Alex Leonardi

QuEra Challenge (Scrapwork)

iQuHack 2025

Native gateset

Z-rotation $$R_z(\varphi) = \begin{pmatrix} e^{-i\frac{\varphi}{2}} & 0 \\ 0 & e^{i\frac{\varphi}{2}} \end{pmatrix}$$

XY-rotation $$U_{xy}(\alpha,\theta) := \cancel{\exp\left(-i\frac{\pi}{4}\left(\sigma_j^x\sigma_k^x + \sigma_j^y\sigma_k^y\right)\right)} = \begin{pmatrix} \cos\frac{\theta}{2} & -ie^{-i\alpha}\sin\left(\frac{\theta}{2}\right) \\ -ie^{i\alpha}\sin\left(\frac{\theta}{2}\right) & \cos\frac{\theta}{2} \end{pmatrix}$$

CZ gate $$CZ = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

$U_3(\theta,\varphi,\lambda)$ set $\lambda = -\varphi$

$$R(\hat{n},\theta) = \begin{pmatrix} \cos\theta + \cos^2\alpha(1-\cos\theta) & \cos\alpha\sin\alpha(1-\cos\theta) & \sin\alpha\sin\theta \\ \cos\alpha\sin\alpha(1-\cos\theta) & \cos\theta + \sin^2\alpha(1-\cos\theta) & -\cos\alpha\sin\theta \\ -\sin\alpha\sin\theta & \cos\alpha\sin\theta & \cos\theta \end{pmatrix}$$

$\sin\theta_n\cos\phi_n,\ \sin\theta_n\sin\phi_n,\ \cos\theta_n$

$\cos\alpha,\ \sin\alpha,\ 0$

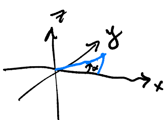
$\theta_n = \frac{\pi}{2} \Rightarrow \cos\theta_n = 0$

$\sin\theta_n = 1$

$\phi_n = \alpha$

$R\left(\theta_n = \frac{\pi}{2},\ \phi_n = \alpha,\ \theta\right) = R(z,\hat{\alpha})R(y,\hat{\beta})R(z,\hat{\gamma})$

$$R(\hat{n},\theta) = \begin{pmatrix} \cos\theta + n_1^2(1-\cos\theta) & n_1n_2(1-\cos\theta) - n_3\sin\theta & n_1n_3(1-\cos\theta) + n_2\sin\theta \\ n_1n_2(1-\cos\theta) + n_3\sin\theta & \cos\theta + n_2^2(1-\cos\theta) & n_2n_3(1-\cos\theta) - n_1\sin\theta \\ n_1n_3(1-\cos\theta) - n_2\sin\theta & n_2n_3(1-\cos\theta) + n_1\sin\theta & \cos\theta + n_3^2(1-\cos\theta) \end{pmatrix}$$



$\cos\left(\frac{\beta}{2}\right) = 0$

$\frac{\beta}{2} = 0 \Rightarrow \beta = 0$

$(\cos\alpha,\ \sin\alpha,\ 0)$

$$R(\hat{n},\theta) = \begin{pmatrix} \cos\theta + n_1^2(1-\cos\theta) & n_1n_2(1-\cos\theta) & n_2\sin\theta \\ n_1n_2(1-\cos\theta) & \cos\theta + n_2^2(1-\cos\theta) & -n_1\sin\theta \\ -n_2\sin\theta & n_1\sin\theta & \cos\theta \end{pmatrix}$$

$n_3 = 0$

$n_1 = \cos\alpha$

$n_2 = \sin\alpha$

$$R(\hat{n},\theta) = \begin{pmatrix} \cos\theta + \cos^2\alpha(1-\cos\theta) & \cos\alpha\sin\alpha(1-\cos\theta) & \sin\alpha\sin\theta \\ \cos\alpha\sin\alpha(1-\cos\theta) & \cos\theta + \sin^2\alpha(1-\cos\theta) & -\cos\alpha\sin\theta \\ -\sin\alpha\sin\theta & \cos\alpha\sin\theta & \cos\theta \end{pmatrix}$$

$$R(\hat{n},\theta)=\begin{pmatrix} \cos\theta+\cos^2\alpha\,(1-\cos\theta) & \cos\alpha\sin\alpha\,(1-\cos\theta) & \sin\alpha\sin\theta \\ \cos\alpha\sin\alpha\,(1-\cos\theta) & \cos\theta+\sin^2\alpha\,(1-\cos\theta) & -\cos\alpha\sin\theta \\ -\sin\alpha\sin\theta & \cos\alpha\sin\theta & \cos\theta \end{pmatrix}$$

$$=\begin{pmatrix} \cos\alpha\cos\beta\cos\gamma-\sin\alpha\sin\gamma & -\cos\alpha\cos\beta\sin\gamma-\sin\alpha\cos\gamma & \cos\alpha\sin\beta \\ \sin\alpha\cos\beta\cos\gamma+\cos\alpha\sin\gamma & -\sin\alpha\cos\beta\sin\gamma+\cos\alpha\cos\gamma & \sin\alpha\sin\beta \\ -\sin\beta\cos\gamma & \sin\beta\sin\gamma & \cos\beta \end{pmatrix}$$

$$\boxed{U_{xy}(\alpha,\theta)=\begin{pmatrix} \cos\frac{\theta}{2} & -ie^{-i\alpha}\sin\left(\frac{\theta}{2}\right) \\ -ie^{i\alpha}\sin\left(\frac{\theta}{2}\right) & \cos\frac{\theta}{2} \end{pmatrix}}$$

$U_3(\theta,\varphi,\lambda)$ set $\lambda=-\varphi$ $\;=R(z,\hat{\alpha})R(y,\hat{\beta})R(z,\hat{\gamma})$

$$\begin{pmatrix} \cos(0) & -ie^{-i\alpha}\sin(0) \\ -ie^{0}\sin(0) & \cos(0) \end{pmatrix}=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$R\left(\theta_n=\tfrac{\pi}{2},\ \phi_n=\alpha,\ \theta\right)\rightarrow R_{xy}(\alpha,\theta)=R_z(\alpha)\,R_y(\theta)\,R_z(-\alpha)$$

$$R_y(\theta)=R_{xy}(\alpha=0,\ \theta)$$

$$U_3(\theta,\varphi,\lambda)=R_z(\theta)\,R_y(\varphi)\,R_z(\lambda)$$
$$=R_z(\theta)\,R_{xy}(\alpha=0,\varphi)\,R_z(\lambda)$$

$U_3(\theta,\varphi,\lambda)$ set $\lambda=-\varphi$

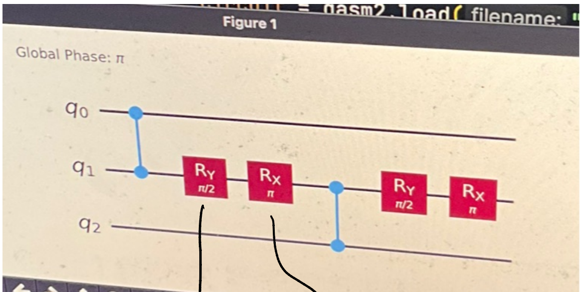
$$U_3(\theta,\varphi,-\varphi)=R_z(\theta)R_x(\varphi)R_z(-\varphi)$$
$$=R_z(\theta)U_{xy}(\alpha=0,\varphi)R_z(-\varphi)$$

$Z^{z} \; Z^{a} \; X^{x} \; Z^{-a}$

$Z^{0} \; Z^{0.5} \; X^{0.5} \; Z^{-0.5}$

$R_z(0.5) \; R_x(0.5) \; R_z(+0.5)$

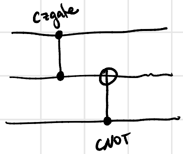
If doesn't work, double the R angles, ie $R_z(2\alpha)$ and $R_x(2\theta)$



ry(state, [1], $\frac{pi}{2}$)

rx(state, [1], pi)

