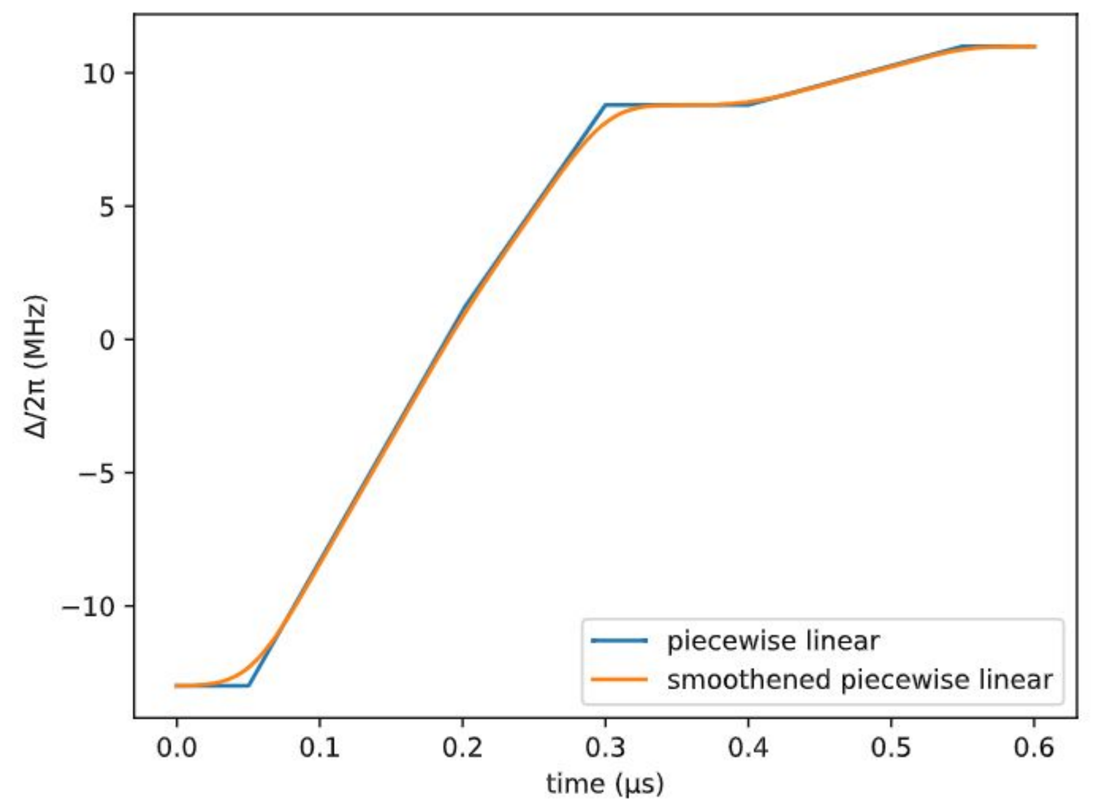
$$\mathbb{H} = \frac{D_{\mathrm{MIS}-1}}{|\mathrm{MIS}| \cdot D_{\mathrm{MIS}}},$$

