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Periodic Noise

Periodic Noise in an image is like a regular patterns of unwanted fluctuation that repeat at fixed interval across the image

Periodic Noise is all about repetition. It creates stripes or grids across the image.

Periodic Noise occurs at regular interval. In image, every few pixels or across certain areas you'll see the same pattern repeating.

Causes- It can occur due to sensor defects, electrical interference or even during image acquistion.

Periodic Noise can be removed by - Band pass
Band Reject
Notch filter

Band pass filter

Band pass filter allow frequencies within a particular range to pass through & attenuate all other frequencies.

This filter allows the frequencies if they fall in the range $D_L - D_H$

This range is known as Band.

1 D Transfer func^n is given by

$$H(D) = \begin{cases} 1 & \text{for } D_L \le D \le D_H \\ 0 & \text{for } D > D_0 \end{cases}$$

where D_0 - Cut off frequency

2-D transfer func^n is given by.

Ideal Band pass filter

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) < D_0 - \frac{W}{2} \\ 1 & \text{if } D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 0 & \text{if } D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

where D_0 - Cut off frequency
$D(u,v)$ - Distance of (u,v) from the center.
W - width of band.

A butterworth band pass filter of order n is given as

$$H_{bp}(u,v) = \frac{\left[\frac{D(u,v)\,W}{D^2(u,v) - D_0^2}\right]^{2n}}{1 + \left[\frac{D(u,v)\,W}{D^2(u,v) - D_0^2}\right]^{2n}}$$

Where n = order
W = width
$D(u,v)$ = Distance of (u,v) from center.

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Band 1

A Gaussian band ~~pass~~ pass filter, is given by

$$H(u,v) = e^{-1/2\left[\frac{D^2(u,v) - D_0^2}{D(u,v)\,w}\right]^2}$$

Band ~~pa~~ Reject filter

Band Reject filter is a filter to attenuate limited range frequencies while ~~leaving~~ allowing the other frequencies to pass.

Band-reject ~~filter~~ filter is a complement of Band pass

$$\boxed{H_{br}(u,v) = 1 - (H_{bp}(u,v))}$$

These filters are very effective in removing ~~per~~ periodic noise.

This filter reject the frequencies if ~~the~~ they fall in the range of $D_L - D_H$.

1-D Transfer func^n

$$H(D) = \begin{cases} 0 & \text{if } D_L \le D_0 \le D_H \\ 1 & \text{fo } D > D_0 \end{cases}$$

~~Ideal pass band Reject~~

Ideal band Reject is given by.

$$H(u,v) = \begin{cases} 1 & \text{if } D(u,v) < D_0 - \frac{W}{2} \\ 0 & \text{if } D_0 - \frac{W}{2} \le D(u,v) \le D_0 + \frac{W}{2} \\ 1 & \text{if } D(u,v) > D_0 + \frac{W}{2} \end{cases}$$

<u>A butterworth Band Reject filter</u>

$$H(u,v) = \frac{1}{1 + \left[\frac{D(u,v)W}{D^2(u,v) - D_0^2}\right]^{2n}}$$

<u>A Gaussian band reject filter</u>

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D^2(u,v) - D_0^2}{D(u,v)W}\right]^2}$$

<u>Notch filter</u> :- It is a special form of Band Reject filter. Instead of removing the entire range of frequencies. It removes only selective frequency components.

It is useful in removing a periodic signal of a clearly identified defined frequency like the interference patterns caused by electrical disturbances.

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An Ideal Notch filter

$$H(u,v) = \begin{cases} 0 & \text{if } D_1(u,v) \le D_0 \text{ or } D_2(u,v) \le D_0 \\ 1 & \text{otherwise} \end{cases}$$

$$D_1(u,v) = \left[\left[\left(u - \frac{M}{2}\right) - u_0\right]^2 + \left[\left(v - \frac{N}{2}\right) - v_0\right]^2\right]^{1/2}$$

$$D_2(u,v) = \left[\left(\left(u - \frac{M}{2}\right) - u_0\right)^2 + \left(\left(v - \frac{N}{2}\right) - v_0\right)^2\right]^{1/2}$$

Butterworth Notch filter

$$H(u,v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u,v)\, D_2(u,v)}\right]^n}$$