Circuit 1



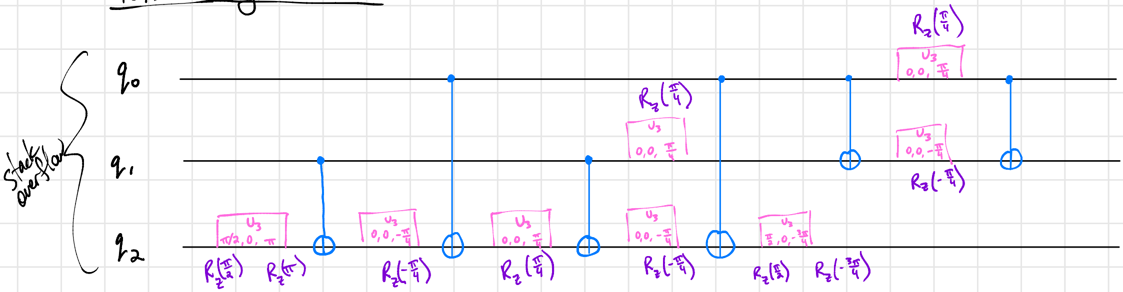
$$\left(\begin{array}{cc|cccc} 0 & 0 & & & & \\ 0 & 0 & & & & \\ \hline & & 1 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 1 \\ & & 0 & 0 & 1 & 0 \\ & & 0 & 1 & 0 & 0 \end{array}\right) \left(\begin{array}{cccc|cc} 1 & 0 & 0 & 0 & & \\ 0 & 1 & 0 & 0 & & \\ 0 & 0 & 1 & 0 & & \\ 0 & 0 & 0 & -1 & & \\ \hline & & & & 0 & 0 \\ & & & & 0 & 0 \end{array}\right)$$

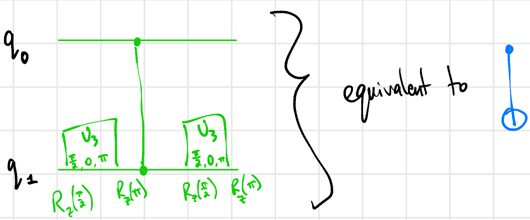
upside down CNOT

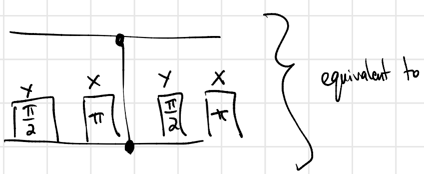
CZ gate

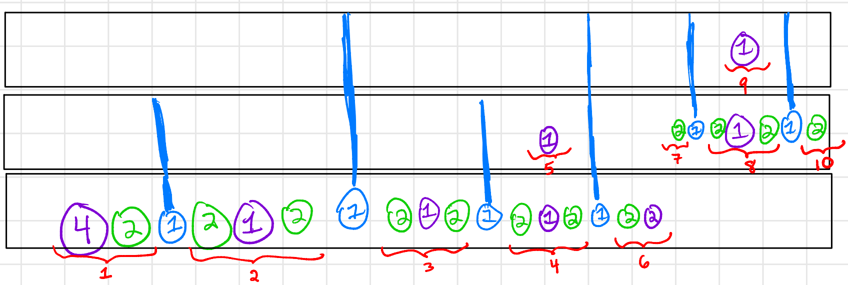
$$U_3(\theta,\varphi,\lambda) = R_z(\theta)\, R_y(\varphi)\, R_z(\lambda)$$
$$= R_z(\theta)\, R_{xy}(\alpha=0,\varphi)\, R_z(\lambda)$$

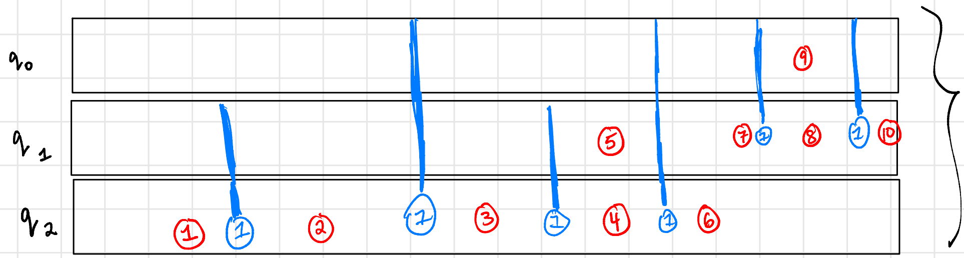
Toffoli using 6 CNOTs











Toffoli:

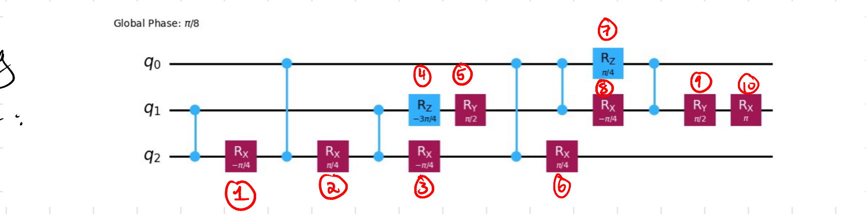
Blue is CZ

Red is R_z

$$U_3(\theta,\varphi,\lambda) = R_z(\theta)\, R_y(\varphi)\, R_z(\lambda)$$
$$= R_z(\theta)\, R_{xy}(\alpha=0,\varphi)\, R_z(\lambda)$$

Toffoli using 6 CNOTs

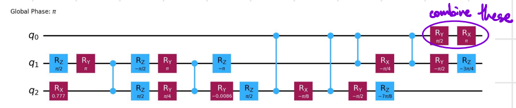
Alan using qiskit:



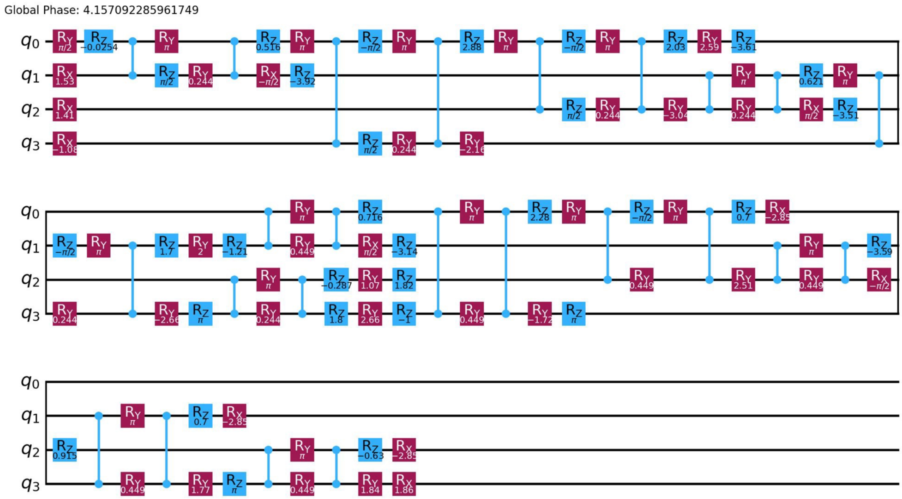
Bottom one is better for GHZ, since staircase is suboptimal for moving procedure

$$\text{PhaseXZ}(a=0.5,\ x=0.5,\ z=-8.88e^{-16})$$

Circuit 2 (Quantum Fourier)

