Контрольный вопрос №1

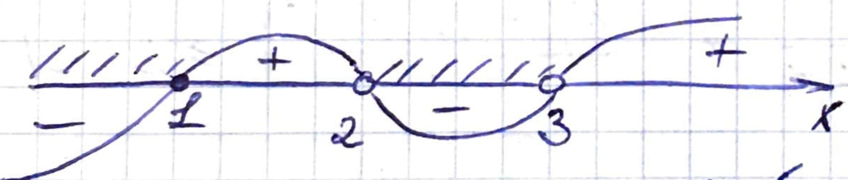
$$\frac{x^2-2x-1}{x-2}+\frac{2}{x-3}\le x$$

$$\frac{(x^2-2x-1)(x-3)+2(x-2)-x(x-2)(x-3)}{(x-2)(x-3)}\le 0$$

$$\frac{x^3-2x^2-x-3x^2+6x+3+2x-4-x^3+3x^2+2x^2-6x}{(x-2)(x-3)}\le 0$$

$$\frac{x-1}{(x-2)(x-3)}\le 0$$

ОДЗ:
$x \neq 2$
$x \neq 3$



$x \in (-\infty; 1] \cup (2; 3)$

Ответ: $x \in (-\infty; 1] \cup (2; 3)$

Контрольный вопрос №2

$$\left(\frac{2}{25x^2-10x-8}+\frac{25x^2-10x-8}{2}\right)^2 \geq 4 \iff$$

$$\iff \left[\begin{array}{l} \dfrac{2}{25x^2-10x-8}+\dfrac{25x^2-10x-8}{2} \geq 2 \\ \dfrac{2}{25x^2-10x-8}+\dfrac{25x^2-10x-8}{2} \leq -2 \end{array}\right.$$

$$25x^2-10x-8 = (25x^2-10x+1)-1-8 =$$
$$= (5x-1)^2-9 = (5x-1-3)(5x-1+3) =$$
$$= (5x-4)(5x+2) = 5(x-0{,}8)\cdot 5(x+0{,}4) =$$
$$= 25(x-0{,}8)(x+0{,}4)$$

$$\left[\begin{array}{ll} \dfrac{2}{25(x-0{,}8)(x+0{,}4)}+\dfrac{25(x-0{,}8)(x+0{,}4)}{2} \geq 2 & (*) \\ \dfrac{2}{25(x-0{,}8)(x+0{,}4)}+\dfrac{25(x-0{,}8)(x+0{,}4)}{2} \leq -2 & (**) \end{array}\right.$$

Замена: $t = 25(x-0{,}8)(x+0{,}4)$

$(*)$ $$\frac{2}{t}+\frac{t}{2} \geq 2$$

$$\frac{2\cdot 2+t^2-2t\cdot 2}{2t} \geq 0$$

$$\frac{4+t^2-4t}{2t} \geq 0$$

$$\frac{(t-2)^2}{2t} \geq 0 \iff \begin{cases} t \neq 0 \\ 2t(t-2)^2 \geq 0 \end{cases}$$

$(t-2)^2 \geq 0$ для всех t $\Rightarrow$

$$\Rightarrow \begin{cases} t \neq 0 \\ 2t(t-2)^2 \geq 0 \end{cases} \iff t > 0$$

$$25(x-0{,}8)(x+0{,}4) > 0$$

$x \in (-\infty; -0{,}4) \cup (0{,}8; +\infty)$

$(**)$ $\frac{2}{t} + \frac{t}{2} \leq -2$

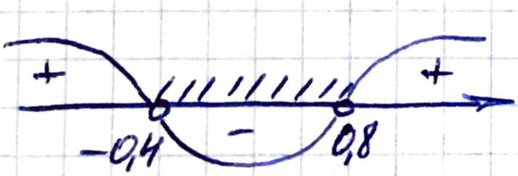
$$\frac{4 + t^2 + 4t}{2t} \leq 0$$

$$\frac{(t+2)^2}{2t} \leq 0 \iff \begin{cases} t \neq 0 \\ (t+2)^2 \cdot 2t \leq 0 \end{cases},$$

$(t+2)^2 \geq 0$ для всех t $\Rightarrow$

$$\Rightarrow \begin{cases} t \neq 0 \\ (t+2)^2 \cdot 2t \leq 0 \end{cases} \iff t < 0$$

$$25(x-0{,}8)(x+0{,}4) < 0$$



$x \in (-0{,}4; 0{,}8)$

В итоге:

$$\left[\begin{array}{l} x\in(-\infty;-0{,}4)\cup(0{,}8;+\infty) \\ x\in(-0{,}4;0{,}8) \end{array}\right. \Rightarrow$$

$$\Rightarrow \quad x\in(-\infty;-0{,}4)\cup(-0{,}4;0{,}8)\cup(0{,}8;+\infty)$$

Ответ:

$x\in(-\infty;-0{,}4)\cup(-0{,}4;0{,}8)\cup(0{,}8;+\infty)$.

