CS 61A
Fall 2022

Midterm Review

Discussion 9: October 24, 2022

All Questions Are Optional

The questions in this assignment are not graded, but they are highly recommended to help you prepare for the upcoming exam. You will receive credit for this lab even if you do not complete these questions.

Recursion and Tree Recursion

Q1: Subsequences

A subsequence of a sequence `S` is a subset of elements from `S`, in the same order they appear in `S`. Consider the list `[1, 2, 3]`. Here are a few of it's subsequences `[]`, `[1, 3]`, `[2]`, and `[1, 2, 3]`.

Write a function that takes in a list and returns all possible subsequences of that list. The subsequences should be returned as a list of lists, where each nested list is a subsequence of the original input.

In order to accomplish this, you might first want to write a function `insert_into_all` that takes an item and a list of lists, adds the item to the beginning of each nested list, and returns the resulting list.

def insert_into_all(item, nested_list): """Return a new list consisting of all the lists in nested_list, but with item added to the front of each. You can assume that nested_list is a list of lists.

```
>>> nl = [[], [1, 2], [3]]
>>> insert_into_all(0, nl)
[[0], [0, 1, 2], [0, 3]]
"""
"*** YOUR CODE HERE ***"
```

def subseqs(s): """Return a nested list (a list of lists) of all subsequences of S. The subsequences can appear in any order. You can assume S is a list.

```
>>> seqs = subseqs([1, 2, 3])
>>> sorted(seqs)
[[], [1], [1, 2], [1, 2, 3], [1, 3], [2], [2, 3], [3]]
>>> subseqs([])
[[]]
"""
if ________________:
    ________________
else:
    ________________
    ________________
```

Q2: Non-Decreasing Subsequences

Just like the last question, we want to write a function that takes a list and returns a list of lists, where each individual list is a subsequence of the original input.

This time we have another condition: we only want the subsequences for which consecutive elements are *nondecreasing*. For example, `[1, 3, 2]` is a subsequence of `[1, 3, 2, 4]`, but since $2 < 3$, this subsequence would *not* be included in our result.

You may assume that the list passed in as `s` contains only nonnegative elements.

Fill in the blanks to complete the implementation of the `non_decrease_subseqs` function. You may assume that the input list contains no negative elements.

You may use the provided helper function `insert_into_all`, which takes in an `item` and a list of lists and inserts the `item` to the front of each list.

def non_decrease_subseqs(s): """Assuming that S is a list, return a nested list of all subsequences of S (a list of lists) for which the elements of the subsequence are strictly nondecreasing. The subsequences can appear in any order.

```
>>> seqs = non_decrease_subseqs([1, 3, 2])
>>> sorted(seqs)
[[], [1], [1, 2], [1, 3], [2], [3]]
>>> non_decrease_subseqs([])
[[]]
>>> seqs2 = non_decrease_subseqs([1, 1, 2])
>>> sorted(seqs2)
[[], [1], [1], [1, 1], [1, 1, 2], [1, 2], [1, 2], [2]]
"""
def subseq_helper(s, prev):
    if not s:
        return ____________________
    elif s[0] < prev:
        return ____________________
    else:
        a = ______________________
        b = ______________________
        return insert_into_all(________, ______________) + ________________
return subseq_helper(____, ____)
```

