Isak_Berntsson_lab4.rmd

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#contribution

Used Isak Berntsson as base, minor changes made.

Answers given after group discussions. isabe723, erija971, ludkn080, linbe580

Task1

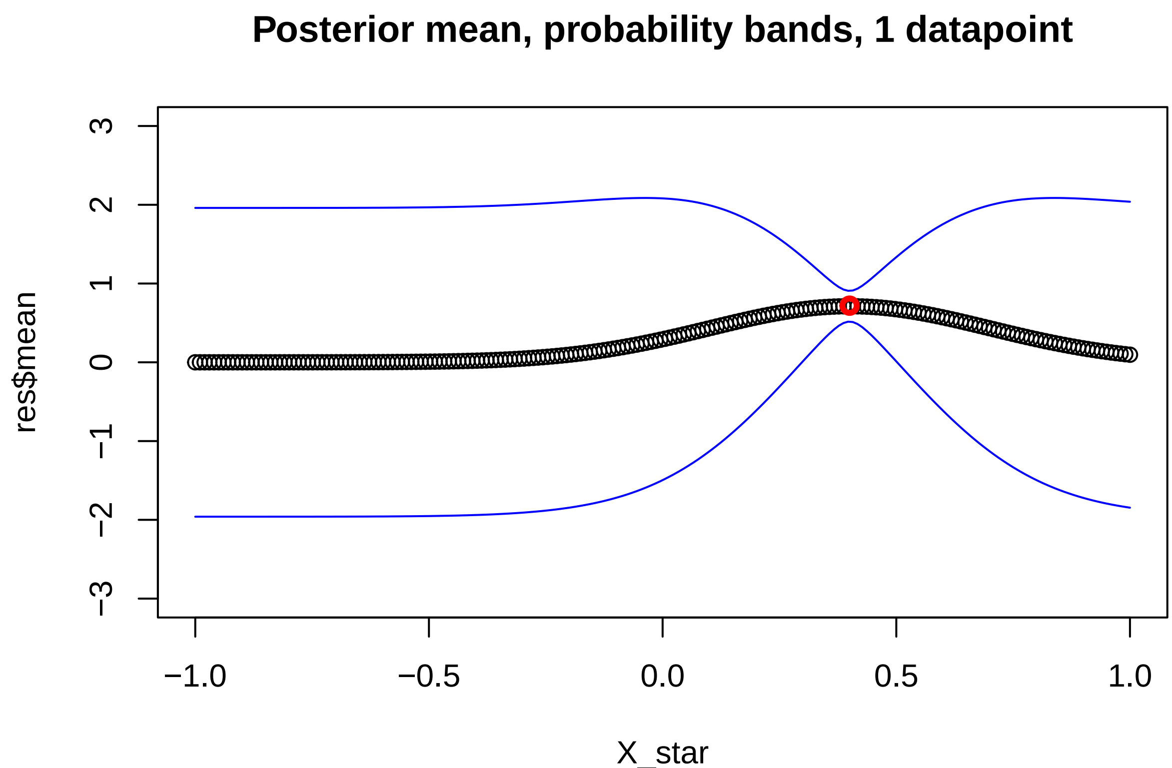
Subtask 1

```
posteriorGP = function(X, y, X_star, sigmaNoise, k, ...){
  n = length(X)

  K = k(X,X, ... )

  k_star = k(X,X_star, ... )

  L = t(chol(K + sigmaNoise^2*diag(n)))

  alpha = solve(t(L),solve(L,y))

  mean_pred = t(k_star) %*% alpha #predictive mean

  v = solve(L,k_star)

  var_pred= k(X_star, X_star) - t(v) %*% v # predictive var

  logmarglik = -.5 * t(y) %*% alpha - sum(log(diag(L))) - .5*n*log(2*pi)

  return(list(mean=mean_pred, var= var_pred, logmarglik = logmarglik))

}
```

Subtask 2

```
#priors
sigma_f = 1
sigma_n = .1
```

```
l = .3


#k = SquaredExpKernel #using func from given script

X = .4
y = .719
X_star = seq(-1,1,length=200)

res = posteriorGP(X,y,X_star, sigma_n,SquaredExpKernel,sigmaF= sigma_f, l)

plot(X_star, res$mean, ylim = c(-3,3), main="Posterior mean, probability bands, 1 datapoint")
lines(X_star, res$mean + 1.96*sqrt(diag(res$var)), col="blue")
lines(X_star, res$mean - 1.96*sqrt(diag(res$var)), col="blue")
points(X,y, col="red", lwd=3)
```