Контрольный вопрос №3

$$\frac{x^4-5x^3+3x-25}{x^2-5x} \geq x^2-\frac{1}{x-4}+\frac{5}{x}$$

$$\frac{x^4-5x^3+3x-25}{x(x-5)}-x^2+\frac{1}{x-4}-\frac{5}{x} \geq 0$$

$$\frac{x^4-5x^3+3x-25-x^3(x-4)(x-5)+x(x-5)-5(x-5)(x-4)}{x(x-5)(x-4)} \geq 0$$

При $x(x-5)(x-4)>0$:

$(x-4)(x^4-5x^3+3x-25)-x^3(x-5)(x-4)+x(x-5)-$

$-5(x-5)(x-4) \geq 0$

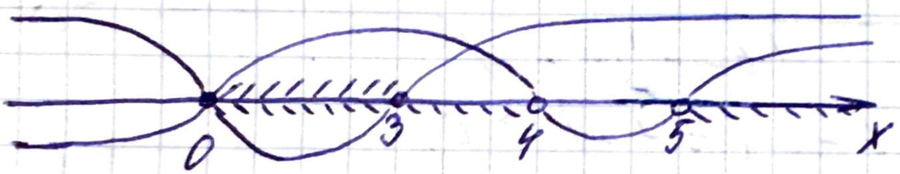
$x^5 - 5x^4 + 3x^2 - 25x - 4x^4 + 20x^3 - 12x + 100 -$

$-x^5 + 9x^4 - 20x^3 + x^2 - 5x - 5x^2 + 45x - 100 \geq 0$

$-x^2 + 3x \geq 0$

$x^2 - 3x \leq 0$

$$\begin{cases} x(x-3) \leq 0 \\ x(x-5)(x-4) > 0 \end{cases}$$

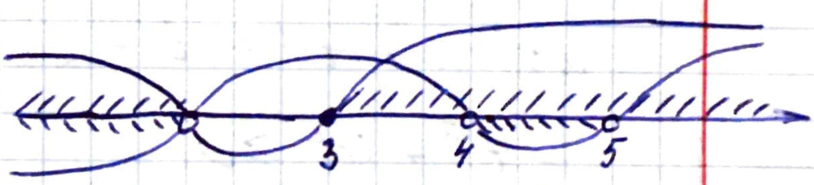


$x \in [0; 3]$

При $x(x-5)(x-4) < 0$:

$-x^2 + 3x \leq 0$

$x^2 - 3x \geq 0$



$$\begin{cases} x(x-3) \geq 0 \\ x(x-5)(x-4) < 0 \end{cases}$$

$x \in (-\infty; 0) \cup (4; 5)$

В итоге:

$$\left[\begin{array}{l} x \in [0; 3] \\ x \in (-\infty; 0) \cup (4; 5) \end{array}\right. \Rightarrow x \in (-\infty; 0) \cup [0; 3] \cup (4; 5)$$

Ответ: $x \in (-\infty; 0) \cup [0; 3] \cup (4; 5)$.

Контрольный вопрос №4

$$\frac{2}{0,5\sqrt{5}\cdot x-1}+\frac{0,5\sqrt{5}x-2}{0,5\sqrt{5}x-3}\geq 2$$

Замена: $t=0,5\sqrt{5}x-2$

$$\frac{2}{t+1}+\frac{t}{t-1}-2\geq 0 \qquad t\neq -1;\ t\neq 1$$

$$\frac{2(t-1)+t(t+1)-2(t+1)(t-1)}{(t-1)(t+1)}\geq 0$$

При $(t-1)(t+1)>0$,

$t\in(-\infty;-1)\cup(1;+\infty)$:

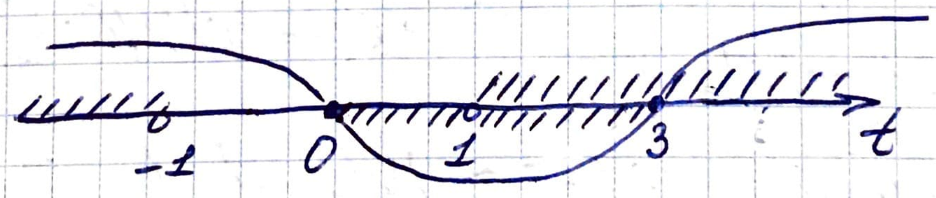
$$2(t-1)+t(t+1)-2(t+1)(t-1)\geq 0$$

$$2t-2+t^2+t-2t^2+2\geq 0$$

$$-t^3+3t\geq 0$$

$$t^3-3t\leq 0$$

$$\begin{cases} t(t-3)\leq 0 \\ t\in(-\infty;-1)\cup(1;+\infty) \end{cases}$$



$t\in(1;3]$

Обр. замена:

$0{,}5\sqrt{5}x-2 \in (1;3] \Leftrightarrow \begin{cases} 0{,}5\sqrt{5}x-2>1 \\ 0{,}5\sqrt{5}x-2\le 3 \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} 0{,}5\sqrt{5}x>3 \\ 0{,}5\sqrt{5}x\le 5 \end{cases} \Leftrightarrow \begin{cases} x>\dfrac{3}{0{,}5\sqrt{5}} \\ x\le\dfrac{5}{0{,}5\sqrt{5}} \end{cases} \Leftrightarrow$

$\Leftrightarrow \begin{cases} x>\dfrac{3\sqrt{5}}{0{,}5\cdot5} \\ x\le\dfrac{5\sqrt{5}}{0{,}5\cdot5} \end{cases} \Leftrightarrow \begin{cases} x>1{,}2\sqrt{5} \\ x\le 2\sqrt{5} \end{cases} \Leftrightarrow$

$\Leftrightarrow x\in(1{,}2\sqrt{5};\,2\sqrt{5}]$

При $(t+1)(t-1)<0$; $t\in(-1;1)$:

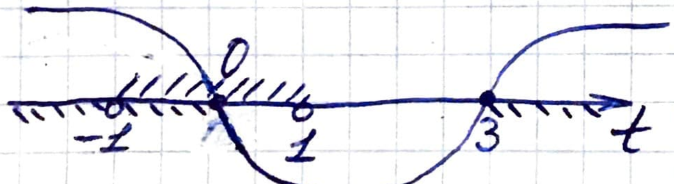
$2(t-1)+t(t+1)-2(t-1)(t+1)\le 0$

$2t-2+t^2+t-2t^2+2\le 0$

$-t^2+3t\le 0$

$t^2-3t\ge 0$

$\begin{cases} t(t-3)\ge 0 \\ t\in(-1;1) \end{cases}$



$\underline{t\in(-1;0]}$

Обр. замена:

$$0{,}5\sqrt{5}x-2 \in (1; 0] \Leftrightarrow \begin{cases} 0{,}5\sqrt{5}x-2 > \\ 0{,}5\sqrt{5}x-2 \leq 2 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 0{,}5\sqrt{5}x > -1 \\ 0{,}5\sqrt{5}x \leq 2 \end{cases} \Leftrightarrow \begin{cases} x > \dfrac{1}{0{,}5\sqrt{5}} \\ x \leq \dfrac{2}{0{,}5\sqrt{5}} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x > \dfrac{\sqrt{5}}{0{,}5\cdot 5} \\ x \leq \dfrac{2\sqrt{5}}{0{,}5\cdot 5} \end{cases} \Leftrightarrow \begin{cases} x > 0{,}4\sqrt{5} \\ x \leq 0{,}8\sqrt{5} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow x \in (0{,}4\sqrt{5}; 0{,}8\sqrt{5}]$$

В итоге: $\left[\begin{array}{l} x \in (1{,}2\sqrt{5}; 2\sqrt{5}] \\ x \in (0{,}4\sqrt{5}; 0{,}8\sqrt{5}] \end{array}\right. \Leftrightarrow$

$$\Leftrightarrow x \in (0{,}4\sqrt{5}; 0{,}8\sqrt{5}] \cup (1{,}2\sqrt{5}; 2\sqrt{5}]$$

<u>Ответ</u>: $x \in (0{,}4\sqrt{5}; 0{,}8\sqrt{5}] \cup (1{,}2\sqrt{5}; 2\sqrt{5}]$

Контрольный вопрос №5

$$\frac{8x^2+7}{x^2+x+1} \geqslant \frac{x}{x+5} + 7$$

$$\frac{8x^2+7}{x^2+x+1} - \frac{x}{x+5} - 7 \geqslant 0$$

$$\frac{(8x^2+7)(x+5) - x(x^2+x+1) - 7(x^2+x+1)(x+5)}{(x^2+x+1)(x+5)} \geqslant 0$$

$x^2+x+1=0$

$\left.\begin{array}{l} D = 1-4 = -3 < 0 \\ 1 > 0 \end{array}\right\} \Rightarrow x^2+x+1 > 0$

для любых x.