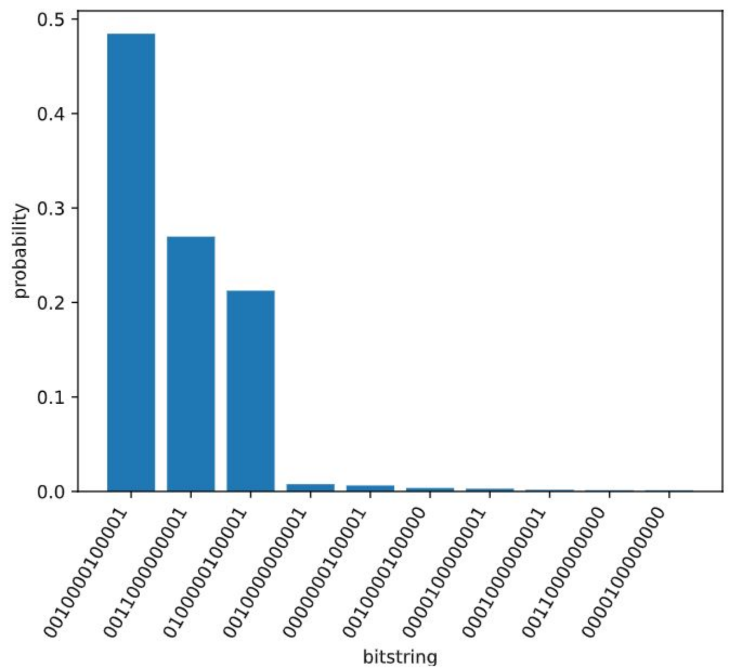
Andrist et al. 2023

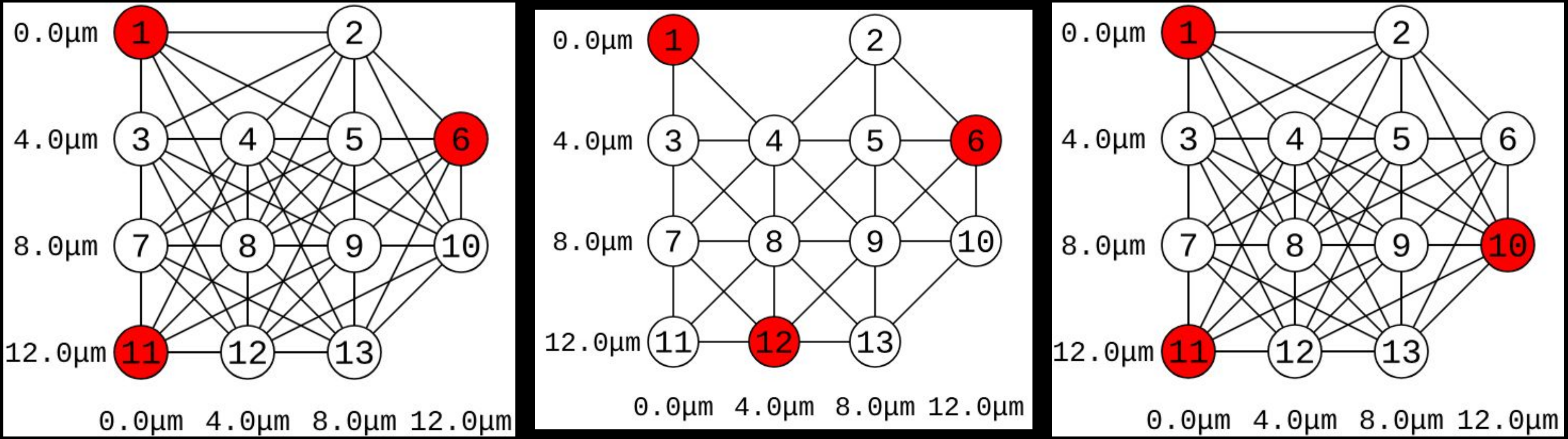




Easy Nelder Mead (4x4, Rb/a = 2.51)






