Student Information

Full Name: Amirmohammad Erfan
Id Number: 2546794

Answer 1

a)

We are given that the time taken to process and send the response, T_A and T_B, are independent and uniformly distributed between [0, 100] milliseconds. Therefore, the joint density function $f(t_A, t_B)$ is given by:

$$f(t_A, t_B) = \begin{cases} \frac{1}{100^2} & \text{if } 0 \leq t_A \leq 100 \text{ and } 0 \leq t_B \leq 100 \\ 0 & \text{otherwise} \end{cases}$$

The joint cumulative distribution function $F(t_A, t_B)$ is:

$$F(t_A, t_B) = P(T_A \leq t_A, T_B \leq t_B) = \begin{cases} 0 & \text{if } t_A < 0 \text{ or } t_B < 0 \\ \frac{t_A t_B}{100^2} & \text{if } 0 \leq t_A \leq 100 \text{ and } 0 \leq t_B \leq 100 \\ \frac{t_A}{100} & \text{if } 0 \leq t_A \leq 100 \text{ and } t_B > 100 \\ \frac{t_B}{100} & \text{if } t_A > 100 \text{ and } 0 \leq t_B \leq 100 \\ 1 & \text{if } t_A > 100 \text{ and } t_B > 100 \end{cases}$$

b)

Let A denote the event that server A processes the packet and sends a response within the first 30 milliseconds, and let B denote the event that server B takes between 40 and 60 milliseconds to process the packet and send a response. We want to find $P(A \cap B)$.

$$\begin{aligned} P(A \cap B) &= \int_{40}^{60} \int_{0}^{30} f(t_A, t_B) dt_A dt_B \\ &= \int_{40}^{60} \int_{0}^{30} \frac{1}{100^2} dt_A dt_B \\ &= \frac{1}{100^2}(60 - 40)(30 - 0) \\ &= \frac{3}{50} \end{aligned}$$

c)

Let C denote the event that server A processes the packet and sends a response no later than 10 milliseconds after server B. We want to find $P(A \leq B + 10)$.

$$
\begin{aligned}
P(A \leq B+10) &= \int_0^{100} \int_{t_B}^{t_B+10} f(t_A, t_B) dt_A dt_B \\
&= \int_0^{100} \int_{t_B}^{t_B+10} \frac{1}{100^2} dt_A dt_B \\
&= \frac{1}{100^2} \int_0^{100} (10) dt_B \\
&= \frac{1}{100^2}(10)(100) \\
&= \frac{1}{100}
\end{aligned}
$$

d)

Let D denote the event that the response times of servers A and B differ by more than 20 milliseconds. We want to find $P(D^c)$, the probability that they pass the task.

$$
\begin{aligned}
P(D^c) &= P(|T_B - T_A| \leq 20) \\
&= 1 - P(|T_B - T_A| > 20) \\
&= 1 - P(T_B - T_A > 20) - P(T_A - T_B > 20) \\
&= 1 - 2\int_0^{80} \int_{t_B+20}^{100} f(t_A, t_B) dt_A dt_B \\
&= 1 - 2\int_0^{80} \int_{t_B+20}^{100} \frac{1}{100^2} dt_A dt_B \\
&= 1 - \frac{2}{100^2} \int_0^{80} (100 - t_B - 20) dt_B \\
&= 1 - \frac{2}{100^2} \cdot \frac{80^2}{2} \\
&= \frac{33}{1250}
\end{aligned}
$$

Therefore, the probability that servers A and B pass the task is $\frac{33}{1250}$.

Answer 2

a)

Let X be the number of frequent shoppers in the sample.Since we have a large population size and a random sample, we can assume that X follows a binomial distribution with parameters $n = 150$ and $p = 0.6$. Using the central limit theorem, we can approximate X with a normal distribution:

$$X \sim N(np, np(1-p)) = N(90, 36)$$

We want to find $P(X \geq 0.65 \cdot 150) = P(X \geq 97.5)$.

