UNIT-1

Automata theory:- It is the study of abstract computing devices or machine.

* It is the model for both h/w & s/w.

why we need to study automata theory?

Automata is an useful model for many important kinds h/w & s/w.

Some kinds of automata are:-

i) S/w for designing & checking the behaviour of digital circuits.

ii) lexical analysis of compiler that breaks input text into logical units such as keywords identifies valid or not, sematics error ⇒ s/o.

iii) s/w for scanning large bodies of text, collection of webpages to find occurances of words, phases & other patterns.

iv) Also s/w for verifying systems of all types that have a finite no.of distinct states.

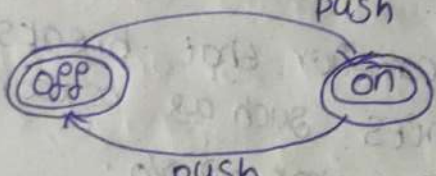
Classification of Automata:-

1. Finite Automata (FA)
2. push Down Automata (PDA)
3. Linear Bounded Automata (LBA)
4. Turing machine (TM)

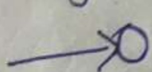
Finite Automata:-

* Finite Automata is simplest machine to recognize patterns.
* Finite Automata / finite state machine is an abstract machine that have '5' elements or Tuples.
* It has a set of states & rules for moving states from one to another, but it depends upon the applied input symbol.

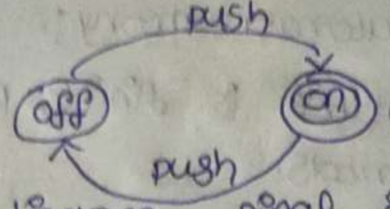
Ex:- on or off switch



* finite automata states are represented by circles labled by some i/p symbol.
* In the above diagram total no. of states are "2", named as on & off & input symbol is "push".
* In finite Automata one of the state disignated as start rate state which is represented by forward arrow followed by circle.



* In the above diagram starting state or initial state is "off".
* It is necessary to indicate one or more states as final or accepting states which is represented by double circle.



In the above diagram final state is "on".

Structural Representation of an Automata:-

there are 2 important notations plays an important role in study of Automata & their applications.

① **Grammer:-**

Grammers are useful model that process the data with recursive structure.

ex:- $E \longrightarrow aE/bE$

$E \longrightarrow a/b$

2. **Regular expression:-**

RE also denotes structure of the data especially Regular expression text string. the unique style of Regular expression is $[A-Z][a-z]^{*}[\ ][0-9]^{+}$

where

$[A-Z]$ represents range of characters from A-Z.

$[a-z]^{*}$ represents range of any no.of lower case characters from a-z.

$[*]$ represents any no.of times

$[+]$ represents atleast one occurance.

$[0-9]^{+}$ represents atleast one digit from 0-9.

Note:- paranthesis are used to group the contents of an expression.

Central concepts of Automata theory:-

① **Alphabet (Σ)**:- Alphabet is finite, non-empty collection of symbols.

To represent an alphabet symbol "Σ" (sigma) is used

Ex:- 1. $\Sigma = \{0, 1\}$ → binary alphabet

2. $\Sigma = \{a, b, c, \cdots z\}$ → represents set of all lower case alphabet.

3. $\Sigma = \{0, 1, 2, \cdots, 9\}$ → set of all digits.

4. $\Sigma = \{A, B, \cdots Z\}$ → Represents set of all upper case alphabet.

5. set of ASCII characters.

② **String**:- It is a finite collection of symbols from alphabet (or)

string is a finite sequence of character choosen from some alphabet.

Ex:- i, 0101 is a string choosen from binary alphabet

ii, aba is a string which is choosen from $\Sigma = \{a, b\}$

③ Empty **string** (ϵ):-

It is a string with zero occurrance of symbol denoted by "ϵ" (epsilon)

Empty string is a string of length "zero".

Ex:- $|0101| = 4$ $|\epsilon| = 0$

4. length of string:-
the no.of positions for symbols in string is called length of the string (or) no.of symbols in string gives length of the string.
standard notation for length of the string ω is "$|\omega|$".

Ex:- $\omega = a1obc$
$|\omega| = 5$

5. Power of an alphabet:- If "Σ" is an alphabet then we defined Σ^k to be the set of strings of length "k".
Each of whose symbols is in "Σ".
Ex:- if $\Sigma = \{a,b\}$ then $\Sigma^0 = \{\epsilon\}$
where "ϵ" is the only string whose length is "zero".
$\Sigma^1 = \{a,b\}$, $\Sigma^2 = \{aa, ab, ba, bb\}$
$\Sigma^3 = \{aaa, aab, aba, baa, bba, bab, baa, bbb\}$
$\Sigma^* \Rightarrow$ set of length of strings 0, 1, 2, 3, ...
$\Sigma^* \Rightarrow \{\epsilon, a, b, aa, ab, ba, bb, aaa, \ldots\}$
$= \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \ldots$

Note:- ① set of all strings over the alphabet Σ is denoted by Σ^*
② set of non-empty strings from the alphabet Σ is denoted by Σ^+
$\Sigma^+ = \Sigma^* - \{\epsilon\} = \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \ldots$

3. set of all strings over the alphabet Σ is

$$\Sigma^* = \Sigma^+ \cup \{\epsilon\} = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \cdots$$

⑥ **concatenation of strings:-**

let x & y be two strings then xy denotes the concatenation of x and y where x is a string composed of 'i' symbol.

i.e $x = \{a_0, a_1, a_2, a_3, \ldots a_i\}$ and

y is string composed of 'j' symbol.

i.e $y = \{b_0, b_1, b_2, \ldots, b_j\}$

then $xy = a_0 a_1 a_2 \cdots a_i b_0 b_1 b_2 \cdots b_j$

ex:-

If $x = 0110$ & $y = 100$ then $xy = 0110100$

Note:-

For any string ω, $\epsilon \cdot \omega = \omega \cdot \epsilon = \omega$

Here ϵ is identity for concatenation.

$\boxed{\text{epsilon} = \epsilon}$

$\boxed{x = a, y = b \quad xy = ab}$

$\boxed{x = a \quad y = \epsilon \quad xy = a\epsilon = |a\epsilon| = 1}$

⑦ **language:-**

language is a collection of appropiate strings (or) language is a set of strings all are which is choosen from an alphabet Σ^* where Σ is particular alphabet.

language of an automata is denoted by 'L'
If Σ is an alphabet then $L \subseteq \Sigma^*$

ex1:-
The language of all strings consists of n zeros followed by n is $\Sigma = \{0,1\}$ where $n \geq 1$
$L = \{01, 001, 011, \cdots\}$

ex3:-
set of all strings which contains equal no.of 0's & 1's over the alphabet $\Sigma = \{0,1\}$
$L = \{\epsilon, 01, 10, 0011, 1100, 0101, 1010, 1001, 0110, \cdots\}$

Note:-
1. ϕ is an empty language i.e it has no strings
$\phi = \{\}$
2. $L = \{\epsilon\}$ is not an empty language i.e it contains one string 'ϵ'.
3. $\phi \neq \{\epsilon\}$ b/z ϕ has no strings where as $\{\epsilon\}$ has one string named epsilon 'ϵ'.

It is common to describe set using formula

i.e $\{w \mid \text{something about } w\}$

ex:-

$L = \{w \mid$ consists of equal no. of zeroes & ones$\}$ over

$\Sigma = \{0, 1\}$

ex:-

$L = \{w \mid w$ is a binary integer whose value is prime$\}$

ex:-

$L = \{w \mid w$ is a symmetrically correct 'C' pgm.

Describe the following language over input set $A = \{a, b\}$

1. $L_1 = \{a, ab, ab^2, ab^3, \dots\}$

   $L_1 = \{w \mid w$ is string which contains only one a followed by any no. of b's$\}$.

2. $L_2 = \{\epsilon, a, aa, aaa, \dots\}$

   $L_2 = \{w \mid w$ is string which contains any no. of a's & 0 no. of b's$\}$

2nd essay

Deterministic finite automata (DFA):-

DFA consists of :-

1. a finite set of states denoted by 'P'.
2. finite set of i/p symbols denoted by sigma(Σ)
3. The transition function (δ) that takes argument as state & i/p symbol and return its state which can be defined as

Data

$$\boxed{\delta : Q \times \Sigma \rightarrow Q}$$

4. A start state 'q_0' i.e one of the state in Q.
5. set of final/accepting states that in denoted by 'F'
6. Deterministic ~~can~~ finite automata will be refered by it's acronym DFA.
7. DFA fin ~~sign~~table notation $A = \{Q, \Sigma, \delta, q_0, F\}$
five table

Ex:- Design DFA for all the strings which ends with zero over the alphabets $\Sigma = \{0, 1\}$.

Sol:- ends with '0'

Sol:-

q_0

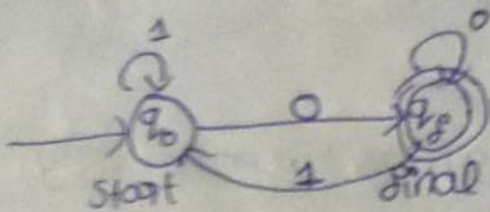
$Q = \{q_0, q_f\}$

$\Sigma = \{0, 1\}$

$\delta(q_0, 0) = q_f$ | $\delta(q_f, 0) = q_f$

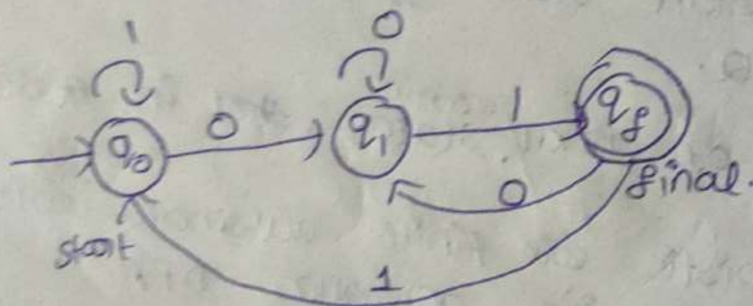
$\delta(q_0, 1) = q_0$ | $\delta(q_f, 1) = q_0$

ends with '0'



Ex-2:
Design DFA for all the strings which ends with 0,1 and over the alphabets $\Sigma = \{0,1\}$.

Sol:



$Q = \{q_0, q_1, q_f\}$

$\Sigma = \{0,1\}$

$\delta(q_0, 0) = q_1$

$\delta(q_0, 1) = q_0$

$\delta(q_1, 1) = q_f$

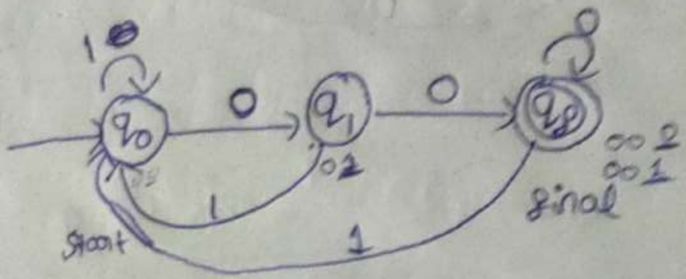
$\delta(q_1, 0) = q_1$

$\delta(q_f, 0) = q_1$

$\delta(q_f, 1) = q_0$

Ex-3:- Design DFA for all strings which ends with '00' over the alphabets $\Sigma=\{0,1\}$

Sol:-



$Q=\{q_0, q_1, q_f\}$

$\Sigma=\{0,1\}$

$\delta(q_0,0)=q_1$

$\delta(q_0,1)=q_0$

$\delta(q_1,0)=q_f$

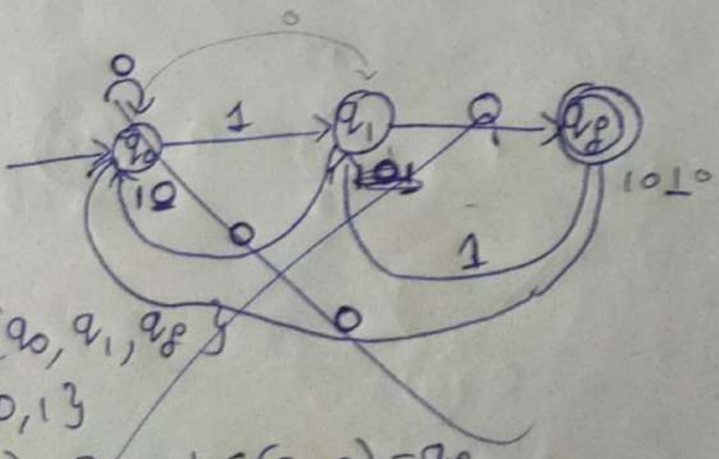
$\delta(q_1,1)=q_0$

$\delta(q_f,0)=q_f$

$\delta(q_f,1)=q_0$

Ex-4:- Design DFA for all strings which ends with 10 over the alphabets $\Sigma=\{0,1\}$

Sol:-

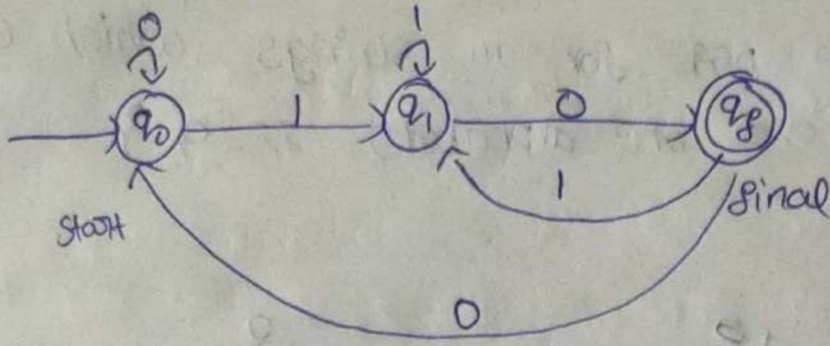


$Q=\{q_0, q_1, q_f\}$

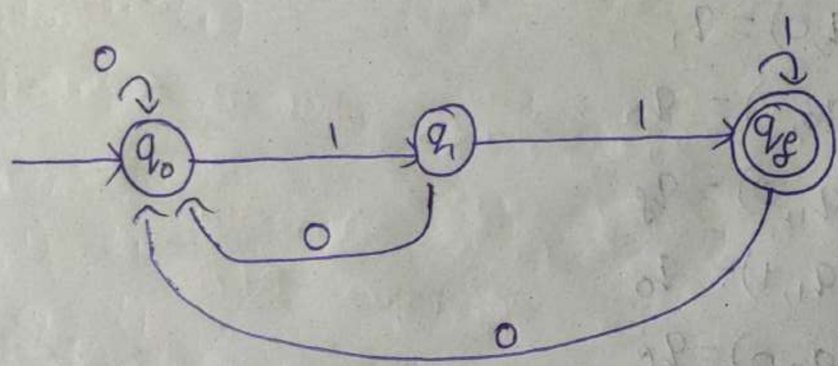
$\Sigma=\{0,1\}$

$\delta(q_0,0)=q_0$ | $\delta(q_1,0)=q_f$

$\delta(q_0,1)=q_1$ | $\delta(q_1,1)=$

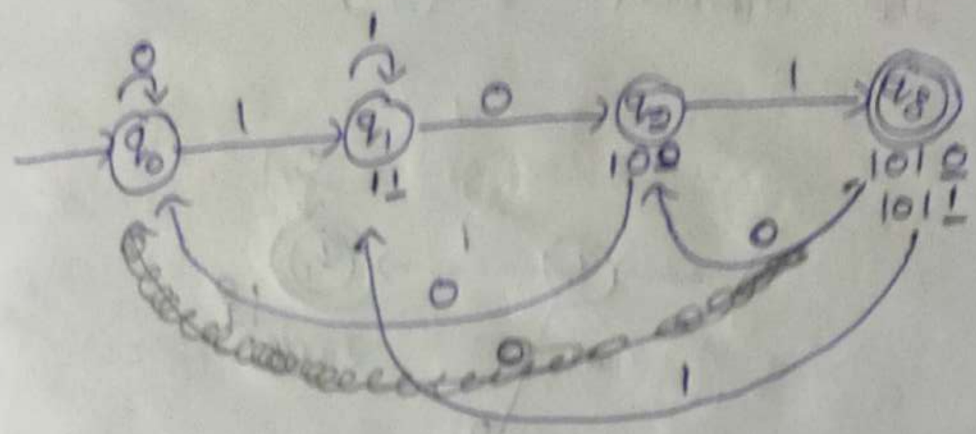


Ex-5: Design DFA for all strings which ends with 11 over the alphabets $\Sigma = \{0,1\}$



Q.6: Design DFA for all strings which ends with 101 over the alphabets $\Sigma = \{0,1\}$

Sol:



$Q = \{q_0, q_1, q_2, q_f\}$

$\Sigma = \{0, 1\}$

$\delta(q_0, 0) = q_0$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_2$

$\delta(q_1, 1) = q_1$

$\delta(q_2, 0) = q_0$

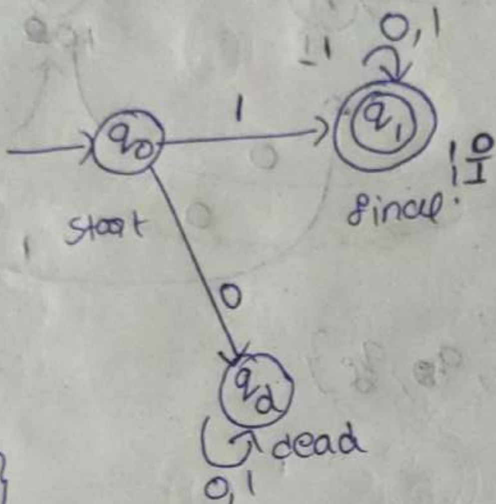
$\delta(q_2, 1) = q_f$

$\delta(q_f, 0) = q_2$

$\delta(q_f, 1) = q_1$

Ex-7:- Design DFA for all strings which accepts for all strings that starts with 1 over the alphabet $\Sigma = \{0,1\}$.