0.0µm
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12.0µm
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8.0µm
12.0µm
1 2 3 4 5 6 7 8 9 10 11 12 13
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0.0µm
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12.0µm
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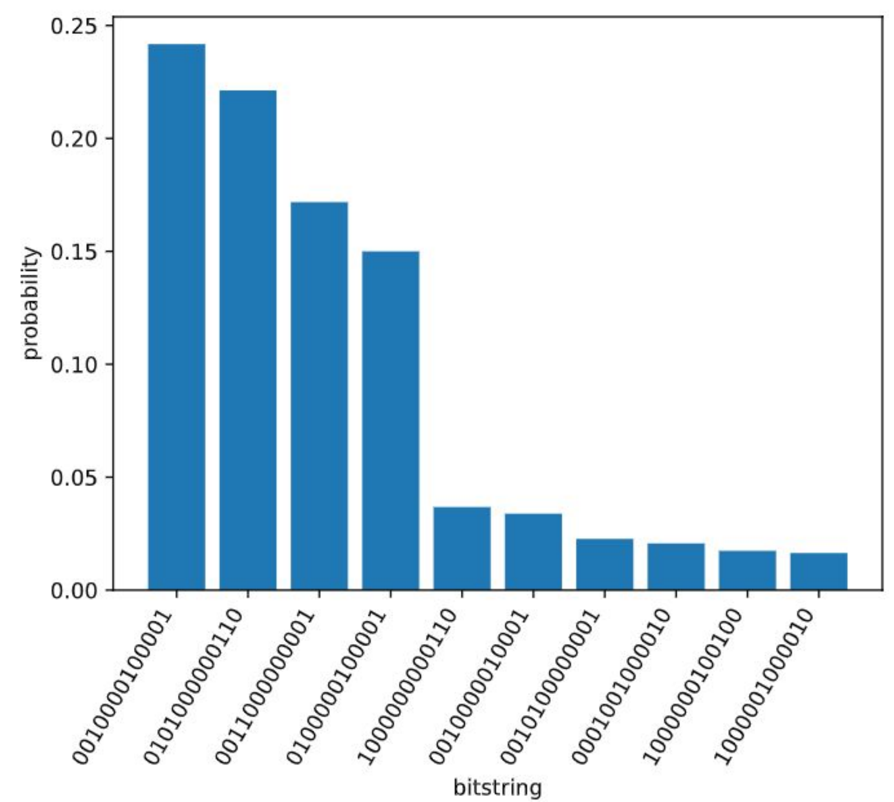
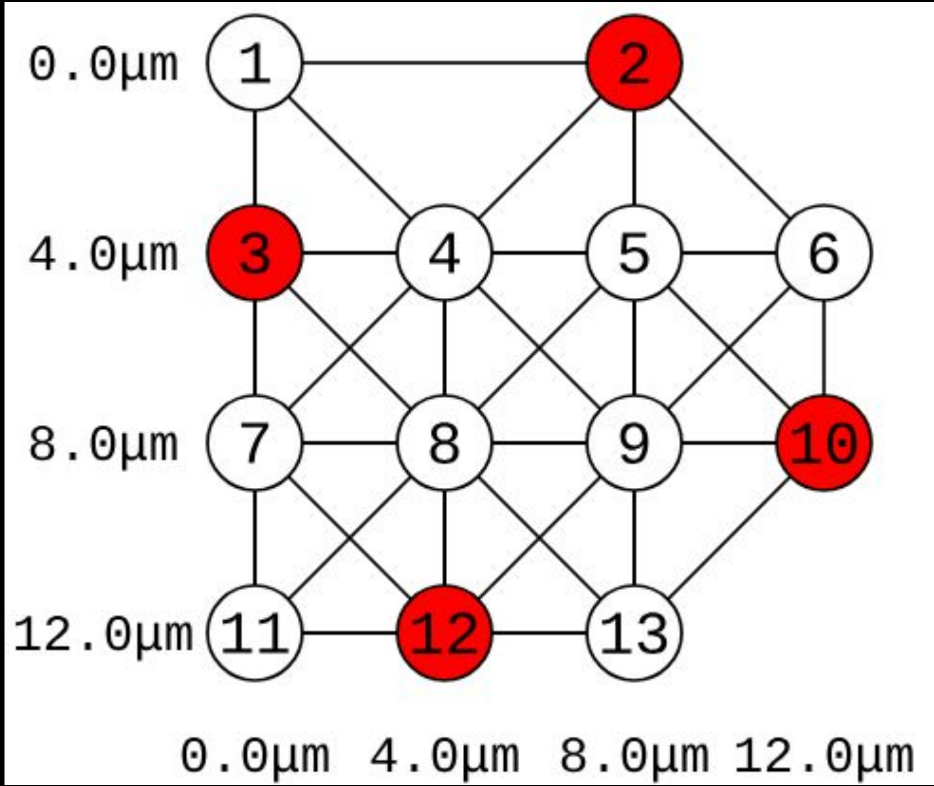
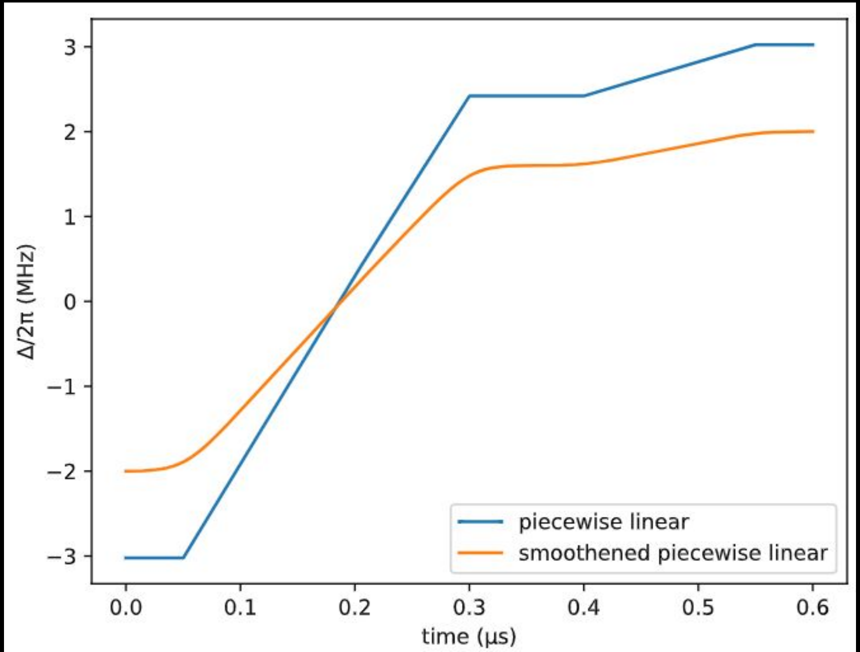
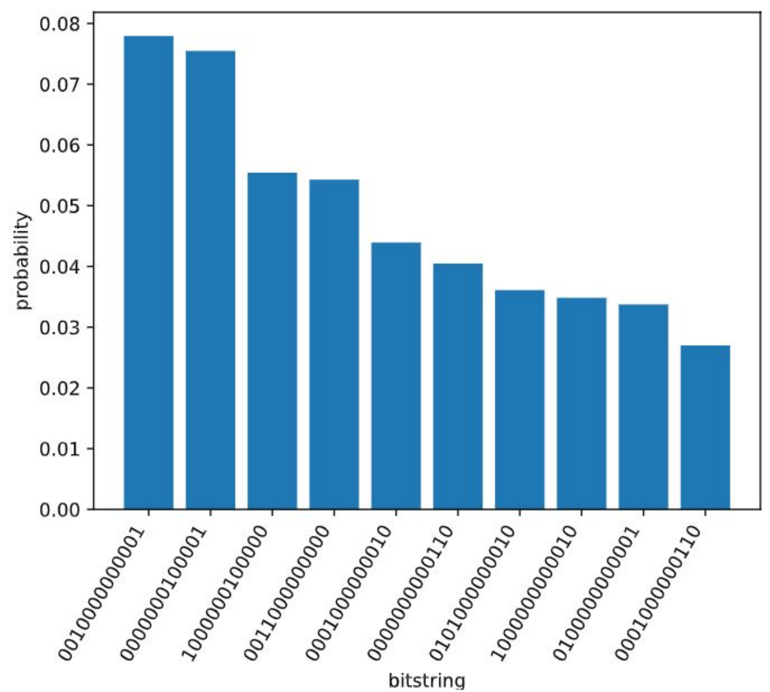
Hard Nelder Mead (4x4, Rb/a = 2.11)