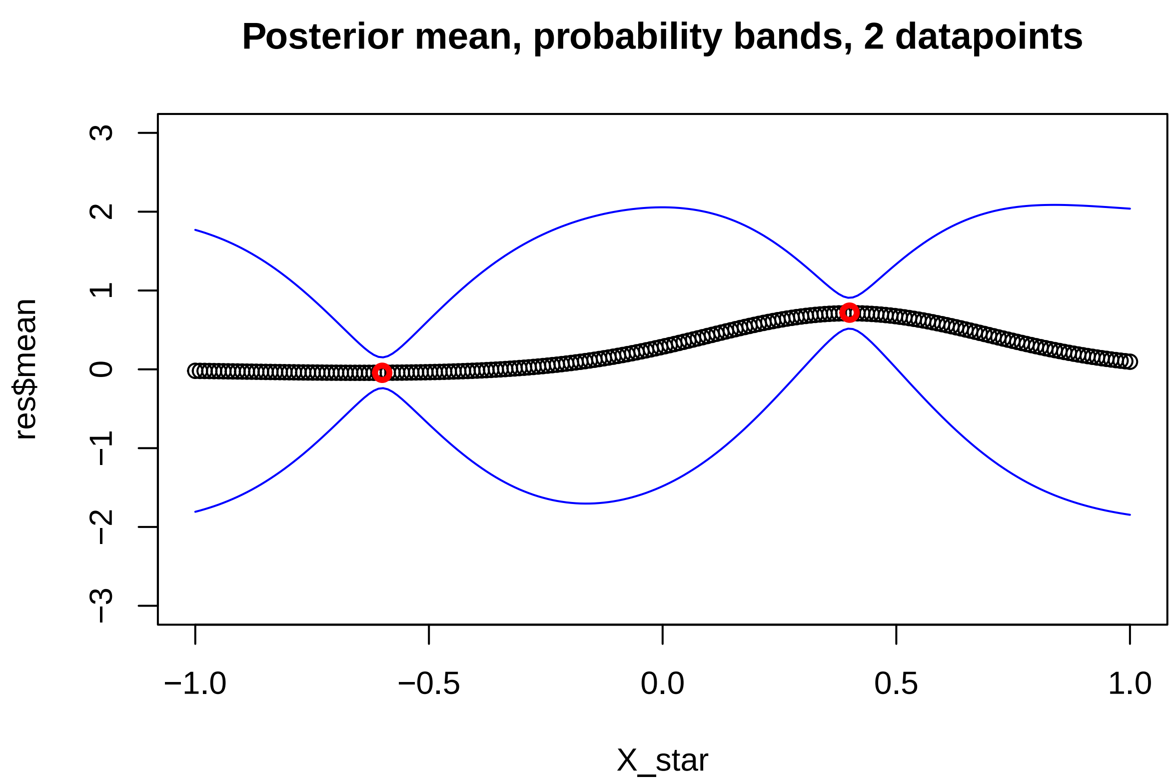


Subtask 3

```
sigma_f = 1
sigma_n = .1
l = .3

dat = data.frame(X=c(X,-.6), y = c(y,-.044))
dat
```

```
##      X      y
## 1  0.4  0.719
## 2 -0.6 -0.044
```

```
res = posteriorGP(dat$X,dat$y,X_star, sigma_n,SquaredExpKernel,sigmaF= sigma_f, l)

plot(X_star, res$mean, ylim = c(-3,3), main="Posterior mean, probability bands, 2 datapoints")
lines(X_star, res$mean + 1.96*sqrt(diag(res$var)), col="blue")
lines(X_star, res$mean - 1.96*sqrt(diag(res$var)), col="blue")
points(dat, col="red", lwd=3)
```



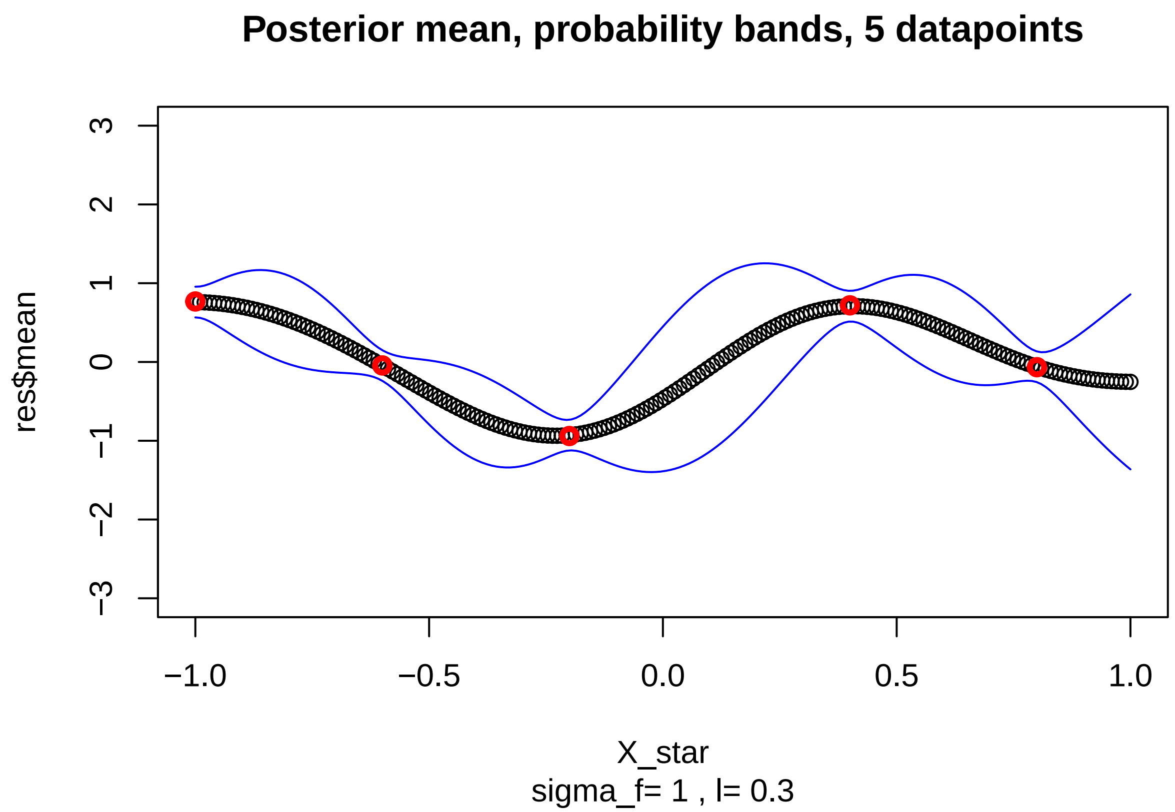
Subtask 4

```
sigma_f = 1
sigma_n = .1
l = .3

dat = data.frame(
  X = c(-1, -.6, -.2, .4, .8),
  y = c(.768, -.044, -.940, .719, -.0664)
  )
dat
```

```
##      X       y
```

```
## 1 -1.0  0.7680
## 2 -0.6 -0.0440
## 3 -0.2 -0.9400
## 4  0.4  0.7190
## 5  0.8 -0.0664
```