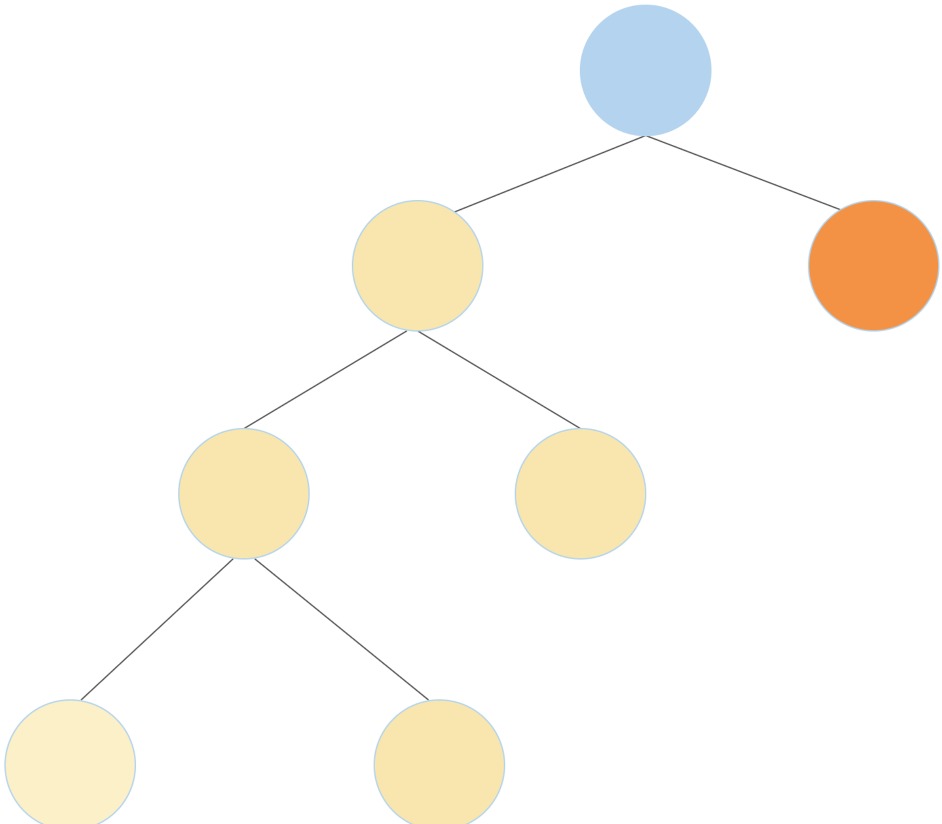
Q3: Number of Trees

A **full binary tree** is a tree where each node has either 2 branches or 0 branches, but never 1 branch.

Write a function which returns the number of unique full binary tree structures that have exactly n leaves. See the doctests for visualizations of the possible full binary tree sturctures that have 1, 2, and 3 leaves.

> **Hint**: A full binary tree can be constructed by connecting two smaller full binary trees to a root node. If the two smaller full binary trees have `a` and `b` leaves, the new full binary tree will have `a + b` leaves. For example, as shown in the first diagram below, a full binary tree with 4 leaves can be constructed by connecting a full binary tree that has three leaves (yel-

low) with a full binary tree that has one leaf (orange). A full binary tree with 4 leaves can also be constructed by connecting two full binary trees with 2 leaves each (second diagram)



For those interested in combinatorics, this problem does have a closed form solution):

