Политропный процесс (постоянная теплоемкость)

I з. Термодинамики $dQ = dU + \delta A$

$$c\nu\,dT = c_V \nu\,dT + p\,dV$$

$$p\,dV = \nu\,dT\,(c - c_V) \quad (1)$$

$$pV = \nu RT$$

$$p\,dV + V\,dp = \nu R\,dT$$

$$\nu\,dT\,(c - c_V) + V\,dp = \nu R\,dT$$

$$V\,dp = \nu\,dT\,(\underbrace{R + c_V}_{=c_p} - c)$$

$$V\,dp = \nu\,dT\,(c_p - c) = -\nu\,dT\,(c - c_p) \quad (2)$$

$$\frac{(2)}{(1)}: \quad \frac{V\,dp}{p\,dV} = -\boxed{\frac{c - c_p}{c - c_V}} = -n \quad (c \neq c_V), \quad \frac{c - c_p}{c - c_V} \equiv n$$

$$\int_{p_0}^{p} \frac{dp}{p} = -n \int_{V_0}^{V} \frac{dV}{V}$$

$$\ln p \Big|_{p_0}^{p} = -n \cdot \ln V \Big|_{V_0}^{V}$$

$$\ln p - \ln p_0 = -n \ln V + n \ln V_0$$

$$\ln p + n \ln V = \ln p_0 + n \ln V_0 \equiv \mathrm{const}$$

$$\ln p + \ln V^n = \mathrm{const}$$

$$\ln (pV^n) = \mathrm{const}$$

$$pV^n = e^{\mathrm{const}} \equiv \mathrm{const}$$

$$pV^{\frac{c - c_p}{c - c_V}} = \mathrm{const}$$

1) Адиабатный процесс:

$dQ = 0 \Rightarrow c = 0$

$pV^{\frac{c_p}{c_v}} = const$ (з-н Пуассона)

2) Изотермический: $dT = 0 \Rightarrow c \to \infty$

$\lim\limits_{c \to \infty} \frac{c - c_p}{c - c_v} = 1$

$pV^1 = const$ (з-н Бойля-Мариотта)

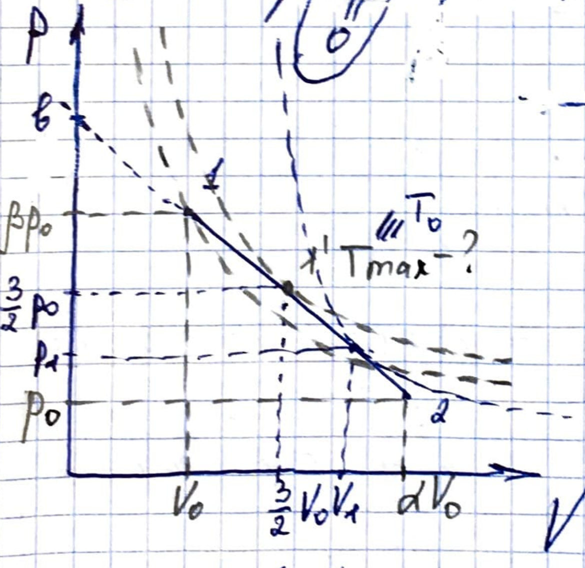
3) Изобарный: $dp = 0$ $p = const$;

$c = c_p \Rightarrow pV^0 = const \Rightarrow p = const$

4) Изохорный: $dV = 0$ $V = const$; $c = c_v$!

(по усл. $c \neq c_v$)

Но: $p\underbrace{dV}_{0}(c_p - c) = V dp \underbrace{(c - c_v)}_{0}$



$p(V) = b - aV$

$a = \frac{\beta - 1}{\alpha - 1} \cdot \frac{p_0}{V_0}$

$\frac{b - p_0}{p_0(\beta - 1)} = \frac{\alpha V_0}{(\alpha - 1)V_0}$

$b - p_0 = \frac{\alpha}{\alpha - 1} p_0(\beta - 1)$

$b = p_0\left(\frac{\alpha}{(\alpha - 1)}(\beta - 1) + 1\right)$

$b = \frac{\alpha\beta - 1}{\alpha - 1} p_0$

$p = \frac{\alpha\beta - 1}{\alpha - 1} p_0 - \frac{\beta - 1}{\alpha - 1} \frac{p_0}{V_0} \cdot V$

