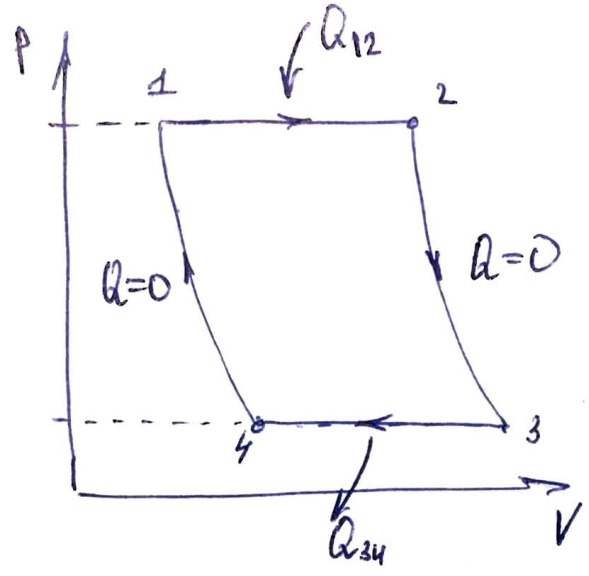
(1)



Дано:

$(A'_{34} =) A$, $A > 0$ (раб. над газом!)

A_0 (= Аунал)

Примечание:

A – работа над газом (без штрика)

A' – работа газа (со штриком)

Решение:

1) $Q_{41} = Q_{23} = 0$, т.к. 2–3 и 4–1 процессы адиабатические.

2) Найдем Q_{34}:

Пусть у газа изм температура на процессе 3–4 на ΔT_{34}. Тогда:

$$Q_{34} = A'_{34} + \Delta U_{34} = -A + \frac{i}{2}\nu R \Delta T_{34}$$

($i = 3$, газ одноатомный)

3) Т.к. 34 процесс изобарический, то

$$Q_{34} = \nu C_p \Delta T_{34} = \nu \cdot \frac{5}{2} R \Delta T_{34} \Rightarrow \nu R \Delta T_{34} = \frac{2}{5} Q_{34}$$

$$\left(C_p = \frac{i+2}{2} = \frac{5}{2}\right)$$

4) $$Q_{34} = -A + \frac{i}{2}\frac{2}{5}Q_{34} = -A + \frac{3}{5}Q_{34}$$

$$\frac{2}{5}Q_{34} = -A$$

$$\boxed{Q_{34} = -\frac{5A}{2}}$$

5) Аналогично найдем Q_{12}:

$$Q_{12} = A'_{12} + \Delta U_{12} = A'_{12} + \frac{3}{2}\nu R \Delta T_{12}$$

$$Q_{12} = \nu C_p \Delta T_{12} = \frac{5}{2}\nu R \Delta T_{12}$$

$$Q_{12} = +\frac{5A'_{12}}{2}$$

6) КПД цикла равен

$$\eta = \frac{A_{цикл}}{Q_н} = \frac{\sum A_i'}{Q_н} = \frac{\sum Q_i}{Q_н} = \frac{Q_н + Q_x}{Q_н}$$

Для этой задачи:

$$\eta = \frac{A_{цикл}}{Q_н} = \frac{A_0}{Q_{12}}$$

Заметим, что $A_{цикл} = A_0 = A_{12}' + A_{23}' + A_{34}' + A_{41}' =$

$$= Q_{12} + Q_{34} + \underbrace{Q_{41} + Q_{23}}_{0} =$$

$$= Q_{12} + Q_{34} \Rightarrow$$

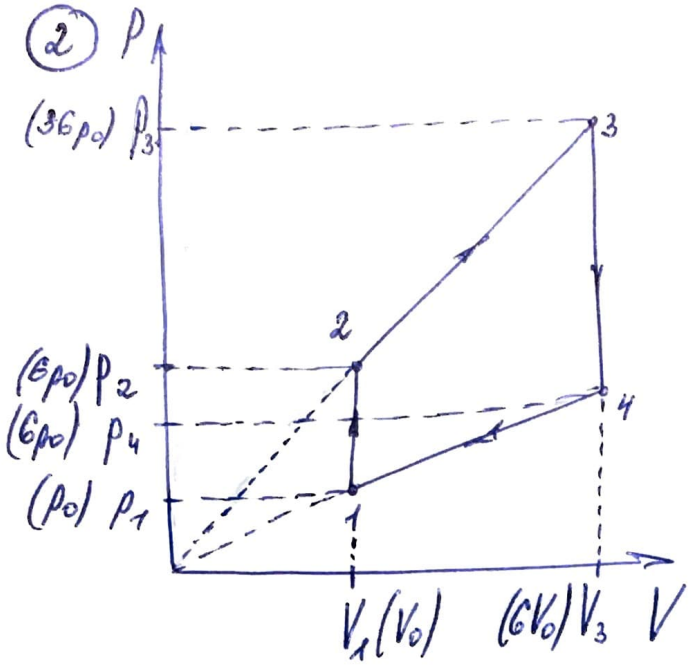
$$\Rightarrow Q_{12} = A_0 - Q_{34} = A_0 - \left(-\frac{5A}{2}\right) = A_0 + \frac{5}{2}A$$

$$\eta = \frac{A_0}{Q_{12}} = \frac{A_0}{A_0 + \frac{5}{2}A} = \frac{2A_0}{2A_0 + 5A}$$

Ответ: $Q_{34} = -\frac{5A}{2}$

$$\eta = \frac{2A_0}{2A_0 + 5A}$$

(2)



Дано:

1-2, 3-4 – изохорные

2-3, 4-1 – $p \sim V$

$p_2/p_1 = V_4/V_2 = 6$

Найти: η – ?

Решение:

1) $\frac{p_2}{p_1} = \frac{V_4}{V_2} = 6$

Пусть. $p_1 = p_0$; $p_2 = 6p_0$

$V_2 = V_1 = V_0$; $V_4 = V_3 = 6V_0$.

2) Т.к. на 2-3 и 4-1 $p \sim V$, то

2-3 $\left.\begin{matrix} p_2 = \alpha V_2 \\ p_3 = \alpha V_3 \end{matrix}\right\} \Rightarrow \frac{6p_0}{p_3} = \frac{\alpha}{\alpha} \cdot \frac{V_0}{6V_0} = \frac{1}{6}$

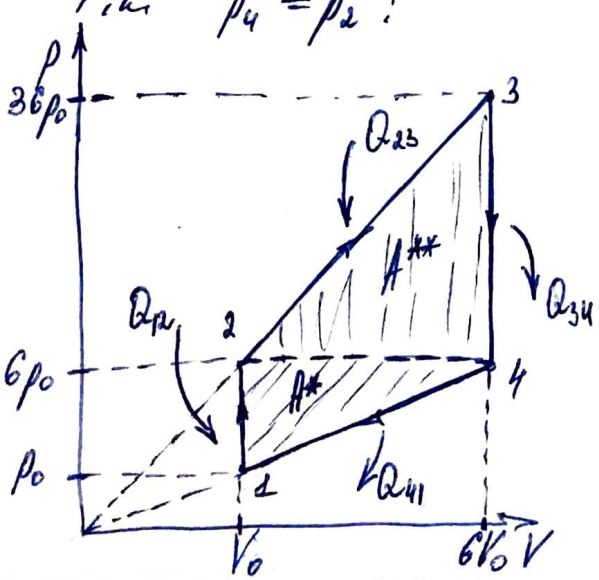
$p_3 = 36p_0$

Найдём еще p_4:

4-1 $\left.\begin{matrix} p_1 = \alpha V_1 \\ p_4 = \alpha V_4 \end{matrix}\right\} \Rightarrow \frac{p_0}{p_4} = \frac{\alpha}{\alpha} \cdot \frac{V_0}{6V_0} = \frac{1}{6}$