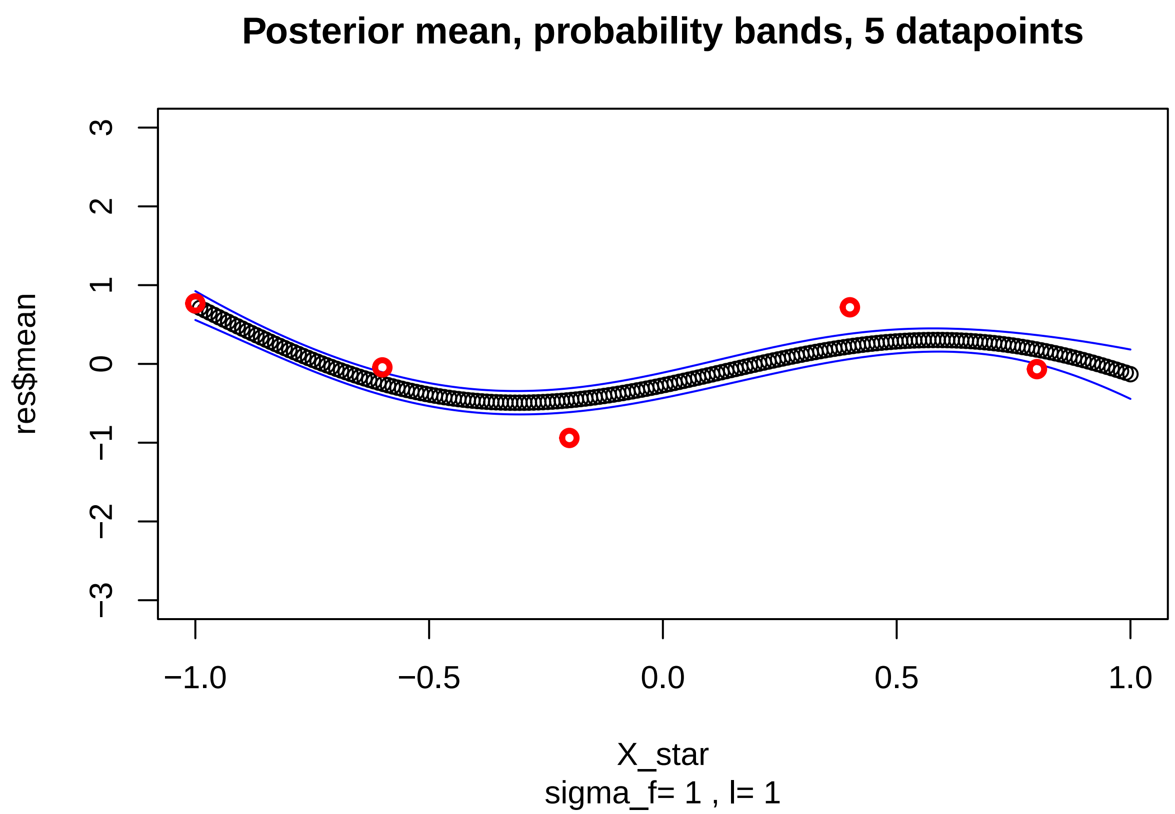
```
subtext = paste("sigma_f=", sigma_f, ", l=",l)

res = posteriorGP(dat$X,dat$y,X_star, sigma_n,SquaredExpKernel,sigmaF= sigma_f, l)

plot(X_star, res$mean, ylim = c(-3,3), main="Posterior mean, probability bands, 5 datapoints", sub=subte
lines(X_star, res$mean + 1.96*sqrt(diag(res$var)), col="blue")
lines(X_star, res$mean - 1.96*sqrt(diag(res$var)), col="blue")
points(dat, col="red", lwd=3)
```



Subtask 5

```
sigma_f = 1
l = 1

res = posteriorGP(dat$X,dat$y,X_star, sigma_n,SquaredExpKernel,sigmaF= sigma_f, l)

subtext = paste("sigma_f=", sigma_f, ", l=",l)
```

```
plot(X_star, res$mean, ylim = c(-3,3), main="Posterior mean, probability bands, 5 datapoints", sub=subt
lines(X_star, res$mean + 1.96*sqrt(diag(res$var)), col="blue")
lines(X_star, res$mean - 1.96*sqrt(diag(res$var)), col="blue")
points(dat, col="red", lwd=3)
```

