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IMPACT OF ANTENNA ARRAY GEOMETRY ON THE CAPACITY OF MIMO COMMUNICATION SYSTEM

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ABSTRACT

MIMO systems, employing array of antennas at both ends of the transmission link, provide significant increase in the achievable capacity. The ability of achieving this high capacity is greatly influenced by the geometry of antenna array used and the correlation between the channel gains on each antenna element in the array. In this paper, the impact different array configurations have on capacity is observed and compared. Five different array configurations - Uniform linear array, Non-uniform linear array, Uniform circular array, Hexagonal array and Star array are used to compare their performances. The expression of capacity of the communication system, derived from information-theoretic approach, is altered to introduce the impact of channel matrix on the system capacity. The influence of different array geometry is modelled by using the concept of 'array manifold vector' in the expression of channel matrix and in turns in the expression of capacity. Using this expression, capacities with different antenna array geometries are compared.

KEY WORDS

AWGN	Additive White Gaussian Noise
MIMO	Multiple Input Multiple Output
BW	Bandwidth
SNR	Signal to Noise Ratio
AoA	Angle of Arrival
ULA	Uniform Linear Array
NULA	Non-Uniform Linear Array
UCA	Uniform Circular Array
HA	Hexagonal Array
SA	Star Array
$\mathbf{H}^H$	Hermitian Transpose oh matrix **H**
"o"	Hadamard Product

1. MIMO SYSTEMS & ANTENNA ARRAYS

MIMO system is a significant technological breakthrough towards ubiquitous next generation communication systems that meet the demand of higher access speeds, quality of service assurance and conducive multi-user environment. The improvement in achievable capacity is largely due to antenna gain, diversity gain, spatial multiplexing gain and interference reduction. Recent researches have demonstrated that the capacity of most wireless systems can be significantly increased by making use of spatial diversity [2, 3, 4]. To overcome the problem of limited bandwidth (BW), multiple antennas can be employed in one or both side of the transmission link. If the propagation medium is rich scattering, MIMO structure can achieve much greater capacity with no increase in BW or transmitted power. The ability of achieving this high capacity is influenced by the available channel information at the receiver and/or transmitter, channel environment, geometry of array used and the correlation between the channel gains on each antenna element.

An array system is a collection of antennas which are spatially distributed at judicious locations in the 3-dimentional real spare, with a common reference point [5]. The overall quality of systems performance is naturally a function of the array structure in conjunction with the geometrical characteristics of the signal environment, as well as algorithms employed. The impact of array is modelled into the expression of capacity using the concept of manifold vector. The expression of capacity is altered to introduce the influence of channel matrix, which can be shown to be a function of the manifold vector of antenna array. For different array configurations manifold vectors are generated and their impact on system capacity is observed and compared.

2. EXPRESSION OF CAPACITY AND ANTENNA ARRAY

In this paper the analysis is conducted in an idealized context for a quasi-static channel. The channel BW and transmitted power is being constrained. The channel characteristic is not known to the transmitter but the receiver knows the characteristic which is subject to Rayleigh fading. The analysis is restricted to narrowband case where the bandwidth is taken to be narrow enough that the channel can be treated as flat over frequency. The assumption of independent Rayleigh paths make sure that for antenna elements placed on a rectangular lattice with half wavelength spacing, the path losses roughly tend to decorrelate [2]. The noise is assumed to be additive white Gaussian noise (AWGN) and inter-element spacing for the array elements is assumed to be half wavelength ($\lambda/2$).

2.1 Model for wireless channel

- Number of antenna at the transmitter, M.
- Number of antenna at the receiver, N.
- Transmitted signal is M dimensional and total power is constrained to P_S regardless of the value of M.
- Where there is only one antenna, it radiates power P_S and the average power at the output of each of the receiving antennas is P.
- The noise at receiver is complex N dimensional AWGN with statistically independent power P_n in each of the N receiving antennas.
- The channel matrix, **H**, is Rayleigh distributed with complex, zero mean and unit variance.

2.2 Capacity and channel matrix

From the seminal papers by Fochini [2,3] and Teletar [4], with some simple modifications, the expression of capacity of a MIMO communication system, subjected to Rayleigh fading, for the above mentioned assumptions is,

$$C = \frac{1}{2}\log_2\{\det(\mathbf{I}_N + \frac{SNR}{M}.\mathbf{H}.\mathbf{H}^H)\} \tag{1}$$

Here,

H is the channel matrix

$\mathbf{I}_N$ is $N \times N$ Identity Matrix

$SNR =$ Signal to noise ratio $= \frac{P_S}{P_n}$

Using beamforming techniques (Spatial Correlation Weight, $\underline{w}$) to decrease the dependence on the propagation conditions, the expression of capacity becomes,

$$C = \frac{1}{2}\log_2\{\det(\mathbf{I}_N + \frac{SNR}{M}.\frac{\underline{w}^H.H.H^H.\underline{w}}{\underline{w}^H.\underline{w}})\} \tag{2}$$

2.3 Introducing array geometry in the expression of capacity

The impact of antenna array geometry is modelled into the expression of the capacity by making channel matrix a function of array manifold vector. In this case the over all channel comprises of transmit and receive manifold vectors in addition to fading coefficients.