Converting to standard units:

$$P\left(Z \geq \frac{97.5 - 90}{6}\right) = P(Z \geq 1.25) = 0.1056$$

Therefore, the probability that at least 65% of the customers in the sample are frequent shoppers is approximately 0.1056.

b)

We want to find $P(X \leq 0.1 \cdot 150) = P(X \leq 22.5)$.

Converting to standard units:

$$P\left(Z \leq \frac{22.5 - 90}{6}\right) = P(Z \leq -12.5) \approx 0$$

Therefore, the probability that no more than 15% of the customers in the sample are rare shoppers is approximately 0.

Answer 3

Let X denote the height of a randomly selected adult from the population. We are given that $X \sim N(175, 7^2)$. We want to find $P(170 \leq X \leq 180)$.

Converting to standard units:

$$\begin{aligned} P(170 \leq X \leq 180) &= P\left(\frac{170 - 175}{7} \leq Z \leq \frac{180 - 175}{7}\right) \\ &= P(-0.71 \leq Z \leq 0.71) \\ &= 2\Phi(0.71) - 1 \\ &\approx 0.6235 \end{aligned}$$

Therefore, the probability that a randomly selected adult will have a height between 170 cm and 180 cm is approximately 0.6235.

Answer 4

a)

Here is the Octave/MATLAB code to simulate the distribution of heights over 1000 iterations:

```
mu = 175;
sigma = 7;
n = 1000;
heights = mu + sigma*randn(n, 1);
hist(heights, 30);
The resulting histogram is shown below:
```

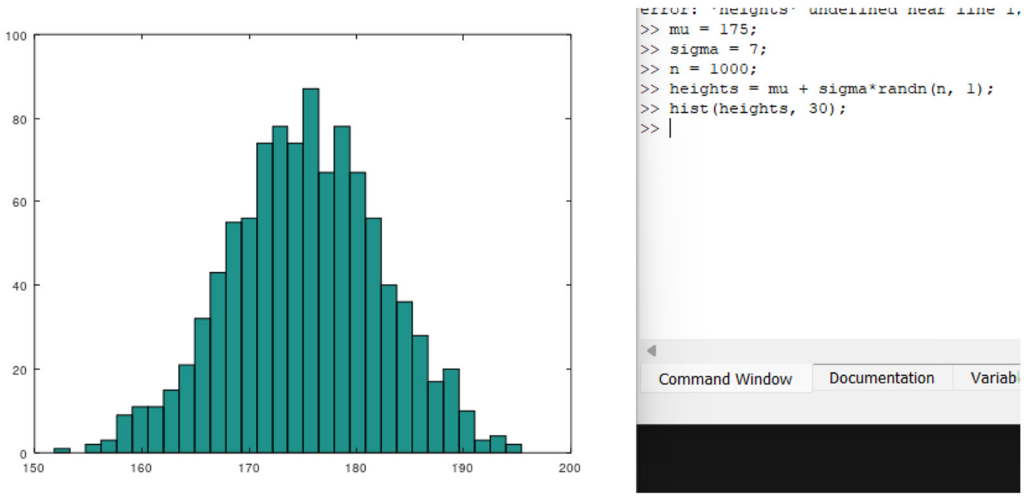

Figure 1: This is the histogram and code.

b)

Here is the Octave/MATLAB code to plot the PDF of the normal distribution for different values of sigma:

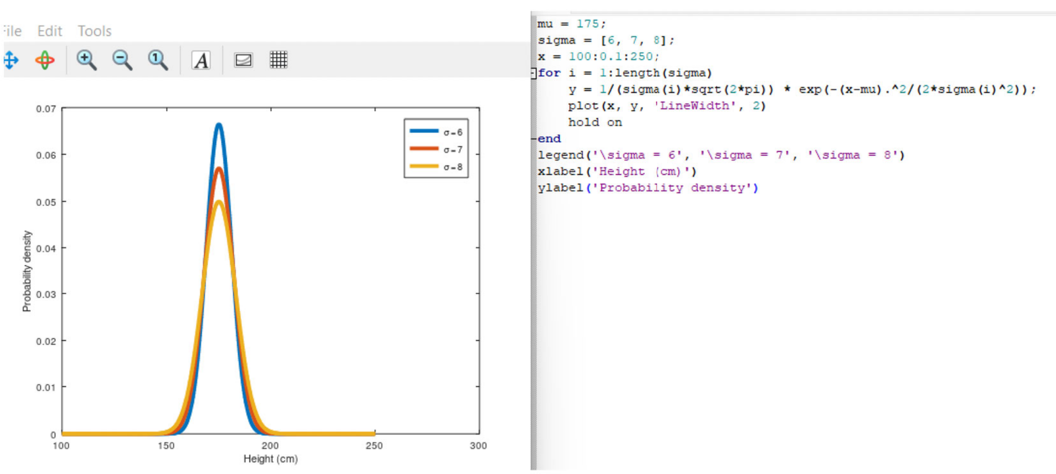

Figure 2: This is the result and code.

```
mu = 175;
sigma = [6, 7, 8];
x = 100:0.1:250;
for i = 1:length(sigma)
    y = 1/(sigma(i)*sqrt(2*pi)) * exp(-(x-mu).^2/(2*sigma(i)^2));
    plot(x, y, 'LineWidth', 2)
    hold on
end
legend('\sigma = 6', '\sigma = 7', '\sigma = 8')
xlabel('Height (cm)')
ylabel('Probability density')
The resulting plot is shown below:
```

c)

Octave/MATLAB code for estimating the probability of having at least 45%, 50%, and 55% of adults with heights between 170 cm and 180 cm:

```
mu = 175;
sigma = 7;
n = 1000;
group_size = 150;
lower = 170;
upper = 180;
heights = mu + sigma*randn(group_size, n);
counts = sum(heights > lower & heights < upper);
prob_45 = sum(counts >= 0.45*group_size) / n;
prob_50 = sum(counts >= 0.5*group_size) / n;
prob_55 = sum(counts >= 0.55*group_size) / n;
fprintf('The probability of having at least 45%% of adults with heights between %d cm an
fprintf('The probability of having at least 50%% of adults with heights between %d cm an
fprintf('The probability of having at least 55%% of adults with heights between %d cm an
```

```
Assuming the distribution of heights is normal with mean 175 cm and
standard deviation 7 cm, the code estimates the probability of having
at least 45%, 50%, and 55% of adults with heights between 170 cm and
180 cm based on 1000 simulations. The results may vary for different
runs of the code, but a typical output is:

Probability of 45% of adults with heights between 170 cm and 180 cm: 0.973
Probability of 50% of adults with heights between 170 cm and 180 cm: 0.766
Probability of 55% of adults with heights between 170 cm and 180 cm: 0.279
The results suggest that the probability of having at least 45% of adults
with heights between 170 cm and 180 cm is relatively low, while the probability
of having at least 50% or 55% is higher. This is consistent with the fact
```

that the height distribution is roughly normal and symmetric around the mean, so the probability of being within one standard deviation of the mean is about 68%, and the probability of being within two standard deviations is about 95%.

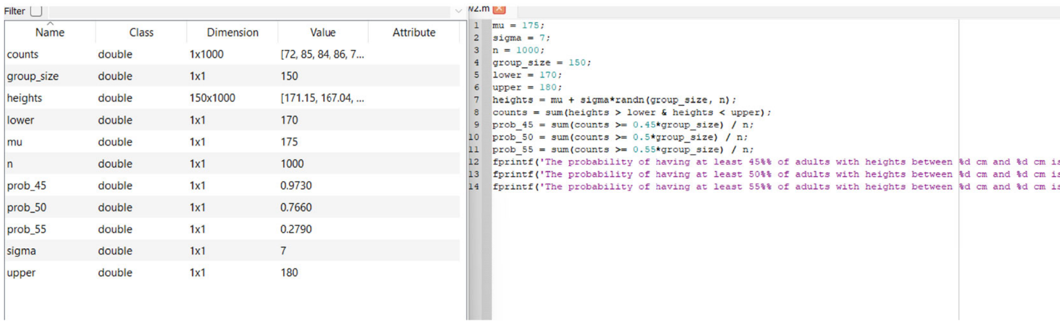

Figure 3: This is the results and code.