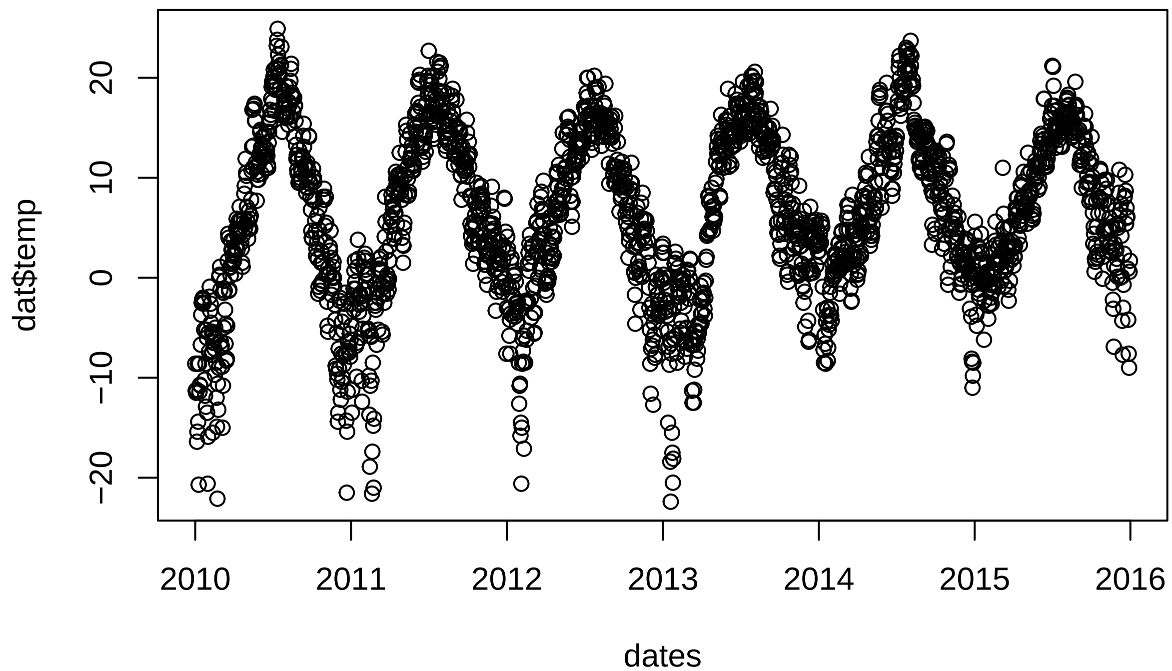


THe higher value of ell/l mean the function is overly smoothed and cannot fit the data as well as in (4)

Task 2

setup

```
dat = read.csv("https://github.com/STIMALiU/AdvMLCourse/raw/master/GaussianProcess/
Code/TempTullinge.csv", header=TRUE, sep=";")

dates = as.Date(dat$date, "%d/%m/%y")
plot(dates, dat$temp)
```




```
n = length(dates)
time = 1:n
day = time %% 365
day[365:n] = day[365:n]+1

sample_ind = time %% 5 == 1

sample_time = time[sample_ind]
sample_day = day[sample_ind]
sample_temp = dat$temp[sample_ind]


sample_dat = data.frame(time = sample_time, day  = sample_day, temp = sample_temp)
```

subtask 1

```
library(kernlab)
```

```
## Warning: package 'kernlab' was built under R version 4.1.3
```

```
#library(AtmRay)




SEK = function(ell, sigmaf){

  SEK_kernel = function(X,X_star){

    X = as.matrix(X)
    X_star = as.matrix(X_star)

    #dimX_star = dim(X_star)

    res = matrix(0,nrow=length(X),ncol=length(X_star))

    #print(dim(res))

    for( i in 1:length(X)){
      vals = sigmaf^2 * exp(-.5 * ((X-X_star[i,])/ell)^2)
      #print(vals)
      res[i,] = vals
    }
    class(res) = "kernel"
    return(res)
  }
  return(SEK_kernel)
}

mu = 1
X = runif(100,-5,5)

kern= SEK(ell=1,sigmaf=1)

print("evaluated at (1,2)")
```

```
## [1] "evaluated at (1,2)"
```

```
kern(1,2)
```

```
##           [,1]
## [1,] 0.6065307
## attr(,"class")
## [1] "kernel"
```

```r
X = c(1,3,4)
Xprim = c(2,3,4)

kmat = kernelMatrix(kern,X,Xprim)

colnames(kmat) = X
rownames(kmat) = Xprim
print(kmat)
```

```
## An object of class "kernelMatrix"
##           1         3         4
## 2 0.6065307 0.1353353 0.0111090
## 3 0.6065307 1.0000000 0.6065307
## 4 0.1353353 0.6065307 1.0000000
```

subtask 2

```r
df_fit = data.frame(temp = sample_dat$temp, const = 1, time= sample_dat$time, time2 = (sample_dat$time)
head(df_fit)
```

```
##    temp const time time2
## 1  -8.6     1    1     1
## 2 -15.4     1    6    36
## 3 -11.4     1   11   121
## 4  -2.5     1   16   256
## 5  -2.5     1   21   441
## 6 -12.9     1   26   676
```

```r
linmod = lm("temp ~ const + time + time2 - 1 ",data = df_fit)
#mod = lm("y ~ time + time2 ",data = df_fit)
linmod
```