Sol:-



$Q = \{q_0, q_1, q_d\}$

$\Sigma = \{0,1\}$

$\delta(q_0,0) = q_d$

$\delta(q_0,1) = q_1$

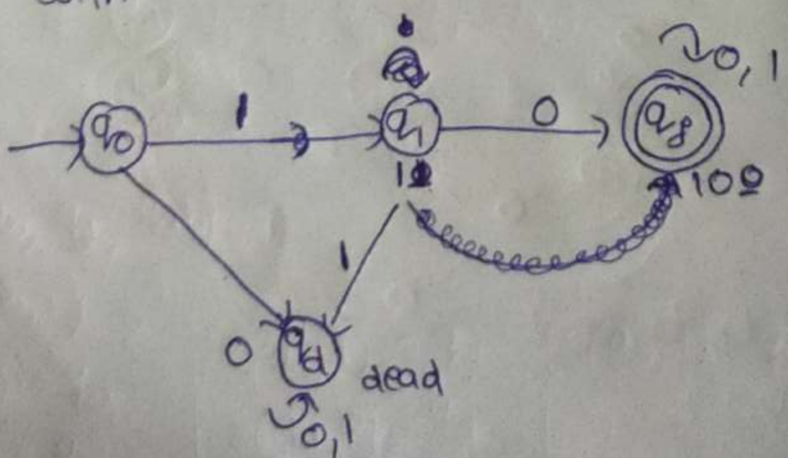
$\delta(q_1,0) = q_1$

$\delta(q_1,1) = q_1$

$\delta(q_d,0) = q_d$

$\delta(q_d,1) = q_d$

Ex-8:- Design DFA which accepts for all strings that starts with 10 over the alphabet $\Sigma = \{0,1\}$.



$Q = \{q_0, q_1, q_f\}$

$\Sigma = \{0, 1\}$

$\delta(q_0, 0) = q_d$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_f$

$\delta(q_1, 1) = q_d$

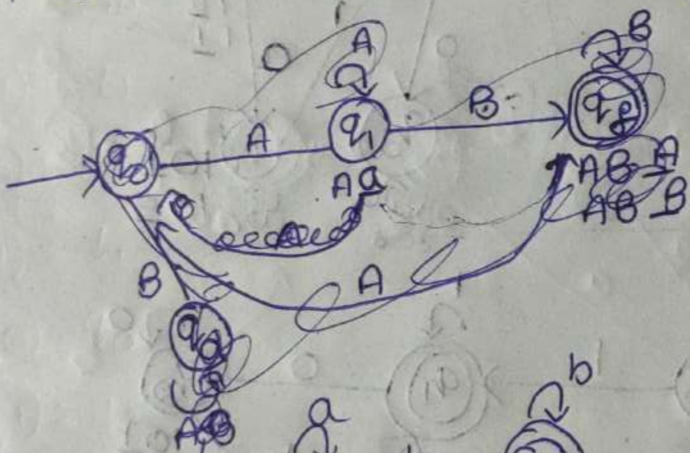
$\delta(q_f, 0) = q_f$

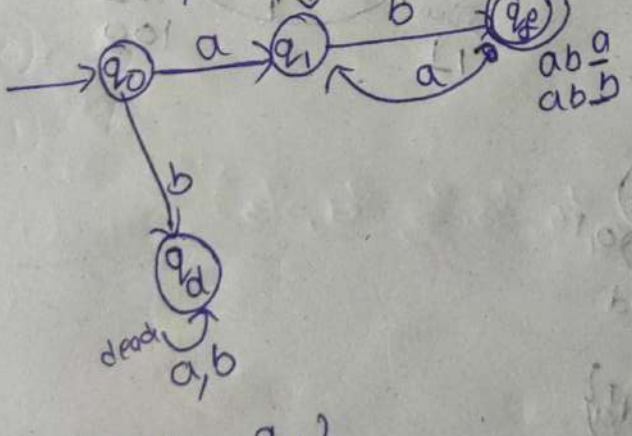
$\delta(q_f, 1) = q_f$

Ex-8: Design DFA which accepts all strings starts with A & ends with B over the alphabet $\Sigma = \{a, b\}$

$L = \{ab, aab, abb,$

Sol:-





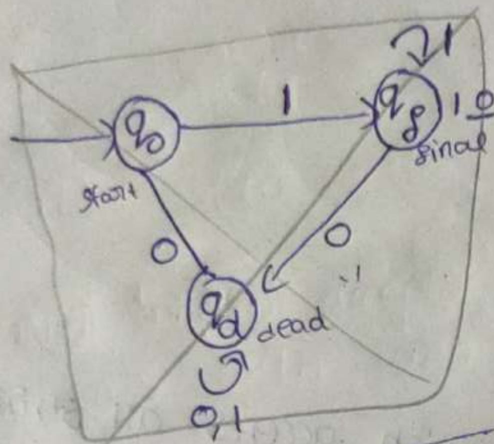
$Q = \{q_0, q_1, q_f, q_d\}$

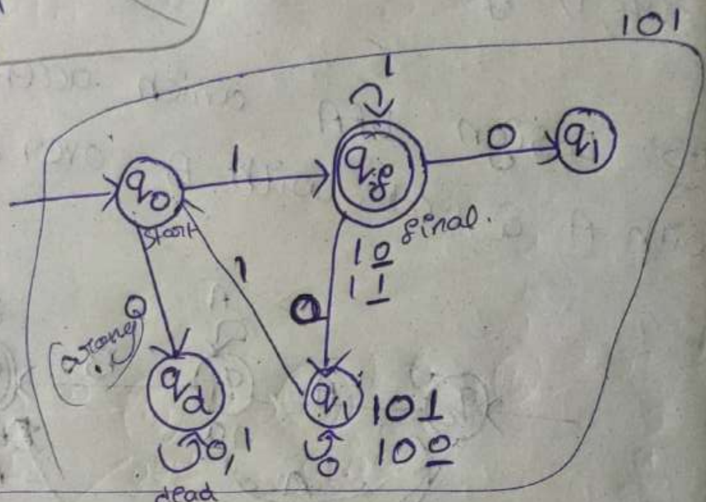
$\Sigma = \{0, 1\}$

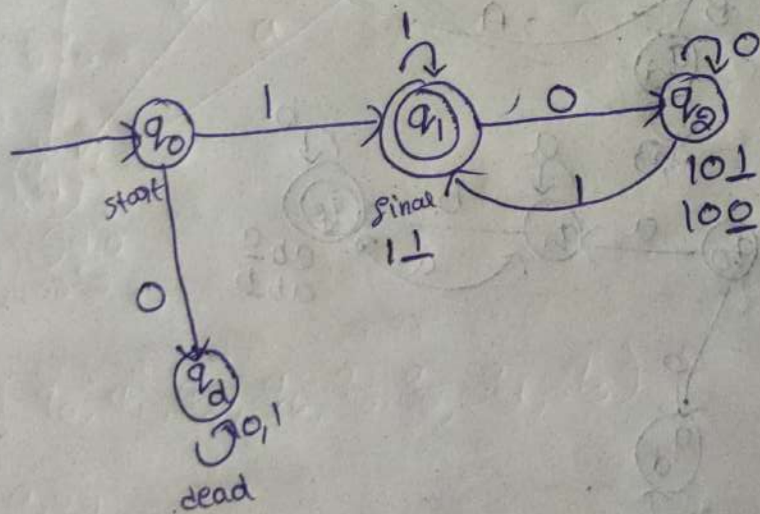
Ex-9:- Design DFA starts with 1 & ends with 1

over

Sol:-







$Q = \{q_0, q_2, q_d, q_1\}$

$\Sigma = \{0, 1\}$

$\delta(q_0, 0) = q_d$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_2$

$\delta(q_1, 1) = q_1$

$\delta(q_d, 0) = q_d$

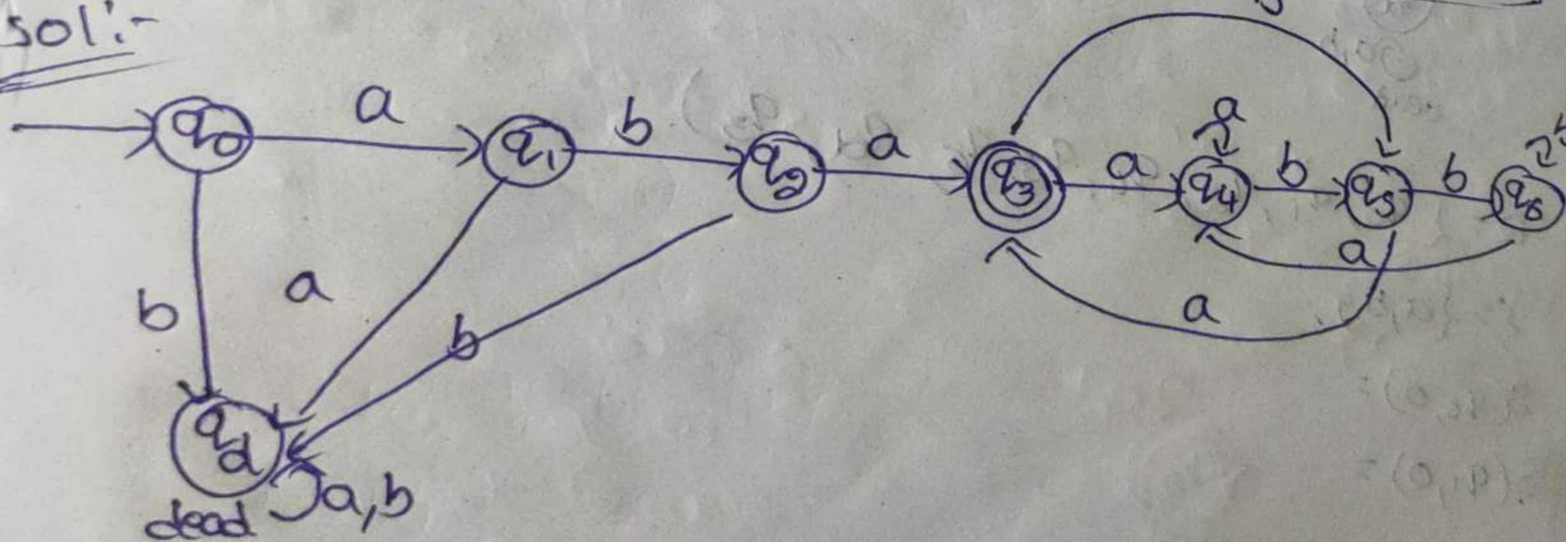
$\delta(q_d, 1) = q_d$

$\delta(q_2, 0) = q_2$

$\delta(q_2, 1) = q_1$

Ex-10: Design DFA which accepts all the string starts with ABA and ends with ABA dead

sol:-

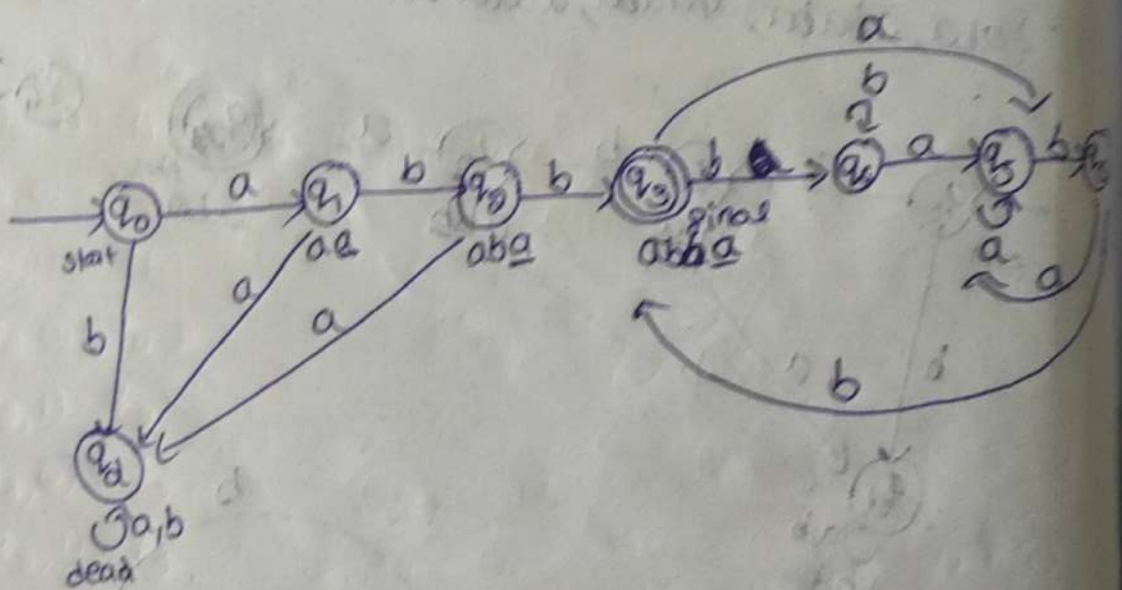


Q-11:

Design DFA which accepts all strings that starts & ends with abb over the alphabets

$\Sigma = \{a, b\}$

Sol:-



$Q = \{q_0, q_1, q_2, q_4, q_5, q_6, q_d, q_3)$

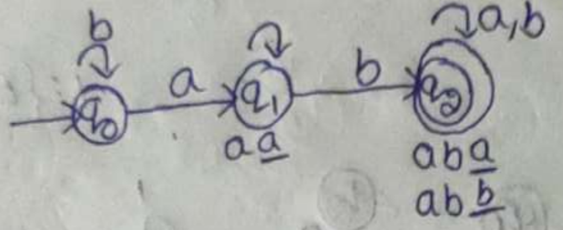
$\Sigma = \{a, b\}$

$\delta(q_0, 0) =$

$\delta(q_1, 0) =$

Ex-12:- Design DFA which accepts all the strings which contains ab as substring over the alphabets $\Sigma = \{a,b\}$

Sol:-



aaab
aaa
aba

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b\}$

$\delta(q_0, a) = q_1$

$\delta(q_0, b) = q_0$

$\delta(q_1, a) = q_1$

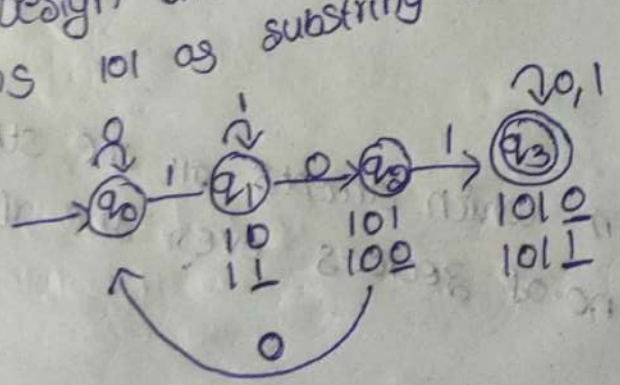
$\delta(q_1, b) = q_2$

$\delta(q_2, a) = q_2$

$\delta(q_2, b) = q_2$

Ex-13:- Design DFA which accepts all the string which contains 101 as substring over the alphabet $\Sigma = \{0,1\}$

