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Sequential suppression of quarkonia and high-energy nucleus–nucleus collisions

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According to the usual application of the sequential-suppression picture to the dynamics of heavy quarkonia in the hot medium formed in ultrarelativistic nuclear collisions, quark–antiquark pairs created in a given bound or unbound state remain in that same state as the medium evolves. We argue that this scenario implicitly assumes an adiabatic evolution of the quarkonia and we show that the validity of the adiabaticity assumption is questionable.

Keywords: Quarkonia; heavy-ion collisions; adiabaticity.

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More than 25 years ago, finite-temperature gauge-theory studies on the lattice of the screening of static (color) charges prompted Matsui and Satz to suggest that in a quark–gluon plasma (QGP), the formation of bound charmonia may be prevented.[1] It was quickly realized that the various $c\bar{c}$ or $b\bar{b}$ states might, in fact, be dissociated at different temperatures.[2,3] Over the years, paralleling progress in theoretical studies of quarkonia properties (see Refs. 4 and 5 for recent reviews), led to the "sequential-suppression" picture of heavy quarkonia as QGP thermometer.[6] According to the latter, a given state will be totally suppressed above a threshold temperature — which might actually be smaller than the transition temperature to a QGP.

This prediction is supported by several approaches. First, spectral functions are extracted from lattice QCD computations of correlators of quantum numbers for heavy quark–antiquark pairs.[7–10] The disappearance of a peak in the spectral function then signals the suppression of a bound state. Alternatively, one resorts to an effective in-medium quark–antiquark ($Q\bar{Q}$) potential — either derived from lattice-QCD computations of spatial correlators[1–3,11–15] or derived within finite-temperature field theory[16–18] — which then enters a Schrödinger or Bethe–Salpeter equation, whose bound states model the quarkonia in the medium. The suppression of a given state takes place when it is no longer bound by the potential although the precise criterion for dissociation might be open to discussion.[19]

In either description, the in-medium bound states of heavy quark–antiquark pairs are, be it explicitly stated or not, *eigenstates* of a Hamiltonian. Note that as the effective potential might actually possess an imaginary part,[15–17] these eigenstates are not necessarily stable, but might have only a finite lifetime. That notwithstanding, the generally accepted picture is that of a temperature-dependent suppression pattern, in which at a given energy density of the medium some states survive, while more excited ones are not bound and thus do not form.

This picture is seemingly supported by experimental measurements in ultrarelativistic nucleus–nucleus collisions. At the SPS, the NA50 collaboration reported that the anomalous suppression of the ψ' in Pb–Pb collisions sets in at a smaller average in-medium path crossed by the $c\bar{c}$ pair than for the J/ψ.[20] At the much higher large hadron collider (LHC) energy, the CMS collaboration studied bottomonia and observed yields consistent with the idea that the excited $\Upsilon(2S)$ and $\Upsilon(3S)$ states are more suppressed than the deeper bound $\Upsilon(1S)$.[21]

The usual explanation for such observations in high-energy nucleus–nucleus collisions is the following, where for the sake of simplicity we leave aside so-called "initial-state effects".[a] At an early stage after the collision, say some instant t_0, the created deconfined medium reaches a high enough energy density that a given quarkonium state, which we shall refer to as "excited", is suppressed, while another state of the same system, hereafter the "ground state", is bound. The common lore is then that, as the medium expands and cools down ($t > t_0$), the ground state stays unaffected, whereas the depopulated excited state remains suppressed, even when the medium temperature has dropped below its dissociation threshold. The only possibility left to the excited state for being recreated is at the transition to the hadronic phase, through the "recombination" of heavy quarks and antiquarks uncorrelated till then.[22,23] That scenario constitutes the standard implementation of the idea of sequential suppression of heavy quarkonia in heavy-ion collisions. Note that this description totally ignores the possible finite lifetime of the ground state due to the imaginary part of the in-medium potential.

