Shridhar Patil

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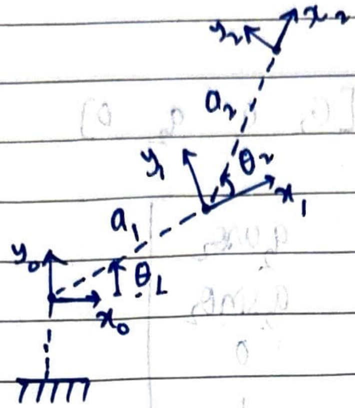
MRT practice problems:

Q1) Planar Elbow Manipulator.



i) link 0 to link 1

A_0^1 = inverse of A_1^0

A_1^0 characterised by 4 D-H parameters.

$[\theta_i \;\; d_i \;\; a_i \;\; \alpha_i]$

$= [\theta_1 \;\; 0 \;\; a_1 \;\; 0]$

$$A_1^0 = \begin{bmatrix} \cos\theta_i & -\sin\theta_i\cos\alpha_i & \sin\theta_i\sin\alpha_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\theta_i\cos\alpha_i & -\cos\theta_i\sin\alpha_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \underbrace{R}_{3\times3} & \underbrace{P}_{3\times1} \\ 0 & 1 \end{bmatrix}$$

$$A_1^0 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & a_1\cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & a_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_0^1 = \begin{bmatrix} R^T & -R^T P \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 & -a_1 \\ -\sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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ii) link 1 to link 2.

A_1^2 : inverse A_2^1

A_2^1 parameters (D-H) $\Rightarrow [\theta_2 \; 0 \; a_2 \; 0]$

$$A_2^1 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} R^T & -R^T P \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & -a_2 \\ -\sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

iii) link 0 → link 2

$$A_0^2 = A_1^2 A_0^1$$

$$= \begin{bmatrix} \cos\theta_2 & \sin\theta_2 & 0 & -a_2 \\ -\sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 & -a_1 \\ -\sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

for origin of link 2 in base frame we require

$$A_2^0 = A_1^0 A_2^1$$

$$\vec{x}_f = A_1^0 A_2^1 (O_{frame\,2})$$

4x1 matrix $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

$$\vec{x}_f = \begin{bmatrix} a_2\cos\theta_2\cos\theta_1 - a_2\sin\theta_2\sin\theta_1 + a_1\cos\theta_1 \\ a_2\cos\theta_2\sin\theta_1 + a_2\sin\theta_2\cos\theta_1 + a_1\sin\theta_1 \\ 0 \\ 1 \end{bmatrix}$$

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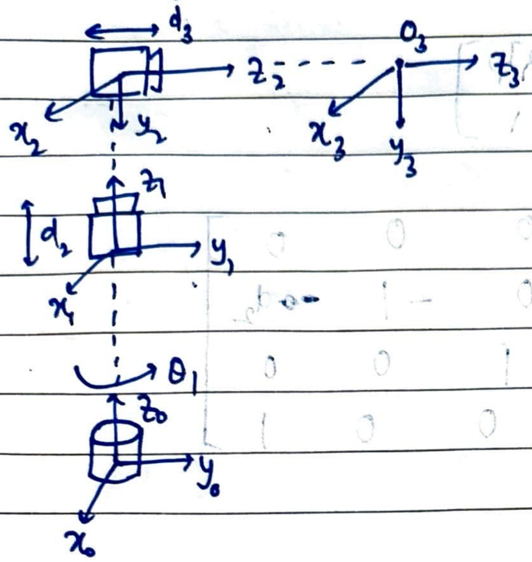
Q2) Three link cylindrical robot.



To find $A_W^R = A_2^R \, A_1^2 \, A_W^1$

i) $A_W^1 =$ inverse A_1^W

$A_1^W \Rightarrow [\theta_i \quad d_i \quad a_i \quad \alpha_i] = [\theta_1 \quad 0 \quad 0 \quad 0]$

$$A_1^W = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_W^1 = \begin{bmatrix} R^T & -R^T P \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & \sin\theta_1 & 0 & 0 \\ -\sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$A_1^2 =$ inverse A_2^1

$A_2^1 \Rightarrow [0 \quad d_2 \quad 0 \quad -\frac{\pi}{2}]$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$A_1^2 = \text{inverse } A_2^1$$

$$= \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix}$$

$$A_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^R \rightarrow \text{Inverse } A_R^2$$

$$A_R^2 \Rightarrow [0 \quad d_3 \quad 0 \quad 0]$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_w^R = A_2^R A_1^2 A_w^1 = T_w^R$$ (overall transformation matrix).

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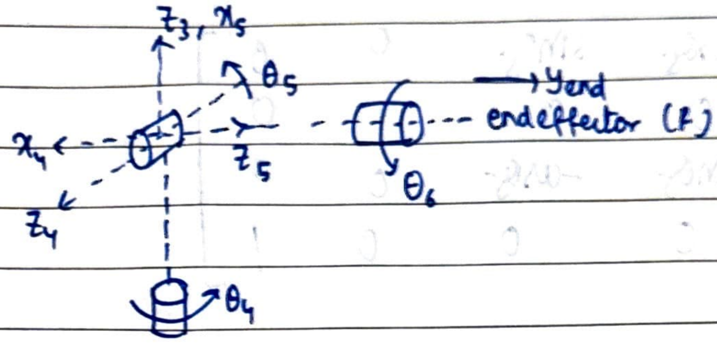
Q3) Spherical unit:-

Expanded view of spherical joint:



$$T_3^{5R} = T_5^R \, T_4^5 \, T_3^4$$

$$T_5^R = \text{inverse } T_R^5$$

$$T_R^5 \Rightarrow [\theta_6 \quad 0 \quad 0 \quad 0]$$

$$= \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5^R = \begin{bmatrix} \cos\theta_6 & \sin\theta_6 & 0 & 0 \\ -\sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R^T & -R^Tp \\ 0 & 1 \end{bmatrix}$$

$$T_4^5 = \text{inverse } T_5^4$$

$$T_5^4 \Rightarrow [\theta_5 \quad 0 \quad 0 \quad \tfrac{\pi}{2}]$$

$$T_5^4 = \begin{bmatrix} \cos\theta_5 & 0 & \sin\theta_5 & 0 \\ \sin\theta_5 & 0 & -\cos\theta_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$T_4^5 = \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix}$$

$$T_4^5 = \begin{bmatrix} \cos\theta_5 & \sin\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin\theta_5 & -\cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T_3^4 = inverse of T_4^3

$T_4^3 \Rightarrow [\ \theta_4 \quad 0 \quad 0 \quad -\frac{\pi}{2}\ ]$

$$T_4^3 = \begin{bmatrix} \cos\theta_4 & 0 & -\sin\theta_4 & 0 \\ \sin\theta_4 & 0 & \cos\theta_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^4 = \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_4 & \sin\theta_4 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ -\sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^R = T_5^R \, T_4^5 \, T_3^4$$

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Q4) Cylindrical manipulator with spherical unit.

$$T_W^R = T_{R_1}^{R_0} T_W^{R_1} \quad (T_{R_1}^R T_W^{R_1})$$

Here R_1 is end effector of cylindrical manipulator.

$$\therefore T_W^R = T_{R_1}^R T_W^{R_1}$$

here R_1 acts as W for spherical unit.

$$T_W^R = (T_5^R T_4^5 T_{R_1}^4)(A_2^{R_1} A_1^2 A_W^1)$$

$(R_1 = 3 \text{ here})$