Sol:-

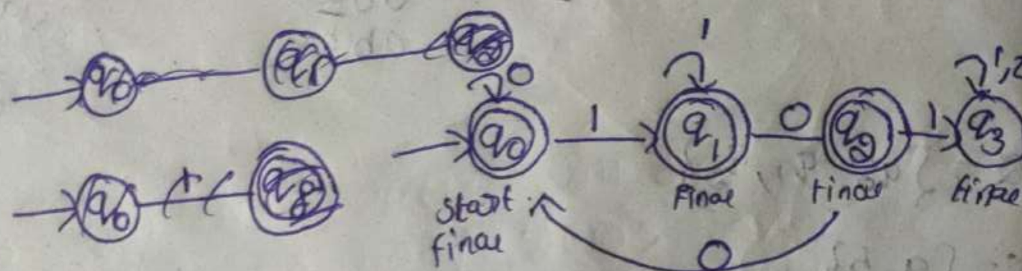


100101
1010
1110

Ex-14:- Design DFA which accepts which doesn't contains 101 as substring over the alphabets
$\Sigma = \{0,1\}$

Sol:-

$L = \{\epsilon, 0, 1$



$Q = \{q_0, q_1, q_2, q_3\}$

$\Sigma = \{0,1\}$

$\delta(q_0, 0) = q_0$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_2$

$\delta(q_1, 1) = q_1$

$\delta(q_2, 0) = q_0$

$\delta(q_2, 1) = q_3$

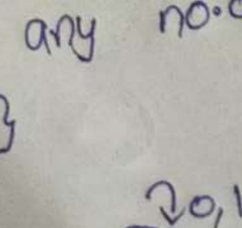
$\delta(q_3, 0) = q_3$

$\delta(q_3, 1) = q_3$

Ex-15:- Design DFA which accepts all the strings over the alphabet contains any no. of zeroes

$\Sigma = \{0,1\}$

Sol:-



$Q = \{q_0\}$

$\Sigma = \{0,1\}$

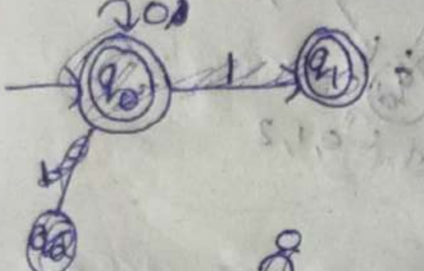
$\delta(q_0, 0) = q_0$

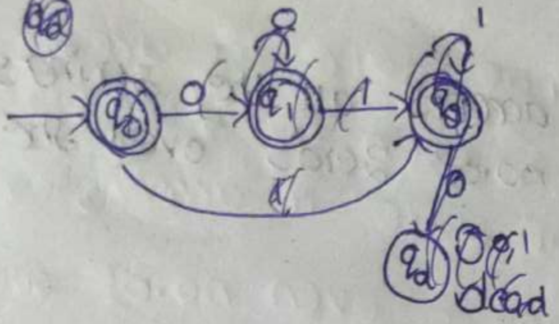
$\delta(q_0, 1) = q_0$

Ex-16:- Design DFA which accepts all the strings ~~an~~ which contains any no.of zeroes followed by any no.of one's over the alphabet $\Sigma = \{0,1\}$

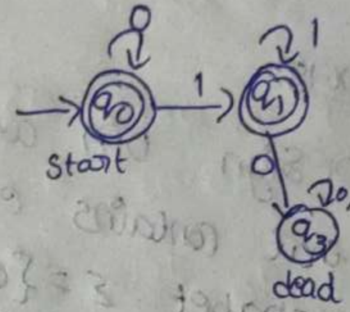
$L = \{\epsilon, 0, 1, 00, 11, 01, 000,$
$111, 001, 011,$
~~100~~, ~~110~~, ~~010~~, ~~010~~

Sol:-





(or)



$Q = \{q_0, q_1\}$

$\Sigma = \{0,1\}$

$\delta(q_0, 0) = q_0$

$\delta(q_0, 1) = q_1$

$\delta(q_1, 0) = q_3$

$\delta(q_1, 1) = q_1$

$\delta(q_3, 0) = q_3$

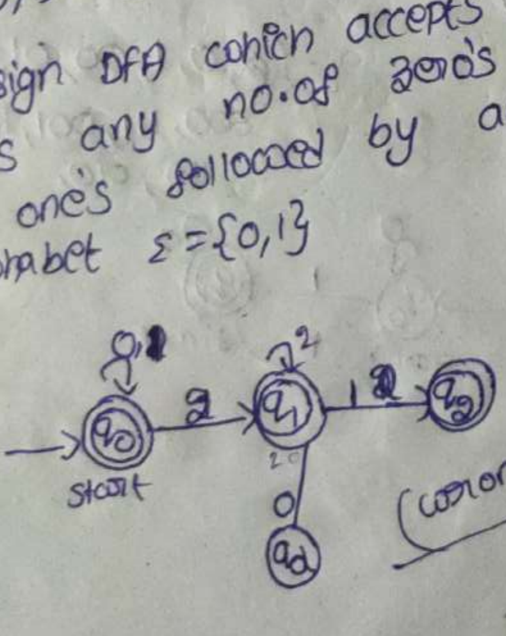
$\delta(q_3, 1) = q_3$

Ex-17:- Design DFA which accepts all strings which contains any no.of zero's followed by any no.of one's followed by any no.of 2's over the alphabet $\Sigma = \{0,1\}$

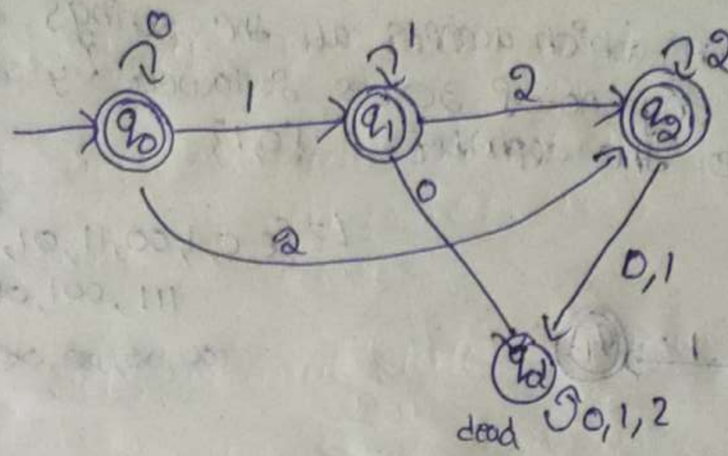
$L = \{\epsilon, 0, 1, 2,$

012

Sol:-



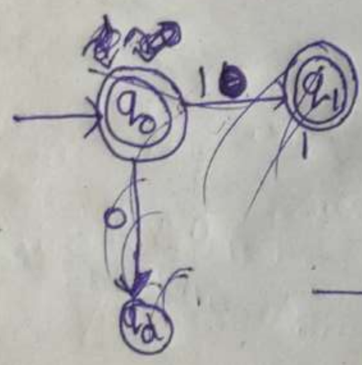
(wrong)

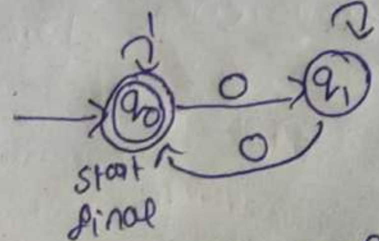


Ex-18:-
Design DFA which accepts all the strings that contains even no. of zero's over the alphabet $\Sigma = \{0,1\}$

Sol:-

$L = \{\epsilon, 1, 00, 11, \cancel{01}, \cancel{10}, \cancel{000}, 111, \cancel{001}, 100, \cancel{101}, \cancel{110}, 010,$

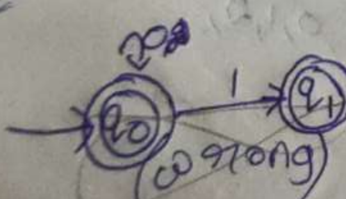


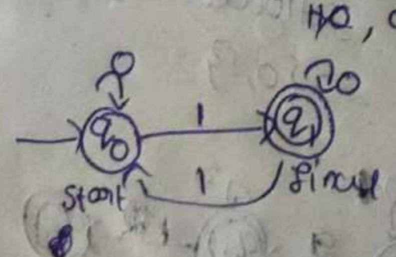


Ex-19:-
Design DFA which contain odd no. of one's over the alphabet $\Sigma = \{0,1\}$

Sol:-

$L = \{\cancel{\epsilon}, 0, 1, 00, 01, 10, \cancel{11}, 000, 111, 001, 100, \cancel{101}, \cancel{110}, 010,$

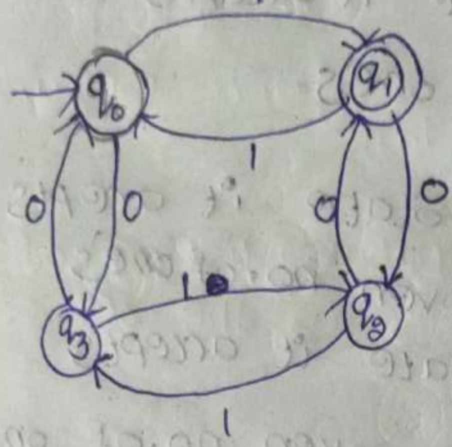




Ex-20:- Design DFA which accepts all the strings that contains even no. of zero's (&) odd no. of one's over the alphabet $\Sigma = \{0,1\}$.

$L = \{1, 00, 0$

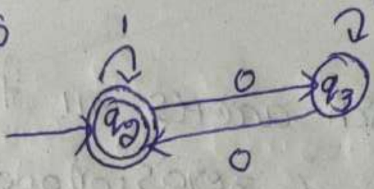
Sol:-



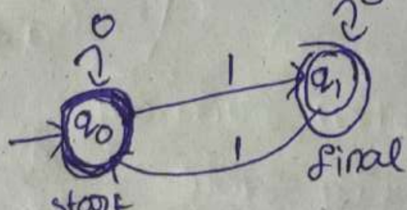
00111

(or)

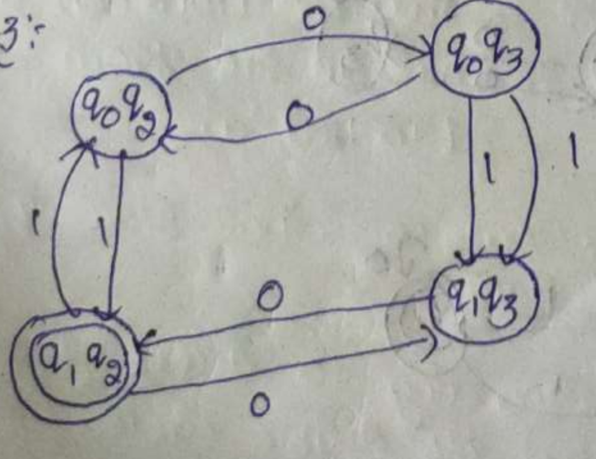
step-1:- even 0's



step-2:- odd no. of 1's

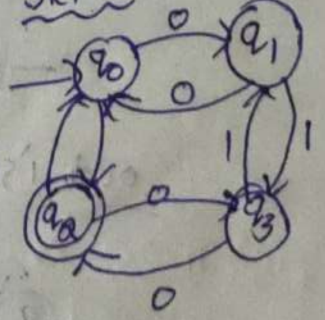


step-3:-



$\Rightarrow$

step-4:-



consider the daigram they are 4 interpretations.

1. q_2 acts as final state it accept even no. of zero's & odd no. of one's

2. (q_3 canto

2. q_3 acts as final state it accepts odd no. of zeros & odd no. of one's.

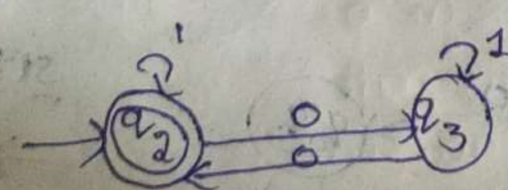
3. q_1 acts as final state it accepts odd no. of zeroes & even no. of one's.

4. q_0 acts as final state it accepts even no. of zeros & even no. of ones.

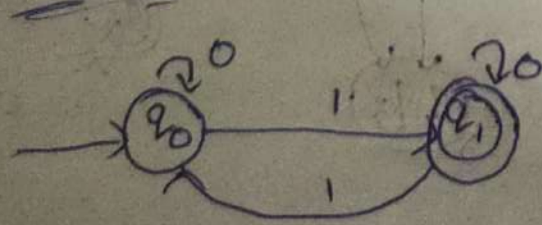
Ex-21:-
Design DFA which accepts all the strings which contain even no. of zero's (or) odd no. of one's

Sol:- step-1:-

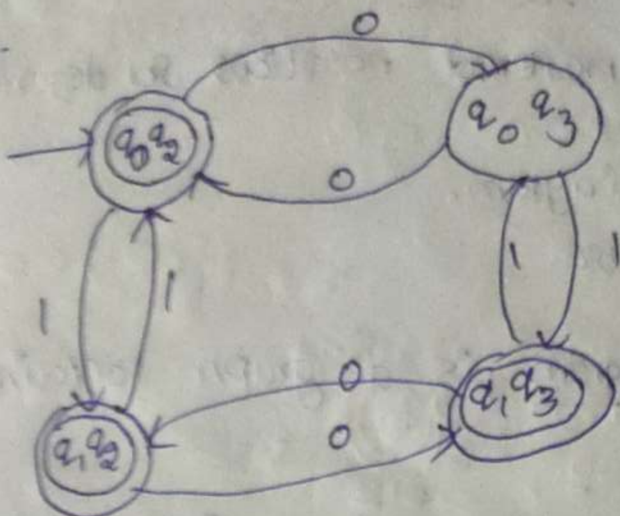
even 0's



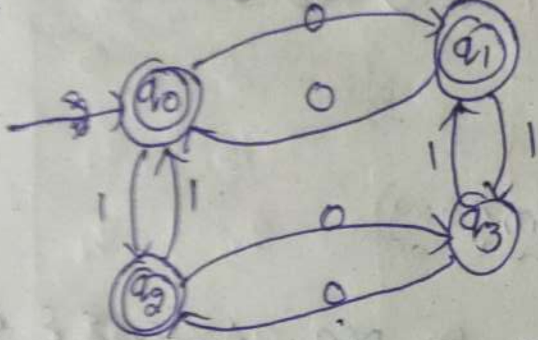
odd 1's



Step-3:



Step 4:



101 0101

From the above diagram there are 4 interpretations

1. If q_1 is not final state & remaining are final state, it accepts even no.of zero's (or) odd no.of one's.

2. If q_3 is not final state & remaining are final states it accept's even no.of zero's (or) even no.of ones

3. If q_2 is not final state & remaining are final states then it accept's even no.of 1's (or) odd no.of zero's.

4. If q_0 is not final state & remaining are final states then it accept's odd no.of 0's & even no.of 1's.

Simple notations for DFA:-

There are 2 preferred notations for describing DFA:-

1. Transition diagram.
2. transition table.

* Transition diagram is a graph contains states & i/p symbols.

* Transition table is tabular listing of delta function (δ) which tells us the set of states & i/p symbols.

1. Transition diagram:- (long question)

Transition diagram for DFA $A=\{Q, \Sigma, \delta, q_0, q_F\}$ is a graph defined as follows:-

Q = set of states
Σ = i/p symbol
δ = transition function
q_0 = starting state
q_F = Final state

i, for each state Q there is a node denoted by circle (O)

ii, for each state q in Q there, for each i/p symbol a in Σ that $\delta(q,a)=P$ then the transition diagram has an arc from q to P labelled 'a'.

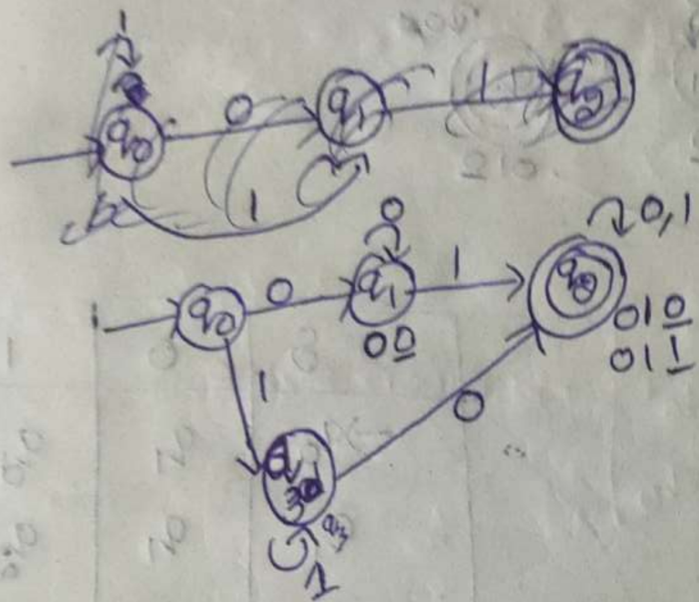
iii, There is an arrow to initial state q_0 labelled Start state

iv, The node corresponding to accepting states (F) are marked by double circle (◎). states not in F have single circle (O).

