From: owner-phenix-p-l@bnl.gov on behalf of Paul Stankus [stankus@mail.phy.ornl.gov]
Sent: Wednesday, February 18, 2004 3:03 PM
To: phenix-p-l@bnl.gov
Cc: Paul Stankus
Subject: notes on x_F and x_Bj, for clarity

Greetings,

Since it has become clear in a recent thread that there is some confusion in some people's understanding of the various "x" variables used in discussing parton physics, in the interest of saving global manpower I thought I would offer a quick note to remind non-experts of the basic notation. I hope this is of some benefit primarily to our younger viewers and others without a background in partons (spinners and other experts will doubtless spot some mistakes, and so are not invited to read too closely...).

Paul

The Bottom Line:

If you listen in on discussions of parton physics you may hear the names of two "x" variables: "Feynman x", also referred to as "X-F" and written x_F or x_{Feynman}; and "Bjorken x", written x_{Bj} and usually referred to as just "x".

For non-experts a useful rule is: when you hear people throwing around "x", as in "that's only true at low x", 99% of the time the "x" they're talking about is the "Bjorken x" which is a central, defining quantity in the parton description of hadrons. The other "x", "Feynman x", arises from a different tradition, and while it has a place in parton physics, particularly for certain measurements, it is _not_ a central quantity on the same level with "Bjorken x".

Now, in historical order:

"Feynman x", usually written x_F

"Feynman x" is a scaling variable defined to describe hadron production in p+p collisions, well before people even talked about quarks or partons (yes, there was a time before quarks!). In the inclusive production of some particle A, ie p+p -> A + anything, we define x_F as

x_F = P^A_Z / P^A_{Z Max}

where P^A_Z is the longitudinal momentum of particle A in the p+p CM frame and P^A_{Z Max} is the maximum momentum that A could ever have in this frame, based on the energy of the collision and the masses of the particles. It should be clear that x_F can vary between -1 and 1, and if you view it as identifying a spot on a scale between minimum and maximum possible momenta (not the same in all frames, of course) then it's even boost-invariant.

The original idea was that if you looked at data for A production at different p+p beam energies they might show a common behavior if you plotted them against x_F, and this would factor out the beam-energy dependence of the shape of the spectrum. Data which follow this are said to show "x_F scaling" which is why x_F is referred to as a scaling variable. [Plots of this type for p-bar production at/near AGS energies, for example, can be found in fig 4.11 of my thesis, on page 185, so I've been through this drill at least once.]

"Bjorken x", officially written x_{Bj} but usually just x

The main "x" that appears in the parton description of hadrons was originally defined in deeply inelastic scattering, where it is one of a handful of relativistically-invariant variables describing the final state of the electron in high-energy e+p scattering. (DIS started with e+p scattering and later expanded to e+A, mu+p/A and even neutrino+p/A.) In low-brow terms, the great contribution of Bjorken was to show that x was a good scaling variable, ie the spectra of outgoing electrons in DIS showed the same shape when plotted against x, even when the other final-state variables and the beam energy were changed. This property of DIS spectra is referred to as "Bjorken scaling", or just "scaling" for short, and the x of DIS became known as "Bjorken x" or x_{Bj}.

The original DIS definition of x is not very enlightening if you're interested in hadron+hadron collisions, however, and I will not even show it here. More interesting for us is the interpretation of x_Bj as a momentum fraction: if you view a DIS event in a frame in which the proton is very highly boosted (such as the electron's rest frame, also called the "infinite momentum frame") then it turns out kinematically that the x_Bj of the event is the fraction of the proton's total momentum which was involved in the scattering. If you imagine the electron as having had a hard interaction with some object within the proton --- a "parton" -- then x_Bj can be seen as the parton's share of the proton's total momentum.

This then leads to the view that the DIS spectra plotted vs x_Bj are reflecting the probability of finding a parton within the proton at that momentum fraction. Thus, the distribution of partons over x_Bj is an intrinsic property of the proton! which explains why it is a good scaling variable independent of the energy of the electron beam. This view is now quite standard, and the distributions of partons (now recognized as quarks, antiquarks, and gluons) over x_Bj are treated as _the_ most fundamental description of the proton (and other hadrons) as seen in high-energy interactions. (It should be clear that x_Bj varies between 0 and 1; the shapes of parton distributions are decidedly non-trivial, and their behavior at very low x is a subject of great study and interest in particular.)