For the MIMO channel model given in Fig.1, when the channel matrix comprises fading coefficient and

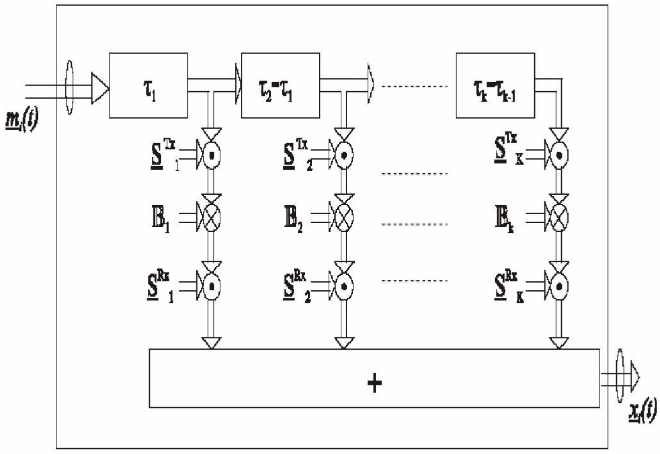

Fig. 1 MIMO Channel Model.

Both the transmitter and receiver array geometry, the overall channel matrix of a MIMO system can be expressed as,

$$\mathbf{H} = (\underline{S}^{Rx}.\underline{S}^{Tx^H})\circ\mathbf{B} \tag{3}$$

Here,

$\underline{S}^{Rx}$	is the array manifold vector at the receiver
$\underline{S}^{Tx}$	is the array manifold vector at the transmitter
B	is the fading coefficient matrix

Using this value of overall channel matrix we have the expression of the capacity of a MIMO communication system as a function of the antenna array geometry of the transmitter and receiver,

$$C = \frac{1}{2}\log_2\{ \det (\mathbf{I}_N + \frac{SNR}{M} \cdot \frac{\underline{w}^H.((\underline{S}^{Rx}.\underline{S}^{Tx})\circ B).((\underline{S}^{Rx}.\underline{S}^{Tx})\circ B)^H.\underline{w}}{\underline{w}^H.\underline{w}})\} \tag{4}$$

3. CAPACITY USING DIFFERENT ARRAY CONFIGURATIONS

Using equation (4), simulations are run to observe and compare the achievable capacity in MIMO communication systems for different array configurations. Uniform linear array (ULA), Non-uniform linear array (NULA), Uniform circular array (UCA), Hexagonal array (HA) and Star array (SA) configurations are used to compare there performances. Number of antenna array element is taken to be 7 and inter-element spacing is 1 unit (in half wavelength). The angle of arrival of the signal at the receiver is taken to be 70^0 and the mainlobe of the antenna array pattern is directed at 60^0 by using 'spatial correction weight'. The variation in achievable capacity, w.r.t. SNR and angle of arrival (AoA), for Hexagonal array configuration is given here,

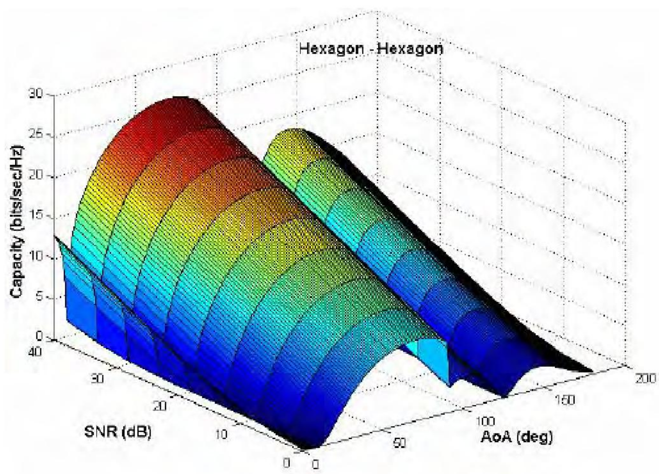

Fig. 2 MIMO system with Hexagonal array using beamforming techniques: Variation in capacity with SNR and AoA.

The above figure shows that for Hexagonal array configuration, capacity is maximum in the direction which coincides with that of the mainlobe of antenna pattern (assumed to be at 60^0 in this case). The effect of using beamforming technique is obvious from this result. As expected, capacity of the system is increasing with SNR and varying with AoA depending on the position of mainlobe of the antenna array pattern and the direction in which signal arrives at the receiver. All the remaining four array configurations were also tried to observe and compare their performances. Each time capacity was influenced by the direction of the mainlobe and angle of arrival of the received signal. For all configurations maximum capacity was in the direction where direction of signal arrival coincides with the mainlobe of the array.