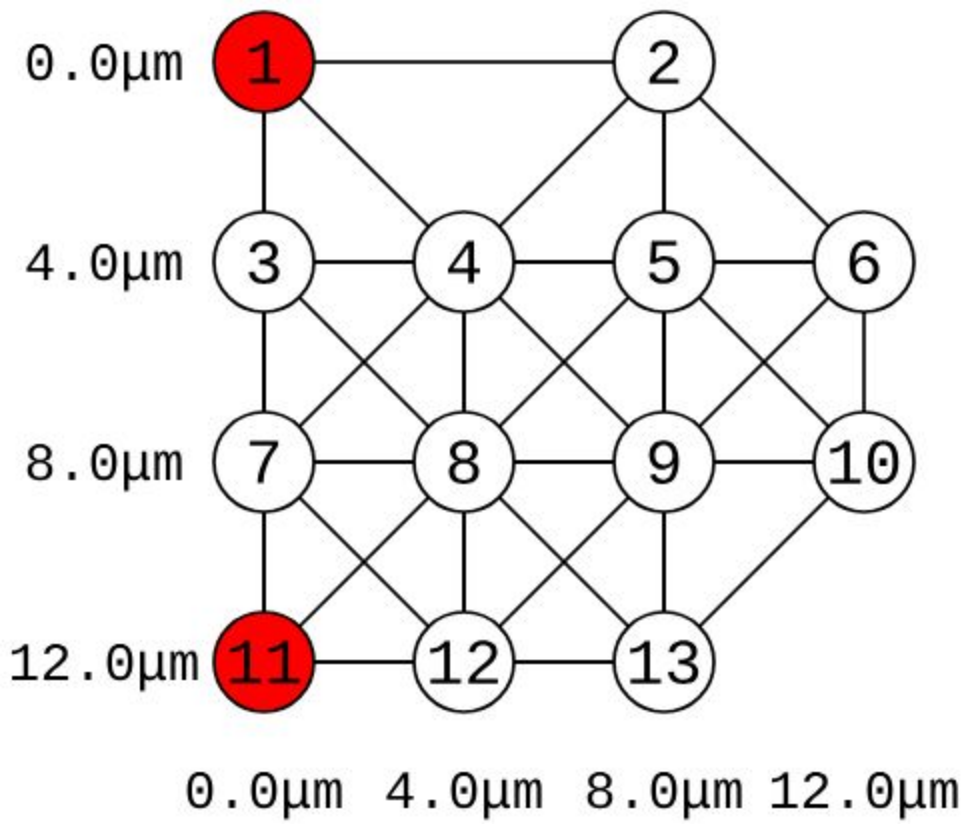




Hard Nelder Mead with Limitations






0.0µm
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12.0µm

Loss Function: Rydberg Density Sum

$$\left\langle \sum_{i \in \mathrm{IS}} |1\rangle_i \langle 1| \right\rangle$$

=

Weighted average number of Rydberg atoms in eigenstates corresponding to IS configurations.