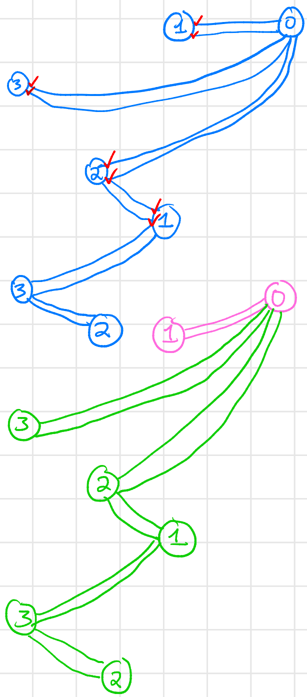


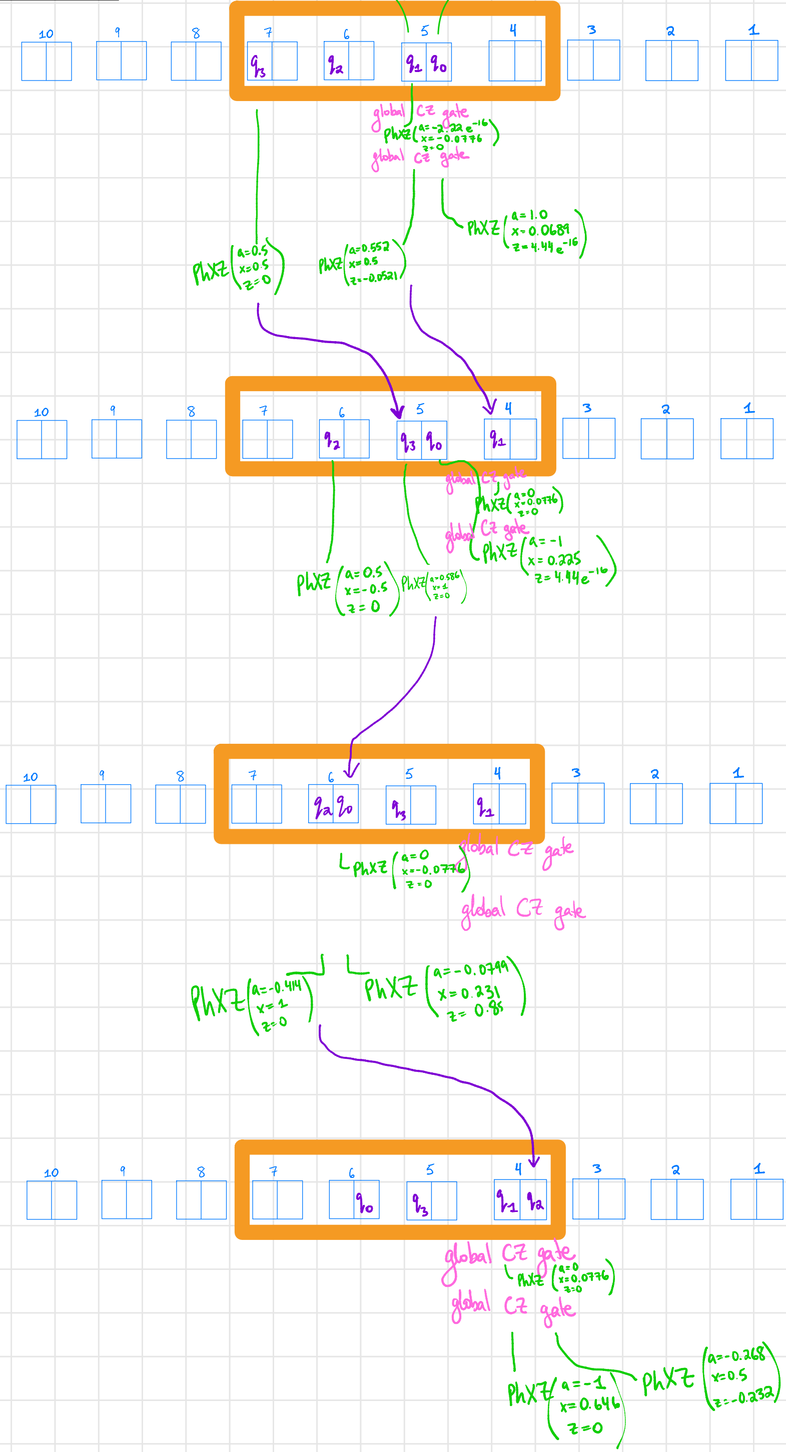
Circuit 3



Circuit 3 (Most Optimal)

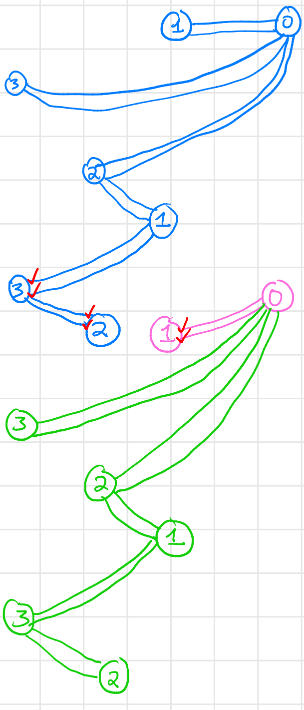
PART 1

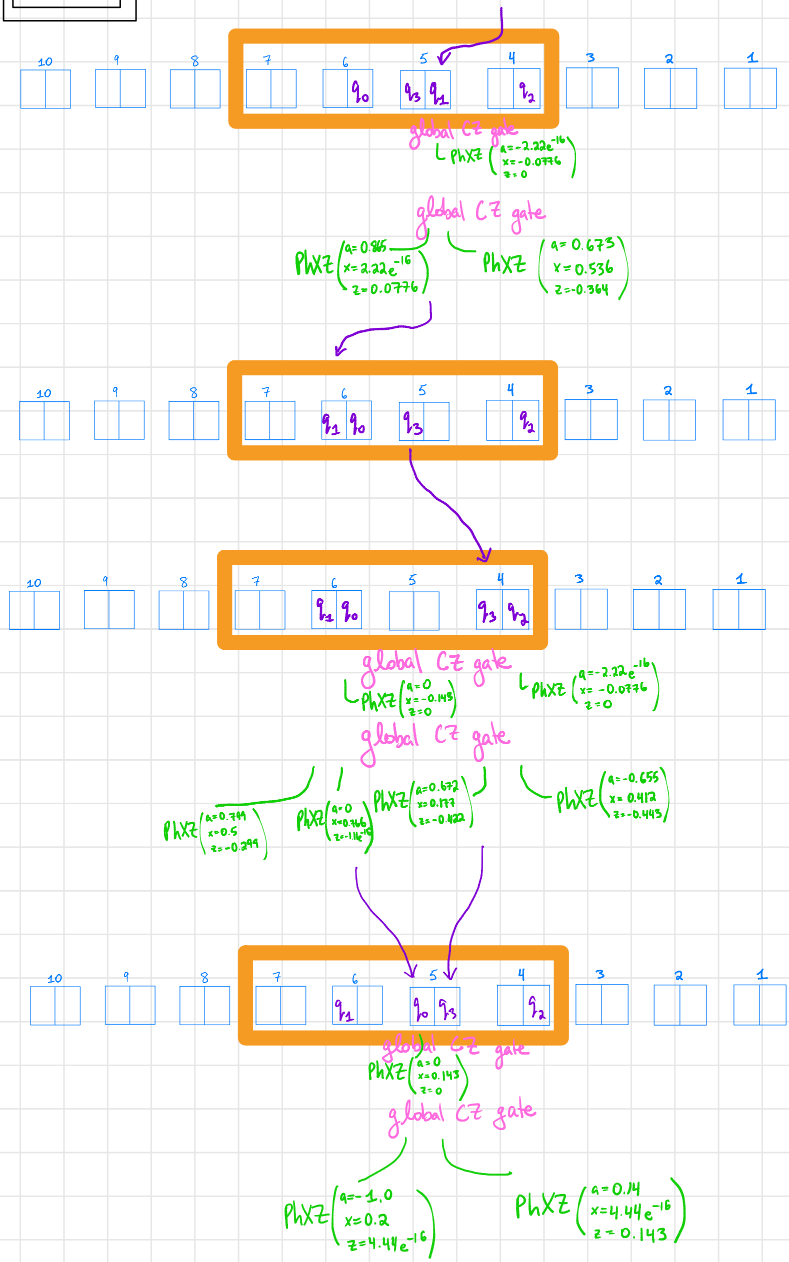




Circuit 3 (Most Optimal)

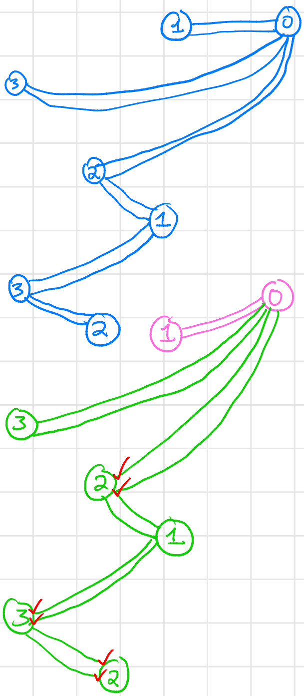
PART 2

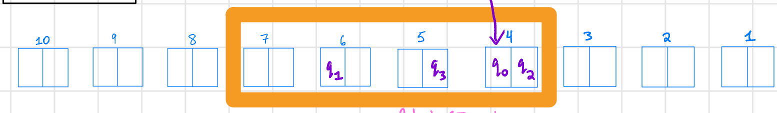




Circuit 3 (Most Optimal)

PART 3





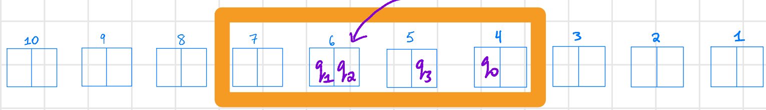
global CZ gate

PhXZ(a=0, x=0.143, z=0)

global CZ gate

PhXZ(a=0.411, x=0.473, z=0.181)

PhXZ(a=-0.0943, x=2.22e-16, z=0.143)