При $x > -5$:

$(8x^2+7)(x+5) - x(x^2+x+1) - 7(x+5)(x^2+x+1) \geqslant 0$

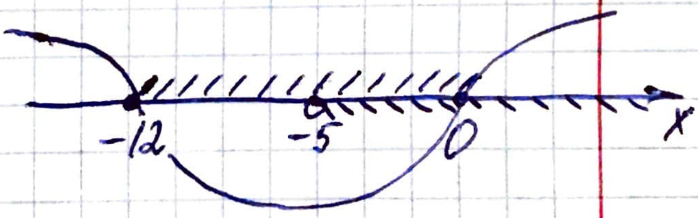
$8x^3 + 40x^2 + 7x + 35 - x^3 - x^2 - x - 7x^3 - 7x^2 - 7x -$

$-35x^2 - 35x - 35 \geqslant 0$

$-3x^2 - 36x \geqslant 0$

$x^2 + 12x \leqslant 0$

$\begin{cases} x(x+12) \leqslant 0 \\ x > -5 \end{cases}$

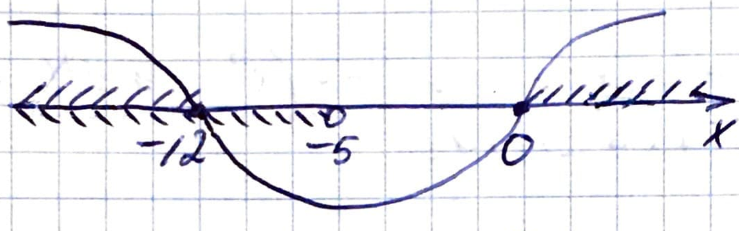


$x \in (-5; 0]$

При $x < -5$:

$-3x^2 - 36x \leq 0$

$\begin{cases} x^2 + 12x \geq 0 \\ x < -5 \end{cases}$ $\begin{cases} x(x+12) \geq 0 \\ x < -5 \end{cases}$



$x \in (-\infty; -12]$

<u>Ответ:</u> $x \in (-\infty; 12] \cup (-5; 0]$

Контрольный вопрос №6

$$(\sqrt{x-4}-1)(x+5)(2x-9)=0 \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x-4 \geq 0 \\ \left[\begin{array}{l} \sqrt{x-4}-1=0 \\ x+5=0 \\ 2x-9=0 \end{array}\right. \end{cases} \Leftrightarrow \begin{cases} x \geq 4 \\ \left[\begin{array}{l} \sqrt{x-4}=1 \\ x=-5 \\ 2x=9 \end{array}\right. \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x \geq 4 \\ \left[\begin{array}{l} x-4=1 \\ x=-5 \\ x=4{,}5 \end{array}\right. \end{cases} \Leftrightarrow \begin{cases} x \geq 4 \\ \left[\begin{array}{l} x=5 \\ x=-5 \\ x=4{,}5 \end{array}\right. \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} x=5 \\ x=4{,}5 \end{array}\right. \quad x \in \{5;\ 4{,}5\}$$

<u>Ответ:</u> $x \in \{5;\ 4{,}5\}$

Контрольный вопрос №7

$$\left(\sqrt{x^3+x-1}-\sqrt{x+2}\right)(x^2-9)=0 \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} x^2-9=0 \\ \sqrt{x^3+x-1}-\sqrt{x+2} \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} x^2=9 \\ \left\{\begin{array}{l} x+2\geq 0 \\ x^3+x-1=x+2 \end{array}\right. \end{array}\right. \Leftrightarrow \left[\begin{array}{l} \left[\begin{array}{l} x=3 \\ x=-3 \end{array}\right. \\ \left\{\begin{array}{l} x\geq -2 \\ x^3=3 \end{array}\right. \end{array}\right. \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{l} \left[\begin{array}{l} x=3 \\ x=-3 \end{array}\right. \\ \left\{\begin{array}{l} x\geq -2 \\ x=\sqrt[3]{3} \end{array}\right. \end{array}\right. \Leftrightarrow \left[\begin{array}{l} x=3 \\ x=\sqrt[3]{3} \end{array}\right.$$

<u>Ответ:</u> $x \in \{3; \sqrt[3]{3}\}$

Контрольный вопрос №8

$$2x^2+7x-12+\sqrt{x^2-25}=x^2-11x+\sqrt{x^2-25}$$

ОДЗ: $x^2 \geq 25 \Leftrightarrow \left[\begin{array}{l} x \geq 5 \\ x \leq -5 \end{array}\right.$