Ex: Design DFA which accepts all strings that consist atleast one "zero" & one "one" over the alphabet $\Sigma\{0,1\}$

$L = \{01,$

Sol:-



2. Transition table:-

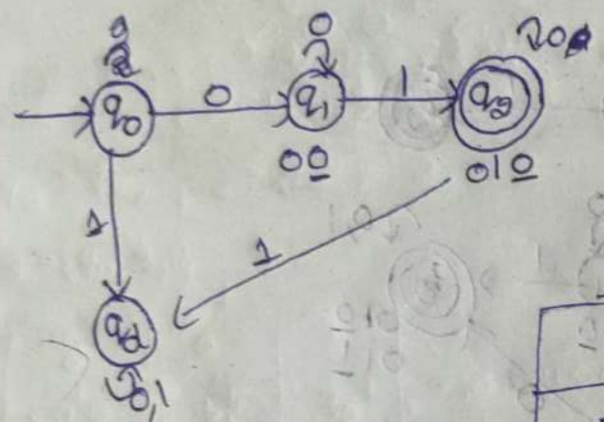
- Transition table is a conventional, tabular representation of function like delta(δ) that take two arguments has state & i/p symbol & returns it's state.
- The rows of the table corresponds to states & coloumns corresponds to i/p symbol.

Ex:- The transition table for above DFA is:-

δ	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_1	q_2
q_3	q_2	q_3

Ex: Design DFA which accepts all the strings that contains atleast one zero followed by atleast ~~1~~ one one.

$L = \{01, 010,$



δ	0	1
$\rightarrow q_0$	q_1	q_d
q_1	q_1	q_2
q_d	q_d	q_d
q_2	q_2	q_d

Extended Transition function:- ($\hat{\delta}$)

Extended Transition function describes what happens when we start with any state & follow any sequence of inputs.

* If 'δ' is our transition function then the extended transition function constructed from (δ) will be denoted by '$\hat{\delta}$' (delta cap)

proof:-

Basis of induction:-

let $\delta(q_0, a) = q_1$ and $\delta(q_1, b) = q_2$

$\therefore \hat{\delta}(q_0, ab) = q_2$

Induction:-

suppose $\hat{\delta}(q_0, \epsilon) = q_0$ & consider any string 'wa' where ω is a string belongs to Σ^* & a is last alphabet of string 'wa', $a \in \Sigma$.

To prove $\hat{\delta}(q_0, ab) = q_2$

sol:-

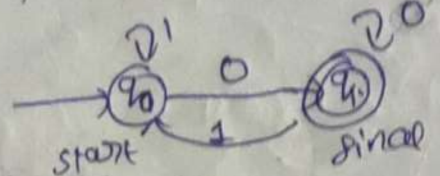
$$
\begin{aligned}
LHS &= \hat{\delta}(q_0, ab) \\
&= \hat{\delta}(\delta(q_0, a), b) \\
&= \hat{\delta}(q_1, b) \\
&= \hat{\delta}(\delta(q_1, b), \epsilon) \\
&= \hat{\delta}(q_2, \epsilon) = q_2 \\
&= \cancel{\hat{\delta}(\delta(q_2, \epsilon)) = q_2}
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
&\because \delta(q_0, a) = q_1 \\
&\delta(q_1, b) = q_2 \\
&\hat{\delta}(q_2, \epsilon) = q_2
\end{aligned}
}
$$

Hence proved//

Design DFA which accepts even binary numbers & also check 101010 is accepted by the DFA or not.

$L = \{\cancel{01}, 00, \cancel{01}, 10, \cdots\}$



$q_0 = q_0$, $\Sigma = \{0, 1\}$

$Q = \{q_0, q_1\}$

δ	0	1
$\rightarrow q_0$	q_1	q_0
$* q_1$	q_1	q_0

Transition table

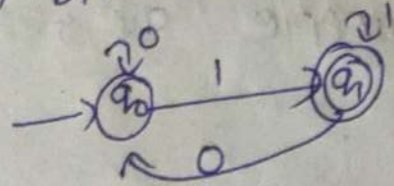
$F = q_1$

check 101010 :-

$$\hat{\delta}(q_0, 101010) = \hat{\delta}(\delta(q_0, 1), 01010)$$
$$= \hat{\delta}(q_0, 01010)$$
$$= \hat{\delta}(\delta(q_0, 0), 1010)$$
$$= \hat{\delta}(q_1, 1010)$$
$$= \hat{\delta}(\delta(q_1, 1), 010)$$
$$= \hat{\delta}(q_0, 010)$$
$$= \hat{\delta}(\delta(q_0, 0), 10)$$
$$= \hat{\delta}(q_1, 10)$$
$$= \hat{\delta}(\delta(q_1, 1), 0)$$
$$= \hat{\delta}(q_0, 0)$$
$$= \hat{\delta}(\delta(q_0, 0), \epsilon)$$
$$= \hat{\delta}(q_1, \epsilon) = q_1$$

given string is accepted by DFA

Design DFA which accepts all odd binary numbers



$Q = \{q_0, q_1\}$

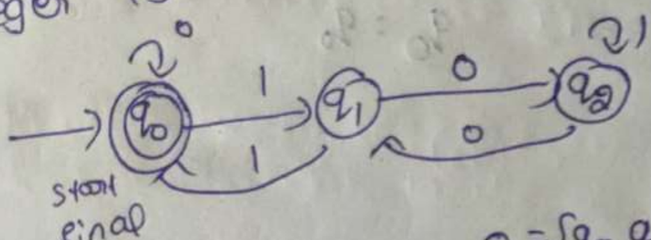
$F = q_1$

δ	0	1
$\rightarrow q_0$	q_0	q_1
$* q_1$	q_0	q_1

$q_0 = q_0$

$\Sigma = \{0, 1\}$

Design DFA which can be interpreted as binary integer is multiple of 3.



$Q = \{q_0, q_1, q_2\}$

$F = q_0$

$\Sigma = \{0, 1\}$

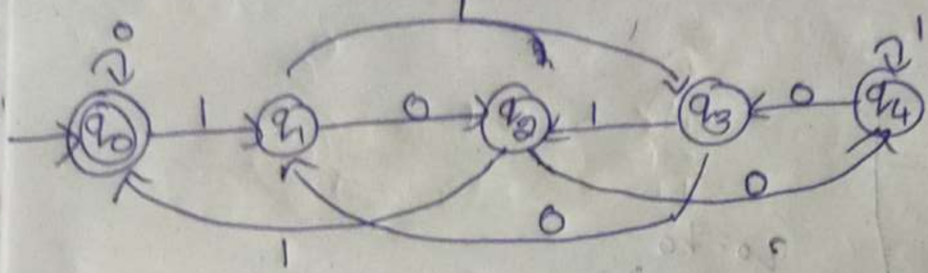
$q_0 = q_0$

δ	0	1
$\rightarrow * q_0$	q_0	q_1
q_1	q_2	q_0
q_2	q_1	q_2

Design DFA which can be interpreted as binary integer is multiple of 5

{0, 5, 10, 15, 20, 25

16	8	4	2	1
1	0	1	0	0



δ	0	1
*→q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

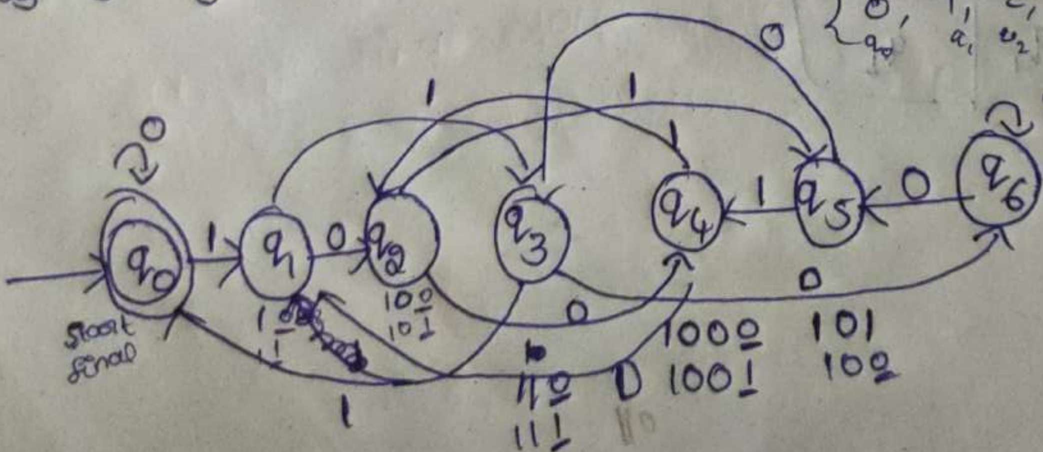
$F = q_0$

$\Sigma = \{0, 1\}$

$q_0 = q_0$

Design DFA for all string which can be interpreted as binary integer divisible by 7.

$\{0, 1, 2, 3, 4, 5, 6\}$ — $q_0, q_1, q_2, q_3, q_4, q_5, q_6$



8 4 2 1

4/7 = 4

2/7 = 7

8/7 = 7

9/7 = 6

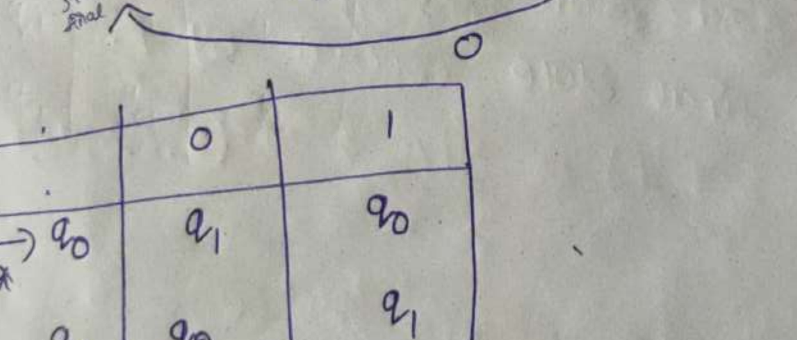
7/7 = 1

8/7 =

	0	1
$\rightarrow *q_0$	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_5
q_3	q_6	q_0
q_4	q_1	q_2
q_5	q_3	q_4
q_6	q_5	q_6

Design DFA for all strings which contains no. of zeros is divisible by 4 over the alphabet $\Sigma = \{0,1\}$

$\{0, 4, 8, 12, \ldots$



$2/4 = 2$
$1/4 = 1$
$3/4 =$
$4/4 = 0$

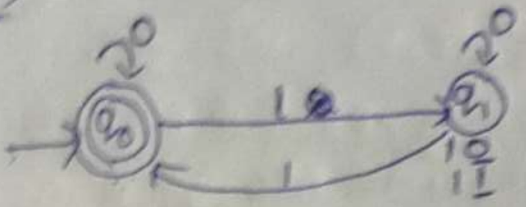
	0	1
$\rightarrow * q_0$	q_1	q_0
q_1	q_2	q_1
q_2	q_3	q_2
q_3	q_0	q_3

→ Transition table.

****

Design DFA which accepts all the strings which contains no. of 1's is even & no. of 0's is multiple of 3.

even 1's



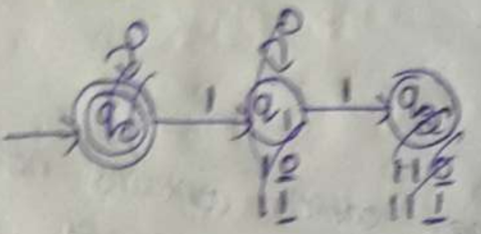
01110

10

0's multiple of 3

0, 3, 6, 9, 12, 15, 18

3/3 = 0

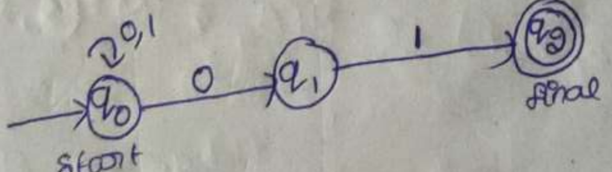


Non-Deterministic finite automata (NFA)

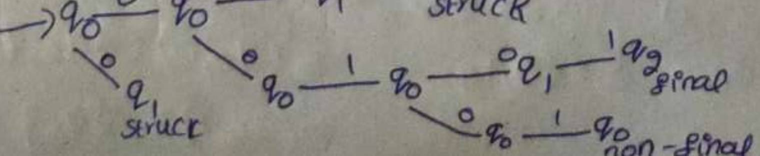
The concept of NFA is exactly the reverse of DFA. The finite automata is called NFA when there exists many paths for specific i/p from current state to next state.

DFA	NFA
* DFA is deterministic finite automata	* NFA is non-deterministic finite automata.
* For a given state on a given i/p it reaches to deterministic & unique state	* For a given state on a given i/p it may reaches to more than one state.
* Transition function for DFA is $\delta: Q \times \Sigma \rightarrow Q$	* Transition function for NFA is $\delta: Q \times \Sigma \rightarrow 2^Q$

Design NFA which accepts all the strings that end with zero.



Ex:- States of NFA designing during a process of i/p symbol



Ex-2:

States of NFA by process of string 11010

$\longrightarrow q_0$

NFA:-(Defintion)

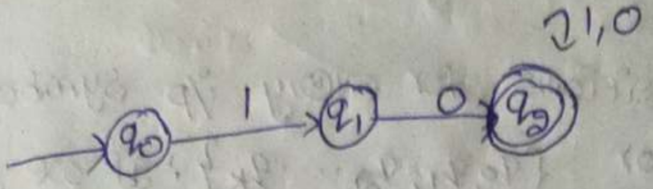
NFA consists of :-

1. Finite set of states denoted by Q.
2. " " " i/p symbols denoted by Σ
3. Transition function data (δ) takes arguments as state & i/p symbol then returns set of states.

   i.e; $\boxed{\delta : Q \times \Sigma \longrightarrow 2^{Q}}$

4. Start ~~start~~ state denoted by q_0
5. set of final state denoted by "F."

Design NFA which accepts all strings starts with 10 over the alphabet $\Sigma = \{0,1\}$



δ or M.

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{0,1\}$

	0	1
$\rightarrow q_0$	ϕ	q_1
q_1	q_2	ϕ
$*q_2$	q_2	q_2

Conversion of NFA into DFA :-

Following procedure specifies

let $M = (Q, \Sigma, \delta, q_0, F)$ is an NFA which accepts the language $L(M)$ there should be equivalent DFA denoted by $M' = (Q', \Sigma', \delta', q_0', F')$ such that $L(M) = L(M')$

Step-1:- Start state of NFA will be the start state of DFA. Hence add (NFA) q_0 of NFA (start state) to Q'

Step-2:- For each state $(q_0, q_1, q_2, \dots q_i)$ in Q' the transition for each i/p symbol Σ can be obtained as ① $\delta'(\{q_0, q_1, q_2, \dots q_i\}, a) = \delta'(q_0, a) \cup \delta'(q_1, a) \cup \dots \delta'(q_i, a)$

$= \{q_0, q_1, q_2 \dots q_k\}$ (some state)

Step-

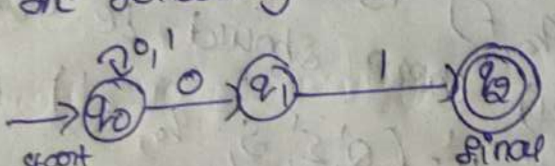
(2) Hence add $\{q_0, q_1, q_2 \cdots q_k\}$ to DFA if it is not already present in Q'.

Step (3)- Find transitions for every i/p symbol from Σ for $\{q_0, q_1, q_2, \cdots q_k\}$. If we get some states $\{q_0, q_1, \cdots q_n\}$ which is not in Q' of DFA then add this state to Q'.

Step (4)- If there is no new state generated then rest of the process of finding all transitions can be stopped.

Step-3:- For the state $\{q_0, q_1, \cdots q_n\}$ in Q' of DFA, if any one state q_i is a final state of NFA then $\{q_0, q_1, \cdots q_n\}$ becomes final state. Thus belongs to F' of the set of all final state DFA

Ex:- Convert the following NFA to DFA.



Sol:- Let $M = (Q, \Sigma, \delta, q_0, F)$ be NFA & $M' = (Q', \Sigma', \delta', q_0', F')$ be DFA.

Step-1:- Start state of NFA (q_0) will be start state of DFA then add q_0 to Q'

i.e $Q' = \{q_0\}$

Step-2:- Find transitions for states in Q' of DFA given $\Sigma = \{0,1\}$ $\therefore$ $\delta'(q_0, 0) = \{q_0, q_1\}$ → it is new state.