Inspecting the scenario sketched above critically, it relies on two basic ingredients. There is first the sequential-suppression pattern in the "initial condition" at t_0, whose theoretical foundations are discussed above. The second element in the scenario is the implicit assumption that "the quarkonium ground state remains the ground state" over the duration of the medium evolution albeit the binding potential changes with time. To our knowledge, the validity of that statement has not been examined before. Recasting this hypothesis more mathematically, a $Q\bar{Q}$ pair

[a]When comparing *relative* yields of different states of a given system, say S-channel charmonia or bottomonia, for a fixed type of nuclear collisions, these effects should play a minor role.

initially in the eigenstate with lowest energy of the (effective) Hamiltonian describing in-medium quarkonia remains in the lowest-energy eigenstate. More generally, the same will hold for every initially bound state — up to late electroweak decays which take place outside the medium. That is, it is assumed that heavy quarkonia are continuously evolving eigenstates of an adiabatically changing instantaneous Hamiltonian. Accordingly, the scenario for the sequential suppression of quarkonia in the medium created in high-energy nucleus–nucleus collisions relies on the hypothesis that the effective in-medium quark–antiquark potential varies slowly enough that each $Q\bar{Q}$ pair is at every successive instant in an energy eigenstate. We now wish to investigate the validity of this assumption.

Before going any further and to dispel any confusion, let us note that the adiabaticity we discuss here is neither that of the medium evolution — related to the production of entropy — nor the adiabatic assumption à la Born–Oppenheimer which allows one to separate gluons from the non-relativistic heavy quarks when writing down an effective potential for the latter.[24]

Let $|n(t)\rangle$ denote the eigenstates of an instantaneous Hamiltonian $\mathsf{H}(t)$ with respective energies $E_n(t)$. Following the approach of Ref. 25, a common criterion for the validity of the adiabatic theorem is the requirement that for every pair of states $|n(t)\rangle$, $|n'(t)\rangle$ and at every instant in the evolution[b]

$$\frac{|\langle n'(t)|\dot{\mathsf{H}}(t)|n(t)\rangle|}{[E_n(t)-E_{n'}(t)]^2} \ll 1 \tag{1}$$

with $\dot{\mathsf{H}}(t)$, the time derivative of the Hamiltonian. In the case of interest for us, $\dot{\mathsf{H}}(t)$ coincides with the time derivative $\mathrm{d}\mathsf{V}/\mathrm{d}t$ of the effective $Q\bar{Q}$ potential.[c] In turn, the latter is simply the product of the rate of change $\dot{T}$ of the medium temperature times the derivative $\mathrm{d}\mathsf{V}/\mathrm{d}T$ of the potential with respect to T, where for the sake of simplicity we have assumed that the medium is (locally) thermalized. For $\dot{T}$, we took the results from a simulation of central Pb–Pb collisions at the LHC within dissipative hydrodynamics,[26] considering the evolution of temperature at the center of the fireball: within the first 7 fm/c of the evolution (that is, as long as $T >$ 200 MeV), the magnitude of $\dot{T}$ always remains larger than about 30 MeV per fm/c and up to 50 MeV per fm/c in the early stages.

For the $Q\bar{Q}$ potential, we considered the lattice QCD results of Ref. 27, including the parametrization

$$V(r) \sim \frac{\frac{4}{3}\alpha_s(T)}{r}\,\mathrm{e}^{-A\sqrt{1+N_f/6}\,T g_{2\text{-loop}}(T) r} \tag{2}$$

with $A \simeq 1.4$ and $g_{2\text{-loop}}(T)$ the 2-loop perturbative coupling, where $0.5 \lesssim \alpha_s(T) \lesssim 1.0$. One then finds as typical amplitude for a matrix element of $\mathrm{d}\mathsf{V}/\mathrm{d}T$ between eigenstates of the instantaneous Hamiltonian

[b]We use a system of units in which $\hbar = c = 1$.
[c]The effective masses of the heavy constituent quarks may also evolve, yet this should constitute a subleading effect, to be considered only in an accurate description.

$$\left| \left\langle n'(t) \middle| \frac{\mathrm{d}\mathsf{V}}{\mathrm{d}T} \middle| n(t) \right\rangle \right| \approx 200\text{–}500\ \mathrm{MeV} \cdot \mathrm{fm}\,.$$

