Tarea 2

Lenguajes

2. Using induction on i, prove that $(w^R)^i = (w^i)^R$ for any string w and all $i \geq 0$.

3. Prove, using induction on the length of the string, that $(w^R)^R = w$ for all strings $w \in \Sigma^*$.

4. Let $X = \{aa, bb\}$ and $Y = \{\lambda, b, ab\}$.

   a) List the strings in the set XY.

   b) How many strings of length 6 are there in X^*?

   c) List the strings in the set Y^* of length three or less.

   d) List the strings in the set X^*Y^* of length four or less.

5. Let L be the set of strings over $\{a, b\}$ generated by the recursive definition

   i) Basis: $b \in L$.

   ii) Recursive step: if u is in L then $ub \in L$, $uab \in L$, and $uba \in L$, and $bua \in L$.

   iii) Closure: a string v is in L only if it can be obtained from the basis by a finite number of iterations of the recursive step.

   a) List the elements in the sets L_0, L_1, and L_2.

   b) Is the string $bbaaba$ in L? If so, trace how it is produced. If not, explain why not.

   c) Is the string $bbaaaabb$ in L? If so, trace how it is produced. If not, explain why not.

6. Give a recursive definition of the set of strings over $\{a, b\}$ that contain at least one b and have an even number of a's before the first b. For example, bab, aab, and $aaaababababab$ are in the set, while aa, abb are not.

7. Give a recursive definition of the set $\{a^i b^j \mid 0 \leq i \leq j \leq 2i\}$.

8. Give a recursive definition of the set of strings over $\{a, b\}$ that contain twice as many a's as b's.

For Exercises 14 through 38, give a regular expression that represents the described set.

14. The set of strings over $\{a, b, c\}$ in which all the a's precede the b's, which in turn precede the c's. It is possible that there are no a's, b's, or c's.

15. The same set as Exercise 14 without the null string.

16. The set of strings over $\{a, b, c\}$ with length three.

17. The set of strings over $\{a, b, c\}$ with length less than three.

18. The set of strings over $\{a, b, c\}$ with length greater than three.

19. The set of strings over $\{a, b\}$ that contain the substring ab and have length greater than two.

20. The set of strings of length two or more over $\{a, b\}$ in which all the a's precede the b's.

39. Use the regular expression identities in Table 2.1 to establish the following identities:

   a) $(ba)^+(a^*b^* \cup a^*) = (ba)^*ba^+(b^* \cup \lambda)$

   b) $b^+(a^*b^* \cup \lambda)b = b(b^*a^* \cup \lambda)b^+$