Grid Search Results

<u>Unrestricted</u>

Searched

Ω from $3*2\pi$ to $5*2\pi$

Δ from $10*2\pi$ to $15*2\pi$

Optimal Values

$\Omega = 4*2\pi$

$\Delta = 11.75*2\pi$

<u>Limited to Aquila Specs</u>

Searched

Ω from 2 MHz to 2.5 MHz

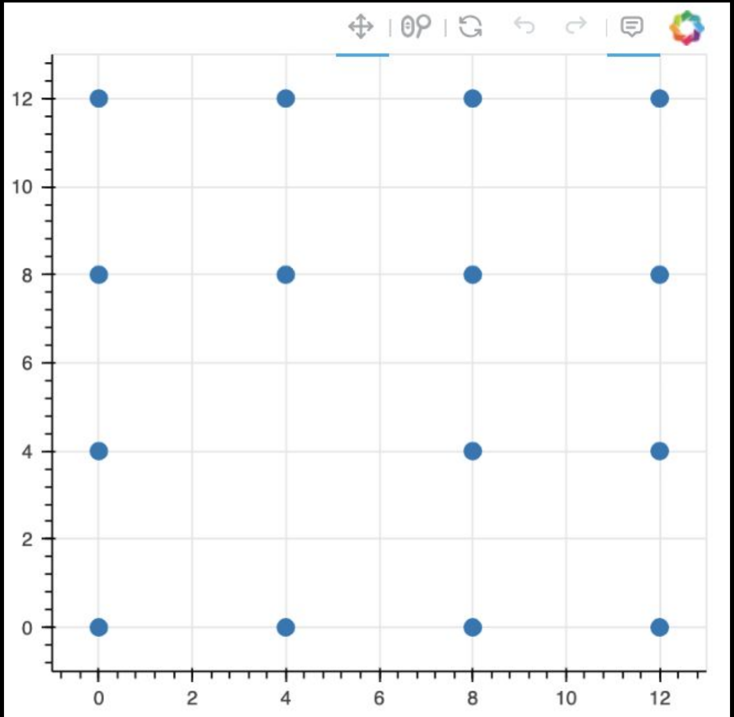
Δ from 0 MHz to 20 MHz

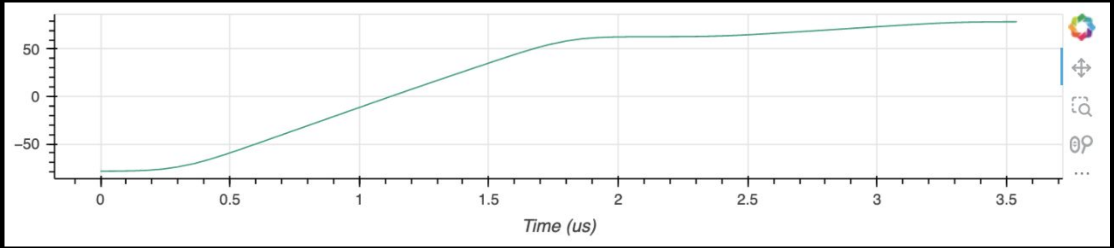
Optimal Values

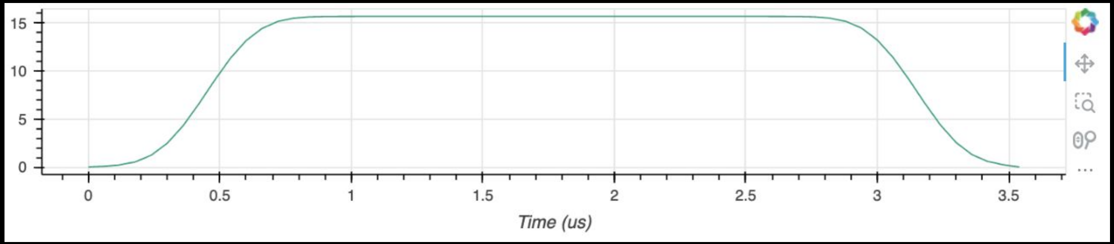
Ω = 2.5 MHz

Δ = 6.8 MHz

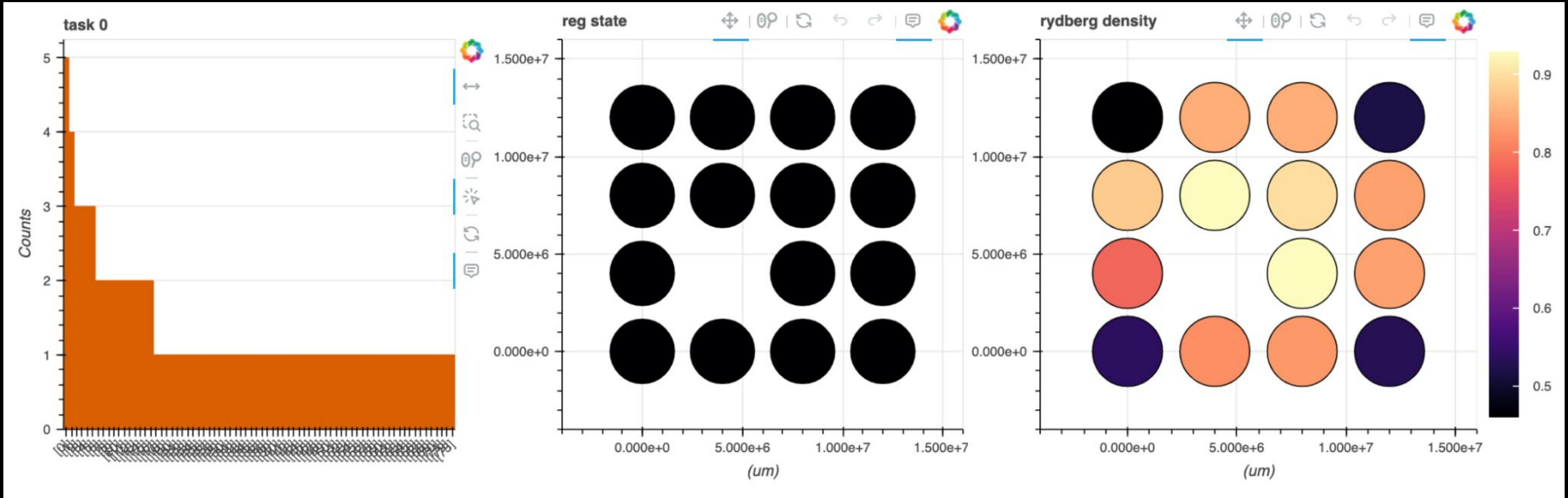
Solving a 4x4 Lattice on Aquila - Setup







Solving a 4x4 Lattice on Aquila - Results



Highest Probability State (5%):

0 1 1 0
1 1 1 1
1 1 1 1
0 1 1 0

Rydberg Density Sum: 3.5

Plan For Larger Grids (when Aquila is back)

1) Use parameters (Ω and Δ) from classical optimization (Since we couldn't use variational algorithm on Aquila)
2) Choose problem hardness based on classical annealing simulation.
3) Run on Aquila
4) (Optional) Postprocessing of Aquila solution using greedy algorithm

Business Applications

Business Applications: **Selecting Features in ML model**

- Each feature in a dataset can be thought of as a vertex in a graph.
- An edge between two vertices (features) could represent a redundancy or a strong correlation between those features.
- The objective of finding an MIS in this graph would be akin to selecting a subset of features that are as independent from each other as possible (i.e., not connected by edges).
- This "independence" minimizes redundancy and might help in building more generalized models.