Gaussian Notch filter

$$H(u,v) = 1 - e^{-\frac{1}{2}\left[\frac{D_1(u,v)\, D_2(u,v)}{D_0^2}\right]}$$

Similar to the Band-pass / Band Reject filters the Notch reject filter can be turned into Notch pass filters with relation.

$$\boxed{H_{np}(u,v) = 1 - H_{nr}(u,v)}$$

if $u_0 = v_0 = 0$ Then

Notch reject filter becomes high pass filter
Notch pass filter becomes low pass filter

Image Restoration Methods

- Complete Knowledge Available
- Partial Knowledge available
- No prior Knowledge is available

Categorization is done on the basis of ~~Knowleg~~ Knowledge available for blurring function

→ Complete knowledge ~~avail~~ of blurring func^n.

↳ Simplest ~~case~~ approach

↳ Original Image can be retrieved by applying original Inverse filter. This approach is also known as Deconvolution.

Partial Knowledge of blurring function is available.

↳ Weiner filter is applicable.

→ No prior knowledge of blurring function

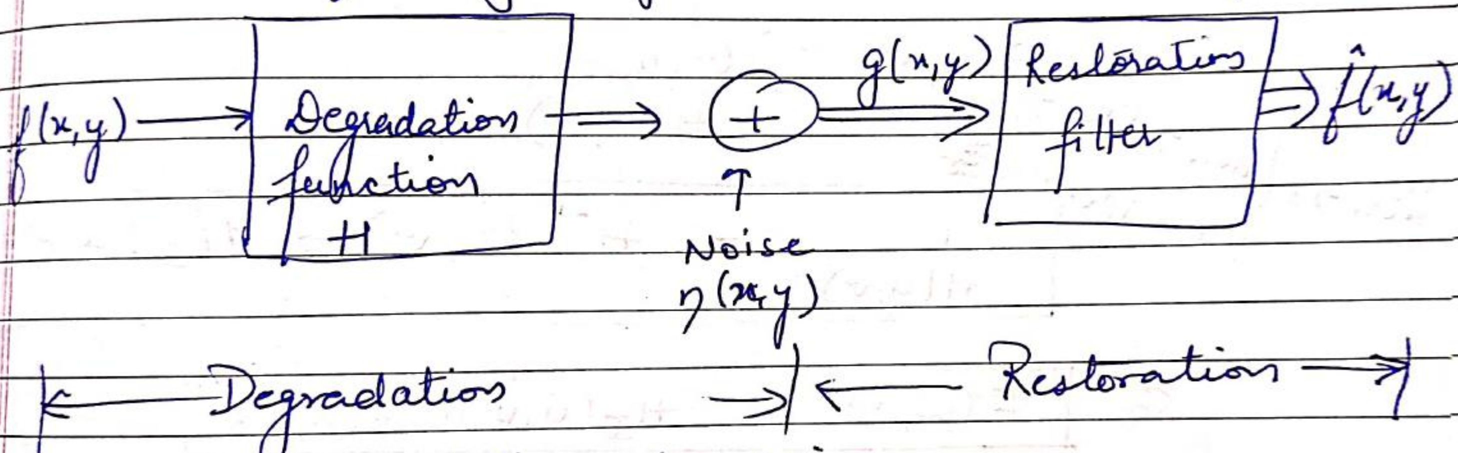
↳ Blind Restoration filter or Blind Deconvolution filter is applicable

~~The Simplest approach~~

Inverse filtering

Inverse Filtering is a technique used in image restoration to attempt to recover the original image from degraded image.

Model of Image Degradation & Restoration process



Applying Convolution

$$g(x,y) = \boxed{h(x,y) * f(x,y)} + n(x,y)$$

$\Downarrow$ F.T.

$$G(u,v) = H(u,v) * F(u,v) + N(u,v)$$

$$F(u,v) = \frac{G(u,v)}{H(u,v)} - \frac{N(u,v)}{H(u,v)}$$

Undegraded image

If $N(u,v) = 0$

then ~~there~~ ~~will~~ we can obtain $F(u,v)$ by inverse filtering.

blur

The process of removing ~~blur~~ and Noise is known as deconvolution or Inverse filtering.

Blurring → Smoothing → Low pass filter.
De-blurring → Sharpening → High pass filter.

