

Computer Simulation

Assignment 2

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Problem 1

 \mathbf{a}

$$1 - P(X \le x) = P(X > x)$$

We want to know how many $P(x_i)$ s are in this summation; it is repeated in every sequence until we reach $x = x_i$ and thus, it is repeated x_i times; so we can rewrite the given statement as follows:

$$\sum_{x=0}^{\infty} x_i P(X = x_i) = \mathbb{E}(X)$$

b

We need to show that $P(Z=z) = \frac{e^{-\lambda}\lambda^z}{z!}$ where $\lambda = \lambda_1 + \lambda_2$:

Since
$$Z = X + Y$$
: $P(Z = z) = \sum_{i=0}^{z} P(X = i, Y = z - i)$; X and Y are independent thus:
$$= \sum_{i=0}^{z} P(X = i) P(Y = z - i) = \sum_{i=0}^{z} \frac{1}{i!(z - i)!} e^{-\lambda_1} \lambda_1^i e^{-\lambda_2} \lambda_2^{z - i} = \sum_{i=0}^{z} \frac{z!}{i!(z - i)!} \frac{e^{-\lambda_1} \lambda_1^i e^{-\lambda_2} \lambda_2^{z - i}}{z!}$$

$$= \sum_{i=0}^{z} {z \choose i} \frac{e^{-\lambda_1} \lambda_1^i e^{-\lambda_2} \lambda_2^{z - i}}{z!} = \sum_{i=0}^{z} {z \choose i} \frac{e^{-(\lambda_1 + \lambda_2)}}{z!} \lambda_1^i \lambda_2^{z - i} = \frac{e^{-\lambda}}{z!} \sum_{i=0}^{z} {z \choose i} \lambda_1^i \lambda_2^{z - i}$$

(using binomial expansion) = $\frac{e^{-\lambda}}{z!}(\lambda_1 + \lambda_2)^z = \frac{e^{-\lambda}\lambda^z}{z!}$ and the proof is concluded.

Problem 2

Problem 3

Problem 4

TBD.

Problem 5

Code.