

Term-by-term calculation of partition functions for each species in the ammonia synthesis reaction $\text{N}_2 + 3\text{H}_2 \rightleftharpoons 2\text{NH}_3$ compared to RASPA manual (section 8.3, table 8.2 pp. 158-161). Calculations correspond to $T = 573 \text{ K}$.

$$\frac{q}{V} = \frac{q_{trans}}{V} q_{rot} q_{vib} q_{elec} e^{\frac{D_0}{RT}} \quad (1)$$

$$\frac{q_{trans}}{V} = \left(\frac{2\pi mk_b T}{h^2} \right)^{\frac{3}{2}} \quad (2)$$

$$q_{rot}(\text{diatomic}) = \frac{T}{\sigma \Theta_{rot}} \quad (3)$$

$$q_{rot}(\text{polyatomic}) = \frac{\pi^{\frac{1}{2}}}{\sigma} \left(\frac{T^3}{\Theta_{rot,A} \Theta_{rot,B} \Theta_{rot,C}} \right)^{\frac{1}{2}} \quad (4)$$

$$q_{vib}|_{V=0} = \prod_{j=1}^{\alpha} \frac{1}{1 - e^{-\frac{\Theta_{vib,j}}{T}}} \quad (5)$$

$$q_{elec} \approx g_{e1} \quad (6)$$

V = volume [m^3]

h = Planck's constant [$6.626 \cdot 10^{-34} \text{ J s}$]

D_0 = atomization/dissociation energy [kcal mol^{-1}]

σ = symmetry number

R = gas constant [$1.987 \cdot 10^{-3} \text{ kcal mol}^{-1} \text{ K}^{-1}$]

Θ_{rot} = rotational temperature [K]

T = temperature [573 K]

α = number of vibrational modes

m = mass [kg]

Θ_{vib} = vibrational temperature [K]

k_b = Boltzmann constant [$1.381 \cdot 10^{-23} \text{ J K}^{-1}$]

g_{e1} = ground state degeneracy

Table 1. Molecular constants from *Statistical Mechanics*, McQuarrie. Tables also reproduced in *Molecular Thermodynamics*, McQuarrie and Simon, Table 4.2 (p. 151), Table 4.4 (p. 162). Parentheses in Θ values indicate degeneracy (omitted if 1).

Species	m (u)	Θ_{rot} (K)	Θ_{vib} (K)	σ	D_0 , absolute (kcal mol ⁻¹)	Reference
N_2	28.0134	2.88	3374	2	225.1	McQuarrie, Table 6-1, p. 95
H_2	2.016	85.3	6332	2	103.2	McQuarrie, Table 6-1, p. 95
NH_3	17.031	13.6(2), 8.92	4800, 1360, 4880(2), 2330(2)	3	276.8	McQuarrie, Table 8-1, p. 132

N₂:

$$m = 28.0134 u * \frac{1 kg}{6.022 * 10^{26} u}$$

$$= 4.65 * 10^{-26} kg$$

$$\frac{q_{trans}}{V} = \left(\frac{2\pi(4.65 * 10^{-26} kg)(1.381 * 10^{-23} J * K^{-1})(573 K)}{(6.626 * 10^{-34} J * s)^2} \right)^{\frac{3}{2}}$$

$$= 3.82 * 10^{32} m^{-3} * \frac{1 m^3}{10^{30} \text{Å}^3}$$

$$= 382 \text{Å}^{-3}$$

$$q_{rot} = \frac{573 K}{2(2.88 K)}$$

$$= 99.5$$

$$q_{vib}|_{v=0} = \frac{1}{1 - e^{-\frac{3374 K}{573 K}}}$$

$$= 1.00$$

$$q_{elec} \approx 1$$

D₀ = 0 (N₂ is element at standard state, and RASPA uses D₀ relative to elements)

$$e^{\frac{0}{RT}} = 1$$

$$\frac{q}{V} = (382 \text{Å}^{-3})(99.5)(1.00)(1)(1)$$

$$= 3.81271 * 10^4 \text{Å}^{-3} \text{ (matches RASPA manual, Table 8.2, p. 160)}$$