Hence Inverse filtering act as High Pass filter (HPF). Noise is high frequency components. Hence Inverse filtering will increase ~~v~~ noise

$$F(u,v) = \frac{G(u,v)}{H(u,v)}$$

$$(u,v) = \begin{cases} \frac{1}{H(u,v)} & |H(u,v)| > 0 \\ 1 & |H(u,v)| \leq 0 \end{cases}$$

$$\boxed{\frac{1}{H(u,v)} = H^{-1}(u,v) = H_I(u,v)}$$

So $\boxed{F(u,v) = H_I(u,v) \cdot G(u,v)}$ — (1)

This shows that if we convolve the blurred Image with inverse of blurring function we get back the original image.

Here $G(u,v)$ = Blurred image.
$H_I(u,v)$ = Inverse of blurring func^n

But if noise term is also present.

$$F(u,v) = \frac{G(u,v)}{H(u,v)} - \frac{N(u,v)}{H(u,v)}$$

$H(u,v)$ ~~c~~ may be zero or close to zero.

$H(u,v)$ $\qquad$ 0

if $H(u,v) = 0$ / very very small then $N(u,v)$ ~~& G(u,v)~~ can have large values resulting in amplication of noise as it tends to infinity.

∴ ~~inferse~~ Inverse filtering is not used in its original form. then we have to go for Wiener filter.

In Case of Eqⁿ 1

$$\boxed{F(u,v) = H_I(u,v) \cdot G(u,v)}$$

$$H_I(u,v) = 0$$

$$F(u,v) = \frac{G(u,v)}{H(u,v)}$$

Inverse of $H(u,v) = \infty$
then Original image Can be obtained.
We'll use
~~F~~ Tr Transfer funcⁿ $\hat{H}(u,v)$

$$\hat{H}(u,v) = \begin{cases} \dfrac{1}{H(u,v)} & \text{if } |H(u,v)| \geq \epsilon \\ 1 & \text{if } |H(u,v)| < \epsilon \end{cases}$$

$\boxed{\epsilon \rightarrow \text{threshold Value.}}$

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~~N.~~ Wiener filter

→ Partial knowlege of blurring func^n is ~~un~~ availabe.

→ N. Wiener proposed the concept of Wiener filter in 1942.

→ Also Known as Minimum Mean Square Error filter or Least Square Error filter.

Limitation of Inverse filter

$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} - \frac{N(u,v)}{H(u,v)}$$

It may possible that $|H(u,v)|$ is very very small or ~~close~~ to zero. then Noise will increases. & i.e. tends to infinity.
Hence ~~inver~~ Inverse filter ~~in very~~ is not applicable when noise is present.
In that case Wiener filter is applicable.

This Approach incorporates both degradation function and statistical characteristics of noise into restoration process.

The Objective is to find out the estimate of the uncorrupted image such that mean square error between them is minimized.

It minimizes the overall mean square ~~err~~ error in the process of inverse filtering

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Objective – Optimize Mean Square Error

$$e^2 = E\{(f - \hat{f})^2\}$$

f = uncorrupted image.
$\hat{f}$ = estimate of f.
e = error measure.
$E(\,)$ = estimat expected value

To estimate error, the correlation matrices of f and n are required.

$R_f = E[f \cdot f^T]$ ~~$R_n = E[f$~~

$R_n = E[n \cdot n^T]$

$$\hat{f} = [H^T \cdot H + \gamma R_f^{-1} R_n]^{-1} H^{-1} g$$

$H \rightarrow$ degradation func^n
$g \rightarrow$ degraded image.

$$\boxed{\gamma = \frac{1}{\alpha}}$$ where α = Lagrange multiplier

This filter in frequency domain.

$$F(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \gamma \left| \frac{S_n(u,v)}{S_f(u,v)} \right|} \right] G(u,v)$$

Spectral Power density = of equal frequency at each interval.

$S_n(u,v)$ = spectral power density of noise.
$S_f(u,v)$ = spectral power density of image.

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Signal to noise ratio

$$\text{Ratio} = \frac{S_n(u,v)}{S_f(u,v)} = \text{constant} = K$$

$$\boxed{F(u,v) = \frac{1}{H(u,v)} \cdot \frac{|H(u,v)|^2}{|H(u,v)|^2 + rK}}$$

K = low freq. aspect of filter.
If K = 0, behave like inverse filter.