$$2x^2+7x-12=x^2-11x$$

$$x^2+18x-12=0$$

$$\frac{D}{4}=81+12=93$$

$$x=\frac{-9\pm\sqrt{93}}{1}$$

$$\left[\begin{array}{l} x=-9+\sqrt{93} \\ x=-9-\sqrt{93} \end{array}\right.$$

$$\sqrt{81}<\sqrt{93}<\sqrt{100}$$

$$9<\sqrt{93}<10$$

$$-9+9<-9+\sqrt{93}<-9+10$$

$$0<-9+\sqrt{93}<1 \Rightarrow x \neq -9+\sqrt{93}$$

$$9<\sqrt{93}<10$$

$$-9>-\sqrt{93}>-10$$

$$-9-9>-9-\sqrt{93}>-9-10$$

$$-18>-9\sqrt{93}>-19 \Rightarrow$$

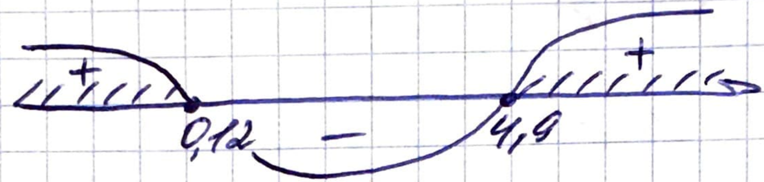
$$\Rightarrow x=-9-\sqrt{93}$$

Ответ: $x=-9-\sqrt{93}$

Контрольный вопрос №9

$$x^2-8x+1+\sqrt{(x-0,12)(x-4,9)}=\sqrt{(x-0,12)(x-4,9)}$$

ОДЗ: $(x-0,12)(x-4,9)\geq 0$



$$x\in(-\infty;0,12]\cup[4,9;+\infty)$$

$$x^2-8x+1=0$$

$$\frac{D}{4}=16-1=15$$

$$x=4\pm\sqrt{15}$$

$4+\sqrt{15}>4,9$ $(\sqrt{15}>0,9)$ $\Rightarrow x=4+\sqrt{15}$

$4-\sqrt{15}\ \overset{?}{\ }\ 0,12$

$\sqrt{15}\ ?\ 3,88$

$3,88^2=15,0544 \Rightarrow \sqrt{15}<3,88 \Rightarrow$

$\Rightarrow 4-\sqrt{15}>0,12 \Rightarrow x\neq 4-\sqrt{15}$

Ответ: $x=4+\sqrt{15}$

Контрольный вопрос №10

$$|6x^3+9x-6|=|6x^3+6x^2-13x+6| \Leftrightarrow$$

$$\Leftrightarrow \left[\begin{array}{ll} 6x^3+9x-6=6x^3+6x^2-13x+6 & (1) \\ -6x^3-9x+6=6x^3+6x^2-13x+6 & (2) \end{array}\right.$$

(1) $6x^3+9x-6=6x^3+6x^2-13x+6$

$$9x-6=6x^2-13x+6$$

$$6x^2-22x+12=0$$

$$3x^2-11x+6=0$$

$$D=121-4\cdot3\cdot6=121-72=49$$

$$x=\frac{11\pm7}{6}$$

$$\left[\begin{array}{l} x=3 \\ x=\frac{2}{3} \end{array}\right.$$

(2) $-6x^3-9x+6=6x^3+6x^2-13x+6$

$$12x^3+6x^2-4x=0$$

$x=0$ – корень

при $x\neq0$:

$$12x^2+6x-4=0$$

$$6x^2+3x-2=0$$

$$D=9+4\cdot6\cdot2=9+48=57$$

$x=\dfrac{-3\pm\sqrt{57}}{12}$ <u>Ответ</u>: $x\in\left\{\dfrac{-3-\sqrt{57}}{12};\ \dfrac{-3+\sqrt{57}}{12};0;\dfrac{2}{3};3\right\}$

Задача 1

$$\sqrt{x^2-6x+9}-\sqrt{x^2+2x+1}=5-3x$$

$$\sqrt{(x-3)^2}-\sqrt{(x+1)^2}=5-3x$$

- При $x \geq 3$:

$$x-3-x-1=5-3x$$

$$3x=9$$

$$\underline{x=3}$$

- При $-1 \leq x < 3$:

$$3-x-x-1=5-3x$$

$$x=3 \Rightarrow x \in \varnothing$$

- При $x < -1$:

$$3-x+x+1=5-3x$$

$$4=5-3x$$

$$3x=1$$

$$x=\frac{1}{3} \Rightarrow x \in \varnothing$$

Ответ: $x=3$

Задача 2

$$\sqrt{x^2-4x+4}+\sqrt{4x^2-17x+15}=2-x$$

$$4x^2-17x+15=0$$

$$D=289-4\cdot4\cdot15=289-240=49$$

$$x=\frac{17\pm7}{8}$$

$$\left[\begin{array}{l}x=3\\x=1{,}25\end{array}\right.\Rightarrow\ 4x^2-17x+15=4(x-3)(x-1{,}25)$$

$$\sqrt{(x-2)^2}+\sqrt{4(x-3)(x-1{,}25)}=2-x$$

Т.к. $\sqrt{(x-2)^2}+\sqrt{4(x-3)(x-1{,}25)}\geq0$, то $x\leq2$

$$|x-2|+2\sqrt{(x-3)(x-1{,}25)}=2-x$$

$$2-x+2\sqrt{(x-3)(x-1{,}25)}=2-x$$

$$2\sqrt{(x-3)(x-1{,}25)}=0$$

$$(x-3)(x-1{,}25)=0$$

$\left[\begin{array}{l}x=3\\x=1{,}25\end{array}\right.$ (не входит в ОДЗ)