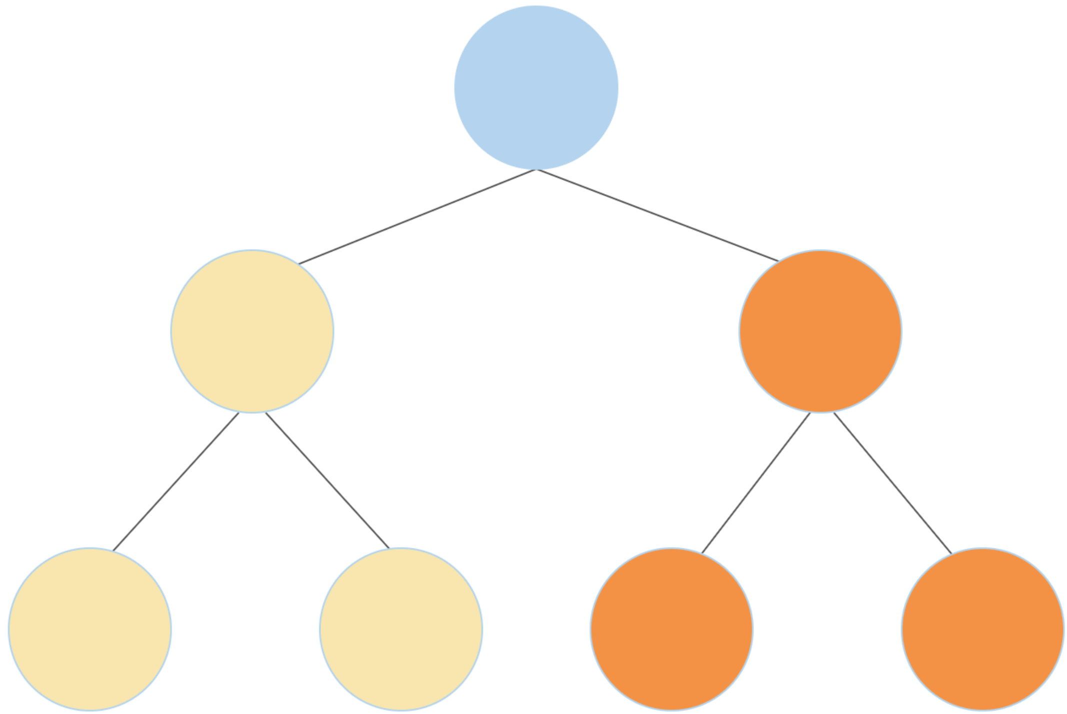
def num_trees(n): """Returns the number of unique full binary trees with exactly n leaves. E.g.,

```
1   2        3       3    ...
*   *        *       *
   / \      / \     / \
  *   *    *   *   *   *
          / \         / \
         *   *       *   *

>>> num_trees(1)
1
>>> num_trees(2)
1
>>> num_trees(3)
2
>>> num_trees(8)
429

"""
"*** YOUR CODE HERE ***"
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.



4-leaf Full Binary Tree 2

Generators

Q4: Partition Generator

Construct the generator function `partition_gen`, which takes in a number `n` and returns an *n-partition iterator*. An *n-partition iterator* yields partitions of `n`, where a partition of `n` is a list of integers whose sum is `n`. The iterator should only return unique partitions; the order of numbers within a partition and the order in which partitions are returned does not matter.

Important: The skeleton code is only a suggestion; feel free to add or remove lines as you see fit.

def partition_gen(n): """ >>> for partition in partition_gen(4): # note: order doesn't matter ... print(partition) [4] [3, 1] [2, 2] [2, 1, 1] [1, 1, 1, 1] """ def yield_helper(j, k): if j == 0: ________________________
elif ________________________: for small_part in ________________________
yield ________________________ yield ________________________
yield from yield_helper(n, n)

Objects

Q5: Vending Machine

In this question you'll create a vending machine that only outputs a single product and provides change when needed.

Create a class called `VendingMachine` that represents a vending machine for some product. A `VendingMachine` object returns strings describing its interactions. Remember to match *exactly* the strings in the doctests – including punctuation and spacing!

Fill in the `VendingMachine` class, *adding attributes and methods as appropriate*, such that its behavior matches the following doctests:

> You may find Python's formatted string literals, or f-strings useful. A quick example: >>> feeling = 'love' >>> course = '61A!' >>> f'I {feeling} {course}' 'I love 61A!'
>
> If you're curious about alternate methods of string formatting, you can also check out an older method of Python string formatting. A quick example:

```
>>> ten, twenty, thirty = 10, 'twenty', [30]
>>> '{0} plus {1} is {2}'.format(ten, twenty, thirty)
'10 plus twenty is [30]'
```

class VendingMachine: """A vending machine that vends some product for some price.

```
>>> v = VendingMachine('candy', 10)
>>> v.vend()
'Nothing left to vend. Please restock.'
>>> v.add_funds(15)
'Nothing left to vend. Please restock. Here is your $15.'
>>> v.restock(2)
'Current candy stock: 2'
>>> v.vend()
'Please update your balance with $10 more funds.'
>>> v.add_funds(7)
'Current balance: $7'
>>> v.vend()
'Please update your balance with $3 more funds.'
>>> v.add_funds(5)
'Current balance: $12'
>>> v.vend()
'Here is your candy and $2 change.'
>>> v.add_funds(10)
'Current balance: $10'
>>> v.vend()
'Here is your candy.'
>>> v.add_funds(15)
'Nothing left to vend. Please restock. Here is your $15.'