We who study hadron-hadron collisions have inherited the momentum-fraction description, often completely unaware (!) of its origins in DIS experiments, and would describe an elementary process within a p+p or A+A collision as: "a parton in a proton(/nucleon) in beam 1 with momentum fraction x_1 interacts with a parton in a proton(/nucleon) in beam 2 with a momentum fraction x_2, which produces thus-and-such in the final state". These x_1 and x_2 are each x_Bj's; and since we know the momenta of protons(/nucleons) in the beams, we can relate x_1 and x_2 immediately to the total energy and total momentum of the parton-parton system, which we can (sometimes) see in the final-state products of the interaction.

x_F in the parton picture

It should be clear from the foregoing that "Bjorken x" is absolutely central to the parton picture; and while Feynman's name does come up in the story of partons (see below) the "Feynman x" variable is not a basic part of the parton picture in any way. Still, we can translate the original Feynman x into the parton language, where it does turn out to have some utility.

Consider a "2->1 process", by which we mean two partons in the initial state combining to form a single particle in the final state. As a concrete example you can picture gluon+gluon -> J/Psi (even though this technically doesn't actually happen! it will suffice as an example). The single final-state particle will have some longitudinal momentum P_Z; and by total momentum conservation this must be equal to P_Z = (x_1 - x_2) root{s}/2 in the collider's nucleon+nucleon CM frame. If the mass of the single final-state particle is small compared to root{s}, as would be true of J/Psi's at RHIC, then the maximum P_Z the particle could have would be essentially root{s}/2. If we keep the same original defintion of x_F as above, then we have the simple result:

x_F = P_Z / P_{Z Max} = x_1 - x_2

as was pointed out by David D. and Mike T. earlier in the thread.

This use of x_F to describe a 2->1 process has some obvious utilities; for example, if x_Bj is a good scaling variable, then x_F will be also. And it provides a handy shorthand, so "high x_F" corresponds to high lab energies (and high rapidities) for the J/Psi, "zero x_F" corresponds to production at mid-rapidity in the collider frame, and "positive x_F"("negative x_F") indicate the J/Psi moving in the same(opposite) direction as whichever is defined as beam 1.

Since x_F is a property of the final-state particle -- or equivalently a property of the two-parton system -- it is _not_ a good way to refer to the properties of a proton(/nucleon). For example, you would almost never say "I'm going to look at the low-x_F behavior of the proton", since low x_F's can come from a variety of x_Bj's in the protons(/nucleons) and fixing low x_F does not focus usefully on any particular part of the proton.

You can make the argument that very high x_F does require one of the x_Bj's to be low (specifically x_2 in the sign convention above), and so "high x_F" processes must somehow involve "low x-Bj" partons, in at least one of the protons(/nucleons). However, this is still a very misleading way to talk about an investigation of low-x parton distributions, because it still mixes together very different parts of the proton(/nucleon). For example, if you fix x_F = 0.6, say, then you are not distinguishing between the cases (x_1=0.59, x_2=10^-2) and (x_1=0.5999, x_2=10^-4) even though the partons may show very different behavior at these two x_2 values.

So, to finally wind up, if you want to identify your interest in the low-x_Bj part of the proton(/nucleon) then you should just say "I'm looking at low-x physics" and leave it at that (the identification of x, with no notation, as x_Bj is now standard). Using x_F as a way of talking about x_Bj is usually misleading, and should be avoided unless you have a very specific reason for it.

OK, I hope the difference between x_F and x_Bj should be clear now! as should be when/why you would use one over the other.

History: why the confusion?

I am not sure why there is so often confusion between Bjorken x and Feynman x, though there certainly is: even people who should know better will say one when they clearly mean the other.

I can venture a guess, though, based on a story that was told to me [I'd like to know of some old-timer can confirm or deny this history; I'm not very good at history, and I was still in grade school when this was going on].

The story is that even though Bjorken identified the central role of the x scaling variable, the parton picture was not taken so seriously until Feynman started to "popularize" it as something that made sense. The scene is Feynman, the great man, lending his credibility to support Bjorken, then a bright but not well-known young man. [Everyone who works in relativistic heavy-ion physics can be grateful that Bjorken, the great man, lent his credibility to us very early on in things like measuring initial thermalized energy densities.] The momentum-fraction interpretation of x_Bj in particular I have heard attributed to Feynman. If true, this may have fixed an association between "Feynman" and "parton x" in many people's minds, from which it is only a short (mis-)step to identifying "parton x" as "Feynman x". This is wrong not once but twice, since Bjorken's name belongs on x_Bj, and x_Feynman was already defined as a different quantity.

Anyway, this is just under-informed speculation on my part, and I would be curious to hear from anyone who knows the actual history in detail.