Ответ: $x=1{,}25$

Задача 3

$$\sqrt{15x^2+2x+8}+4x=0$$

$$\sqrt{15x^2+2x+8}=-4x \iff$$

$$\iff \begin{cases} -4x \geq 0 \\ 15x^2+2x+8=(-4x)^2 \end{cases} \iff$$

$$\iff \begin{cases} x \leq 0 \\ 15x^2+2x+8=16x^2 \end{cases}$$

$$15x^2+2x+8=16x^2$$

$$x^2-2x-8=0$$

$$D=4+4\cdot 8=36$$

$$x=\frac{2\pm 6}{2}$$

$$\left[\begin{array}{l} x=4 \\ x=-2 \end{array}\right. , \; 4>0 \Rightarrow x=-2$$

Ответ: $x=-2$

Задача 4

$$\sqrt{x+2} = |x-1| \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} |x-1| \geq 0 \\ x+2 = (|x-1|)^2 \end{cases} \Leftrightarrow x+2 = x^2-2x+1$$

$$x^2 - 3x - 1 = 0$$

$$D = 9 + 4 = 13$$

$$x = \frac{3 \pm \sqrt{13}}{2}, \quad x_1 = \frac{3+\sqrt{13}}{2}, \quad x_2 = \frac{3-\sqrt{13}}{2}$$

$$x_1 + x_2 = \frac{3+\sqrt{13}}{2} + \frac{3-\sqrt{13}}{2} = \frac{6}{2} = 3$$

Ответ: $x_1 + x_2 = 3$

Задача 5

$$\sqrt{|x^2+14x+47|-1} = |x+7|-1$$

$$|x^2+14x+47|-1 = (|x+7|-1)^2$$

$$|x^2+14x+47|-1 = (x+7)^2-2|x+7|+1 =$$

$$= x^2+14x+49-2|x+7|+1$$

$$|x^2+14x+47| - x^2-14x+2|x+7| = 51$$

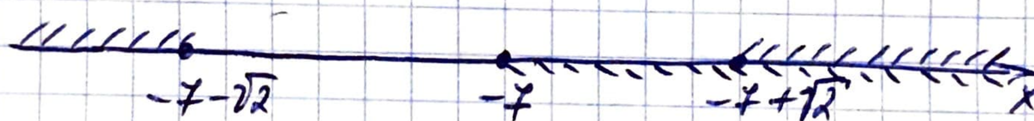
$$x^2+14x+47=0$$

$$\frac{D}{4} = 49-47=2$$

$$x = \frac{-7\pm\sqrt{2}}{1} \Rightarrow$$

$$\Rightarrow x^2+14x+47 = (x-(-7-\sqrt{2}))\cdot(x-(-7+\sqrt{2}))$$

При $(x-(-7-\sqrt{2}))(x-(-7+\sqrt{2})) \geq 0$, $x \geq -7$:



$$x^2+14x+47-x^2-14x+2(x+7)=51$$

$$2x+14+47=51$$

$$2x=-10$$

$$x=-5$$

При $(x-(-7-\sqrt{2}))(x-(-7+\sqrt{2}))\geq 0,\ x<-7$:

$x^2+14x+47-x^2-14x-2(x+7)=51$

$-2x-14+47=51$

$2x=-18$

$x=-9$

При $(x-(-7-\sqrt{2}))(x-(-7+\sqrt{2}))<0,\ x\geq -7$:

$-x^2-14x-47-x^2-14x+2(x+7)=51$

$-2x^2-28x+2x+14-47=51$

$2x^2+26x+84=0$

$x^2+13x+42=0$

$D=169-4\cdot 42=169-168=1$

$x=\frac{-13\pm 1}{2}$

$\left[\begin{array}{l} x=-6 \\ x=-7 \end{array}\right.$, не входит в ОДЗ.

При $(x-(-7-\sqrt{2}))(x-(-7+\sqrt{2}))<0$, $x<-7$:

$$-x^2-14x-47-x^2-14x-2(x+7)=51$$

$$-2x^2-28x-2x-14-47=51$$

$$2x^2+30x+112=0$$

$$x^2+15x+56=0$$

$$D=225-4\cdot 56=1$$

$$x=\frac{-15\pm 1}{2}$$

$$\left[\begin{array}{l} x=-8 \\ x=-7 \end{array}\right.$$, не входит в ОДЗ

В итоге:

$$\left[\begin{array}{l} x=-9 \\ x=-8 \\ x=-6 \\ x=-5 \end{array}\right.$$

<u>Ответ:</u> $x_1+x_2+x_3+x_4=$

$=-28$

Задача 7

$$\begin{cases} 4x^2-5x+1>0, & (1) \\ \dfrac{(5x^2-2x+1)-(4x^2-5x+1)}{4x^2-5x} \leq 0 & (2) \end{cases}$$