Hence add $\{q_0, q_1\}$ to Q' of DFA

$\therefore Q' = (\{q_0\}, \{q_0, q_1\})$

Step-3 :- find transitions for $\{q_0, q_1\}$ over $\Sigma = \{0,1\}$

$\delta'(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0) = \{q_0, q_1\} \cup \phi = \{q_0, q_1\}$

it is in Q'

$\delta'(\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1) = \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$

it is new state

Hence add $\{q_0, q_2\}$ to Q' of DFA

$\therefore Q' = (\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\})$

Step-4 :- find transitions for $\{q_0, q_2\}$ over $\Sigma = \{0,1\}$

$\delta'(\{q_0, q_2\}, 0) = \delta(q_0, 0) \cup \delta(q_2, 0) = \{q_0, q_1\} \cup \phi = \{q_0, q_1\}$

$\delta'(\{q_0, q_2\}, 1) = \delta(q_0, 1) \cup \delta(q_2, 1) = \{q_0\} \cup \phi = \{q_0\}$

It is ~~new state~~ already in Q'

~~Hence add $\{q_2\}$ to Q' of DFA~~

~~$\therefore Q' = (\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_2\})$~~

~~Step-5 :- find transititions for $\{q_2\}$ over $\Sigma = \{0,1\}$~~

~~$\delta'(q_2, 0) = \phi$~~

~~$\delta'(q_2, 1) = \phi$~~

$\therefore$ There are no new states generated.

Hence $\{q_0, q_2\}$ becomes final state of DFA b/z final state of NFA is q_2

step-2:

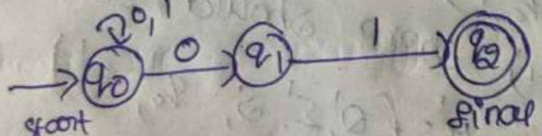
(2) Hence add $\{q_0, q_1, q_2 \cdots q_k\}$ to DFA if it is not already present in Q'.

step-(3):- Find transitions for every i/p symbol from Σ for $\{q_0, q_1, q_2, \cdots q_k\}$. If we get some states $\{q_0, q_1, \cdots q_n\}$ which is not in Q' of DFA then add this state to Q'.

step-(4):- If there is no new state generated then rest of the process of finding all transitions can be stopped.

step-3:- For the state $\{q_0, q_1, \cdots q_n\}$ in Q' of DFA, if any one state q_i is a final state of NFA then $\{q_0, q_1, \cdots q_n\}$ becomes final state. thus belongs to F' of the set of all final state DFA

Ex:- convert the following NFA to DFA.



sol:- Let $M = (Q, \Sigma, \delta, q_0, F)$ be NFA & $M' = (Q', \Sigma', \delta', q_0', F')$ be DFA.

step-1:- start state of NFA (q_0) will be start state of DFA then add q_0 to Q'

i.e $Q' = \{q_0\}$

step-2:- find transitions for states in Q' of DFA over $\Sigma = \{0,1\}$ $\therefore$ $\delta'(q_0, 0) = \{q_0, q_1\}$ → it is new state.

$\delta'(q_0, 1) = \{q_0\}$ → it is already in Q'.

Hence add $\{q_0, q_1\}$ to Q' of DFA

$\therefore Q' = (\{q_0\}, \{q_0, q_1\})$

Step-3:- find transitions for $\{q_0, q_1\}$ over $\Sigma = \{0,1\}$

$\delta'(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0) = \{q_0, q_1\} \cup \phi = \{q_0, q_1\}$ It is in Q'

$\delta'(\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1) = \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$ It is new state

Hence add $\{q_0, q_2\}$ to Q' of DFA

$\therefore Q' = (\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\})$

Step-4:- find transitions for $\{q_0, q_2\}$ over $\Sigma = \{0,1\}$

$\delta'(\{q_0, q_2\}, 0) = \delta(q_0, 0) \cup \delta(q_2, 0) = \{q_0, q_1\} \cup \phi = \{q_0, q_1\}$

$\delta'(\{q_0, q_2\}, 1) = \delta(q_0, 1) \cup \delta(q_2, 1) = \{q_0\} \cup \phi = \{q_0\}$ It is ~~new state~~ already in Q'

~~Hence add $\{q_2\}$ to Q' of DFA~~

~~$\therefore Q' = (\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_2\})$~~

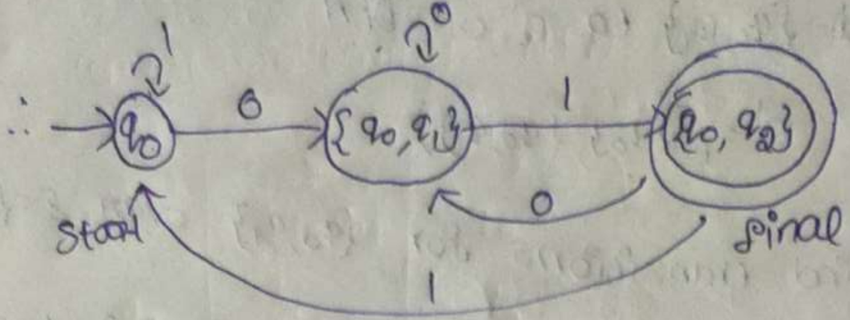
~~Step-5:- find transititions for $\{q_2\}$ over $\Sigma = \{0,1\}$~~

~~$\delta'(q_2, 0) = \phi$~~

~~$\delta'(q_2, 1) = \phi$~~

$\therefore$ There are no new states generated.

Hence $\{q_0, q_2\}$ becomes final state of DFA b/z final state of NFA is q_2

∴ 

transition table

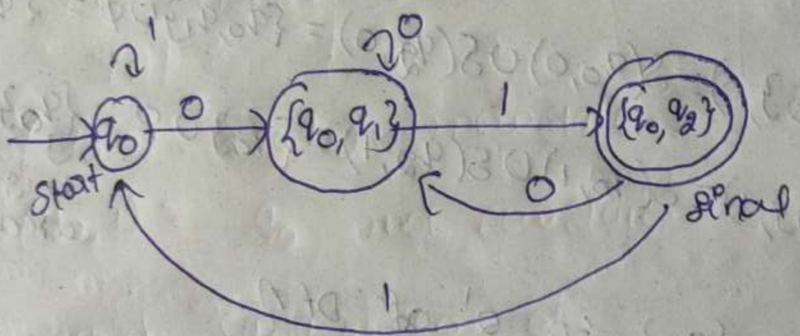
NFA

	0	1
$\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_0\}$
$\{q_0,q_1\}$	$\{q_1,q_0\}$	$\{q_0,q_2\}$
q_1	ϕ	$\{q_2\}$
$*q_2$	ϕ	ϕ

DFA

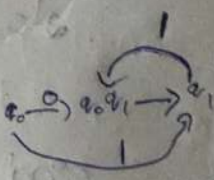
	0	1
$\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_0\}$
$\{q_0,q_1\}$	$\{q_0,q_1\}$	$\{q_0,q_2\}$
$*\{q_0,q_2\}$	$\{q_0,q_1\}$	$\{q_0\}$

sol:



Ex-2:- convert the following NFA to DFA

	0	1
$\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_1\}$
$*q_1$	ϕ	$\{q_0,q_1\}$



Step-1:- Start state of NFA (q_0) will be start state of DFA then add q_0 to Q'

i.e $Q' = \{q_0\}$

Step-2:- Find transitions for states in Q' of DFA over $\Sigma=\{0,1\}$

$\delta'(q_0,0)=\{q_0,q_1\}$ } It is new states
$\delta'(q_0,1)=\{q_1\}$

Hence add $\{q_0,q_1\}$ & $\{q_1\}$ to Q' of DFA

$\therefore Q'=(\{q_0\},\{q_0,q_1\},\{q_1\})$

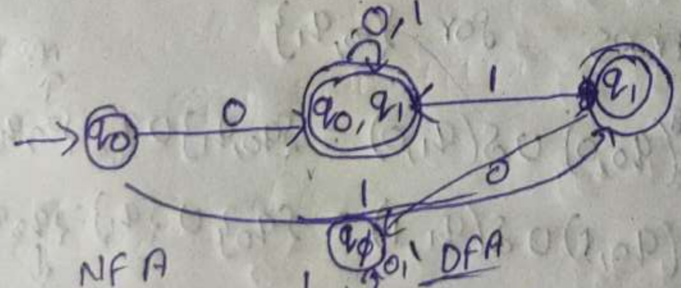
Step-3:- Find transitions for $\{q_0,q_1\}$ & $\{q_1\}$

$\delta'(\{q_0,q_1\},0)=\delta(q_0,0)\cup\delta(q_1,0)=\{q_0,q_1\}\cup\phi=\{q_0,q_1\}$
$\delta'(\{q_0,q_1\},1)=\delta(q_0,1)\cup\delta(q_1,1)=\{q_1\}\cup\{q_0,q_1\}=\{q_0,q_1\}$
$\delta'(\{q_1\},0)=\phi \rightarrow$ newstate
$\delta'(\{q_1\},1)=\{q_0,q_1\}$

Step-4:- add ϕ to Q'
$\therefore Q'=\{\{q_0\},\{q_0,q_1\},\{q_1\},\{\phi\}\}$
$\delta'(\phi,0)=\phi$
$\delta'(\phi,1)=\phi$

$\therefore$ there are no new states.

Hence $\{q_0,q_1\},\{q_1\}$ are becomes final states of DFA b/z final state of NFA is q_1



NFA

	0	1
$\rightarrow q_0$	q_0q_1	q_1
$*q_1$	ϕ	$\{q_0q_1\}$

DFA

	0	1
$\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_1\}$
$*\{q_0q_1\}$	$\{q_0,q_1\}$	$\{q_0,q_1\}$
$*\{q_1\}$	ϕ	$\{q_0,q_1\}$
ϕ	ϕ	ϕ

Ex-3:- convert following NFA to DFA

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
q_1	q_2	q_1
q_2	q_2	q_3
$* q_3$	ϕ	q_2

Step-1:- Start state of NFA (q_0) will be start state of DFA. then add q_0 to Q'.

$$Q' = (\{q_0\})$$

Step-2:- Find transitions for states in Q' of DFA

$\delta'(q_0, 0) = \{q_0, q_1\} \rightarrow$ new state

$\delta'(q_0, 1) = q_0 \rightarrow$ not new state.

Hence add $\{q_0, q_1\}$ to Q'.

$$\therefore Q' = (\{q_0\}, \{q_0, q_1\})$$

Step-3:- Find transitions for $\{q_0, q_1\}$

$\delta'(\{q_0, q_1\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0) = \{q_0, q_1\} \cup q_2 = \{q_0, q_1, q_2\}$ → new state

$\delta'(\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1) = \{q_0\} \cup \{q_1\} = \{q_0, q_1\}$ → not new

Hence add $\{q_0, q_1, q_2\}$ to Q'

$$Q' = (\{q_0\}, \{q_0, q_1\}, \{q_0, q_1, q_2\})$$

step-4: find transitions for $\{q_0, q_1, q_2\}$

$\delta'(\{q_0,q_1,q_2\},0) = \delta(q_0,0) \cup \delta(q_1,0) \cup \delta(q_2,0) = \{q_0,q_1\} \cup \{q_2\} \cup \{q_2\}$
$= \{q_0,q_1,q_2\}$

$\delta'(\{q_0,q_1,q_2\},1) = \delta(q_0,1) \cup \delta(q_1,1) \cup \delta(q_2,1) = \{q_0\} \cup \{q_1\} \cup \{q_3\}$ old
$= \{q_0,q_1,q_3\}$
newsta

add $\{q_0,q_1,q_3\}$ to Q'

$\therefore Q' = (\{q_0\}, \{q_0,q_1\}, \{q_0,q_1,q_2\}, \{q_0,q_1,q_3\})$

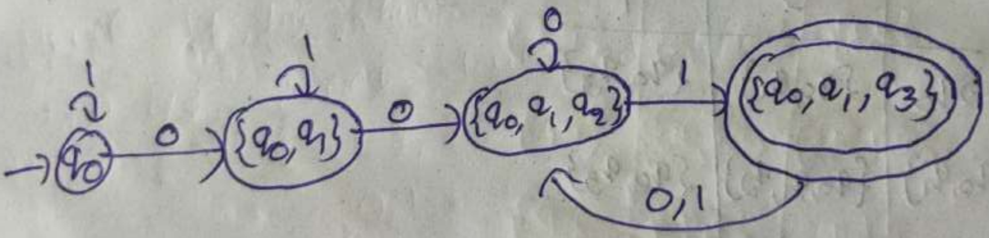
step-5: find transitions fo $\{q_0, q_1 q_3\}$

$\delta'(\{q_0,q_1,q_3\},0) = \delta(q_0,0) \cup \delta(q_1,0) \cup \delta(q_3,0) = \{q_0,q_1\} \cup \{q_2\} \cup \{\phi\} = \{q_0,q_1,q_2\}$

$\delta'(\{q_0,q_1,q_3\},1) = \delta(q_0,1) \cup \delta(q_1,1) \cup \delta(q_3,1) = \{q_0\} \cup \{q_1\} \cup \{q_2\} = \{q_0,q_1,q_2\}$

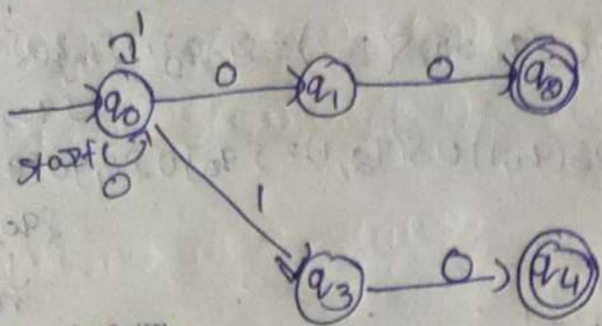
$\therefore$ There is no new states.

Hence $\{q_0,q_1,q_3\}$ is final state of DFA b/z final state of NFA is q_3



	0	1
$\rightarrow q_0$	$\{q_0,q_1\}$	q_0
$\{q_0,q_1\}$	$\{q_0,q_1,q_2\}$	$\{q_0,q_1\}$
$\{q_0,q_1,q_2\}$	$\{q_0,q_1,q_2\}$	$\{q_0,q_1,q_3\}$
$*\{q_0,q_1,q_3\}$	$\{q_0,q_1,q_2\}$	$\{q_0,q_1,q_2\}$

Ex-Q:- convert NFA to DFA



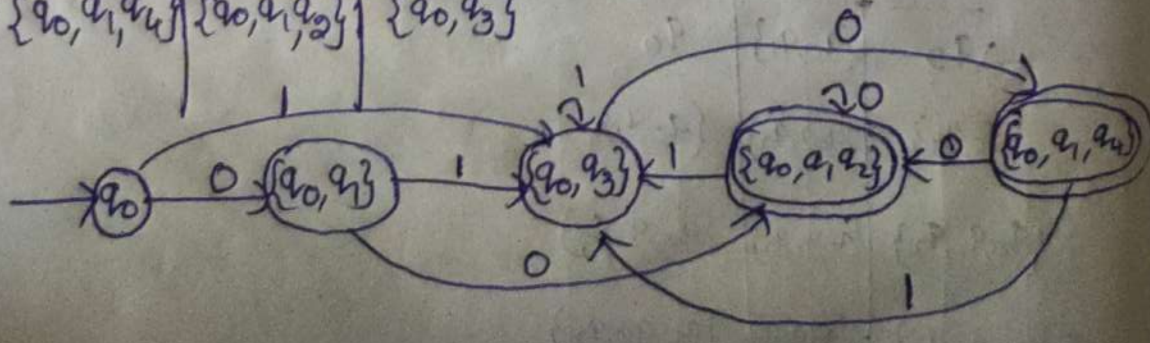
write theory
shortcut

NFA

	0	1
→ q0	{q0, q1}	{q0, q3}
q1	{q2}	φ
* q2	φ	φ
q3	{q4}	φ
* q4	φ	φ

DFA

	0	1
→ q0	{q0, q1}	{q0, q3}
{q0, q1}	{q0, q1, q2}	{q0, q3}
{q0, q3}	{q0, q1, q4}	{q0, q3}
* {q0, q1, q2}	{q0, q1, q2}	{q0, q3}
* {q0, q1, q4}	{q0, q1, q2}	{q0, q3}



ex-5: convert following NFA to DFA

	0	1
$\rightarrow p$	$\{p,q\}$	$\{p\}$
q	$\{r,s\}$	$\{t\}$
$*r$	$\{p,r\}$	t
$*s$	ϕ	ϕ
$*t$	ϕ	ϕ

Step-1:- start state of NF (p) will be start state of DFA then add p to Q'

$$Q' = (\{p\})$$

Step-2:- Find transitions for states in Q' of DFA

$\delta'(p,0) = \{p,q\} \rightarrow$ new state

$\delta'(p,1) = \{p\}$

$\therefore$ add $\{p,q\}$ to Q'

