

### Task 1)

A)

Transitivity: If  $X \rightarrow Y$  and  $Y \rightarrow Z$ , then  $X \rightarrow Z$

Gives us in our case:

$C \rightarrow \{A,D\}$  and  $A \rightarrow \{C,B\}$ ,

This gives us

$\{C\} \rightarrow \{A\}, \{A\} \rightarrow \{B\}$

B)

Transitivity: If  $X \rightarrow Y$  and  $Y \rightarrow Z$ , then  $X \rightarrow Z$

Gives us:

$\{A\} \twoheadrightarrow \{C\}$ ,  $\{C\} \twoheadrightarrow \{D\}$ ,  $\{D,E\} \twoheadrightarrow \{F\}$

$\{C\} \rightarrow \{B\}$

### Task 2)

A)

Given that we know  $\{A\}$  we can use Armstrong's rules to also find  $\{B,C,D\}$

$F^+ = \{A,B,C,D\}$

B) }

From Transitivity we see that  $\{C,E\}$  gives us all attributes in the relation.  $\{C,E\}$  can thereby count as a super key-couple and  $X \twoheadrightarrow R$ .  $F^+ = \{A,B,C,D,E,F\}$ .

### Task 3)

A)  $\{A,B\}$  is a candidate key. We can't find any smaller subset of attributes that is a super key.

Transitivity gives us  $\{D\} \twoheadrightarrow \{B\}$ .

$\{A,D\}$  is also a candidate key. The same relations hold after we use  $\{D\} \twoheadrightarrow \{B\}$ , Transitivity.

B) If R is BCNF we should be able to reach all attributes within R from our super key without taking unnecessary long paths. In our case we can reach all attributes from the Candidate  $\{A,B\}$ . We can also reach all attributes from  $\{A,D\}$  but with a longer path. If we excluded FD2 and FD3 we would have a BCNF with  $\{A,B\}$  as super key.

C)  $\{A,B\}$  is key

- FD1 is bcnf.
- FD2 is not.
- FD3 is not.
  - Using FD2, we decompose R.
    - $R_1(\underline{E}, F)$  with FD2.

- $\{E\} \rightarrow \{F\}$
  - $R2(\underline{A}, B, C, D, E)$  with FD1, FD3
    - $\{A, B\} \rightarrow \{C\}$
    - $\{A, B\} \rightarrow \{D\}$
    - $\{A, B\} \rightarrow \{E\}$
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- R1 is in BCNF but not R2  $\rightarrow$  Decompose R2.
  - R2 is not in BCNF.
    - Using FD3, we decompose R2.
      - $R2A(\underline{D}, B)$  with FD3
        - $\{D\} \rightarrow \{B\}$
      - $R2B(\underline{A}, C, D, E)$  with FD1
        - $\{A\} \rightarrow \{C\}$
        - $\{A\} \rightarrow \{D\}$
        - $\{A\} \rightarrow \{E\}$
  - R2A & R2B is in BCNF (FD1 is in BCNF)
  - The result of decomposing R consists of R1, R2A & R2B.

Task 4)

A)

Relation schema R with a set F of functional dependencies is in BCNF if for every non-trivial FD  $X \rightarrow Y$  in F+ we have that X is a superkey

BC is our candidate key.  $\rightarrow$  FD3 is not in BCNF

- FD1:  $\{A, B, C\} \rightarrow \{D, E\}$
- FD2:  $\{B, C, D\} \rightarrow \{A, E\}$
- FD3:  $\{C\} \rightarrow \{D\}$

B)

- FD1 is bcnf.
- FD2 is bcnf
- FD3 is not bcnf.
  - Using FD1, we decompose R.
    - $R1(\underline{A}, B, C, D, E)$  with FD1.
      - $\{A, B, C\} \rightarrow \{D\}$
      - $\{A, B, C\} \rightarrow \{E\}$
    - $R2(\underline{B}, C, A)$  with FD2, FD3
      - $\{B, C\} \rightarrow \{A\}$

The result of decomposing R consists of R1, R2

