Rajat Kumar Pradhan
19111045
5th semester, Biomed

Gödel's incompleteness theorems

July 18, 2021

Kurt Gödel's incompleteness theorems are important in mathematical logic and philosophy of mathematics. For any such consistent formal system, there will always be statements about natural numbers that are true but unprovable within the system. They were first of several closely related theorems on the limitations of formal systems.

1 Formal systems: completeness, consistency, and effective axiomatization

The incompleteness theorems apply to formal systems of sufficient complexity to express the basic arithmetic of the natural numbers. Theoretical systems that contain a sufficient amount of arithmetic cannot possess all three of these properties, they say. In other systems, such as set theory, only some sentences of the formal system express statements.

2 First incompleteness theorem

"Any consistent formal system F within which a certain amount of elementary arithmetic can be carried out is incomplete; i.e., there are statements of the language of F which can neither be proved nor disproved in F."

3 Second incompleteness theorem

"Assume F is a consistent formalized system which contains elementary arithmetic. Then $F \nvdash Cons(F)$."

4 Examples of undecidable statements

There are two distinct senses of the word "undecidable" in mathematics and computer science. The first is that of a statement being neither provable nor refutable in a specified deductive system. The second applies not to statements but to infinite sets of questions requiring a yes or no answer.

5 Relationship with computability

Matiyasevich proved that there is no algorithm that determines whether there is an integer solution to the equation p = 0. The incompleteness theorem is closely related to several results about undecidable sets in recursion theory. It can be used to obtain a proof to Gödel's first incompleteness theorem.