The numerator in Eq. (1) is thus of order (80–160 MeV)^2. In turn, the denominator is of order (100–350 MeV)^2 for the excited $b\bar{b}$ states, so that the ratio (1) can be in some cases smaller than 0.1, for other channels larger than 1. Because of those channels, it is far from warranted that the adiabaticity assumption holds: the potential evolves so quickly that a quark–antiquark pair which at some time is in a given instantaneous eigenstate will a short while later no longer be in the evolved eigenstate; instead, it will have components over all the new eigenstates — including the new ground state. That is, even if criterion (1) holds for the latter, yet it is populated by contributions from excited states.

We wish to emphasize here that, this "repopulation" mechanism is neither the customary recombination at hadronization, nor the feed down from late decays, but a natural consequence of the "reshuffling" of $Q\bar{Q}$ states due to the rapid medium evolution.

A naive picture of the effect of this rapid evolution is provided by dividing the typical size $r_{\mathrm{rms}} \approx 0.3$–$0.75$ fm of a bound bottomonium by the characteristic velocity $v \sim 0.3c$ of the non-relativistic constituent quark and antiquark, which gives a duration $\tau \approx 1$–2.5 fm/c for an "orbit" of the b-quark. On such a time scale, the QGP cools down by at least 30 MeV to 75 MeV, resulting in a significant change in the effective potential (2), which illustrates why the adiabatic evolution of bottomonia is far from being warranted.

As a final argument against using the hypothesis of an adiabatic evolution of $Q\bar{Q}$ pairs in a QGP, we note that recent studies emphasized the fact that even when criterion (1) is satisfied, i.e. the evolution is slow — the system with evolving Hamiltonian can be driven from one instantaneous eigenstate to a different one at later times by resonant interactions (see e.g. Ref. 28). The latter lead to Rabi oscillations between eigenstates — that is, they are tailored to induce transitions which violate the adiabatic theorem — on a time scale given by the inverse of the Rabi frequency ω_R.

In the case of a $Q\bar{Q}$ pair in a QGP at the temperatures found in high-energy nuclear collisions, there are obviously plenty of degrees of freedom around with energies matching possible transition lines. The corresponding Rabi frequencies, however, depend on the interaction term. Adopting for the sake of illustration a dipolar interaction, one finds values of π/ω_R, which in a two-level system is the time after which a transition has occurred with probability 1, of the order 2 fm/c to 20 fm/c, depending on the transition Bohr frequency, the medium size and the assumed coupling strength. This means that on such a time scale a $Q\bar{Q}$ pair certainly does not remain in the same instantaneous eigenstate, which again hints at the invalidity of the adiabatic theorem for heavy quarkonia in a dynamical QGP.

One might be tempted to argue that in an effective potential approach, the transition-inducing degrees of freedom have been integrated out. Yet the construction of an effective theory ultimately relies on the adiabatic theorem,[29] so that it is inconsistent to use the notion blindly here. More precisely, we surmise, although we have not investigated this idea in detail, that the violation of adiabaticity caused by resonant interactions translates into the imaginary part of the effective in-medium potential, which physically has the same effect of giving a finite lifetime to the Hamiltonian eigenstates.

In summary, we have shown that the usual scenario for the sequential suppression of heavy quarkonia in the hot medium created in ultrarelativistic heavy-ion collisions relies on the hypothesis that $Q\bar{Q}$ pairs evolve adiabatically in the medium. We have then presented several arguments which make us doubt that this assumption holds. In our view, this hints at the idea that a given quark–antiquark pair is in a constantly changing linear combination of instantaneous energy eigenstates, rather than in a smoothly evolving unique eigenstate. As a consequence, one should explicitly follow the evolution of the pair in the QGP, using a dynamical microscopic description, as attempted in Ref. 30. Such an approach should, of course, be suited for rapidly evolving media, so as to eventually be able to compare with experimental results from high-energy nucleus–nucleus collisions. On the other hand, in the regime of a (quasi-)static environment of the quarkonia it should also make contact with equilibrium-based formalisms as lattice QCD or finite-temperature field theory.

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