$\therefore Q' = (\{p\}, \{p,q\})$

Step-3:- Find transitions for $\{p,q\}$

$\delta'(\{p,q\},0) = \delta(p,0) \cup \delta(q,0) = \{p,q\} \cup \{r,s\} = \{p,q,r,s\}$

$\delta'(\{p,q\},1) = \delta(p,1) \cup \delta(q,1) = \{p\} \cup \{t\} = \{p,t\}$ } two state

$\therefore$ add $\{p,q,r,s\}$, $\{p,t\}$ to Q'

$Q' = (\{p\}, \{p,q\}, \{p,q,r,s\}, \{p,t\})$

Step-4:- $\delta'(\{p,q,r,s\},0) = \delta(p,0) \cup \delta(q,0) \cup \delta(r,0) \cup \delta(s,0) =$
$\{p,q\} \cup \{r,s\} \cup \{p,r\} \cup \phi = \{p,q,r,s\}$

$\delta'(\{p,q,r,s\},1) = \delta(p,1) \cup \delta(q,1) \cup \delta(r,1) \cup \delta(s,1) = \{p\} \cup \{t\} \cup \{t\} \cup \phi = \{p,t\}$

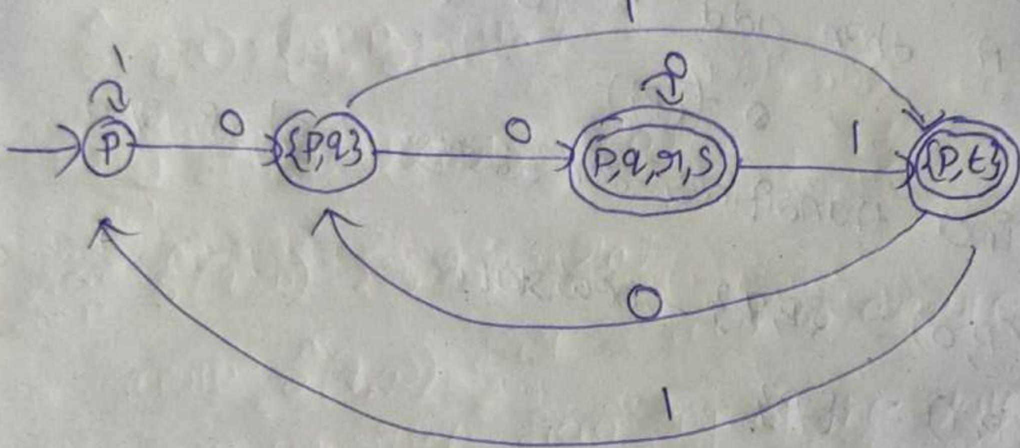
$\delta'(\{p,t\},0) = \delta(p,0) \cup \delta(t,0) = \{p,q\} \cup \phi = \{p,q\}$ ~~no~~ new states

$\delta'(\{p,t\},1) = \delta(p,1) \cup \delta(t,1) = \{p\} \cup \phi = \{p\}$

no new states.

$Q' = (\{p\}, \{p,q\}, \{p,q,r,s\}, \{p,t\})$

final state of NFA is $\{p,q,r,s\}, \{p,t\}$



DFA

	0	1
$\{p\}$	$\{p,q\}$	$\{p\}$
$\{p,q\}$	$\{p,q,r,s\}$	$\{p,t\}$
$\{p,q,r,s\}$	$\{p,q,r,s\}$	$\{p,t\}$
$\{p,t\}$	$\{p,q\}$	$\{p\}$

Ex-6:-

	0	1
$\rightarrow p$	$\{p,q\}$	$\{p\}$
q	r	r
r	s	ϕ
$*s$	s	s

	0	1
$\rightarrow p$	$\{p,q\}$	$\{p\}$
$\{p,q\}$	$\{p,q,r\}$	$\{p,r\}$
$\{p,q,r\}$	$\{p,q,r,s\}$	$\{p,r\}$
$\{p,r\}$	$\{p,q,s\}$	$\{p\}$
$*\{p,q,r,s\}$	$\{p,q,r,s\}$	$\{p,r,s\}$
$*\{p,q,s\}$	$\{p,q,r,s\}$	$\{p,r,s\}$
$*\{p,r,s\}$	$\{p,q,s\}$	$\{p,s\}$
$*\{p,s\}$	$\{p,q,s\}$	$\{p,s\}$

Draw diagram

Extended transition function for NFA denoted by $\hat{\delta}$

extended transition function ($\hat{\delta}$) for NFA takes as arguments as state & string contains a set of i/p symbols denoted by 'w' & return set of states.

Basis of induction:-

$\hat{\delta}(q,\epsilon) = \{q\}$ without treating any i/p symbol we are only in state begin with q

Induction:-

suppose 'ω' is of the form $\omega = xa$ where a is the final symbol of ω such that

$a \in \Sigma$ & $x \in \Sigma^*$

$$\hat{\delta}(q,x) = \{p_1, p_2 \cdots p_k\}$$

let $\bigcup_{i=1}^{k} \delta(p_i, a) = \{r_1, r_2 \cdots r_n\}$

then $\hat{\delta}(q,\omega) = \{r_1, r_2, \cdots r_n\}$

from

To compute $\hat{\delta}(q,\omega)$ first we need to compute $\hat{\delta}(q,x)$ and then any transition from any of the states labeled 'a'

Theorem:- Imp

If $D = (Q_D, \Sigma_D, \delta_D, \{q_0\}, F_D)$ is constructed from $N = (Q_N, \Sigma_N, \delta_N, q_0, F_N)$ then prove $L(D) = L(N)$

Proof:-

let us consider string w of length k+1 is of the form $w = xa$ where a is final alphabet & x is the rest of the string w.

Basis of induction:-

let $|\omega| = 0$ i.e $\omega = \epsilon$

LHS $\hat{\delta}_D(q_0, \epsilon) = \{q_0\}$

RHS $\hat{\delta}_N(q_0, \epsilon) = \{q_0\}$

$\therefore \hat{\delta}_D(q_0, \epsilon) = \hat{\delta}_N(q_0, \epsilon) = \{q_0\}$

Inductive hypothesis:-

Assume that $\hat{\delta}_D(q_0,x) = \hat{\delta}_N(q_0,x)$ is true for $|w| = k$

i.e $w = x$

Inductive step:-

To prove $L(D) = L(N)$ for $|w| = k+1$ i.e, $w = xa$

The definition of $\hat{\delta}$ for NFA is $\hat{\delta}_N(q_0,w) = \bigcup_{i=1}^{N} \delta(P_i,a)$ → (1)

on the other hand

the subset construction $\delta_D(\{P_1,P_2,P_3 \cdots P_k\},a) = \bigcup_{i=1}^{k} \delta_N(P_i,a)$ → (2)

LHS

$$\delta_D(\{q_0\},w) = \delta_D(\delta_D(\hat{\delta}(q_0,x),a)$$
$$= \delta_D(\{P_1,P,\cdots P_k\},a)$$
$$= \bigcup_{i=1}^{k} \delta(P_i,a)$$
$$= \hat{\delta}_N(q_0,w)$$
$$\therefore \hat{\delta}_D(q_0,w) = \hat{\delta}_N(q_0,w)$$

ϵ-NFA (Finite automata with epsilon(ϵ) transition)

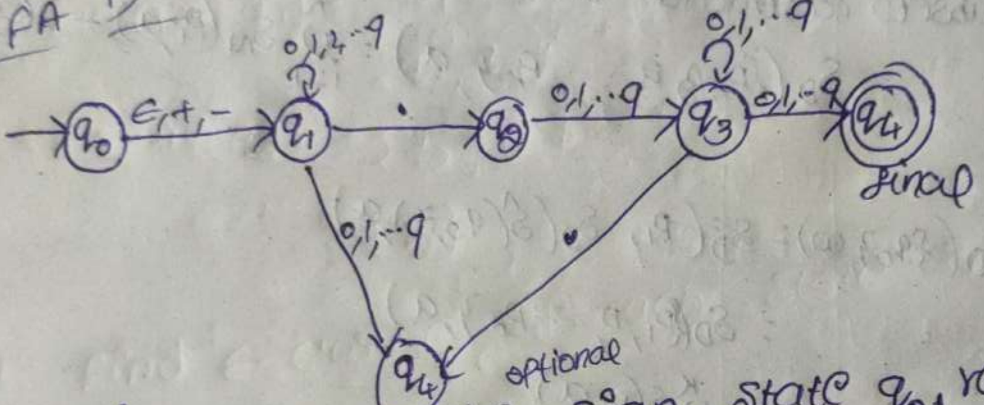
this is another extension of finite automata the new future is that the transition of (ϵ) epsilon that is an emty string.

* NFA is allowed to make transition without receiving an i/p symbol such NFA is called as epsilon NFA (ϵ-NFA)

Ex:- ϵ-NFA that accepts decimal numbers consisting of

1. an optional + or - sign.
2. a string of digits 0,1,2,....9
3. a decimal point (.)
4. another string of digits 0,1,....9

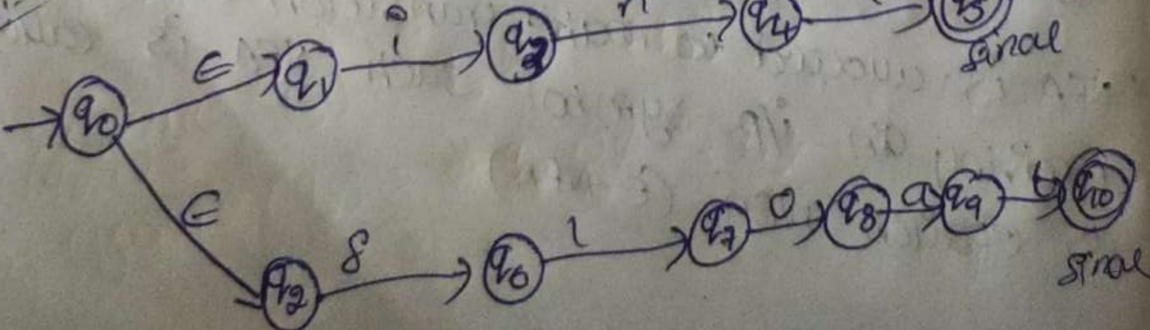
Sol:- NFA is



state q_0 represents the optional sign, state q_1 represents the situate decimal point, state q_5 represepresents definitely one digit followed by decimal point. state q_3 represents at least one digit before the decimal point & after the decimal point.

Ex:- construct ϵ-NFA that recognize set of keywords int & float.

Sol:-



Formal Definition of ϵ-NFA:-

1. ϵ-NFA consists of set of states denoted by Q.
2. set of i/p symbols denoted by Σ
3. Transition function δ takes arguments as state & either i/p symbol or empty state. then that generates set of states

$$\boxed{\delta : Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^{Q}}$$

4. start state denoted by q_0.
5. set of final state denoted by F.

Definition of ϵ-closure(P):-

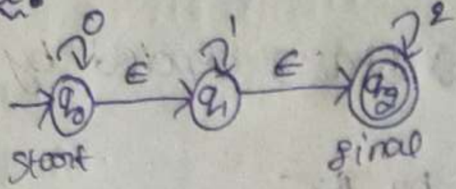
ϵ-closure(P) is set of all states which are reachable from state P on ϵ-transition such that

i, ϵ-closure(P) = {P} $\cup$ ϵ-transitions from 'p' where $p \in Q$

ii, ϵ-closure(P) = q & ϵ-closure(q) $\delta(q, \epsilon) = r$ then

ϵ-closure(P) = {q, r}

Ex:- find ϵ-closure of each state in diagram.



sol

$\epsilon\text{-closure}(q_0) = \{q_0\} \cup \epsilon\text{-closure } q_0$

$= \{q_0\} \cup q_1 \cup q_2$

$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$

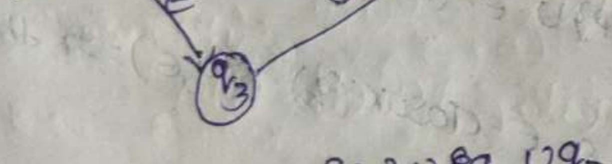
$\epsilon\text{-closure}(q_1) = q_1 \cup q_2$

$= \{q_1, q_2\}$

$\epsilon\text{-closure}(q_2) = \{q_2\} \cup \phi$

$= \{q_2\}$

Ex:- find ϵ-closure of each state in given diagram.



$\epsilon\text{-closure}(q_0) = \{q_0\} \cup q_1 \cup q_3$

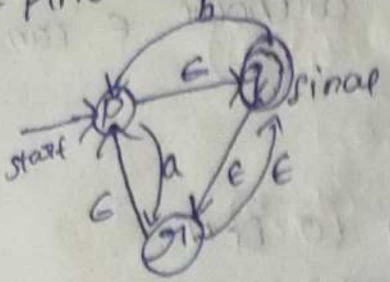
$= \{q_0, q_1, q_3\}$

$\epsilon\text{-closure}(q_1) = q_1 \cup \phi = \{q_1\}$

$\epsilon\text{-closure}(q_3) = q_3 \cup \phi = \{q_3\}$

$\epsilon\text{-closure}(q_2) = \phi\, q_2 \cup \phi = \{q_2\}$

Ex:- Find ϵ-closure for following



ϵ-closure $(p) = \{p\} \cup \{q\} \cup \{r\} \cup \{q\}$
$= \{p, q, r\}$

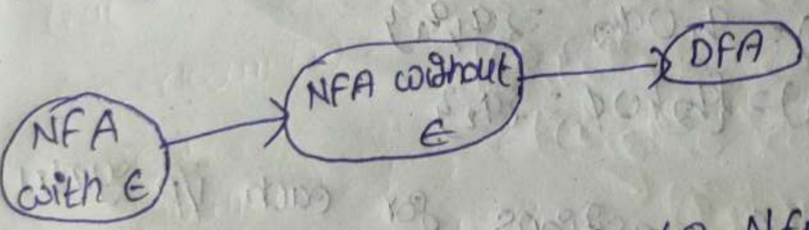
ϵ-closure $(r) = \{r\} \cup \{p\} \cup q \cup r$
$= \{p, q, r\}$

ϵ-closure $(q) = \{q\} \cup r \cup p$
$= \{p, q, r\}$

Convertion from ϵ-NFA to DFA

NFA with ϵ can be converted to NFA without ϵ can be converted to DFA.

Following diagram represents convertion procedure:-



Convertion from ϵ-NFA to NFA:-

Step-1:- Find out all the ϵ transitions from each state in Q that will called as ϵ-closure (q_i) where $q_i \in Q$

Step-2:- Find $\hat{\delta}$ transitions for each i/p symbol & for each state of given ϵ-NFA.