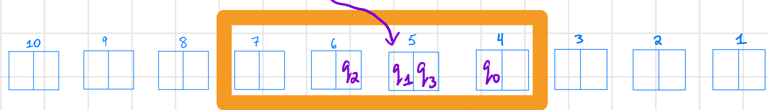
global CZ gate

PhXZ(a=4.44e-16, x=0.143, z=0)

global CZ gate

PhXZ(a=-4e-15, x=0.217, z=4e-15)

PhXZ(a=0.224, x=0.5, z=-0.724)



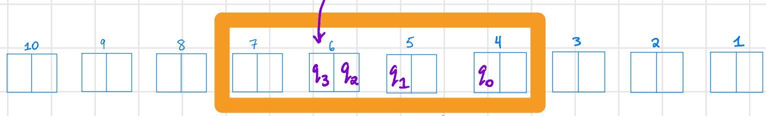
global CZ gate

PhXZ(a=0, x=0.143, z=0)

global CZ gate

PhXZ(a=0.411, x=0.473, z=0.181)

PhXZ(a=0.429, x=1, z=0)



global CZ gate

PhXZ(a=0, x=-0.143, z=0)

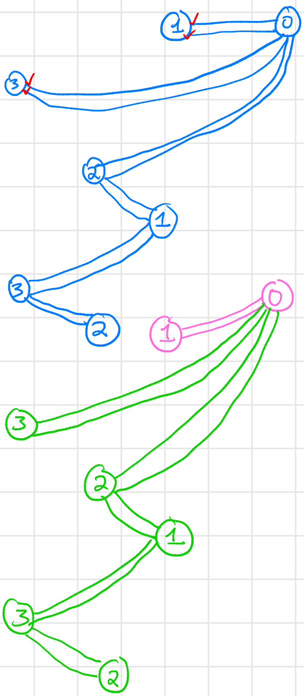
global CZ gate

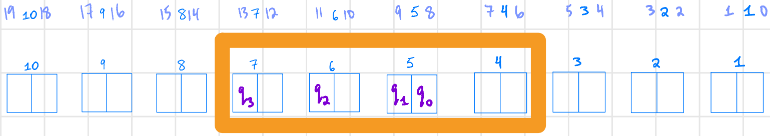
PhXZ(a=-0.107, x=0.907, z=-0.893)

PhXZ(a=0.569, x=0.437, z=0.689)

5AM PART 1

Circuit 3 (Most Optimal)





$R_y(0.5)$ on Slot 8

$R_z(-0.008)$ on slot 8

$R_x(0.486)$ on slot 9

$R_x(0.448)$ on slot 11

$R_x(-0.343)$ on slot 13

Global CZ

Y on SLOT 8 $R_y(\pi)$

S on SLOT 9 $R_z(\frac{\pi}{2})$ ✓

$Y(0.078)$ on SLOT 9 $R_y(0.078)$

GLOBAL CZ ✓

$R_z(0.164)$ on SLOT 8

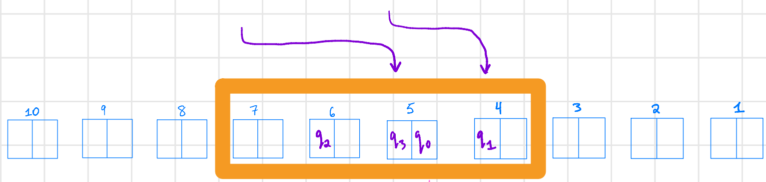
Y on SLOT 8 $R_y(\pi)$

$R_x(-0.5)$ on SLOT 9 ✓

$R_z(0.752)$ on SLOT 9 ✓

move q_1 from slot 9 into slot 7

move q_3 from slot 13 into slot 9



GLOBAL CZ gate

S^{-1} on SLOT 8 $R_z(-\frac{\pi}{2})$

Y on SLOT 8 $R_y(\pi)$

S on SLOT 9 $R_z(\frac{\pi}{2})$ ✓

$Y(0.078)$ on SLOT 9 $R_y(0.078\pi)$

global CZ gate

$R_z(0.915)$ on SLOT 8 ✓

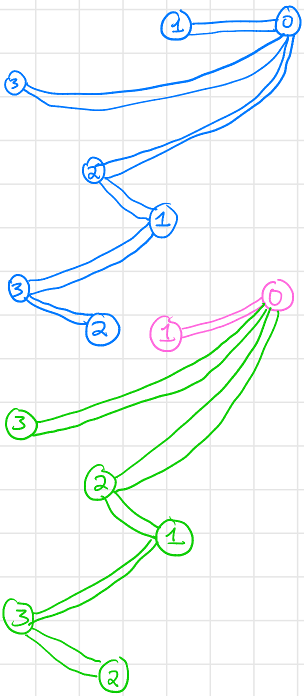
Y on SLOT 8 $R_y(\pi)$

$Y(-11/16)$ on SLOT 9 $R_y(-\frac{11}{16}\pi)$

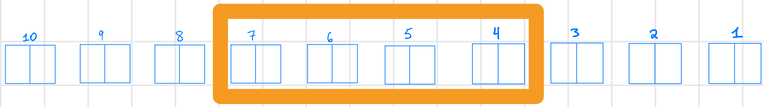
There are 20 more CZ's so there would be 10 more sets of instructions like this page!!!!

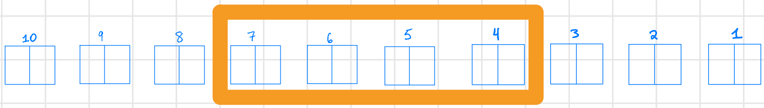
5AM PART 2

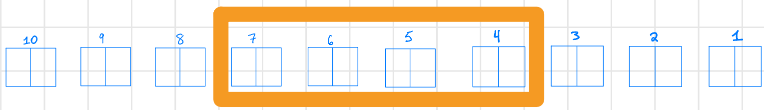
Circuit 3 (Most Optimal)

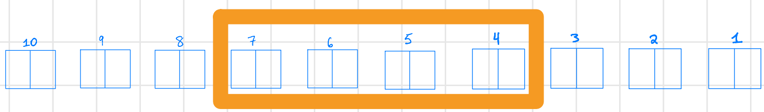


19 10 18 | 17 9 16 | 15 8 14 | 13 7 12 | 11 6 10 | 9 5 8 | 7 4 6 | 5 3 4 | 3 2 2 | 1 1 0