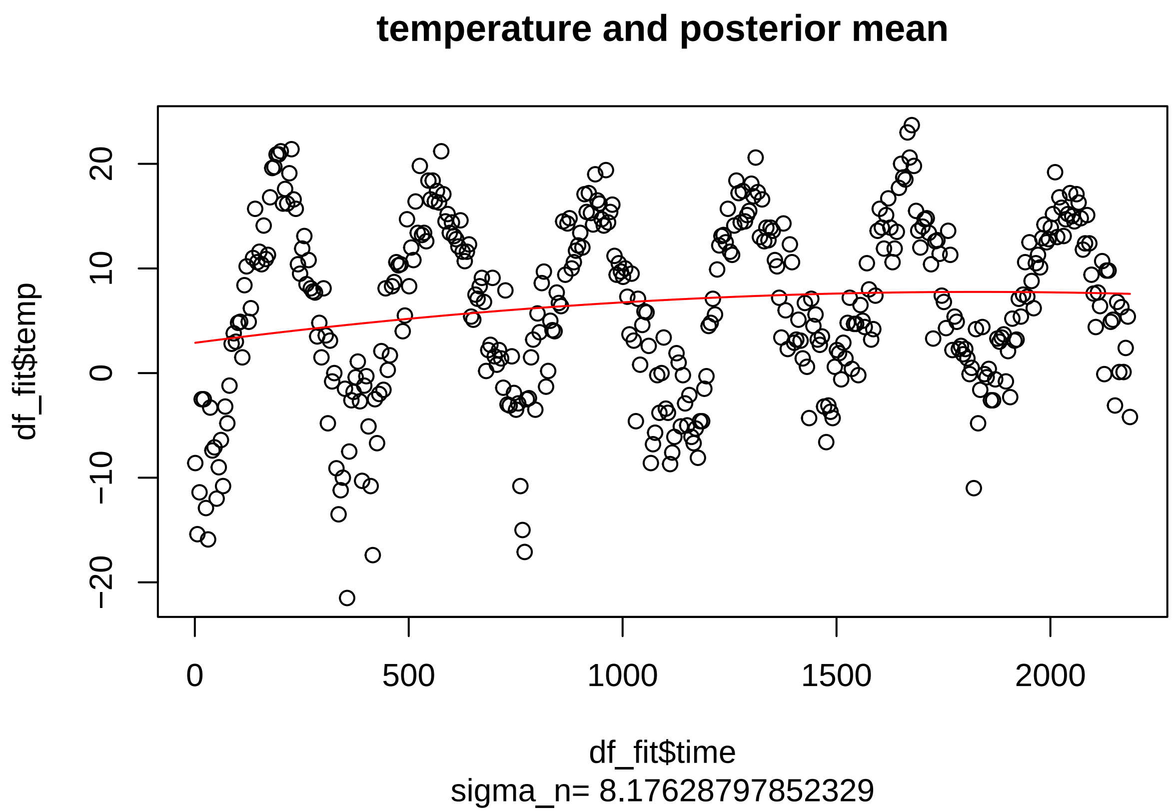
```
##
## Call:
## lm(formula = "temp ~ const + time + time2 - 1 ", data = df_fit)
##
## Coefficients:
##      const        time       time2
##  2.892e+00   5.319e-03  -1.453e-06
```

```r
yhat = predict(object = linmod, data=df_fit)
#head(yhat) # posterior mean

residuals = yhat - df_fit$temp
sigma_n = sd(residuals)

ell = .2
sigma_f = 20

kern = SEK(ell,sigmaf = sigma_f)
```

```r
posterior = posteriorGP(X = df_fit$time, y = df_fit$temp, X_star = df_fit$time, sigmaNoise = sigma_n ,k
#mod = gausspr(x = df_fit$time, y=df_fit$temp, kernel = kern, var= sigma_n^2)

'mod = gausspr(df_fit$time, df_fit$temp , kernel=kern, var = sigma_n^2)
yhat = predict(mod, df_fit$time)
'
```

```
## [1] "mod = gausspr(df_fit$time, df_fit$temp , kernel=kern, var = sigma_n^2)\nyhat = predict(mod, df_f
```

```r
plot(df_fit$time,df_fit$temp, main="temperature and posterior mean", sub=paste("sigma_n=",sigma_n))
lines(df_fit$time,yhat, col="red")
```




```r
#lines(df_fit$time,posterior$mean, col="green")


old_posterior_mean = yhat #used in subtask 4
```

A lower sigma results in a smoother predictive mean, unable to fit the data. A very large sigma results in a perfect fit. Overfit

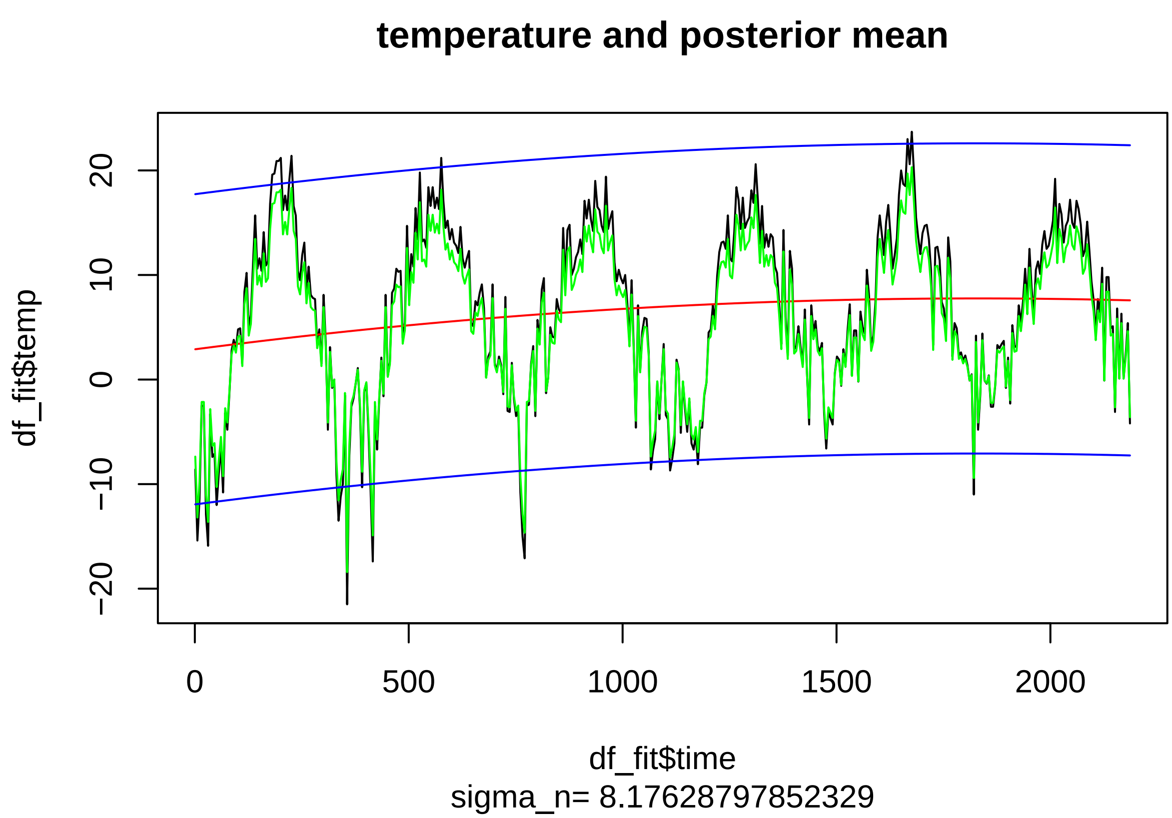
A very large ell makes the predictions static and the function does not fit the data very well

