

Assignment I: Math for Eng II

Year I: IS, IT, CS, CSE

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Q.1. Solve the following first order odes

a)

$$\frac{dy}{dx} - y^2 x e^{3x+1} = 0$$

b)

$$xy' - 4y = x + 1$$

Q.2. Obtain the solutions of the following second order odes

a)

$$y'' + 4y = \sin^2(2x), \quad \text{with } y(\pi) = 0 \quad \text{and } y'(\pi) = 0.$$

b)

$$2x^2 y'' + 5xy' + y = x^2 - x$$

c) Show that $y = x$ is a solution of

$$x^2 y'' - x(2 + x \cot x) y' + (2 + x \cot x) y = 0.$$

Use the method of variation of parameters to obtain the general solution of the ode

$$x^2 y'' - x(2 + x \cot x) y' + (2 + x \cot x) y = x^3 \sin x.$$

Q.3. a) Reduce the equation

$$ty''' - 2y'' + t^2 y' + \frac{1}{t} y = 1 + t$$

into system of first order odes when $y(0) = 1$, $y'(0) = 2$ and $y''(0) = -1$

b) Solve the following system of odes

$$\begin{cases} x' = x + 2y \\ y' = 4x + 3y \end{cases} \quad \text{with } x(0) = 1 \quad \text{and} \quad y(0) = -1$$

Q.4. Use D-operator to solve the following system of first order odes

$$\begin{cases} 2x' + y' + x + y = t^2 + 4t \\ x' + y' + 2x + 2y = 2t^2 - 2t \end{cases}$$

Q.5. a) Find the Laplace transform of the following function

$$f(t) = t^2 e^{-t} \cos(2t) + t \sin(2t)$$

b) Find the inverse Laplace transform of the function

$$F(s) = \frac{1}{(s+1)(s+2)(s^2+2s+1)}$$

Q.6. a) Use the Laplace transform to find the charge function $q(t)$ of *LRC* series circuit governed by

$$L \frac{d^2 q}{dt^2} + R \frac{dq}{dt} + \frac{1}{C} q = E(t)$$

where $L = 1$ henry, $R = 20\Omega$, $C = 0.005$ and $E(t) = 150v$. Consider the initial conditions $q(0) = q'(0) = 0$.

b) Use Laplace transform to solve the following system of second order odes

$$\begin{cases} 2x'' - y'' - x' - y' = 3y - 9x \\ 2x'' - y'' + x' + y' = 5y - 7x \end{cases} \quad \text{subject to} \quad x(0) = x'(0) = 1 \quad \text{and} \quad y(0) = y'(0) = 0.$$

Good Luck!