>>> w = VendingMachine('soda', 2)
>>> w.restock(3)
'Current soda stock: 3'
>>> w.restock(3)
'Current soda stock: 6'
>>> w.add_funds(2)
'Current balance: $2'
>>> w.vend()
'Here is your soda.'
"""
"*** YOUR CODE HERE ***"
```

Mutable Lists

Q6: Trade

In the integer market, each participant has a list of positive integers to trade. When two participants meet, they trade the smallest non-empty prefix of their list of integers. A prefix is a slice that starts at index 0.

Write a function `trade` that exchanges the first `m` elements of list `first` with the first `n` elements of list `second`, such that the sums of those elements are equal, and the sum is as small as possible. If no such prefix exists, return the string `'No deal!'` and do not change either list. Otherwise change both lists and return `'Deal!'`. A partial implementation is provided.

Hint: You can mutate a slice of a list using *slice assignment*. To do so, specify a slice of the list `[i:j]` on the left-hand side of an assignment statement and another list on the right-hand side of the assignment statement. The operation will replace the entire given slice of the list from `i` inclusive to `j` exclusive with the elements from the given list. The slice and the given list need not be the same length.

```
>>> a = [1, 2, 3, 4, 5, 6]
>>> b = a
>>> a[2:5] = [10, 11, 12, 13]
>>> a
[1, 2, 10, 11, 12, 13, 6]
>>> b
[1, 2, 10, 11, 12, 13, 6]
```

Additionally, recall that the starting and ending indices for a slice can be left out and Python will use a default value. `lst[i:]` is the same as `lst[i:len(lst)]`, and `lst[:j]` is the same as `lst[0:j]`.

def trade(first, second): """Exchange the smallest prefixes of first and second that have equal sum.

```
>>> a = [1, 1, 3, 2, 1, 1, 4]
>>> b = [4, 3, 2, 7]
>>> trade(a, b) # Trades 1+1+3+2=7 for 4+3=7
'Deal!'
>>> a
[4, 3, 1, 1, 4]
>>> b
[1, 1, 3, 2, 2, 7]
>>> c = [3, 3, 2, 4, 1]
>>> trade(b, c)
'No deal!'
>>> b
[1, 1, 3, 2, 2, 7]
>>> c
[3, 3, 2, 4, 1]
>>> trade(a, c)
'Deal!'
>>> a
[3, 3, 2, 1, 4]
>>> b
[1, 1, 3, 2, 2, 7]
>>> c
[4, 3, 1, 4, 1]
>>> d = [1, 1]
>>> e = [2]
>>> trade(d, e)
'Deal!'
>>> d
[2]
>>> e
[1, 1]
"""
m, n = 1, 1

equal_prefix = lambda: ______________________
while ________________________________:
    if __________________:
        m += 1
    else:
        n += 1

if equal_prefix():
    first[:m], second[:n] = second[:n], first[:m]
    return 'Deal!'
else:
    return 'No deal!'
```

Note: This worksheet is a problem bank—most TAs will not cover all the problems in discussion section.

Q7: Shuffle

Define a function `shuffle` that takes a sequence with an even number of elements (cards) and creates a new list that interleaves the elements of the first half with the elements of the second half.

To interleave two sequences `s0` and `s1` is to create a new sequence such that the new sequence contains (in this order) the first element of `s0`, the first element of `s1`, the second element of `s0`, the second element of `s1`, and so on.

> **Note:** If you're running into an issue where the special heart / diamond / spades / clubs symbols are erroring in the doctests, feel free to copy paste the below doctests into your file as these don't use the special characters and should not give an "illegal multibyte sequence" error.

def card(n): ""“Return the playing card numeral as a string for a positive n <= 13.""" assert type(n) == int and n > 0 and n <= 13, “Bad card n” specials = {1: ‘A’, 11: ‘J’, 12: ‘Q’, 13: ‘K’} return specials.get(n, str(n))

def shuffle(cards): """Return a shuffled list that interleaves the two halves of cards.