H₂:

$$m = 2.016 u * \frac{1 kg}{6.022 * 10^{26} u}$$

$$= 3.35 * 10^{-27} kg$$

$$\frac{q_{trans}}{V} = \left(\frac{2\pi(3.35 * 10^{-27} kg)(1.381 * 10^{-23} J * K^{-1})(573 K)}{(6.626 * 10^{-34} J * s)^2} \right)^{\frac{3}{2}}$$

$$= 7.38 * 10^{30} m^{-3} * \frac{1 m^3}{10^{30} \text{Å}^3}$$

$$= 7.38 \text{Å}^{-3}$$

$$q_{rot} = \frac{573 K}{2(85.3 K)}$$

$$= 3.36$$

$$q_{vib|v=0} = \frac{1}{1 - e^{-\frac{6332 K}{573 K}}} = 1.00$$

$$q_{elec} \approx 1$$

$D_0 = 0$ (N_2 is element at standard state, and RASPA uses D_0 relative to elements)

$$e^{\frac{0}{RT}} = 1$$

$$\begin{aligned} \frac{q}{V} &= (7.38 \text{ \AA}^{-3})(3.36)(1.00)(1)(1) \\ &= 2.47837 * 10^1 \text{ \AA}^{-3} \text{ (matches RASPA manual, Table 8.2, p. 160)} \end{aligned}$$

NH_3 :

$$\begin{aligned} m &= 17.031 u * \frac{1 kg}{6.022 * 10^{26} u} \\ &= 2.83 * 10^{-26} kg \end{aligned}$$

$$\begin{aligned} \frac{q_{trans}}{V} &= \left(\frac{2\pi(2.83 * 10^{-26} kg)(1.381 * 10^{-23} J * K^{-1})(573 K)}{(6.626 * 10^{-34} J * s)^2} \right)^{\frac{3}{2}} \\ &= 1.81 * 10^{32} m^{-3} * \frac{1 m^3}{10^{30} \text{ \AA}^3} \\ &= 181 \text{ \AA}^{-3} \end{aligned}$$

$$\begin{aligned} q_{rot} &= \frac{\pi^{\frac{1}{2}}}{3} \left(\frac{(573 K)^3}{(13.6 K)^2 (8.92 K)} \right)^{\frac{1}{2}} \\ &= 200 \end{aligned}$$

$$\begin{aligned} q_{vib|v=0} &= \frac{1}{1 - e^{-\frac{1360 K}{573 K}}} \left(\frac{1}{1 - e^{-\frac{2330 K}{573 K}}} \right)^2 \frac{1}{1 - e^{-\frac{4800 K}{573 K}}} \left(\frac{1}{1 - e^{-\frac{4880 K}{573 K}}} \right)^2 \\ &= 1.14 \end{aligned}$$

$$q_{elec} \approx 1$$

$$NH_3 = \frac{1}{2} N_2 + \frac{3}{2} H_2$$

$$\begin{aligned} D_0 &= D_{0,NH_3,absolute} - \sum D_{0,constituent\ elements,absolute} \\ &= 276.8 \frac{kcal}{mol} - \left(\frac{1}{2} \left(225.1 \frac{kcal}{mol} \right) + \frac{3}{2} \left(103.2 \frac{kcal}{mol} \right) \right) \\ &= 9.45 \frac{kcal}{mol} \end{aligned}$$

$$e^{\frac{9.45 \frac{\text{kcal}}{\text{mol}}}{\left(1.987 \frac{10^{-3} \text{kcal}}{\text{mol K}}\right)(573\text{K})}} = 4021$$

$$\begin{aligned} \frac{q}{V} &= (181 \text{ \AA}^{-3})(200)(1.14)(1)(4021) \\ &= 1.66002 * 10^8 \text{ \AA}^{-3} \text{ (RASPAs manual shows } 1.498133 * 10^8 \text{ \AA}^{-3}, \text{ Table 8.2, p. 160)} \end{aligned}$$

Table 2. Summary of term-by-term hand calculations versus RASPAs manual values (Table 8.2, p. 161).

Species	Atkinson				RASPAs	
	$q_{\text{trans}}/V \text{ (\AA}^{-3}\text{)}$	q_{rot}	q_{vib}	$e^{D_0/RT}$	$q/V \text{ (\AA}^{-3}\text{)}$	$q/V \text{ (\AA}^{-3}\text{)}$
N ₂	3.8220E+02	9.9479E+01	1.0028E+00	1	3.81271E+04	3.81253E+04
H ₂	7.3788E+00	3.3587E+00	1.0000E+00	1	2.47837E+01	2.48054E+01
NH ₃	1.8118E+02	1.9951E+02	1.1422E+00	4020.5	1.66002E+08	1.498133E+08