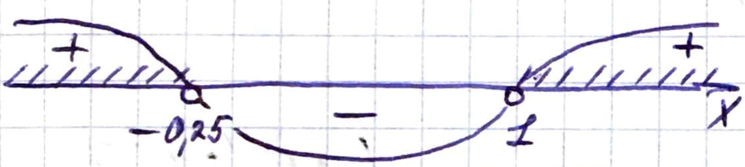
(1) $4x^2-5x+1>0$

$4(x-1)(x+0{,}25)>0$

$4x^2-5x+1=0$

$D=25-16=9$

$x_{1,2}=\dfrac{5\pm3}{3}$



$\left[\begin{array}{l} x=1 \\ x=-0{,}25 \end{array}\right. \Rightarrow$

$x\in(-\infty;-0{,}25)\cup(1;+\infty) \Rightarrow 4x^2-5x+1=4(x-1)(x+0{,}25)$

(2) $\dfrac{(5x^2-2x+1)-(4x^2-5x+1)}{4x^2-5x}\leq 0 \Leftrightarrow$

$$\Leftrightarrow \left[\begin{array}{l} \begin{cases} (5x^2-2x+1)-(4x^2-5x+1)\geq 0 & (2.1.1.) \\ 4x^2-5x<0 & (2.1.2.) \end{cases} \\ \begin{cases} (5x^2-2x+1)-(4x^2-5x+1)\leq 0 & (2.2.1.) \\ 4x^2-5x>0 & (2.2.2.) \end{cases} \end{array}\right.$$

$\boxed{2.1.1}$

$(5x^2-2x+1)-(4x^2-5x+1)\geq 0$

$x^2+3x\geq 0$

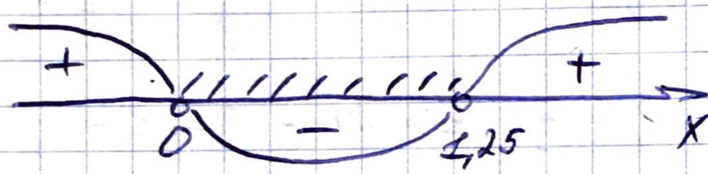
$x(x+3)\geq 0$

$x \in (-\infty; -3] \cup$
$\cup [0; +\infty)$

$\boxed{2.1.2.}$

$4x^2 - 5x < 0$

$4x(x - 1,25) < 0$

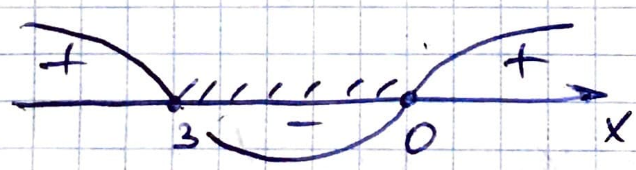


$$\begin{cases} x \in (0; 1,25) \\ x \in (-\infty; -3] \cup \\ \quad \cup [0; +\infty) \end{cases} \Rightarrow$$

$\Rightarrow x \in (0; 1,25)$

$\boxed{2.2.1}$ $(5x^2 - 2x + 1) - (4x^2 - 5x + 1) \leq 0$

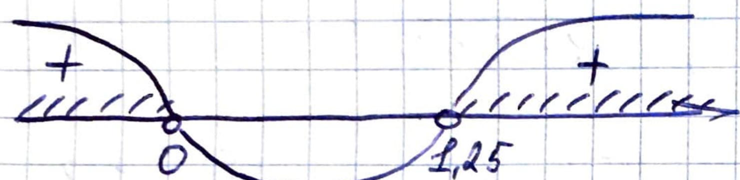
$x^2 + 3x \leq 0$



$x \in [-3; 0)$

$\boxed{2.2.2}$ $4x^2 - 5x > 0$

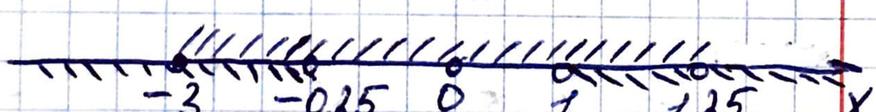
$4x(x - 1{,}25) > 0$



$$\begin{cases} x \in (-\infty; 0) \cup (1{,}25; +\infty) \\ x \in [-3; 0] \end{cases} \Rightarrow x \in [-3; 0)$$

(2) $$\left[\begin{array}{l} x \in (0; 1{,}25) \\ x \in [-3; 0) \end{array}\right. \Rightarrow x \in [-3; 0) \cup (0; 1{,}25)$$

В итоге: $$\begin{cases} x \in (-\infty; -0{,}25) \cup (1; +\infty) \\ x \in [-3; 0) \cup (0; 1{,}25) \end{cases}$$