Subtask 3

```
posterior_sigma = sqrt(posterior$var[1,1])
posterior_sigma
```

```
## [1] 7.568272
```

```
plot(df_fit$time,df_fit$temp, type="l", main="temperature and posterior mean", sub=paste("sigma_n=",sig
lines(df_fit$time,yhat, col="red")
lines(df_fit$time,posterior$mean, col="green")

lines(df_fit$time, yhat + 1.96 * posterior_sigma, col="blue")
lines(df_fit$time, yhat - 1.96 * posterior_sigma, col="blue")
```



Bands seem to capture the variance of the measuremetns weell.

Subtask 4

```
df_fit = data.frame(temp = sample_dat$temp, const = 1, day= sample_dat$day, day2 = (sample_dat$day)^2 )
head(df_fit)
```

```
##    temp const day day2
```

```
## 1  -8.6     1   1    1
## 2 -15.4     1   6   36
## 3 -11.4     1  11  121
## 4  -2.5     1  16  256
## 5  -2.5     1  21  441
## 6 -12.9     1  26  676
```

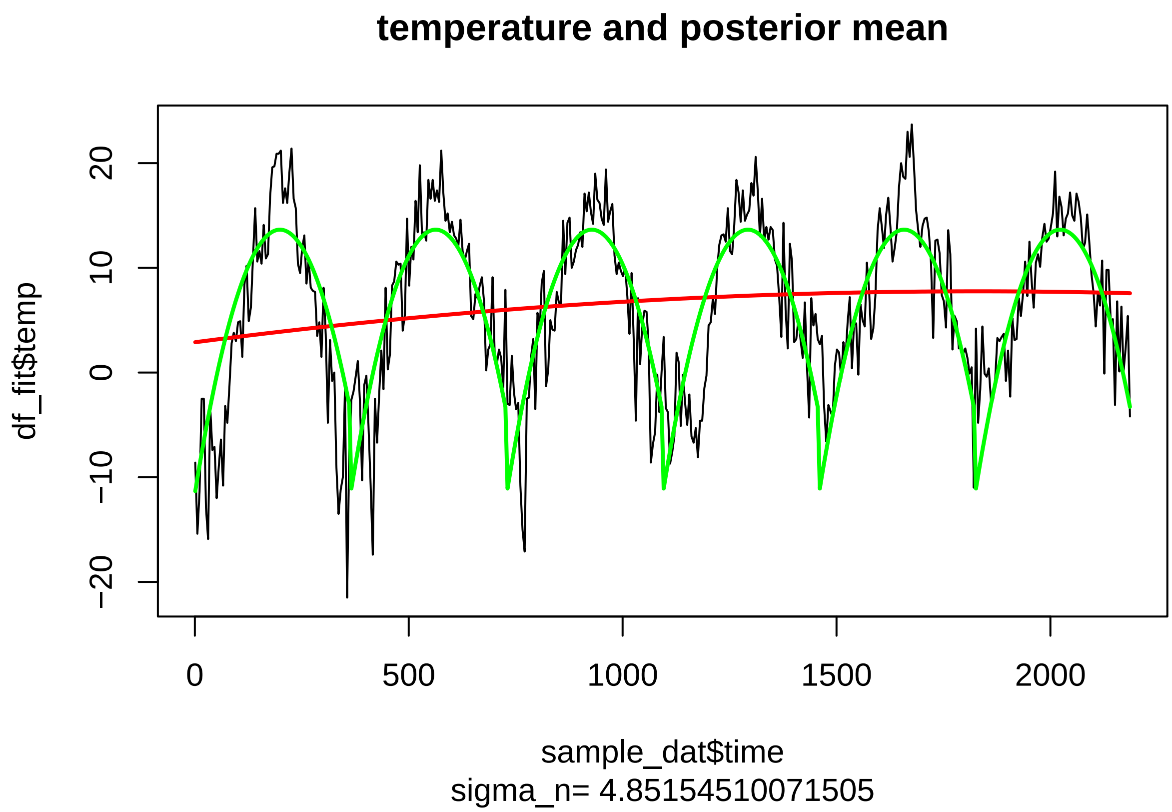
```r
mod = lm("temp ~ const + day + day2 - 1 ",data = df_fit)
mod
```

```
##
## Call:
## lm(formula = "temp ~ const + day + day2 - 1 ", data = df_fit)
##
## Coefficients:
##      const         day        day2
## -1.159e+01   2.536e-01  -6.372e-04
```

```r
yhat = predict(object = mod, data=df_fit)
head(yhat) # posterior mean
```

```
##          1          2          3          4          5          6
## -11.333082 -10.087352  -8.873480  -7.691466  -6.541309  -5.423010
```

```r
residuals = yhat - df_fit$temp
sigma_n = sd(residuals)

ell = .2
sigma_f = 20

kern = SEK(ell,sigmaf = sigma_f)
posterior = posteriorGP(X = df_fit$day, y = df_fit$temp, X_star = df_fit$day, sigmaNoise = sigma_n ,k =

plot(sample_dat$time,df_fit$temp, type="l", main="temperature and posterior mean", sub=paste("sigma_n="
#lines(sample_dat$time,yhat, col="red")
lines(sample_dat$time,old_posterior_mean, col="red", lwd=2)
lines(sample_dat$time,yhat, col="green", lwd =2)
```

