SHARIF UNIVERSITY OF TECHNOLOGY

Computer Simulation

Assignment 2

Author: Kasra Amani
Student No. 98101171

Instructor: **Prof. Bardia Safaei**

Date Last Edited: November 29, 2022

Problem 1

a

$1 - P(X \leq x) = P(X > x)$
We want to know how many $P(x_i)$s are in this summation; it is repeated in every sequence until we reach $x = x_i$ and thus, it is repeated x_i times; so we can rewrite the given statement as follows:
$\sum_{x=0}^{\infty} x_i P(X = x_i) = \mathbb{E}(X)$

b

We need to show that $P(Z = z) = \frac{e^{-\lambda}\lambda^z}{z!}$ where $\lambda = \lambda_1 + \lambda_2$:
Since $Z = X + Y$: $P(Z = z) = \sum_{i=0}^{z} P(X = i, Y = z - i)$; X and Y are independent thus:
$= \sum_{i=0}^{z} P(X = i)P(Y = z - i) = \sum_{i=0}^{z} \frac{1}{i!(z-i)!} e^{-\lambda_1}\lambda_1^i e^{-\lambda_2}\lambda_2^{z-i} = \sum_{i=0}^{z} \frac{z!}{i!(z-i)!} \frac{e^{-\lambda_1}\lambda_1^i e^{-\lambda_2}\lambda_2^{z-i}}{z!}$
$= \sum_{i=0}^{z} \binom{z}{i} \frac{e^{-\lambda_1}\lambda_1^i e^{-\lambda_2}\lambda_2^{z-i}}{z!} = \sum_{i=0}^{z} \binom{z}{i} \frac{e^{-(\lambda_1+\lambda_2)}}{z!} \lambda_1^i \lambda_2^{z-i} = \frac{e^{-\lambda}}{z!} \sum_{i=0}^{z} \binom{z}{i} \lambda_1^i \lambda_2^{z-i}$
(using binomial expansion) $= \frac{e^{-\lambda}}{z!}(\lambda_1 + \lambda_2)^z = \frac{e^{-\lambda}\lambda^z}{z!}$ and the proof is concluded.

Problem 2

Lemma: The nth person necessarily sits in either the first seat or the last one.
Proof by contradiction: If the nth person isn't sitting in either of those two, he must be sitting in the ith seat where $2 \leq i \leq n-1$; so the ith seat was empty when the last person arrived and decided to sit in it; but if it had been empty until the end, it was also empty the moment that ith person himself arrived and he must have sat there but he didn't and the nth person is now sitting there so the lemma must be correct.

We now divide the problem into n steps; each step consists of the ith person arriving and taking a seat. The fate of the last person is decided the moment someone sits in either the last or the first seat (because of the lemma proven earlier) and since the chances of the last or first being chosen is equal in all of the n steps (either 0 when the person deciding to sit down has their designated seat available or equal to each other otherwise), the chances of the last person arriving and finding their seat empty is $\frac{p_1}{2} + \frac{p_2}{2} + ... + \frac{p_n}{2} = \frac{p_1 + p_2 + ... + p_n}{2}$ where we know $\sum_{i=1}^{n} p_i = 1$ and thus, the chances of the last person finding their seat empty upon arrival is $\frac{1}{2}$.

Problem 3

Problem 4
TBD.

Problem 5
Code.