$x \in [-3; -0{,}25) \cup (1; 1{,}25)$

<u>**Ответ:**</u> $x \in [-3; -0{,}25) \cup (1, 1{,}25)$

Задача 8

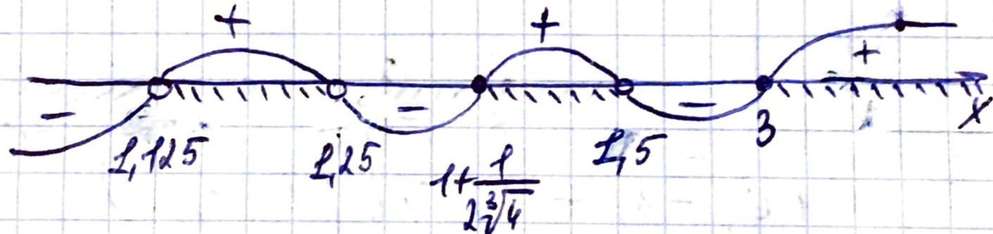
$$\begin{cases} x > 1 \\ \dfrac{(x-3)\left(x-\left(1+\frac{1}{2\sqrt[3]{4}}\right)\right)}{(x-1{,}5)(x-1{,}25)(x-1{,}125)} \geq 0 \end{cases}$$

$$\sqrt[3]{1} < \sqrt[3]{4} < \sqrt[3]{8} \Rightarrow 1 < \sqrt[3]{4} < 2 \Rightarrow$$

$$\Rightarrow 2 < 2\sqrt[3]{4} < 4 \Rightarrow 0{,}5 > \frac{1}{2\sqrt[3]{4}} > 0{,}25 \Rightarrow$$

$$\Rightarrow 1{,}5 > +\frac{1}{2\sqrt[3]{4}} > 1{,}25$$

$$(x-3)\left(x-\left(1+\frac{1}{2\sqrt[3]{4}}\right)\right)(x-1{,}5)(x-1{,}125) \geq 0$$



$$\begin{cases} x \in (1{,}125; 1{,}25) \cup \left[1+\frac{1}{2\sqrt[3]{4}}; 1{,}5\right) \cup [3; +\infty) \\ x > 1 \end{cases} \Rightarrow$$

$$\Rightarrow x \in (1{,}125; 1{,}25) \cup \left[1+\frac{1}{2\sqrt[3]{4}}; 1{,}5\right) \cup [3; +\infty)$$

Ответ: $x \in (1{,}125; 1{,}25) \cup \left[1+\frac{1}{2\sqrt[3]{4}}; 1{,}5\right) \cup [3; +\infty)$

Задача 9

$$\sqrt{1-\sqrt{4x^3-5x+1}}=\sqrt{1+\sqrt{4x^3-5x+1}} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 1-\sqrt{4x^3-5x+1} = 1+\sqrt{4x^3-5x+1} \\ 1-\sqrt{4x^3-5x+1} \geq 0 \\ 1+\sqrt{4x^3-5x+1} \geq 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} -\sqrt{4x^3-5x+1} = \sqrt{4x^3-5x+1} \\ \sqrt{4x^3-5x+1} \leq 1 \\ \sqrt{4x^3-5x+1} \geq 1 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \sqrt{4x^3-5x+1} = 0 \\ \sqrt{4x^3-5x+1} = 1 \end{cases} \Rightarrow x \in \varnothing$$

Ответ: $x \in \varnothing$

Задача 6

$$ax+\sqrt{5-4x-x^2}=3a+3$$

Один корень $\Rightarrow D=0$

$$\sqrt{5-4x-x^2}=3a-ax+3$$

ОДЗ: $5-4x-x^2>0$

$$5-4x-x^2=(3a-ax+3)^2$$

$$5-4x-x^2=(a(3-x)+3)^2$$

$$5-4x-x^2=a^2(9-6x+x^2)+6a(3-x)+9$$

$$5-4x-x^2=9a^2-6a^2x+a^2x^2+18a-6ax+9$$

$$x^2(a^2+1)+x(4-6a)+(9a^2+18a+4)=0$$

$$D=(4-6a)^2-4\cdot(a^2+1)\cdot(9a^2+18a+4)=0$$ (по усл.)

$$(4-6a)^2-4(a^2+1)(9a^2+18a+4)=0$$

$$16-4\cdot 6a\cdot 2+36a^2-4(9a^4+18a^3+4a^2+9a^2+18a+4)=0$$

$$4(4-12a+9a^2-9a^4-18a^3-4a^2-9a^2-18a-4)=0$$

$$-9a^4-18a^3-4a^2-30a=0$$

$$9a^4+18a^3+4a^2+30a=0$$

$$a(9a^3+18a^2+4a+30)=0$$

$a=0$ — первый корень

$$9a^3 + 18a^2 + 4a + 30 = 0$$

$a = \ldots$

Ответ: уравнение имеет
один корень
хотя бы при $a = 0$.