$\hat{\delta}(q_1, b) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), b))$

$= \epsilon\text{-closure}(\delta(\epsilon\text{closure}(q_1), b))$

$= \epsilon\text{-closure}(\delta(\{q_1, q_2\}, b))$

$= \epsilon\text{-closure}(\delta(q_1, b) \cup \delta(q_2, b))$

$= \epsilon\text{-clos}(q_1 \cup \phi)$

$= \epsilon\text{-closure}(q_1)$

$= \{q_1, q_2\}$

$\hat{\delta}(q_1, c) = \epsilon\text{-closure}(\delta(\hat{\delta}(q_1, \epsilon), c))$

$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_1), c))$

$= \epsilon\text{-clos}(\delta(\{q_1, q_2\}, c))$

$= \epsilon\text{-clos}(\delta(q_1, c) \cup \delta(q_2, c))$

$= \epsilon\text{-clos}(\phi \cup q_2)$

$= \epsilon\text{-clos}\{q_2\}$

$= \{q_2\}$

$\hat{\delta}(q_2, a) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_2, \epsilon), a))$

$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_2), a))$

$= \epsilon\text{-clos}(\delta(\{q_2\}, a))$

$= \epsilon\text{-clos}(\delta(q_2, a))$

$= \epsilon\text{-clos}(\phi)$

$= \phi$

$\hat{\delta}(q_2, b) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_2, \epsilon), b))$

$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_2), b))$

$= \epsilon\text{-clos}(\delta(\{q_2\}), b)$

$= \epsilon\text{-clos}(\delta(q_2, b))$

$= \epsilon\text{-clos}(\phi)$

$= \phi$

$\hat{\delta}(q_2, c) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_2, \epsilon), c))$

$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_2), c))$

$= \epsilon\text{-clos}(\delta\{q_2\}, c)$

$= \epsilon\text{-clos}(\delta(q_2, c)$

$= \epsilon\text{-clos}(q_2)$

$= \{q_2\}$

Step-3: NFA without ϵ

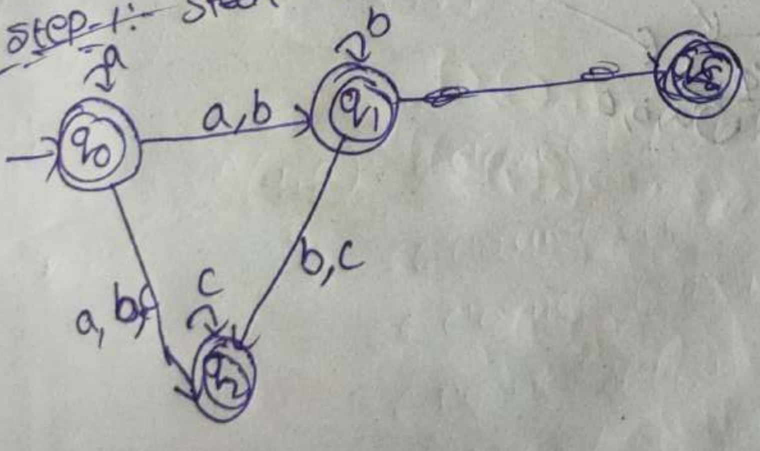
	a	b	c
$\rightarrow * q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
$* q_1$	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
$* q_2$	ϕ	ϕ	$\{q_2\}$

Here q_0, q_1, q_2 acts as final states of NFA b/c containing final states of ϵ-NFA i.e q_2 (ϵ-NFA) ϵ-closure(q_0), ϵ-clos(q_1), & ϵ-clos(q_2)

Draw du

Now ~~convert~~ NFA ~~to DFA~~

Step-1:- ~~Start state~~



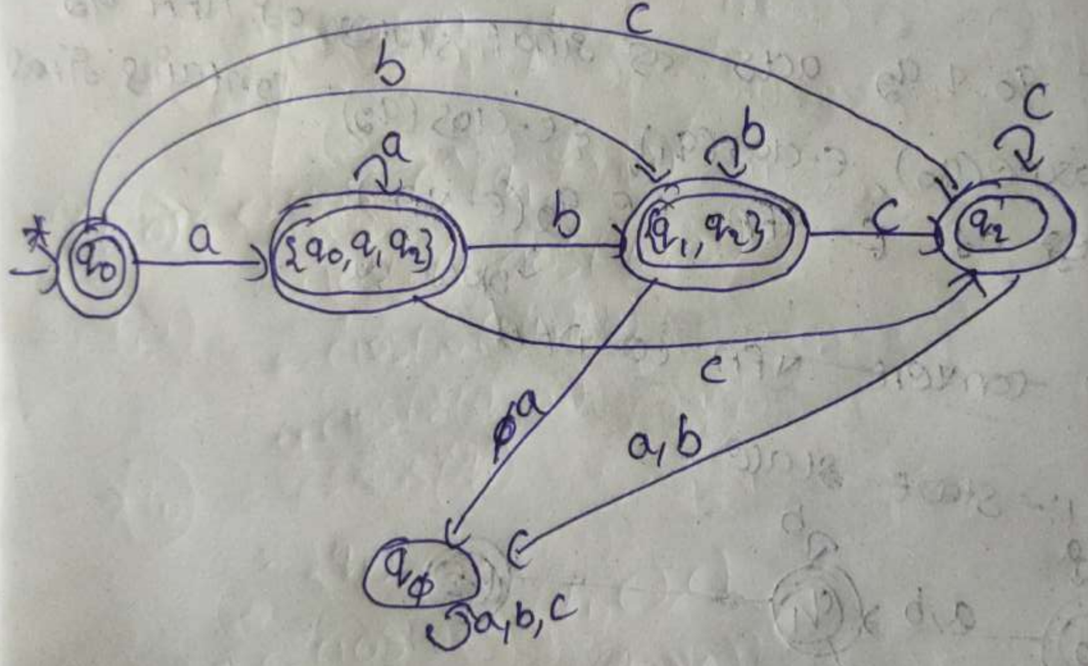
noco

convert NFA to DFA

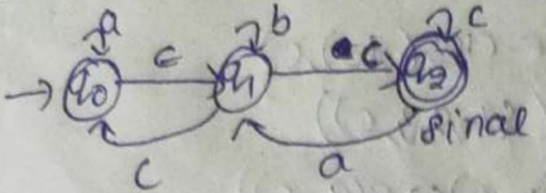
shortcut

step-1: start of state of NFA (q_0) will be start state of DFA

	a	b	c
* → q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
* $\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
* $\{q_1, q_2\}$	ϕ	$\{q_1, q_2\}$	$\{q_2\}$
* $\{q_2\}$	ϕ	ϕ	$\{q_2\}$
ϕ	ϕ	ϕ	ϕ



Q2:- convert following ϵ-NFA to DFA



sol:- convert ϵ-NFA to NFA without ϵ

step-1:- Find ϵ-closure for each state in given ϵ-NA

$Q = \{q_0, q_1, q_2\}$

$\Sigma = \{a, b, c\}$

$F = \{q_2\}$

$q_0 = q_0$

$\epsilon\text{-clos}(q_0) = q_0 \cup q_1 = \{q_0, q_1\}$

$\epsilon\text{-clos}(q_1) = \cancel{\phi}\, q_1 \cup \phi = \{q_1\}$

$\epsilon\text{-clos}(q_2) = \cancel{\phi}\, q_2 \cup \phi = \{q_2\}$

step-2:- Find $\hat{\delta}$ transitions for each i/p symbol

$$
\begin{aligned}
\hat{\delta}(q_0, a) &= \epsilon\text{-clos}(\delta(\hat{\delta}(q_0, \epsilon), a)) \\
&= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_0), a)) \\
&= \epsilon\text{-clos}(\delta(\{q_0, q_1\}, a)) \\
&= \epsilon\text{-clos}(\delta(q_0, a) \cup \delta(q_1, a)) \\
&= \epsilon\text{-clos}(q_0 \cup \phi) \\
&= \epsilon\text{-clos}(q_0) \\
&= \{q_0, q_1\}
\end{aligned}
$$

$$
\begin{aligned}
\hat{\delta}(q_0, b) &= \epsilon\text{-clos}(\delta(\hat{\delta}(q_0, \epsilon), b)) \\
&= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_0), b)) \\
&= \epsilon\text{-clos}(\delta(\{q_0, q_1\}, b)) \\
&= \epsilon\text{-clos}(\delta(q_0, b) \cup \delta(q_1, b)) \\
&= \epsilon\text{-clos}(\phi \cup q_1) \\
&= \epsilon\text{-clos}(q_1) \\
&= \{q_1\}
\end{aligned}
$$

$$\hat{\delta}(q_0,c) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_0,\epsilon),c))$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_0),c)$$
$$= \epsilon\text{-clos}(\delta(\{q_0,q_1\},c))$$
$$= \epsilon\text{-clos}(\delta(q_0,c) \cup \delta(q_1,c))$$
$$= \epsilon\text{-clos}(\phi \cup q_2)$$
$$= \epsilon\text{-clos}(q_2)$$
$$= \{q_2\}$$

$$\hat{\delta}(q_1,a) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_1,\epsilon),a))$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_1),a)$$
$$= \epsilon\text{-clos}(\delta(\{q_1\},a)$$
$$= \epsilon\text{-clos}(\delta(q_1,a))$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi$$

$$\hat{\delta}(q_1,b) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_1,\epsilon),b))$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_1),b))$$
$$= \epsilon\text{-clos}(\delta(\{q_1\},b)$$
$$= \epsilon\text{-clos}(\delta(q_1,b))$$
$$= \epsilon\text{-clos}(q_1)$$
$$= \{q_1\}$$

$$\hat{\delta}(q_1,c) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_1,\epsilon),c)$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(\{q_1\}),c))$$
$$= \epsilon\text{-clos}(\delta(\{q_1\}),c))$$
$$= \epsilon\text{-clos}(\delta(q_1,c))$$
$$= \epsilon\text{-clos}(q_0,q_2)$$
$$= \cancel{\{q_2\}}\ \{q_0,q_2\}$$

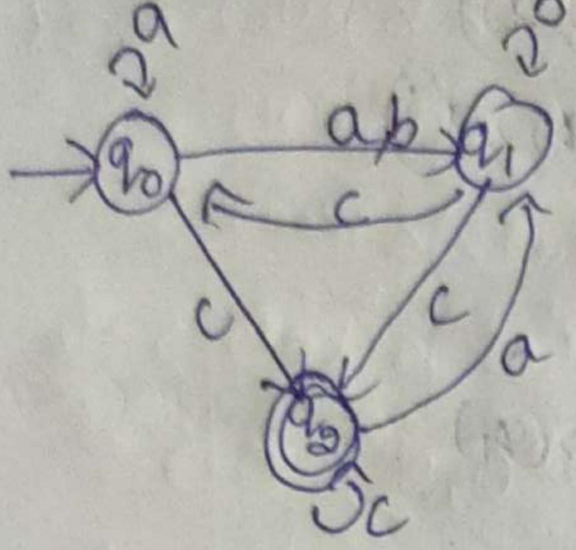
$$\hat{\delta}(q_2,a) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_2,\epsilon),a))$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_2),a))$$
$$= \epsilon\text{-clos}(\delta(\{q_2\},a))$$
$$= \epsilon\text{-clos}(\delta(q_2,a))$$
$$= \epsilon\text{-clos}(\cancel{\phi}) = \epsilon\text{-clos}(q_1)$$
$$\cancel{=\phi} \quad = \{q_1\}$$

$$\hat{\delta}(q_2,b) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_2,\epsilon),b))$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_2),b))$$
$$= \epsilon\text{-clos}(\delta(\{q_2\},b))$$
$$= \epsilon\text{-clos}(\delta(q_2,b))$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi$$

$$\hat{\delta}(q_2,c) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_2,\epsilon),c))$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_2),c))$$
$$= \epsilon\text{-clos}(\delta(\{q_2\},c))$$
$$= \epsilon\text{-clos}(\delta(q_2,c))$$
$$= \epsilon\text{-clos}\{q_2\}$$
$$= \{q_2\}$$

Step-3:- NFA without ϵ

	a	b	c
$\rightarrow q_0$	$\{q_0,q_1\}$	$\{q_1\}$	$\{q_2\}$
q_1	ϕ	$\{q_1\}$	$\{q_0,q_2\}$
$* q_2$	$\{q_1\}$	ϕ	$\{q_2\}$

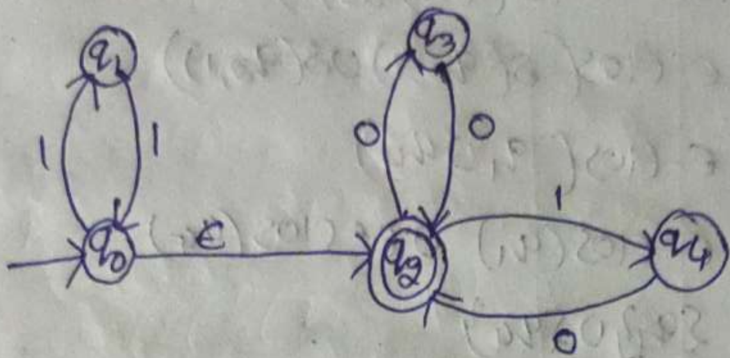


Now convert NFA to DFA

	a	b	c
$\to q_0$	$\{q_0, q_1\}$	$\{q_1\}$	$\{q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1\}$	$\cancel{\{q_2\}}$ $\{q_0, q_2\}$
$\{q_1\}$	ϕ	$\{q_1\}$	$\cancel{\{q_2\}}$ $\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_1\}$	$\{q_2\}$
$* \{q_2\}$	$\{q_1\}$	ϕ	$\{q_2\}$
ϕ	ϕ	ϕ	ϕ

Ex-3 convert following ε-NFA to DFA

(b)



Step-1:-

$Q = \{q_0, q_1, q_2, q_3, q_4\}$

$\Sigma = \{\cancel{a, b}\ 0, 1\}$

$F = \{q_2\}$

$q_0 = q_0$

$\epsilon\text{-clos}\{q_0\} = q_0 \cup q_2 = \{q_0, q_2\}$

$\epsilon\text{-clos}\{q_1\} = \{q_1\}$

$\epsilon\text{-clos}\{q_2\} = \{q_2\}$

$\epsilon\text{-clos}\{q_3\} = \{q_3\}$

$\epsilon\text{-clos}\{q_4\} = \{q_4\}$

Step-2:-

$\hat{\delta}(q_0, 0) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_0, \epsilon), 0))$

$\hat{\delta}(q_0, 1) = \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_0), 0))$

$= \epsilon\text{-clos}(\delta(\{q_0, q_2\}), 0)$

$= \epsilon\text{-clos}(\delta(q_0, 0) \cup \delta(q_2, 0))$

$= \epsilon\text{-clos}(\phi \cup q_3\}$

$= \epsilon\text{-clos}\{q_3\}$

$= \{q_3\}$

$$\hat{\delta}(q_0,1) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_0,\epsilon),1))$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(\{q_0\}),1))$$
$$= \epsilon\text{-clos}(\delta(\{q_0,q_2\},1)$$
$$= \epsilon\text{-clos}(\delta(q_0,1) \cup \delta(q_2,1))$$
$$= \epsilon\text{-clos}(q_1 \cup q_4)$$
$$= \epsilon\text{-clos}(q_1) \cup \epsilon\text{-clos}(q_4)$$
$$= \{q_1\} \cup \{q_4\}$$
$$= \{q_1,q_4\}$$

$$\hat{\delta}(q_1,0) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_1,\epsilon),0))$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(\{q_1\}),0)$$
$$= \epsilon\text{-clos}(\delta(\{q_1\},0)$$
$$= \epsilon\text{-clos}(\delta(q_1,0))$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi$$

$$\hat{\delta}(q_1,1) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_1,\epsilon)1)).$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(\{q_1\}),1)$$
$$= \epsilon\text{-clos}(\delta(\{q_1\}),\phi)$$
$$= \epsilon\text{-clos}(\delta(q_1,1))$$
$$= \epsilon\text{-clos}(q_0)$$
$$= \{q_0,q_2\}$$

$$\hat{\delta}(q_2,0) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_2,\epsilon),0)$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(\{q_2\},0))$$
$$= \epsilon\text{-clos}(\delta(\{q_2\},0))$$
$$= \epsilon\text{-clos}(\delta(q_2,0))$$
$$= \epsilon\text{-clos}(q_3)$$
$$= \{q_3\}$$

$$\hat{\delta}(q_2,1) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_2,\epsilon),1)$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}\{q_2\},1)$$
$$= \epsilon\text{-clos}(\delta\{q_2\},1))$$
$$= \epsilon\text{-clos}(\delta(q_2,1))$$
$$= \epsilon\text{-clos}\{q_4\}$$
$$= \{q_4\}$$

$$\hat{\delta}(q_3,0) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_3,\epsilon),0)$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(\{q_3\},0))$$
$$= \epsilon\text{-clos}(\delta\{q_3\}),0))$$
$$= \epsilon\text{-clos}(\delta(q_3,0))$$
$$= \epsilon\text{-clos}\{q_2\}$$
$$= \{q_2\}$$

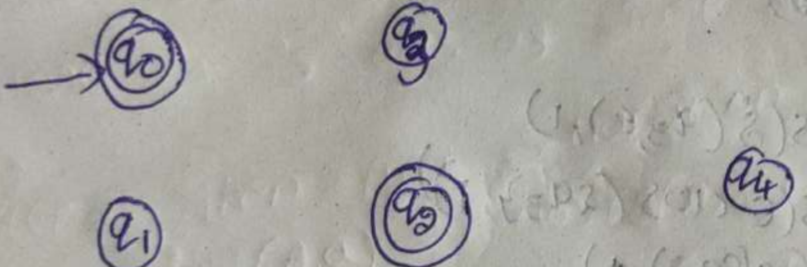
$$\hat{\delta}(q_3,1) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_3,\epsilon),1)$$
$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(\{q_3\},1))$$
$$= \epsilon\text{-clos}(\delta\{q_3\}),1)$$
$$= \epsilon\text{-clos}(\delta(q_3,1))$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi$$

$$\hat{\delta}(q_4,0) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_4,\epsilon),0)$$
$$= \epsilon\text{-clos}(\delta(q_4,0))$$
$$= \epsilon\text{-clos}\{q_2\} \quad - \; q_0$$
$$= \{q_2\}$$

$$\hat{\delta}(q_4,1) = \epsilon\text{-clos}(\delta(\hat{\delta}(q_4,\epsilon),1))$$
$$= \epsilon\text{-clos}(\delta(q_4,1)$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi$$

NFA without ϵ

	0	1
$\rightarrow * q_0$	q_3	$\{q_1, q_4\}$
q_1	ϕ	$\{q_0, q_2\}$
$* q_2$	$\{q_3\}$	$\{q_4\}$
q_3	$\{q_2\}$	ϕ
q_4	$\{q_2\}$	ϕ



Conversion of ϵ-NFA to DFA (Direct method):-

consider $m = (Q, \Sigma, \delta, q_0, F)$ be an ϵ-NFA then its equivalent DFA is $M_D = (Q_D, \Sigma_D, \delta_D, q_0', F_D)$ then

step-1:-

ϵ-closure$(q_0) = \{P_0, P_1 \cdots P_n\}$ be states start state of DFA

i.e $Q_D = (\{P_0, P_1, \cdots P_n\})$

step-2:-

then find δ transition for each state in Q_D on each i/p Σ.