```
>>> shuffle(range(6))
[0, 3, 1, 4, 2, 5]
>>> suits = ['H', 'D', 'S', 'C']
>>> cards = [card(n) + suit for n in range(1,14) for suit in suits]
>>> cards[:12]
['AH', 'AD', 'AS', 'AC', '2H', '2D', '2S', '2C', '3H', '3D', '3S', '3C']
>>> cards[26:30]
['7S', '7C', '8H', '8D']
>>> shuffle(cards)[:12]
['AH', '7S', 'AD', '7C', 'AS', '8H', 'AC', '8D', '2H', '8S', '2D', '8C']
>>> shuffle(shuffle(cards))[:12]
['AH', '4D', '7S', '10C', 'AD', '4S', '7C', 'JH', 'AS', '4C', '8H', 'JD']
>>> cards[:12]  # Should not be changed
['AH', 'AD', 'AS', 'AC', '2H', '2D', '2S', '2C', '3H', '3D', '3S', '3C']
"""
assert len(cards) % 2 == 0, 'len(cards) must be even'
half = _______________
shuffled = []
for i in _____________:
    _________________
    _________________
return shuffled
```

Linked Lists

Q8: Insert

Implement a function `insert` that takes a `Link`, a `value`, and an `index`, and inserts the `value` into the `Link` at the given `index`. You can assume the linked list already has at least one element. Do not return anything – `insert` should mutate the linked list.

> **Note**: If the index is out of bounds, you should raise an `IndexError` with: raise IndexError(‘Out of bounds!’)

def insert(link, value, index): """Insert a value into a Link at the given index.

```
>>> link = Link(1, Link(2, Link(3)))
>>> print(link)
<1 2 3>
>>> other_link = link
>>> insert(link, 9001, 0)
>>> print(link)
<9001 1 2 3>
>>> link is other_link # Make sure you are using mutation! Don't create a new linked list
    .
True
>>> insert(link, 100, 2)
>>> print(link)
<9001 1 100 2 3>
>>> insert(link, 4, 5)
Traceback (most recent call last):
    ...
IndexError: Out of bounds!
"""
"*** YOUR CODE HERE ***"
```

class Link: """A linked list.

```
>>> s = Link(1)
>>> s.first
1
>>> s.rest is Link.empty
True
>>> s = Link(2, Link(3, Link(4)))
>>> s.first = 5
>>> s.rest.first = 6
>>> s.rest.rest = Link.empty
>>> s                                    # Displays the contents of repr(s)
Link(5, Link(6))
>>> s.rest = Link(7, Link(Link(8, Link(9))))
>>> s
Link(5, Link(7, Link(Link(8, Link(9)))))
>>> print(s)                             # Prints str(s)
<5 7 <8 9>>
"""
empty = ()

def __init__(self, first, rest=empty):
    assert rest is Link.empty or isinstance(rest, Link)
    self.first = first
    self.rest = rest

def __repr__(self):
    if self.rest is not Link.empty:
        rest_repr = ', ' + repr(self.rest)
    else:
        rest_repr = ''
    return 'Link(' + repr(self.first) + rest_repr + ')'

def __str__(self):
    string = '<'
    while self.rest is not Link.empty:
        string += str(self.first) + ' '
        self = self.rest
    return string + str(self.first) + '>'
```

Q9: Deep Linked List Length

A linked list that contains one or more linked lists as elements is called a *deep* linked list. Write a function `deep_len` that takes in a (possibly deep) linked list and returns the *deep length* of that linked list. The deep length of a linked list is the total number of non-link elements in the list, as well as the total number of elements contained in all contained lists. See the function's doctests for examples of the deep length of linked lists.

Hint: Use `isinstance` to check if something is an instance of an object.

def deep_len(lnk): """ Returns the deep length of a possibly deep linked list.

```
>>> deep_len(Link(1, Link(2, Link(3))))
3
>>> deep_len(Link(Link(1, Link(2)), Link(3, Link(4))))
4
>>> levels = Link(Link(Link(1, Link(2)), \
        Link(3)), Link(Link(4), Link(5)))
>>> print(levels)
<<<1 2> 3> <4> 5>
>>> deep_len(levels)
5
"""
if _____________:
    return 0
elif _____________:
    return 1
else:
    return _________________________
```

class Link: """A linked list.