Selecting Features in ML

1. Healthcare and Medical Diagnosis

Enhanced feature selection could lead to more accurate and early diagnosis of diseases by pinpointing relevant biomarkers from complex datasets, such as genomic data or medical imaging. It could improve patient outcomes and help in personalized medicine by identifying which treatments are most likely to work for specific patients based on their unique data profiles.

2. Finance and Economics

In financial modeling, better feature selection could lead to more robust risk assessment models, fraud detection, and algorithmic trading strategies. It can help in understanding market trends and economic forecasting by focusing on the most predictive indicators.

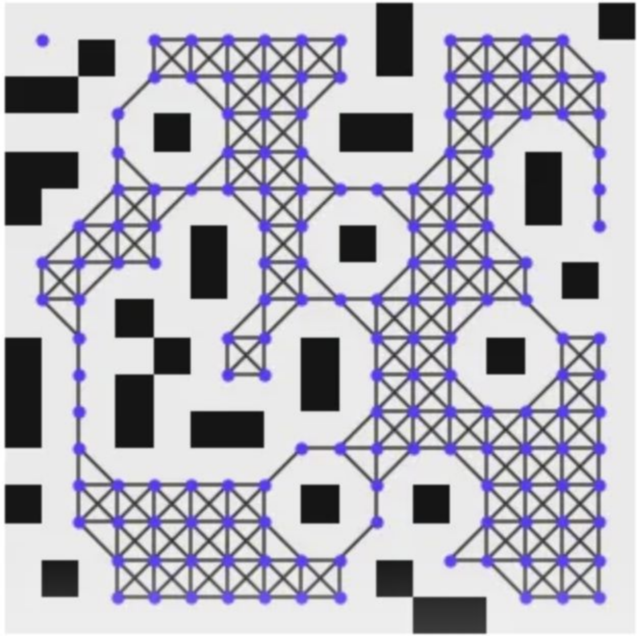
3. Environmental Science

Improved feature selection could enhance climate modeling and the prediction of extreme weather events. It could also improve the monitoring of environmental degradation, like deforestation and pollution, by efficiently analyzing satellite imagery and sensor data.

VLSI

- In VLSI design, floorplanning is the task of arranging large-scale components on a chip.
- The goal is to optimize the layout for various objectives such as minimizing the total area of the chip, reducing the length of interconnects (which impacts the speed and power consumption), and avoiding thermal hotspots.
- The MIS problem can be applied to model component placement where each vertex in a graph represents a component, and edges represent constraints such as incompatibility or interference between components.
- An independent set in this graph could represent a set of components that can be placed together without violating these constraints, aiding in the identification of optimal placement strategies.

VLSI



the 157-node, 420-edge VLSI problem

1. Improved Computer and Electronics Performance

More efficient VLSI designs would lead to faster processors and memory, smaller and more powerful electronics, and reduced power consumption, which is crucial for mobile devices and can lead to advances in everything from smartphones to supercomputers.

2. Advancements in Quantum Computing

VLSI techniques are integral to developing quantum computers. Efficient VLSI solutions could accelerate the creation of large-scale, reliable quantum circuits, potentially revolutionizing computing by solving problems that are currently intractable.

3. Medical Devices and Diagnostics

VLSI is key to the miniaturization of medical devices such as implants and wearables. Improved VLSI design could lead to more sophisticated, smaller, and less invasive medical devices, as well as portable diagnostic equipment with better performance.

Sources

- Github Assets and References
- https://queracomputing.github.io/Bloqade.jl/dev/tutorials/5.MIS/main/

Edabi, A., et al. (2022). Quantum Optimization of Maximum Independent Set using Rydberg Atom Arrays. *arXiv*, arXiv:2202.09372.
Andrist, N., et al. (2023). Hardness of the Maximum Independent Set Problem on Unit-Disk Graphs and Prospects for Quantum Speedups. *arXiv*, arXiv:2307.09442.
Menssen, A. et al. (2023). Scalable photonic integrated circuits for high-fidelity light control. *Optica*, 10(10), 1366. https://doi.org/

Thanks for Listening!