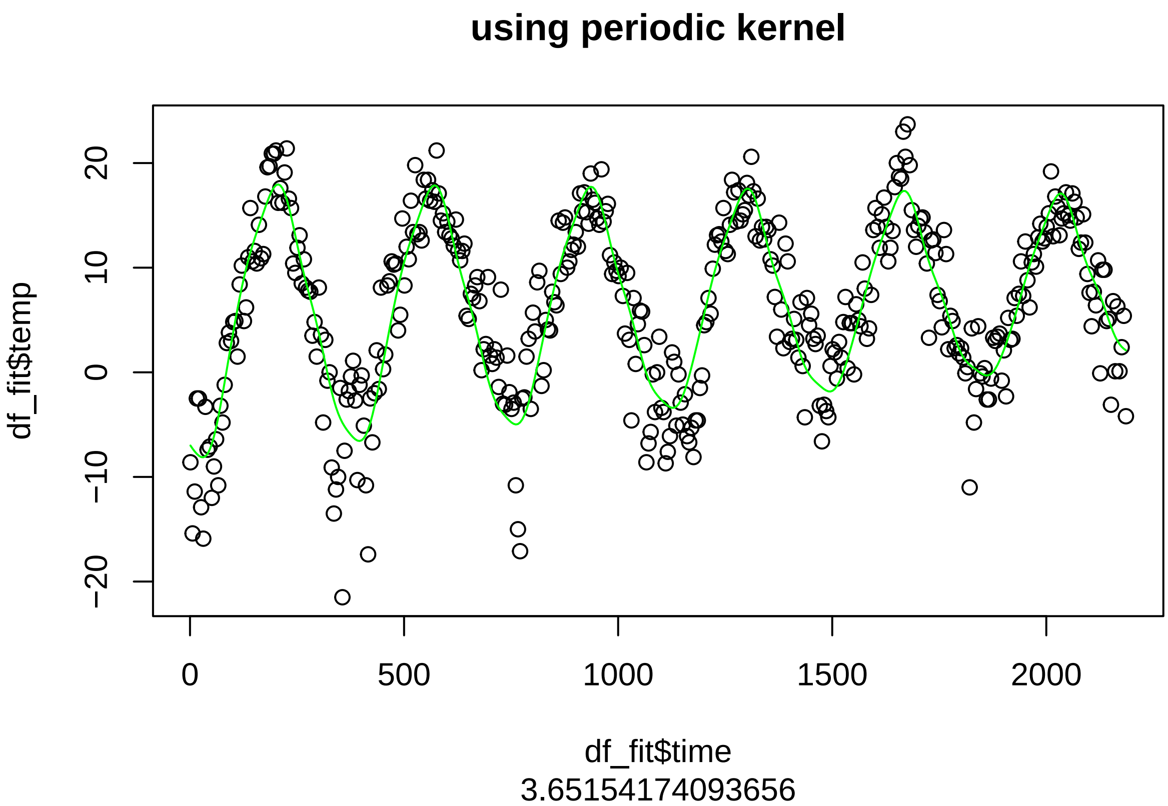


This new model based on day rather than time can handle the predictable annual changes in temperature in a much better way. sigma_n takes on a much smaller value with the new model. 8.17 -> 4.85 because the residuals are much smaller / fit is better.

Subtask 5

```
df_fit = data.frame(temp = sample_dat$temp, const = 1, time= sample_dat$time, time2 = (sample_dat$time)

SEK_periodic = function(ell_1, ell_2, d, sigmaf){

  kernel = function(X,X_star){

    X = as.matrix(X)
    X_star = as.matrix(X_star)

    #dimX_star = dim(X_star)

    res = matrix(0,nrow=length(X),ncol=length(X_star))

    #print(dim(res))

    for( i in 1:length(X)){
```

```
      vals= sigmaf^2 * exp((-2/ell_1^2)*sin(pi * abs(X-X_star[i,])/d)^2)  * exp(-.5 * abs(X-X_star[i,])

      res[i,] = vals
    }
    return(res)
  }
  class(kernel) = "kernel"
  return(kernel)
}



sigma_f = 20
l1 = 1
l2 = 10
d = 365/sd(df_fit$time)

kern = SEK_periodic(l1, l2, d, sigma_f)



mod = gausspr(x = df_fit$time, y=df_fit$temp, kernel = kern, var= sigma_n^2)

yhat = predict(mod, df_fit$time)

residuals = yhat - df_fit$temp
sigma_n = sd(residuals)

plot(df_fit$time, df_fit$temp, sub= sigma_n, main ="using periodic kernel")
lines(df_fit$time, yhat, col="green")
```




```
#contour()
```

Thsi model genereates predictions that are periodic and smoother while still following the general upwards trend.