```
>>> s = Link(1)
>>> s.first
1
>>> s.rest is Link.empty
True
>>> s = Link(2, Link(3, Link(4)))
>>> s.first = 5
>>> s.rest.first = 6
>>> s.rest.rest = Link.empty
>>> s                                 # Displays the contents of repr(s)
Link(5, Link(6))
>>> s.rest = Link(7, Link(Link(8, Link(9))))
>>> s
Link(5, Link(7, Link(Link(8, Link(9)))))
>>> print(s)                          # Prints str(s)
<5 7 <8 9>>
"""
empty = ()

def __init__(self, first, rest=empty):
    assert rest is Link.empty or isinstance(rest, Link)
    self.first = first
    self.rest = rest

def __repr__(self):
    if self.rest is not Link.empty:
        rest_repr = ', ' + repr(self.rest)
    else:
        rest_repr = ''
    return 'Link(' + repr(self.first) + rest_repr + ')'

def __str__(self):
    string = '<'
    while self.rest is not Link.empty:
        string += str(self.first) + ' '
        self = self.rest
    return string + str(self.first) + '>'
```

Q10: Linked Lists as Strings

Kevin and Jerry like different ways of displaying the linked list structure in Python. While Kevin likes box and pointer diagrams, Jerry prefers a more futuristic way. Write a function `make_to_string` that returns a function that converts the linked list to a string in their preferred style.

Hint: You can convert numbers to strings using the `str` function, and you can combine strings together using `+`.

```
>>> str(4)
'4'
>>> 'cs ' + str(61) + 'a'
'cs 61a'
```

def make_to_string(front, mid, back, empty_repr): """ Returns a function that turns linked lists to strings.

```
>>> kevins_to_string = make_to_string("[", "|-]-->", "", "[]")
>>> jerrys_to_string = make_to_string("(", " . ", ")", "()")
>>> lst = Link(1, Link(2, Link(3, Link(4))))
>>> kevins_to_string(lst)
'[1|-]-->[2|-]-->[3|-]-->[4|-]-->[]'
>>> kevins_to_string(Link.empty)
'[]'
>>> jerrys_to_string(lst)
'(1 . (2 . (3 . (4 . ()))))'
>>> jerrys_to_string(Link.empty)
'()'
"""
def printer(lnk):
    if ______________:
        return _________________________
    else:
        return _________________________
return printer
```

class Link: """A linked list.

```
>>> s = Link(1)
>>> s.first
1
>>> s.rest is Link.empty
True
>>> s = Link(2, Link(3, Link(4)))
>>> s.first = 5
>>> s.rest.first = 6
>>> s.rest.rest = Link.empty
>>> s                                   # Displays the contents of repr(s)
Link(5, Link(6))
>>> s.rest = Link(7, Link(Link(8, Link(9))))
>>> s
Link(5, Link(7, Link(Link(8, Link(9)))))
>>> print(s)                            # Prints str(s)
<5 7 <8 9>>
"""
empty = ()

def __init__(self, first, rest=empty):
    assert rest is Link.empty or isinstance(rest, Link)
    self.first = first
    self.rest = rest

def __repr__(self):
    if self.rest is not Link.empty:
        rest_repr = ', ' + repr(self.rest)
    else:
        rest_repr = ''
    return 'Link(' + repr(self.first) + rest_repr + ')'

def __str__(self):
    string = '<'
    while self.rest is not Link.empty:
        string += str(self.first) + ' '
        self = self.rest
    return string + str(self.first) + '>'
```

Trees

Q11: Long Paths

Implement `long_paths`, which returns a list of all *paths* in a tree with length at least `n`. A path in a tree is a list of node labels that starts with the root and ends at a leaf. Each subsequent element must be from a label of a branch of the previous value's node. The *length* of a path is the number of edges in the path (i.e. one less than the number of nodes in the path). Paths are ordered in the output list from left to right in the tree. See the doctests for some examples.

def long_paths(t, n): """Return a list of all paths in t with length at least n.