$pV = \nu R T_0$

$$\frac{\alpha\beta-1}{\alpha-1}p_0 - \frac{\beta-1}{\alpha-1}\frac{p_0}{V_0}\cdot V = \nu R T_0\cdot\frac{1}{V} \quad \Big|\cdot V(\alpha-1)$$

$$(\alpha\beta-1)\cdot p_0 V - (\beta-1)p_0\frac{1}{V_0}V^2 - \nu R T_0(\alpha-1) = 0$$

$$D = \left[(\alpha\beta-1)p_0\right]^2 - 4\nu R T_0(\alpha-1)(\beta-1)\frac{p_0}{V_0} = 0$$

$$(\alpha\beta-1)^2 p_0 = 4\nu R T_0(\alpha-1)(\beta-1)\frac{1}{V_0}$$

$$\boxed{T_0 = \frac{p_0 V_0(\alpha\beta-1)^2}{4\nu R(\alpha-1)(\beta-1)}} \quad T_{max}!$$

$$\alpha = \beta = 2$$

$$T_0 = \frac{p_0 V_0\, 9}{4\nu R} \qquad \frac{3}{2}p_0\,\frac{3}{2}V_0 = \nu R T_0$$

$$p(V) = b - aV$$

$$pV = \nu R T$$

$$bV - aV^2 = \nu R T$$

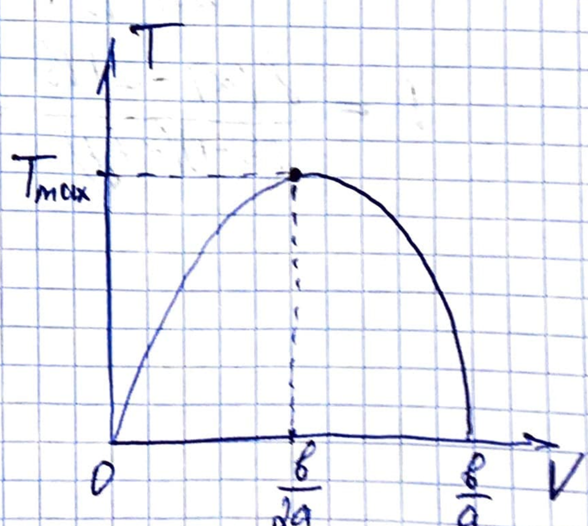
$$-aV^2 + bV - \nu R T = 0 \qquad T = \frac{bV - aV^2}{\nu R} =$$

$$= -\frac{a}{\nu R}V^2 + \frac{b}{\nu R}V$$

$$V = \frac{b}{a}$$

$$T_{max} = -\frac{a}{\nu R}\left(\frac{b}{2a}\right)^2 + \frac{b}{\nu R}\cdot\frac{b}{2a}$$

$$T_{max} = \frac{b^2}{4a\nu R}$$



Рассм. случай:

$$pV^{\gamma} = const \qquad \gamma = \frac{c_p}{c_v} = \frac{i+2}{i}$$

$$pV^{\gamma} = p_1 V_1^{\gamma}$$

$$p = p_1 V_1 \frac{1}{V^{\gamma}} \qquad \boxed{p_1 V_1 \frac{1}{V^{\gamma}} = b - aV}$$

$$p_1 V_1^{\gamma} = b - aV \qquad p = b - aV$$

$$p' = -a$$

$$p' = p_1 V_1^{\gamma} \cdot \frac{1}{V^{\gamma+1}}(-\gamma) = -a \qquad p_1 V_1^{\gamma} = V^{\gamma}(b - aV)$$

$$V^{\gamma}(b - aV) \cdot \frac{-\gamma}{V^{\gamma+1}} = -a$$

$$\frac{\gamma b}{V} - a\gamma = a$$

$$\frac{\gamma b}{V} = a(1+\gamma) \qquad V = \frac{\gamma b}{a(1+\gamma)} \quad (*)$$

Проверим $(*)$ $\beta = \alpha = 2$:

$$V = \frac{\gamma\left(\frac{\alpha\beta - 1}{\alpha - 1}\right) p_0}{\frac{\beta-1}{\alpha-1} \cdot \frac{p_0}{V_0}(1+\gamma)}$$

$$V = \frac{\gamma(\alpha\beta - 1)V_0}{(\beta-1)(1+\gamma)} = \frac{3 \cdot V_0}{1+\gamma} = \frac{3 \cdot \frac{5}{3} V_0}{1 + \frac{5}{3}} = \frac{15}{8} V_0 \quad (< 2V_0)$$