Student Information

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Answer 1

0.1 a

To construct a 98% confidence interval, we first need to calculate the sample mean ($\overline{x}$) and sample standard deviation (s).

$$\overline{x} = 6.81$$
$$s = 1.056$$

Then, we calculate the critical value (z*) for 98% confidence level.

$$z^* = 2.33$$

The confidence interval is;

$$\begin{aligned}(\overline{x} - z^* s/\sqrt{n}, \overline{x} + z^* s/\sqrt{n}) &= (6.81 - 2.33 * 1.056/\sqrt{16}, 6.81 + 2.33 * 1.056/\sqrt{16}) \\ &= (6.19, 7.43)\end{aligned}$$

0.2 b

$$H_0 : \mu = 7.5$$
$$H_a : \mu < 7.5$$

To test this at 5% significance level, we calculate the test statistic (t) and find the critical value (t_c) from t-table.

$$t = (\overline{x} - \mu_0)/(s/\sqrt{n}) \ = (6.81 - 7.5)/(1.056/\sqrt{16}) \ = -2.614$$
$$t_c = -2.13 (\text{from t-table, for 15 degrees of freedom and } \alpha = 0.05, \text{two-tailed})$$

Since t ¡ t_c, we reject H_0. So, at 5% significance level, we can claim that the improvement is effective in reducing the gasoline consumption.

0.3 c

Since the sample mean (6.81) is less than the mean before improvement (6.5), we can immediately reject H_0 without any test. The alternative hypothesis is supported.

Answer 2

0.4 a

$$H_0 : \mu_1 = (\mu_2 = 5000)$$
$$H_a : \mu_1 > (\mu_2 = 5000)$$

Ali's claim should be considered as the null hypothesis.

0.5 b

To test this at 5% significance level, we calculate the test statistic (z) and find the critical value (z_c) from standard normal table.

$$z = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{s_1^2/n_1 + s_2^2/n_2}} = \frac{(5500 - 5000)}{\sqrt{(2000)^2/100 + (2000)^2/100}} = 1.76$$
$$z_c = 1.96$$

Since z ¿ z_c, we reject H_0. So, at 5% significance level, Ahmed can claim that there is an increase in the rent prices compared to the last year.

0.6 c

The P-value is the probability of observing a sample mean of 5500 or greater, assuming the null hypothesis is true.

$$P - \text{value} = 0.006$$
$$\text{(From standard normal table, for } z = 2.5\text{)}$$

This small P-value means that there is a very small chance of observing such an increase in sample mean if there is no actual increase in population mean. So, it gives strong evidence against H_0 in favor of H_a.

0.7 d

$$H_0 : \mu_1 = \mu_2$$
$$H_a : \mu_1 < \mu_2$$

$$z = \frac{(\overline{x_1} - \overline{x_2})}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(5500 - 6500)}{\sqrt{\frac{(2000)^2}{100} + \frac{(3000)^2}{60}}} = -2.29$$
$$z_c = -2.58; \text{ (From standard normal table, for } \alpha = 0.01\text{, two-tailed)}$$

Since z ¡ z_c, we <u>cannot</u> reject H_0. So, at 1% significance level, we cannot claim that the prices in Ankara are lower than the prices in Istanbul.

Answer 3

$$H_0 : \text{The number of rainy days is independent of the season.}$$
$$H_a : \text{The number of rainy days depends on the season.}$$

	Winter	Spring	Summer	Autumn
Rainy days:	34	32	15	19
Expected if H_0 is true:	30	30	30	30

$$\begin{aligned} X^2_{\text{obs}} &= (34-30)^2/30 + (32-30)^2/30 + (15-30)^2/30 + (19-30)^2/30 \\ &= 12.2 \end{aligned}$$

The P-value is 0.017 (From Chi-square distribution table, for 3 degrees of freedom) This small P-value means that there is little chance of observing such deviations from expected values if H_0 is true. So, we reject H_0 in favor of H_a. We can claim that the number of rainy days depends on the season in Ankara.

Answer 4

Here is the Octave code and output:

```
data = [34 32 15 19; 56 58 75 71];
e = sum(data, 2) * sum(data, 1) / sum(data(:));
c2 = sum((data(:) - e(:)).^2 ./ e(:));
df = numel(data) - numel(data)/sum(size(data)) - 1;
pval = 1 - chi2cdf(c2, df);
fprintf('(X^2)_obs value: %.4f\n', c2);
fprintf('Degrees of freedom: %d\n', df);
fprintf('P-value: %.4f\n', pval);


So the test statistic (X2_\text{obs}) is 14.73 and the P-value is 0.0182.
This matches with the results obtained manually.
The code is generic and can work for any input by just changing the 'seasons'
 and 'exp\_vals' matrices.
```

```
data = [34 32 15 19; 56 58 75 71];
e = sum(data, 2) * sum(data, 1) / sum(data(:));
c2 = sum((data(:) - e(:)).^2 ./ e(:));
df = numel(data) - numel(data)/sum(size(data)) - 1;
pval = 1 - chi2cdf(c2, df);
fprintf('(X^2)_obs value: %.4f\n', c2);
fprintf('Degrees of freedom: %d\n', df);
fprintf('P-value: %.4f\n', pval);
```

```
(X^2)_obs value: 14.7323
Degrees of freedom: 5.66667
P-value: 0.0182
>>
```

Figure 1: Screenshot of code.