```
>>> long_paths(Tree(1), 0)
[[1]]
>>> long_paths(Tree(1), 1)
[]
>>> t = Tree(3, [Tree(4), Tree(4), Tree(5)])
>>> left = Tree(1, [Tree(2), t])
>>> mid = Tree(6, [Tree(7, [Tree(8)]), Tree(9)])
>>> right = Tree(11, [Tree(12, [Tree(13, [Tree(14)])])])
>>> whole = Tree(0, [left, Tree(13), mid, right])
>>> print(whole)
0
  1
    2
    3
      4
      4
      5
  13
  6
    7
      8
    9
  11
    12
      13
        14
>>> for path in long_paths(whole, 2):
...     print(path)
...
[0, 1, 2]
[0, 1, 3, 4]
[0, 1, 3, 4]
[0, 1, 3, 5]
[0, 6, 7, 8]
[0, 6, 9]
[0, 11, 12, 13, 14]
>>> for path in long_paths(whole, 3):
...     print(path)
...
[0, 1, 3, 4]
[0, 1, 3, 4]
[0, 1, 3, 5]
[0, 6, 7, 8]
[0, 11, 12, 13, 14]
>>> long_paths(whole, 4)
[[0, 11, 12, 13, 14]]
"""
"*** YOUR CODE HERE ***"
```

class Tree: """ >>> t = Tree(3, [Tree(2, [Tree(5)]), Tree(4)]) >>> t.label 3 >>> t.branches[0].label 2 >>> t.branches[1].is_leaf() True """ def **init**(self, label, branches=[]): for b in branches: assert isinstance(b, Tree) self.label = label self.branches = list(branches)

```
def is_leaf(self):
    return not self.branches

def __repr__(self):
    if self.branches:
        branch_str = ', ' + repr(self.branches)
    else:
        branch_str = ''
    return 'Tree({0}{1})'.format(self.label, branch_str)

def __str__(self):
    def print_tree(t, indent=0):
        tree_str = '  ' * indent + str(t.label) + "\n"
        for b in t.branches:
            tree_str += print_tree(b, indent + 1)
        return tree_str
    return print_tree(self).rstrip()
```

Q12: Reverse Other

Write a function `reverse_other` that mutates the tree such that **labels** on *every other* (odd-depth) level are reversed. For example, `Tree(1,[Tree(2, [Tree(4)]), Tree(3)])` becomes `Tree(1,[Tree(3, [Tree(4)]), Tree(2)])`. Notice that the nodes themselves are *not* reversed; only the labels are.

def reverse_other(t): """Mutates the tree such that nodes on every other (odd-depth) level have the labels of their branches all reversed.

```
>>> t = Tree(1, [Tree(2), Tree(3), Tree(4)])
>>> reverse_other(t)
>>> t
Tree(1, [Tree(4), Tree(3), Tree(2)])
>>> t = Tree(1, [Tree(2, [Tree(3, [Tree(4), Tree(5)]), Tree(6, [Tree(7)])]), Tree(8)])
>>> reverse_other(t)
>>> t
Tree(1, [Tree(8, [Tree(3, [Tree(5), Tree(4)]), Tree(6, [Tree(7)])]), Tree(2)])
"""
"*** YOUR CODE HERE ***"
```

Efficiency

Q13: Efficiency Practice

Choose the term that fills in the blank for the functions defined below: `<function>` runs in `____` time in the length of its input.

- Constant
- Logarithmic
- Linear
- Quadratic
- Exponential
- None of these

Assume that `len` runs in constant time and `all` runs in linear time in the length of its input. Selecting an element of a list by its index requires constant time. Constructing a range requires constant time.

```
def count_partitions(n, m):
    """Counts the number of partitions of a positive integer n,
    using parts up to size m."""
    if n == 0:
        return 1
    elif n < 0:
        return 0
    elif m == 0:
        return 0
    else:
        with_m = count_partitions(n-m, m)
        without_m = count_partitions(n, m-1)
        return with_m + without_m

def is_palindrome(s):
    """Return whether a list of numbers s is a palindrome."""
    return all([s[i] == s[len(s) - i - 1] for i in range(len(s))])

def binary_search(lst, n):
    """Takes in a sorted list lst and returns the index where integer n
    is contained in lst. Returns -1 if n does not exist in lst."""
    low = 0
    high = len(lst)
    while low <= high:
        middle = (low + high) // 2
        if lst[middle] == n:
            return middle
        elif n < lst[middle]:
            high = middle - 1
        else:
            low = middle + 1
    return -1
```

The `is_palindrome` question was reformatted from question 6(d) on fall 2019's final.