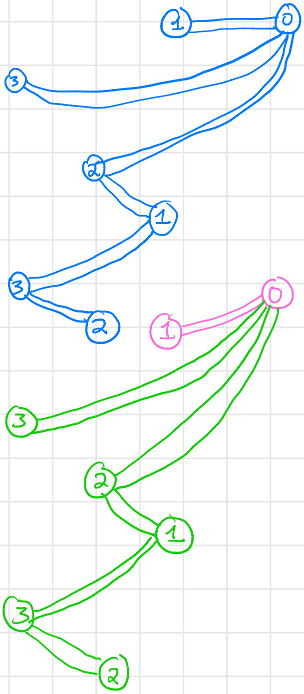




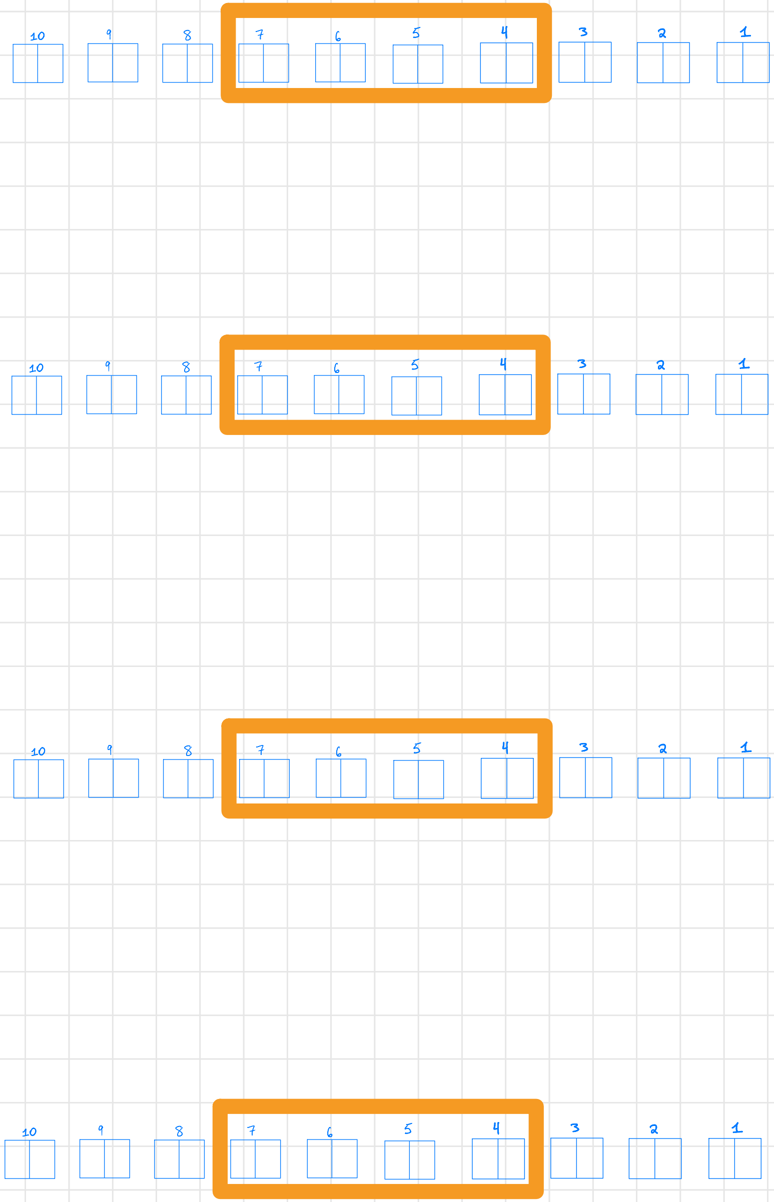


5AM PART 3

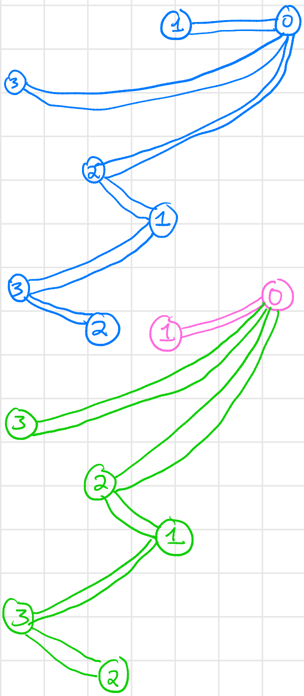
Circuit 3 (Most Optimal)

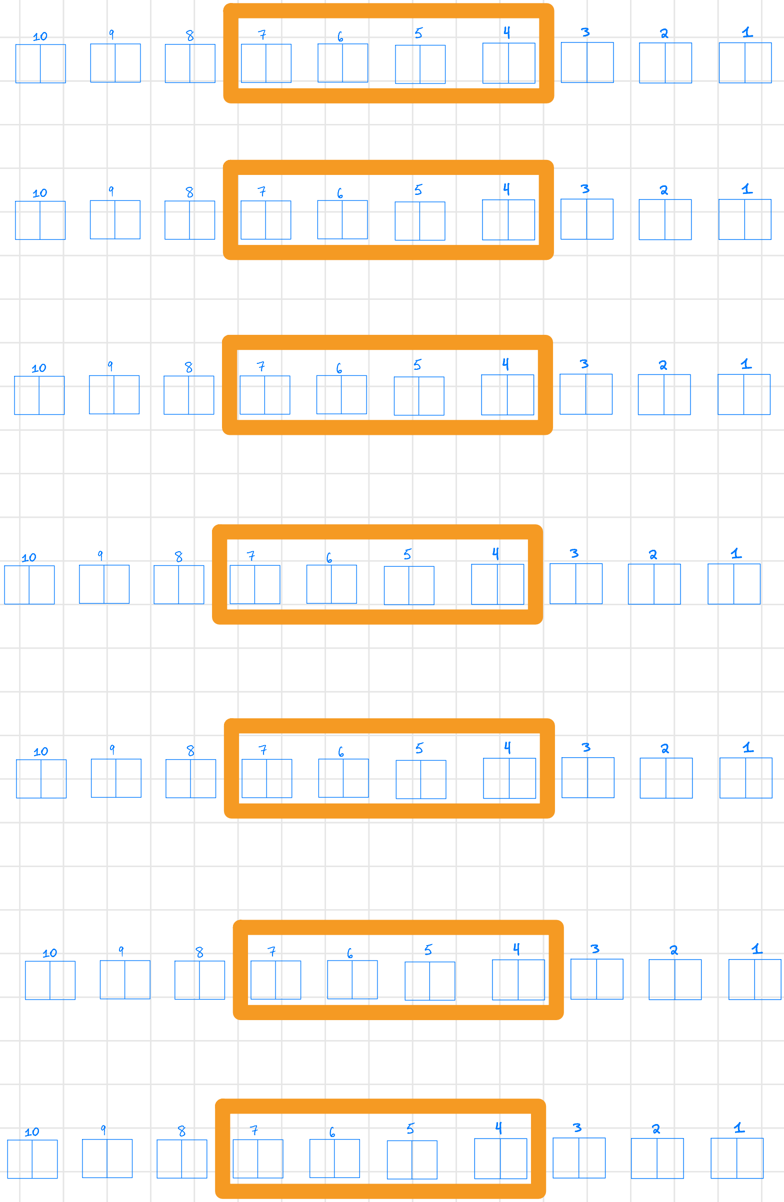


19 10 18 17 9 16 15 8 14 13 7 12 11 6 10 9 5 8 7 4 6 5 3 4 3 2 2 1 1 0

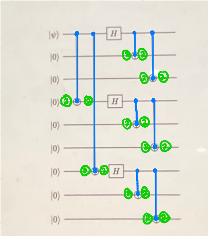


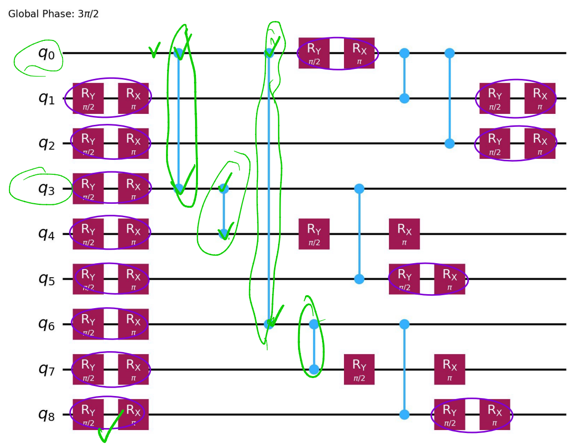
Circuit 3 (Most Optimal)