$\delta_D(\{P_0, P_1, \cdots P_n\}, a) = \epsilon\text{-clos}(\{P_0, P_1, \cdots P_n\}, a)$ where $a \in \Sigma$

$= \{r_1, r_2, \cdots r_n\}$ which not in Q_D

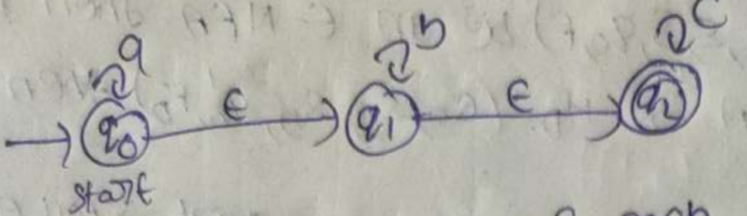
then Add$(r_1, r_2, \cdots r_n)$ to Q_D

$\therefore Q_D = \{\{P_0, P_1, \cdots P_n\}, \{r_1, r_2, \cdots r_n\}\}$

step-3:- Repeat step-2 until no new states found.

step-4:- The states obtained in Q_D, the states containing final state in Q_D acts as final state of DFA.

Ex:- convert following ϵ-NFA to DFA (direct method)



step-1:- find ϵ-closure of each state in NFA

$\epsilon\text{-closure}(q_0) = q_0 \cup q_1 \cup q_2 = \{q_0, q_1, q_2\}$

$\epsilon\text{-closure}(q_1) = q_1 \cup q_2 = \{q_1, q_2\}$

$\epsilon\text{-clos}(q_2) = q_2\} \cup \phi = \{q_2\}$

step-2:- consider $\epsilon\text{-closu}(q_0) = \{q_0, q_1, q_2\} \xrightarrow[\text{as}]{A}$ ~~final~~

here A is acts as start state of DFA $Q_D = (\{q_0, q_1, q_2\})$

step-2:- Then find δ-transition A on each i/p symbol

$\delta_D(A, a) = \epsilon\text{-clos}(\delta(A, a))$

$= \epsilon\text{-clos}(\delta(\{q_0, q_1, q_2\}, a))$

$= \epsilon\text{-clos}(\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a))$

$= \epsilon\text{-clos}(\{q_0 \cup \phi \cup \phi\})$

$= \epsilon\text{-clos}\{q_0\}$

$= \{q_0, q_1, q_2\}$ which is already in Q_D

$\delta_D(A, b) = \epsilon\text{-clos}(\delta(A, b))$

$= \epsilon\text{-clos}(\delta(\{q_0, q_1, q_2\}, b))$

$= \epsilon\text{-clos}(\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b))$

$= \epsilon\text{-clos}(\{\phi \cup q_1 \cup \phi\})$

$= \epsilon\text{-clos}(q_1)$

$= \{q_1, q_2\} \xrightarrow{\text{named as}} B$

$\therefore Q_D = (\underbrace{\{q_0, q_1, q_2\}}_{A}, \underbrace{\{q_1, q_2\}}_{B})$

$$\delta_D(A,C) = \epsilon\text{-clos}(\delta(A,C))$$
$$= \epsilon\text{-clos}(\delta(\{q_0, q_1, q_2\}, c)$$
$$= \epsilon\text{-clos}(\delta(q_0,c) \cup \delta(q_1,c), \delta(q_2,c))$$
$$= \epsilon\text{-clos}(\{\phi \cup \phi \cup q_2\}$$
$$= \epsilon\text{-clos}\,\{q_2\}$$
$$= \{q_2\} \longrightarrow C$$

$$\therefore Q_D = (\underbrace{\{q_0, q_1, q_2\}}_{A}, \underbrace{\{q_1, q_2\}}_{B}, \underbrace{\{q_2\}}_{C})$$

$$\delta_D(B,a) = \epsilon\text{-clos}(\delta(B,a))$$
$$= \epsilon\text{-clos}(\delta(\{q_1, q_2\}, a))$$
$$= \epsilon\text{-clos}(\delta(q_1,a) \cup \delta(q_2,a))$$
$$= \epsilon\text{-clos}(\phi \cup \phi)$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi \longrightarrow D$$

$$\therefore Q_D = (\underbrace{\{q_0, q_1, q_2)}_{A}, \underbrace{\{q_1, q_2\}}_{B}, \underbrace{\{q_2\}}_{C}, \underbrace{\phi}_{D})$$

$$\delta_D(B,b) = \epsilon\text{-clos}(\delta(B,b))$$
$$= \epsilon\text{-clos}(\delta(\{q_1, q_2\}, b))$$
$$= \epsilon\text{-clos}(\delta(q_1, b) \cup \delta(q_2, b))$$
$$= \epsilon\text{-clos}(q_1 \cup \phi)$$
$$= \epsilon\text{-clos}(q_1)$$
$$= \{q_1, q_2\}$$

$$\delta_D(B,C) = \epsilon\text{-clos}(\delta(B,C))$$
$$= \epsilon\text{-clos}(\delta(\{q_1, q_2\}, c)$$
$$= \epsilon\text{-clos}(\delta(q_1, c) \cup \delta(q_2, c))$$
$$= \epsilon\text{-clos}(\phi \cup q_2)$$
$$= \epsilon\text{-clos}\{q_2\}$$
$$= \{q_2\}$$

$$\delta_D(C,a) = \epsilon\text{-clos}(\delta(C,a))$$
$$= \epsilon\text{-clos}(\delta(\{q_2\}, a))$$
$$= \epsilon\text{-clos}(\delta(q_2, a))$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi$$

$$\delta_D(C,b) = \epsilon\text{-clos}(\delta(C,b))$$
$$= \epsilon\text{-clos}(\delta(\{q_2\}, b)$$
$$= \epsilon\text{-clos}(\delta(q_2, b)$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi$$

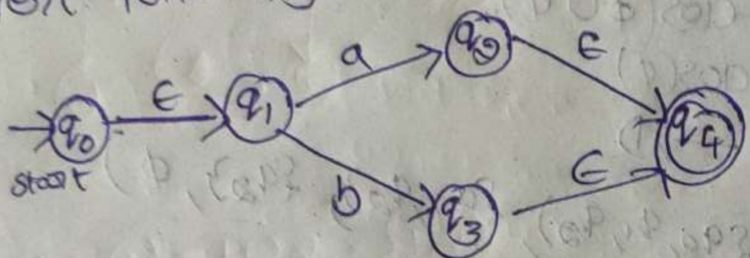
$$\delta_D(C,C) = \epsilon\text{-clos}(\delta(C,C))$$
$$= \epsilon\text{-clos}(\delta\{q_2\}, c))$$
$$= \epsilon\text{-clos}(\delta(q_2, c)$$
$$= \epsilon\text{-clos}\,\{q_2\}$$
$$= \{q_2\}$$

	a	b	c
*→A $\{q_0,q_1,q_2\}$	$\{q_0,q_1,q_2\}$	$\{q_1,q_2\}$	$\{q_2\}$
*B $\{q_1,q_2\}$	ϕ	$\{q_1,q_2\}$	$\{q_2\}$
*C $\{q_2\}$	ϕ	ϕ	$\{q_2\}$
ϕ	ϕ	ϕ	ϕ

In ~~DF~~ Q_D every state contains final state of ε-NFA so all states considered as final state.

Ex-2:-

convert following ε-NFA to DFA.



step-1:-

$\epsilon\text{-clos}(q_0) = q_0 \cup q_1 = \{q_0, q_1\}$

$\epsilon\text{-clos}(q_1) = q_1 \cup \phi = \{q_1\}$

$\epsilon\text{-clos}(q_2) = q_2 \cup q_4 = \{q_2, q_4\}$

$\epsilon\text{-clos}(q_3) = q_3 \cup q_4 = \cancel{\{q_4\}}\ \{q_3, q_4\}$

$\epsilon\text{-clos}(q_4) = q_4 \cup \phi = \{q_4\}$

~~step~~ Here $\epsilon\text{-clos}(q_0) = \{q_0, q_1\} \longrightarrow A$

$Q_D = \{q_0, q_1\}$

A is acts as start state of DFA

step-2:-

$$\delta_D(A,a) = \epsilon\text{-clos}(\delta(A,a))$$
$$= \epsilon\text{-clos}(\delta(\{q_0,q_1\},a))$$
$$= \epsilon\text{-clos}(\delta(q_0,a) \cup \delta(q_1,a))$$
$$= \epsilon\text{-clos}(\phi \cup q_2)$$
$$= \epsilon\text{-clos}(q_2)$$
$$= \{q_2,q_4\} \rightarrow B \text{ (named as)}$$

$$Q_D = (\underbrace{\{q_0,q_1\}}_{A}, \underbrace{\{q_2,q_4\}}_{B})$$

$\delta_D(A,$

$$\delta_D(A,b) = \epsilon\text{-clos}(\delta(A,b))$$
$$= \epsilon\text{-clos}(\delta(\{q_0,q_1\},b))$$
$$= \epsilon\text{-clos}(\delta(q_0,b) \cup \delta(q_1,b))$$
$$= \epsilon\text{-clos}(\phi \cup q_3)$$
$$= \epsilon\text{-clos}(q_3)$$
$$= \{q_3,q_4\} \rightarrow C$$

$$Q_D = (\underbrace{\{q_0,q_1\}}_{A}, \underbrace{\{q_2,q_4\}}_{B}, \underline{\{q_3,q_4\}})$$

$$\delta_D(B,a) = \epsilon\text{-clos}(\delta(B,a))$$
$$= \epsilon\text{-clos}(\delta(\{q_2,q_4\},a))$$
$$= \epsilon\text{-clos}(\delta(q_2,a) \cup \delta(q_4,a))$$
$$= \epsilon\text{-clos}(\phi \cup \phi)$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi$$

$$\delta_D(B,b) = \epsilon\text{-clos}(\delta(B,b))$$
$$= \epsilon\text{-clos}(\delta(\{q_2,q_4\},b))$$
$$= \epsilon\text{-clos}(\delta(q_2,b) \cup \delta(q_4,b))$$
$$= \epsilon\text{-clos}(\phi \cup \phi)$$
$$= \phi$$

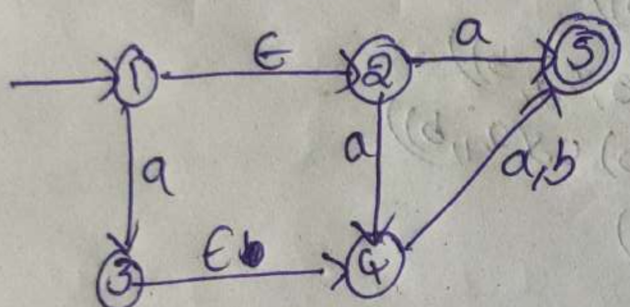
$$\delta_D(C,a) = \epsilon\text{-clos}(\delta(C,a))$$
$$= \epsilon\text{-clos}(\delta(\{q_3,q_4\},a))$$
$$= \epsilon\text{-clos}(\delta(q_3,a) \cup \delta(q_4,a))$$
$$= \epsilon\text{-clos}(\phi \cup \phi)$$
$$= \phi$$

$$\delta_D(C,b) = \epsilon\text{-clos}(\delta(C,b))$$
$$= \epsilon\text{-clos}(\delta(q_3,b) \cup \delta(q_4,b))$$
$$= \epsilon\text{clos}(\phi \cup \phi)$$
$$= \epsilon\text{-clos}(\phi)$$
$$= \phi$$

	a	b
→ A $\{q_0, q_1\}$	$\{q_2, q_4\}$	$\{q_3, q_4\}$
* B $\{q_2, q_4\}$	ϕ	ϕ
* C $\{q_3, q_4\}$	ϕ	ϕ
ϕ	ϕ	ϕ

Ex-3:-

ϵ-NFA to DFA



Sol:-

$\epsilon\text{-clos}(1) = 1 \cup 2 = \{1,2\}$

$\epsilon\text{-clos}(2) = 2 \cup \phi = \{2\}$

$\epsilon\text{-clos}(3) = 3 \cup \phi = \{3,4\}$

$\epsilon\text{-clos}(4) = 4 \cup \phi = \{4\}$

$\epsilon\text{-clos}(5) = 5 \cup \phi = \{5\}$

$\epsilon\text{-clos}(1) = \{1,2\} \longrightarrow A$

$Q_D = \{1,2\}$

A acts as start state of DFA.

step-2 :-

$$\delta_D(A,a) = \epsilon\text{-clos}(\delta(A,a))$$
$$= \epsilon\text{-clos}(\delta(\{1,2\},a))$$
$$= \epsilon\text{-clos}(\delta(1,a) \cup \delta(2,a))$$
$$= \epsilon\text{-clos}(3 \cup \{5,4\})$$
$$= \epsilon\text{-clos}(\{3,5,4\})$$
$$= \epsilon\text{-clo}\{3\} \cup \epsilon\text{-clo}\{5\} \cup \epsilon\text{-clos}\{4\}$$
$$= \{3\} \cup \{5\} \cup \{4\}$$
$$= \{3,4,5\} \rightarrow B$$

$$\delta_D(A,b) = \epsilon\text{-clos}(\delta(A,b))$$
$$= \epsilon\text{-clos}(\delta(1,b) \cup \delta(2,b))$$
$$= \epsilon\text{-clos}(\phi \cup \phi)$$
$$= \phi$$

$$Q_D = \{\underbrace{\{1,2\}}_{A}, \underbrace{\{3,4,5\}}_{B}\}$$

$$\delta_D(B,a) = \epsilon\text{-clos}(\delta(B,a))$$
$$= \epsilon\text{-clos}(\delta(\{3,4,5\},a))$$
$$= \epsilon\text{-clos}(\delta(3,a) \cup \delta(4,a) \cup \delta(5,a))$$
$$= \epsilon\text{-clos}(\phi \cup 5 \cup \phi)$$
$$= \epsilon\text{-clos}(5)$$
$$= \{5\} \rightarrow C$$

$$\delta_D(B,b) = \epsilon\text{-clos}(\delta(B,b))$$
$$= \epsilon\text{-clos}(\delta(3,b) \cup \delta(4,b) \cup \delta(5,b))$$
$$= \epsilon\text{-clos}(\phi \cup 5 \cup \phi)$$
$$= \epsilon\text{-clos}(5)$$
$$= \cancel{\epsilon\text{-clos}\{4\} \cup \epsilon\text{-clos}\{5\}}$$
$$= \cancel{\{4\} \cup \{5\}}$$
$$= \cancel{\{4,5\}} \rightarrow D$$
$$= \{5\}$$

$$Q_D = \{\underbrace{\{1,2\}}_{A}, \underbrace{\{3,4,5\}}_{B}, \underbrace{\{5\}}_{C}, \underbrace{\cancel{\{4,5\}}}_{D}\}$$

$$\delta_D(C,a) = \epsilon\text{-clos}(\delta(C,a))$$
$$= \epsilon\text{-clos}(\delta(\{5\},a))$$
$$= \epsilon\text{-clos}(\delta(5,a))$$
$$= \phi$$

$$\delta_D(C,b) = \epsilon\text{-clos}(\delta(5,b))$$
$$= \phi$$

$$\delta_D(D,a) = \epsilon\text{-clos}(\delta(D,a))$$
$$= \epsilon\text{-clos}(\delta(\{4,5\},a))$$
$$= \epsilon\text{-clos}(\delta(4,a) \cup \delta(5,a))$$
$$= \epsilon\text{-clos}(5 \cup \phi)$$
$$= \{5\}$$

$$\delta_D(D,b) = \epsilon\text{-clos}(\delta(4,b) \cup (5,b))$$
$$= \epsilon\text{-clos}(5 \cup \phi)$$
$$= \{5\}$$

	a	b
A $\{1,2\}$	$\{3,4,5\}$	ϕ
B $\{3,4,5\}$	$\{5\}$	$\{4,5\}$
C $\{5\}$	ϕ	ϕ
D $\{4,$[illegible]$\}$	$\{$[illegible]$\}$	$\{$[illegible]$\}$
ϕ	ϕ	ϕ