Task 3

Subtask1

```
data <- read.csv("https://github.com/STIMALiU/AdvMLCourse/raw/master/
GaussianProcess/Code/banknoteFraud.csv", header=FALSE, sep=",")
names(data) <- c("varWave","skewWave","kurtWave","entropyWave","fraud")
data[,5] <- as.factor(data[,5])

set.seed(111)
SelectTraining <- sample(1:dim(data)[1], size = 1000,
replace = FALSE)

df_train = data[SelectTraining,]
head(df_train)
```

```
##      varWave skewWave  kurtWave entropyWave fraud
## 507  2.09300  8.30610  0.022844    -3.27240     0
## 979  0.75896  0.29176 -1.650600     0.83834     1
## 175  1.87990  2.47070  2.493100     0.37671     0
## 793 -2.73380  0.45523  2.439100     0.21766     1
## 699  3.24030 -3.70820  5.280400     0.41291     0
## 69   1.00090  7.78460 -0.282190    -2.66080     0
```

```
mod = gausspr(x = df_train[, 1:2], y=df_train$fraud, type="classification")
```

```
## Using automatic sigma estimation (sigest) for RBF or laplace kernel
```

```
yhat = predict(mod, df_train[,1:2])

conf_mat = table(yhat,df_train$fraud)
conf_mat
```

```
##
## yhat   0   1
##    0 503  18
##    1  41 438
```

```
acc = sum(diag(conf_mat)) / sum(conf_mat)
acc
```

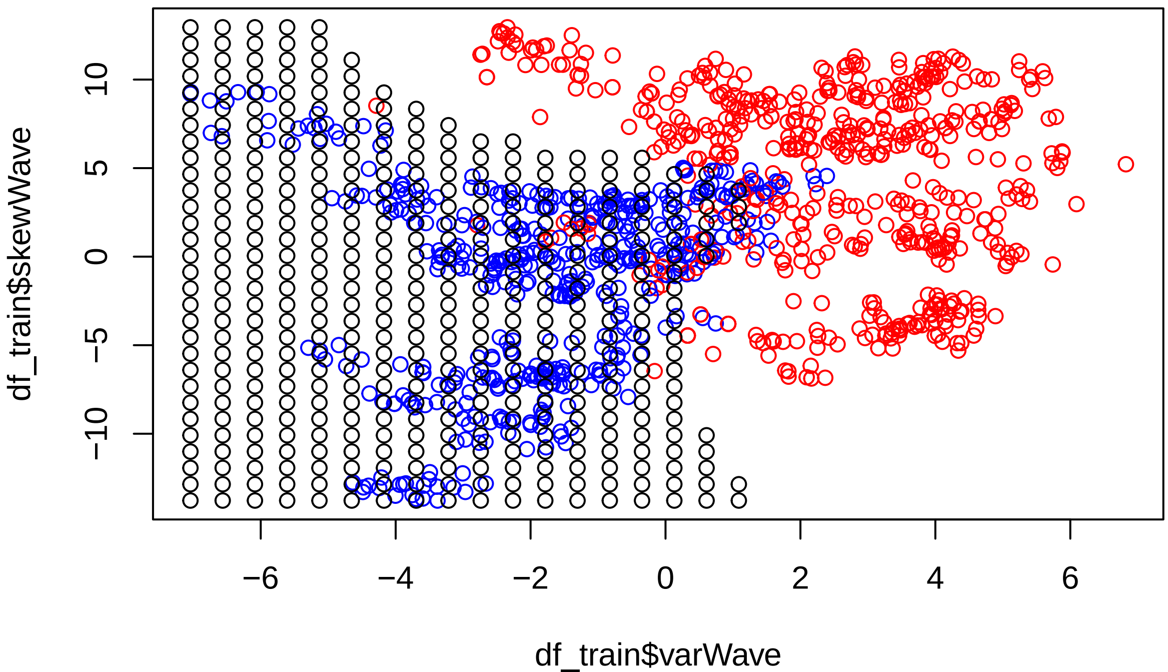
```
## [1] 0.941
```

```
n=30
g1 = seq(min(df_train$varWave),max(df_train$varWave), length.out=n )
g2 = seq(min(df_train$skewWave),max(df_train$skewWave), length.out=n )

grid = expand.grid(g1,g2)
colnames(grid) = c("varWave", "skewWave")
```

```r
yhat_grid = (as.numeric( predict(mod,grid)) - 1) == 1

fraud_bool = (as.numeric(df_train$fraud) - 1) == 1
countour = grid[yhat_grid,]

cols = array("red",length(fraud_bool))

cols[fraud_bool] = "blue"

'plot(countour$varWave, countour$skewWave,col="black")
points(df_train$varWave,df_train$skewWave, col=cols)
'
```

```
## [1] "plot(countour$varWave, countour$skewWave,col=\"black\")\npoints(df_train$varWave,df_train$skewWa
```

```r
plot(df_train$varWave,df_train$skewWave, col=cols)
points(countour$varWave, countour$skewWave,col="black")
```



Subtask 2

```r
df_test = data[-SelectTraining,c(1,2,5)]

yhat = predict(mod,df_test[,1:2])

conf_mat = table(yhat,df_test$fraud)
conf_mat
```

```
##
## yhat   0   1
##    0 199   9
##    1  19 145
```

```r
acc = sum(diag(conf_mat)) / sum(conf_mat)
acc
```

```
## [1] 0.9247312
```

Subtask 3

```r
df_train = data[SelectTraining,]
df_test = data[-SelectTraining,]
mod = gausspr(df_train[,-5], df_train[,5], type="classification")
```

```
## Using automatic sigma estimation (sigest) for RBF or laplace kernel
```

```r
yhat = predict(mod, df_test[,-5])

conf_mat = table(yhat,df_test$fraud)
conf_mat
```

```
##
## yhat   0   1
##    0 216   0
##    1   2 154
```

```r
acc = sum(diag(conf_mat)) / sum(conf_mat)
acc
```

```
## [1] 0.9946237
```

Accuracy using the final model is better (99.46% vs 92.47%) in terms of accuracy