Circuit 4 Part 1





✓ initialize 9 qubits 0, 1, 2, . . . , 8

✓ apply R_{xy} globally

✓ move q_0 and q_3 into 1^{st} slot

✓ apply local R_{xy} to q_0 to undo Global

✓ apply CZ to 1^{st} slot

✓ move q_0 out of 1^{st} slot

✓ move q_4 into 1^{st} slot, q_0 and q_6 into 2^{nd} slot

combine global CZ

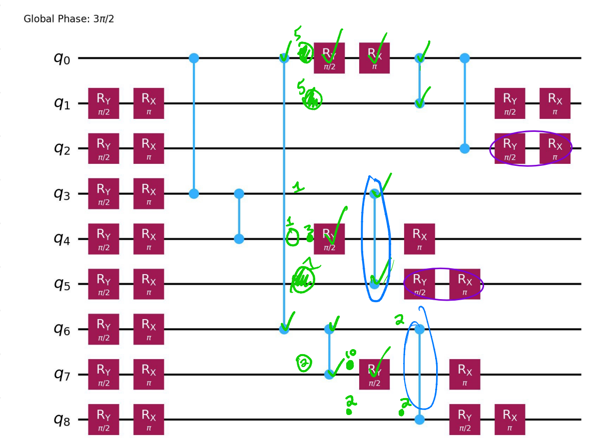
✓ apply CZ to 1^{st} slot

✓ apply CZ gate on 2^{nd} slot

✓ move q_0 out of 2^{nd} slot

✓ move q_7 into 2^{nd} slot

Circuit 4 Part 2



✓ apply CZ to 2nd slot

move q_4 out of 1st slot, move q_0 and q_1 into 5th slot

and move q_5 into 1st slot, move q_7 out of 2nd slot

✓ move q_4 into 3rd slot

✓ apply R_y to q_4 in 3rd slot

✓ apply R_y, R_x to q_0 in 5th slot

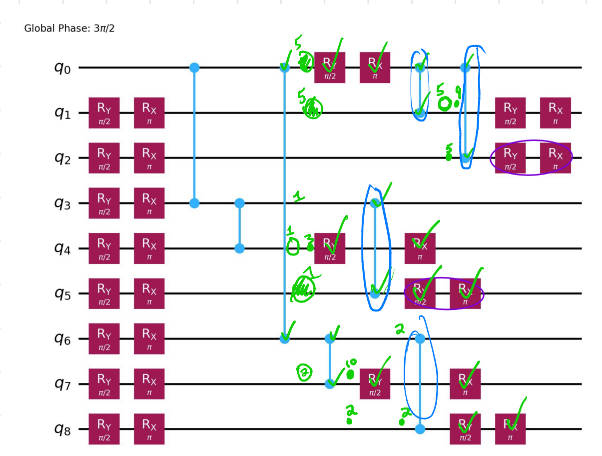
✓ apply CZ to 1st slot

✓ move q_7 into 10th slot, move q_8 into 2nd slot

✓ apply R_y to q_7 in 10th slot

combine in global CZ
✓ apply CZ to 2nd slot
✓ apply CZ to 5th slot

Circuit 4 Part 3



✓ apply R_x to q_4 in 3rd slot

✓ apply R_y, R_x to q_5 in 1st slot

✓ apply R_x to q_7 in 10th slot

✓ apply R_y, R_x to q_8 in 2nd slot

✓ move q_1 out of 5th slot

✓ move q_1 into 9th slot, move q_2 into 5th slot

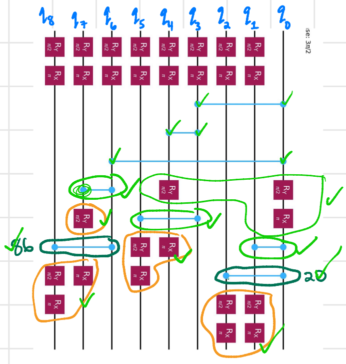
✓ apply CZ to 5th slot

✓ apply R_y, R_x to q_1 in 9th slot

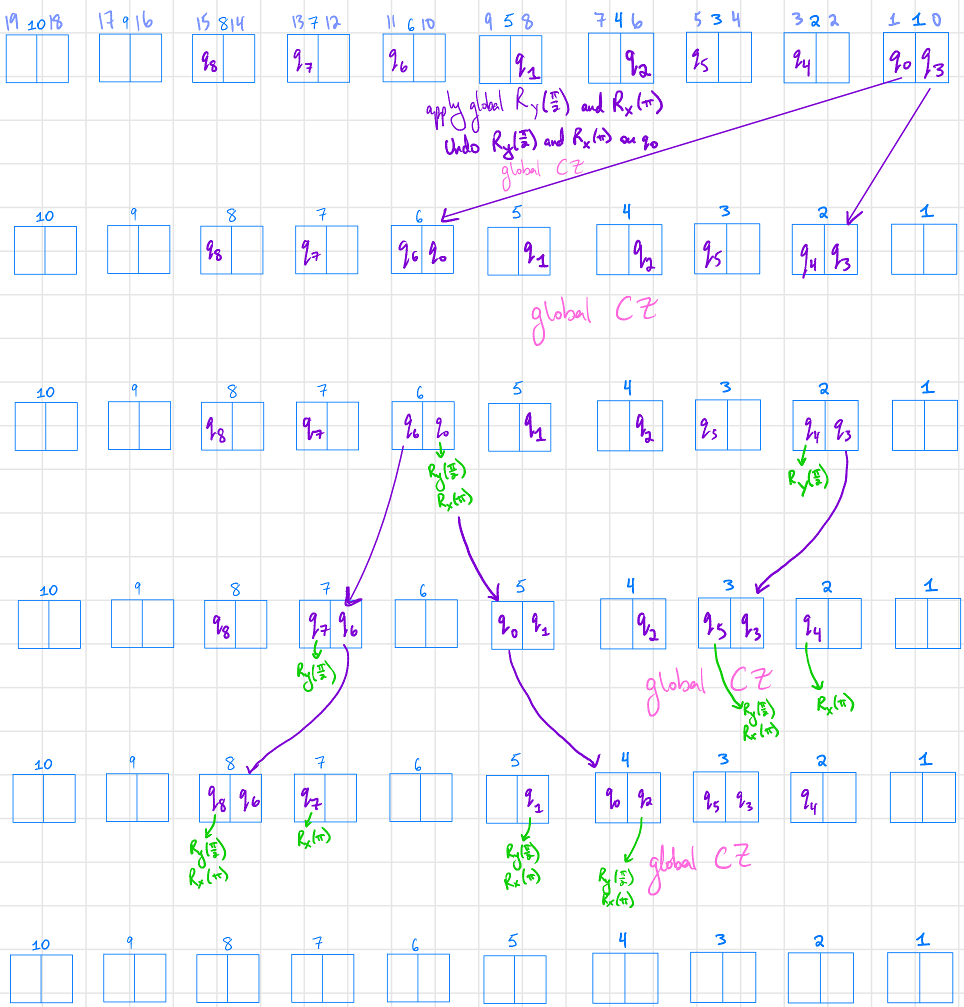
✓ apply R_y, R_x to q_2 in 5th slot

DONE (ie end of psuedo code)

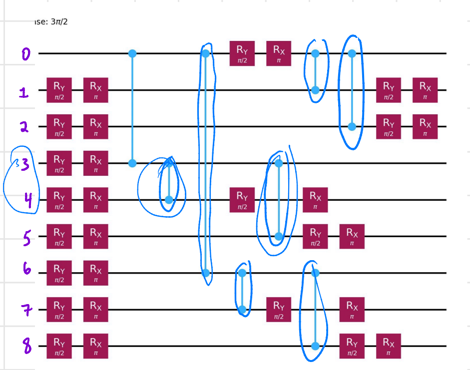
Circuit 4.2



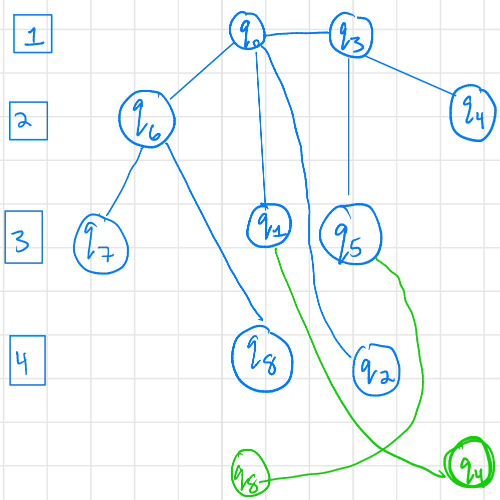
q8 q7 q6 q5 q4 q3 q2 q1 q0
86
20



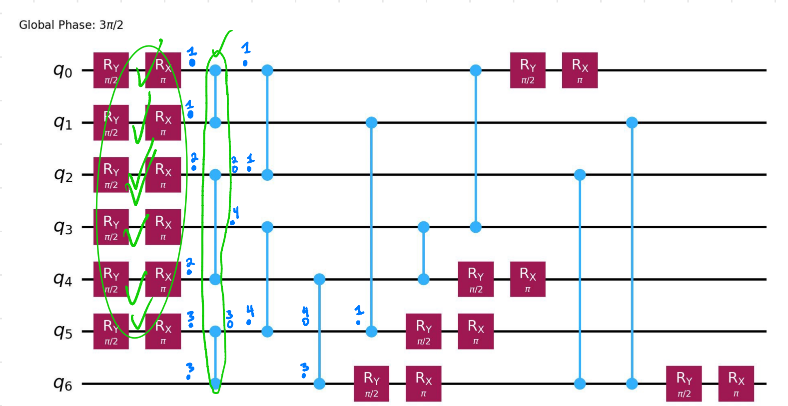
apply global Ry(π/2) and Rx(π)
Undo Ry(π/2) and Rx(π) on q0
global CZ
global CZ
global CZ
global CZ



$$16\left\{\begin{array}{l}|000\rangle\\ \vdots\\ |000\rangle\end{array}\right.$$



Circuit 5



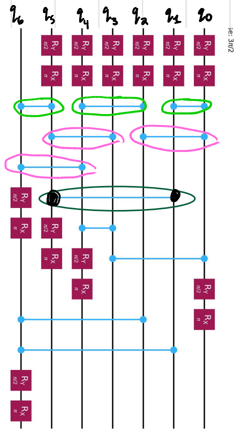
✓ apply R_y & R_x globally

✓ move q_0 & q_1 into 1st slot, move q_2 & q_4 into 2nd slot, move q_5 & q_6 into the 3rd slot