Now to compare the impact of different array configurations (i.e. different geometric shape of array) on the system capacity, the variation as a function of SNR and AoA is given in Fig.3 and Fig. 4, respectively. Fig. 3 presents the change in capacity, as antenna geometry is changed, as a function of SNR while AoA is assumed to be fixed at 70^0. Fig. 4 shows the variation in system capacity as a function of AoA while SNR is being kept fixed at 20dB.

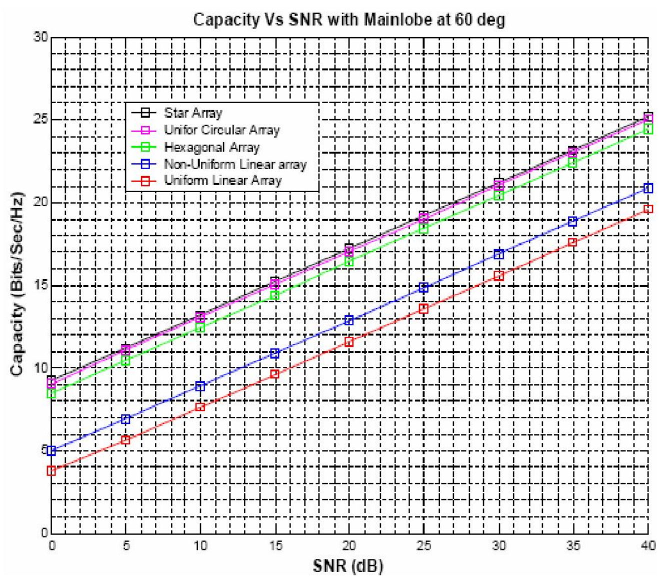

Fig. 3 MIMO system using beamforming techniques: Capacity for different antenna array geometry (AoA 70^0).

Fig.3 and Fig.4 show that when the signal arrives from an angle of 70^0, capacity obtained using ULA configuration at the receiver has the lowest value due to high spatial correlation, compared to the other four array configurations. NULA performance is slightly better than that of ULA and provides lower spatial correlation. For this specific situation, Star array configuration provides the best performance in terms of capacity. Also in Fig.4, the effect of beamforming technique ('Spatial Correction Weight' was used in this paper) is obvious. It can be clearly seen that we have the highest value of capacity where the gain is maximum.

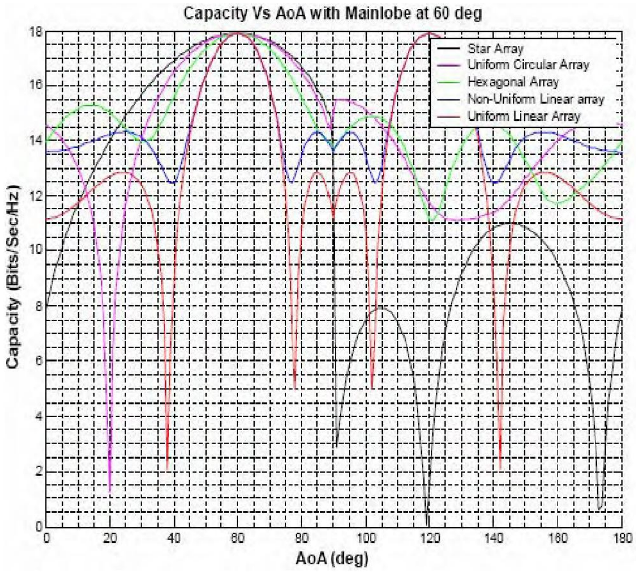

Fig. 4 MIMO system using beamforming techniques: Capacity for different antenna array geometry (SNR 20dB).

4. CONCLUTION AND FUTURE WORK

This paper presents a brief analysis of the impact array geometry has on capacity. Five different array configurations were tried to analyze their influence on capacity. The use of beamforming techniques is also taken into consideration. It was interesting to notice that without the use of beamformer, different array configurations did not provide any significant change in achievable capacity.

In this paper the AoA and direction of the mainlobe of antenna array was taken to be fixed. Taking different combinations of these two factors should provide interesting results to be considered and it remains to be done in future. Also increasing the number of antenna elements keeping or without keeping inter-element spacing fixed, promises to give more insight to the impact array configurations have on system capacity.

MIMO systems using multiple transmit and receive antennas are widely recognized as the vital breakthrough that will allow future wireless systems to achieve higher data rates with limited BW and power resources. However, the capacity benefits depend strongly on how well the channel can be tracked at both ends and whether the fades associated with different transmit and receive antennas are correlated or not. There is a wide range of scopes for future works to analyze the progress we have made towards determining the capacity benefits of multiple antennas under different assumptions about the underlying channel and what is known about this channel at the transmitters and receivers.

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