UNIVERSITY OF RUHUNA

Faculty of Engineering

End-Semester 3 Examination in Engineering: September 2014

Module Number: IS3304 Module Name: Numerical Methods

[Three hours]

[Answer **all questions,** each question carries twelve marks]

Q1.

a) By considering one practical application, briefly explain the importance of the use of numerical methods for solving scientific and/or engineering problems.

b) You are working for a company that makes floats for commodes. The floating ball has a specific gravity of 0.6 and a diameter of 0.11m. You are asked to find the depth to which the ball is submerged when floating in water.

The equation that gives the depth x (m) to which the ball is submerged under water is given by,

$$x^3 - 0.165x^2 + 3.993 * 10^{-4} = 0$$

i.) Considering the physics of the problem (i.e. submerging of a ball with diameter of 0.11m), discuss why the starting interval [0.00, 0.11] would be a good guess to bracket the root.

ii.) Use the bisection method of finding roots of equations to find the depth x to which the ball is submerged under water. Conduct iterations until you get the solution with an accuracy of 0.001.

iii.) If you use the 'Newton-Raphson method' to solve the above equation, discuss why $x = 0.00$ and $x = 0.11$ are not good choices for the starting value, x_0.

Q2.

a) The Lagrangian interpolating polynomial of degree n that passes through $n+1$ data points (x_0,y_0), (x_1,y_1),............, (x_{n-1},y_{n-1}), (x_n,y_n) is defined as $P_n(x) = \sum_{i=0}^{n} y_i L_i(x)$.

where, $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$.

Show that $L_i(x_j) = 1$ when $i = j$ and

$L_i(x_j) = 0$ when $i \neq j$.

b) If (x_0,y_0), (x_1,y_1), (x_2,y_2) and (x_3,y_3) are distinct node points, obtain the

i.) linear interpolation polynomial, $P_1(x)$

ii.) quadratic interpolation polynomial, $P_2(x)$

iii.) cubic interpolation polynomial, $P_3(x)$

by using above node points.

c) The number of bacteria in a test sample taken at every 10 seconds is given bellow.

Time (sec)	0	10	20	30	40	50	60	70	80	90
No. of bacteria	26	225	625	1225	2025	3024	4225	5625	7225	9025

Assuming that the growth of bacteria is quadratic with respect to time and by using Lagrange interpolation, estimate the most accurate bacteria population at 15 and 85 seconds.

Q3.

a) Solve the following set of linear equations using the LU factorization method.

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & -2 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Would you use Gauss-Seidel method to solve this system of equations? Explain.

b) The voltage $V = V(t)$ in an electrical circuit obeys the equation:

$$V(t) = L\left(\frac{dI}{dt}\right) + R\,I(t)$$

where R is resistance and L is inductance. Using $R = 2$, $L = 0.05$ and value for $I(t)$ in the following table,

t	1.0	1.1	1.2	1.3	1.4
$I(t)$	8.2277	7.2428	5.9908	4.5260	2.9122

find $I'(1.2)$ by numerical differentiation with central difference and use it to compute $V(1.2)$.

Q4.

a) Briefly explain the following topics giving an example for each item.

i.) Ordinary differential equations (ODE)
ii.) Partial differential equations (PDE)
iii.) Initial value problem (IVP)
iv.) Boundary value problem (BVP)

b) A polluted lake has an initial concentration of a bacteria of 10^7 parts/m^3, while the acceptable level is only 10^5 parts/m^3. The concentration of the bacteria will reduce as fresh water enters the lake. The differential equation that governs the concentration C of the bacteria as a function of time (in weeks) is given by,

$$\frac{dC}{dt} + 0.06C = 0, \qquad C(0) = 10^7$$

Using the Runge-Kutta 4th order method, find the concentration of the bacteria after 7 weeks. Take a step size of 3.5 weeks.

Q5.

a) Classify the following equations as linear or non-linear, and state their order.

i.) $\frac{\partial^2 u}{\partial x^2} = c^2 \frac{\partial^2 u}{\partial t^2}$

ii.) $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 1$

iii.) $\frac{\partial w}{\partial t} + \frac{\partial^3 w}{\partial x^3} - 6w\frac{\partial w}{\partial x} = 0$

iv.) $2\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial x \partial t} + 3\frac{\partial^2 f}{\partial t^2} + 4\frac{\partial f}{\partial x} + \cos(2t) = 0$

b) Use Crank-Nicolson method to solve the heat equation,

$$2\frac{\partial^2 u(x,t)}{\partial x^2} - \frac{\partial u(x,t)}{\partial t} = 0 \quad \text{for } 0 \le x \le 1 \text{ and } 0 \le t \le 0.4$$

with the initial conditions,

$u(x,0) = f(x) = x - x^2$

and the boundary conditions,

$u(0,t) = 0$
$u(1,t) = t.$

Use, $h = 0.2$ and $k = 0.2$, where h and k are step sizes along x and t axes respectively.