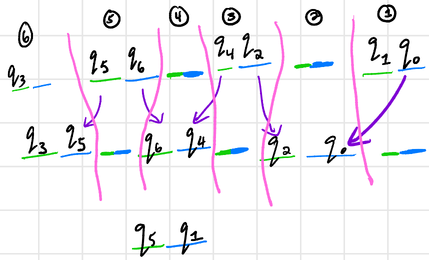
✓ apply inverse R_y & R_z to q_6 in 3rd slot

✓ global CZ { apply CZ to SLOTs 1, 2, and 3

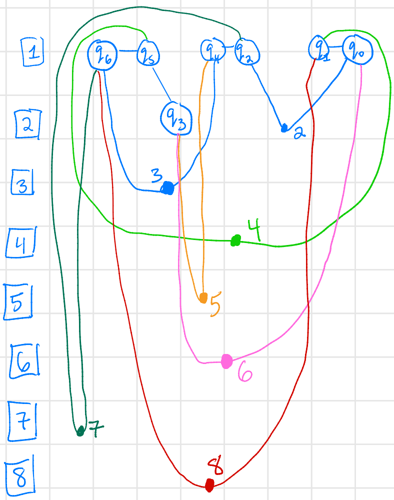
Circuit 5



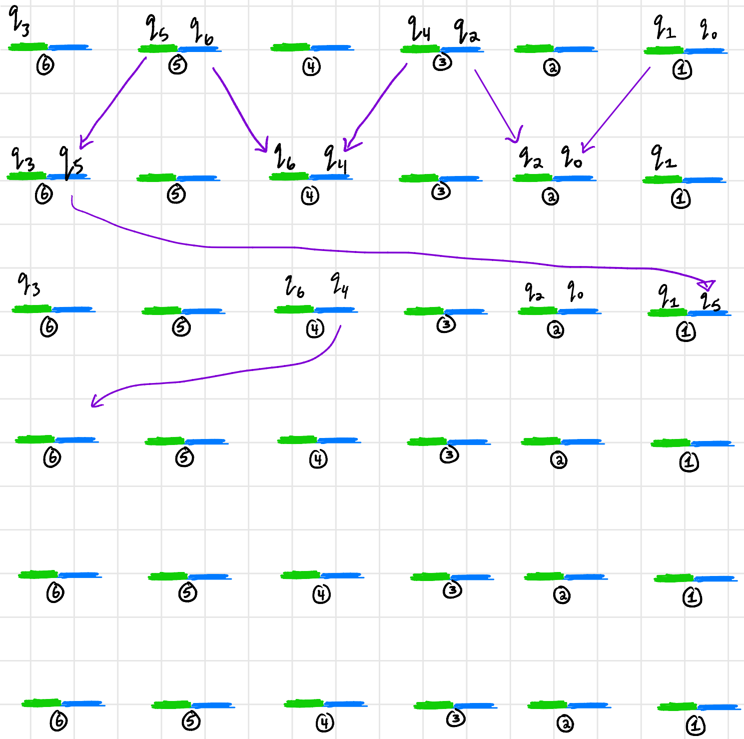
q6 q5 q4 q3 q2 q1 q0
Ry
Rx



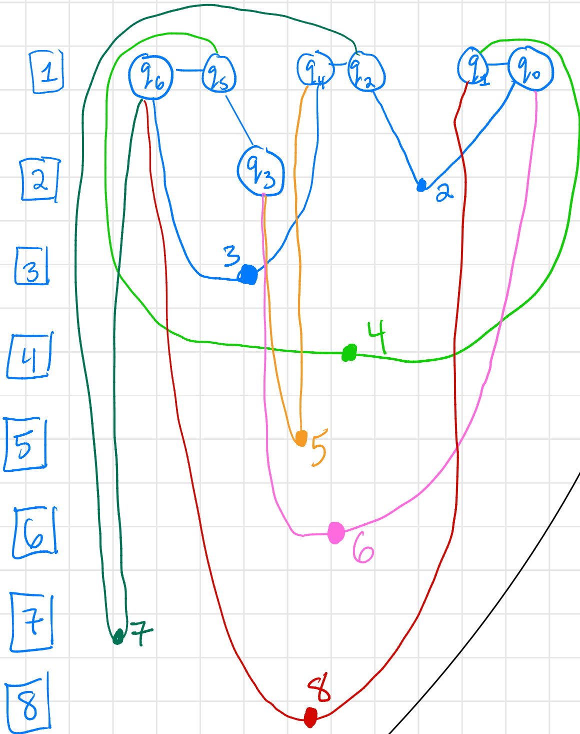
6 5 4 3 2 1
q3
q5 q6
q4 q2
q1 q0
q3 q5
q6 q4
q2
q0
q5 q1



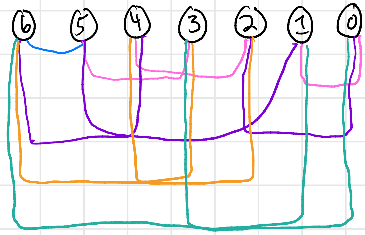
1
2
3
4
5
6
7
8
q6 q5 q4 q2 q1 q0
q3



q3
q5 q6
q4 q2
q1 q0
6 5 4 3 2 1
q3 q5
q6 q4
q2 q0
q1
q6 q4
q2 q0
q1 q5



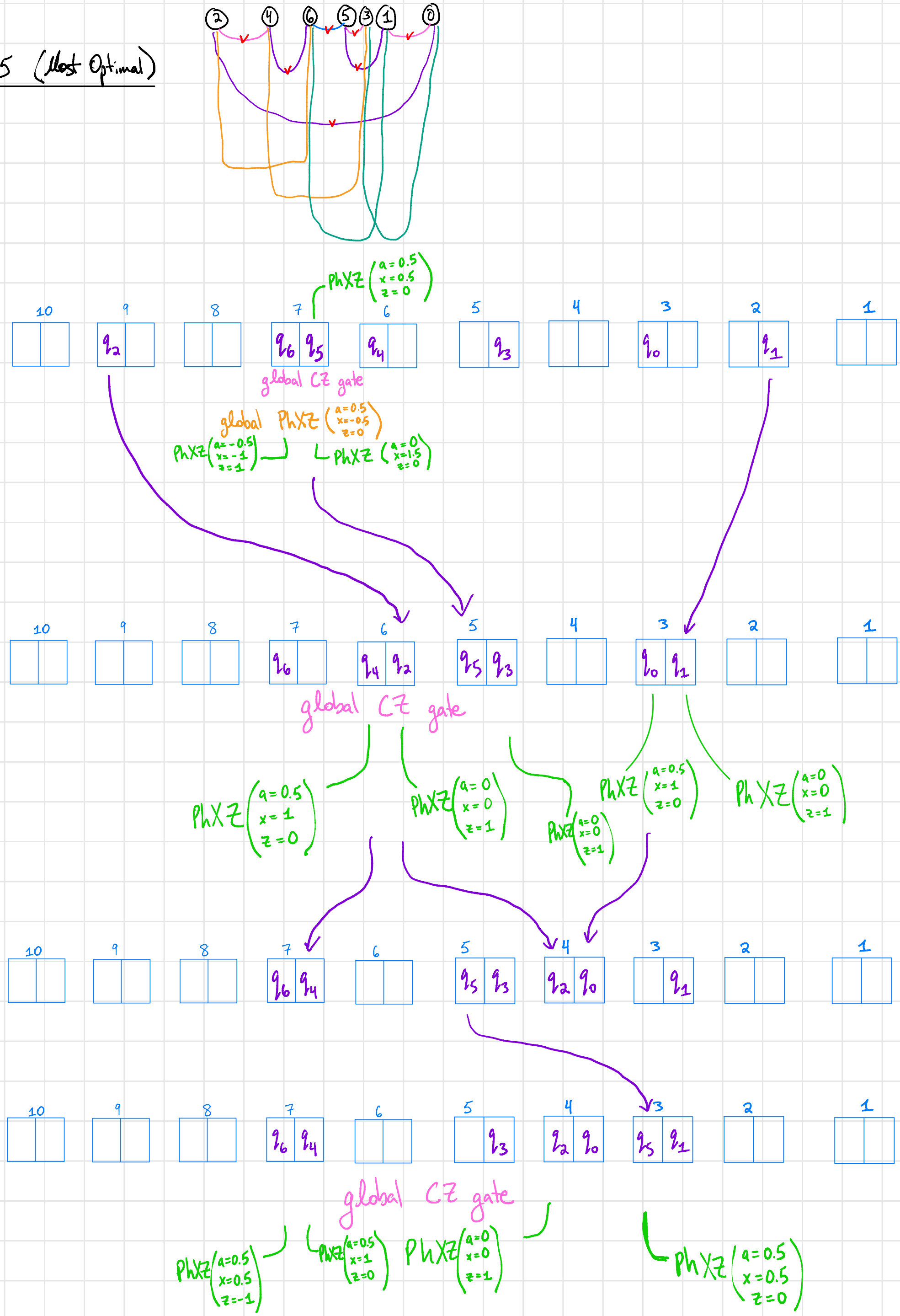
1
2
3
4
5
6
7
8
q_6 q_5 q_4 q_2 q_1 q_0
q_3
2
3
4
5
6
7
8



