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Answer 1

part a)

- If G has an Eulerian circuit, then every vertex of G has even degree.
- After checking the degrees of all vertices in G, we find that all vertices either have degree of 2 or 4
- Therefore, G does have an Eulerian circuit.

part b)

- If G has an Eulerian path, then exactly 0 or 2 vertices of G have odd degrees.
- After checking the degrees of all vertices in G, we find that exactly 0 vertices have odd degrees.
- Therefore, if the question intends to propose a path that doesn't follow the same path as the circuit no G does not have a Distinct Eulerian path otherwise, G does have an Eulerian path that is not a circuit.

part c)

- A Hamilton circuit in G is a closed loop that visits every vertex of G exactly once.
- After trying to find such a circuit in G, we cant
- Therefore, G does not have a Hamilton circuit.

part d)

- A Hamilton path in G is a path that visits every vertex of G exactly once and does not return to the starting vertex.
- After trying to find such a path in G, we find that m-j-f-e-a-c-b-g-k-l-i-d-h
- Therefore, G does have a Hamilton path that is not a circuit.

part e)

- The chromatic number of G, $\chi(G)$, is the minimum number of colors needed to color the vertices of G so that no two adjacent vertices share the same color.
- After trying to color the vertices of G with the minimum number of colors, we find that 3 is the chromatic number. (for example: a-b-k-h-m-f blue, g-d-l-j green, c-e-i yellow)
- Therefore, $\chi(G) = 3$.

part f)

- A graph G is bipartite if its vertices can be divided into two disjoint sets such that every edge connects a vertex in one set to a vertex in the other set.
- After trying to divide the vertices of G into two disjoint sets, we find that There exist an odd cycle within the graph. Meaning it can not be bipartite.

- Chromatic number of graph was 3, bipartite graphs can have chromatic number = 2.
- Therefore, G is (not) bipartite.
- If we delete the following 3 edges we will have a bipartite graph (j, f), (h, i), (b, g).

part g)

- A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.
- After trying to find a complete subgraph with at least four nodes in G, we can not.
- Therefore, G does not have a complete subgraph with at least four nodes.
- If we add edge between d and l then we have a complete subgraph with at least 4 nodes (d-h-l-i).

Answer 2

Yes, these two graphs are isomorphic.

In graph theory, two graphs are said to be isomorphic if there is a one-to-one correspondence between their vertices and edges such that the result preserves both the adjacency and non-adjacency of the vertices.

Here's why these two graphs are isomorphic:

- They have the same number of vertices (8 vertices each).
- They have the same number of edges (14 edges each).
- The vertices can be paired such that the connections between vertices in one graph correspond to the connections between vertices in the other graph. (f(a) = a', f(c) = e',f(d) = g',f(h) = f', f(g) = d', f(e) = b' ,f(b) = c', f(f) =h')

Therefore, we can conclude that these two graphs are isomorphic.

Answer 3

part a) For a cycle graph

$$(C_n)$$

with

$$(n \geq 3)$$

, the chromatic number depends on whether (n) is odd or even.
If (n) is odd, the chromatic number is 3; if even, it's 2. This is because an odd cycle cannot be colored with only 2 colors without adjacent vertices sharing the same color, which violates the rules of graph coloring.
A cycle graph is bipartite if and only if it has an even number of vertices.
This is because you can divide the vertices into two sets where edges only go between the sets and not within them.

part b) For a cube graph

$$(Q_n)$$

with

$$(n \geq 1)$$

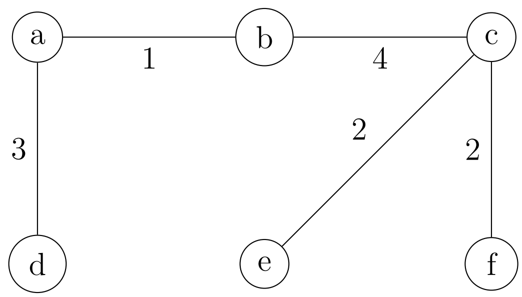
, the chromatic number is 2 because it's always bipartite.
This is because you can divide the vertices into two sets where edges only go between the sets and not within them.
Therefore, you can color the graph with just two colors without any adjacent vertices sharing the same color.

Answer 4

part a) The order in which the edges are added to the tree: (Kruksal)

Start at any vertex. Let's start with 'a'.
The smallest edge is (a, b) with weight 1. Add it to the tree.
The Next smallest are (c, e) and (c, f) with weight 2. Add them to the tree.
The Next smallest is (a, d) with weight 3. Add it to the tree.
The Next smallest is (b, c) with weight 4. Add it to the tree.

part b) The minimum spanning tree is as follows:



part c) The minimum spanning tree is not unique.
For example, instead of adding the edge (c, f) with weight 2, we could add the edge (e, f) with weight 2.
The total weight of the minimum spanning tree would still be the same.
Therefore, there are multiple minimum spanning trees for this graph.

Answer 5

part a) A full binary tree is a binary tree where each branch/internal vertex has exactly two children.
If a binary tree is full, it means every non-leaf node contributes exactly two leaf nodes.
Let us denote the number of Leaf vertices as (L)
So, we denote the number of internal vertices as $(n - L)$, and the number of leaf vertices is $(n - L + 1)$.
Then the number of vertices is sum of internal vertices and leaf verices which is:
$n = n - L + n - L + 1 = 2n - 2L + 1$
Solving for L we can see that we get $L = \frac{n+1}{2}$

part b) The chromatic number of a tree is always 2, except for a single-vertex tree, which has

a chromatic number of 1.
This is because any tree can be colored with at most two colors so that no two adjacent vertices have the same color.
This property is due to the fact that trees do not contain any cycles, and therefore, it is always possible to color a tree with no more than two colors.
Only vertices with different colors are parents and childs (Chromatic 2)

part c) For an $m - ary$ tree (a tree in which each parent node has at most m children), considering a tree with only max of 1 non-leaf node at each level (full tree not complete) the height can be bounded by $\lfloor \frac{n}{m} \rfloor$.
The reason is for each level we have m-nodes so we divide n by m also since we have 1 node only at root level then we need to take the floor of $\frac{n}{m}$.