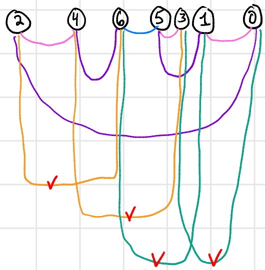
6 5 4 3 2 1 0

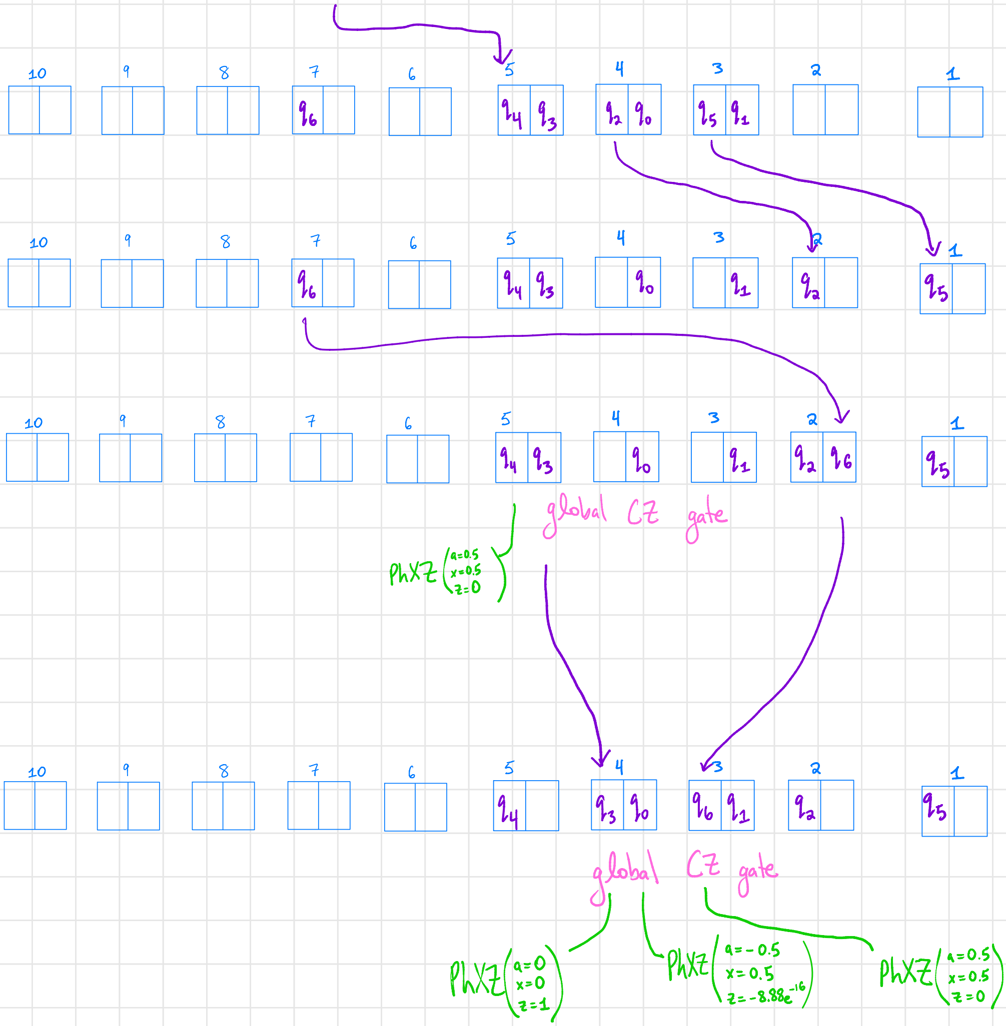
2 4 6 5 3 1 0

Circuit 5 (Most Optimal)



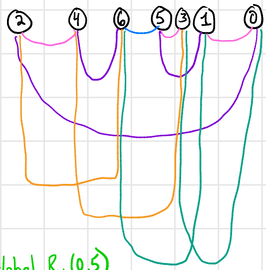
Circuit 5 (Most Optimal)



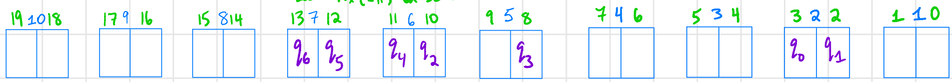


REDEYE PART 1

Circuit 5 (Most Optimal)



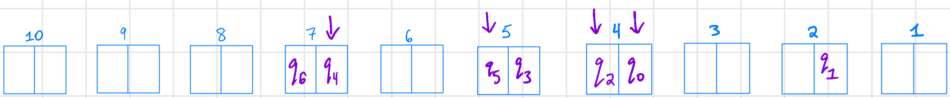
Global $R_y(0.5)$
Global $R_x(\pi)$
Local $R_y(-0.5)$ on SLOT 13
Local $R_x(-\pi)$ on SLOT 13



Global CZ gate

move q_5 from SLOT 12 to SLOT 9
& move q_2 from slot 10 to SLOT 7
move q_0 from SLOT 3 to SLOT 6
& move q_4 from SLOT 11 to SLOT 12

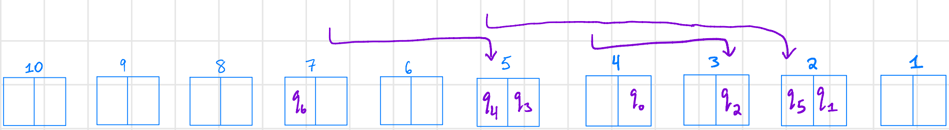
53 20
46



Global CZ gate

~~$R_y(0.5)$ on Slot 13~~

move q_5 from slot 9 to SLOT 3
& move q_4 from slot 12 into SLOT 9
& move q_2 from slot



Global CZ gate
move q_3 from slot 8 into slot 7
Global CZ gate
Global $R_y(0.5)$
local $R_y(-0.5)$ on SLOTS 2 and 4
Global $R_x(\pi)$
local $R_x(-\pi)$ on SLOTS 2, 4, and 7
move q_2 from slot 4 into slot 12 and q_1 from slot 2 into slot 10
global CZ gate

move q_6 from slot 13 into slot 11

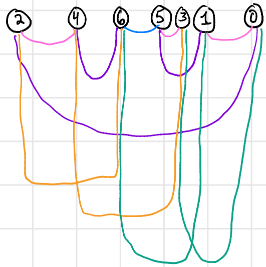
global CZ gate

$R_y(0.5)$ on slot 11

$R_x(\pi)$ on slot 11

REDEYE PART 2

Circuit 5 (Most Optimal)



19 10 18 | 17 9 16 | 15 8 14 | 13 7 12 | 11 6 10 | 9 5 8 | 7 4 6 | 5 3 4 | 3 2 2 | 1 1 0

10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1

10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1

10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1

REDEYE PART 3

Circuit 5 (Most Optimal)

19 10 18 | 17 9 16 | 15 8 14 | 13 7 12 | 11 6 10 | 9 5 8 | 7 4 6 | 5 3 4 | 3 2 2 | 1 1 0

10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1

10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1

10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1

REDEYE PART 4

Circuit 5 (Most Optimal)

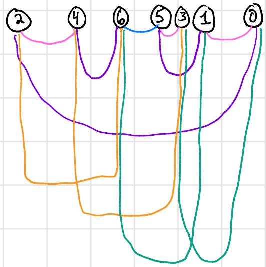
19 10 18 | 17 9 16 | 15 8 14 | 13 7 12 | 11 6 10 | 9 5 8 | 7 4 6 | 5 3 4 | 3 2 2 | 1 1 0

10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1

10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1

10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1

Circuit 5 (Most Optimal)



10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

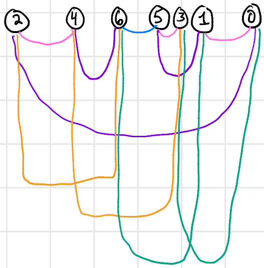
10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

Circuit 5 (Most Optimal)



10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1

10 9 8 7 6 5 4 3 2 1