$p_4 = 6p_0 = p_2$

Т.к. $p_4 = p_2$:



3) Найдем η:

$\eta = \frac{A_{цикл}}{Q_н} = \frac{A^* + A^{**}}{Q_{12} + Q_{23}}$

4) $A^* = (6V_0 - V_0)(6p_0 - p_0)\cdot\frac{1}{2} = \frac{25}{2}p_0V_0$

$A^{**} = (6V_0 - V_0)(36p_0 - 6p_0)\frac{1}{2} = \frac{5\cdot 30}{2}p_0V_0 = 75p_0V_0$

$A_4 = A^* + A^{**} = \frac{25}{2}p_0V_0 + 75p_0V_0 = \frac{175}{2}p_0V_0$

5) $Q_{12} = A_{12} + \Delta U_{12}$

$A_{12} = 0 \Rightarrow Q_{12} = \Delta U_{12} = \frac{3}{2}\nu R\cdot\Delta T_{12} =$

$= \frac{3}{2}(\nu RT_2 - \nu RT_1) = \frac{3}{2}(6p_0V_0 - p_0V_0) = \frac{15}{2}p_0V_0.$

6) $Q_{23} = A_{23} + \Delta U_{23}$

$A_{23} = (6p_0 + 36p_0)\cdot\frac{1}{2}\cdot(6V_0 - V_0) = \frac{42}{2}\cdot 5\,p_0V_0 = 105\,p_0V_0$

$\Delta U_{23} = \frac{3}{2}\nu R\Delta T_{23} = \frac{3}{2}(\nu RT_3 - \nu RT_2) =$

$= \frac{3}{2}(6V_0\cdot 36p_0 - 6p_0\cdot V_0) = \frac{3}{2}\cdot 210\,p_0V_0 = 315\,p_0V_0$

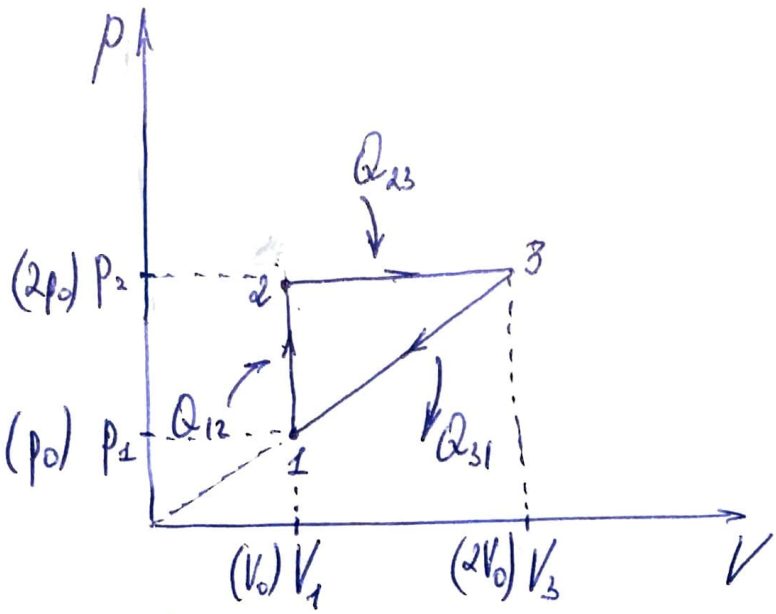
$Q_{23} = A_{23} + \Delta U_{23} = 105p_0V_0 + 315p_0V_0 = 420p_0V_0$

7) $\eta = \frac{A_4}{Q_{\text{н}}} = \frac{\frac{175}{2}p_0V_0}{\frac{15}{2}p_0V_0 + 420p_0V_0} = \frac{175}{15+840} = \frac{175}{855} = \frac{35}{171} \approx$

$\approx 0{,}205$ или $20{,}5\%$

Ответ: $\eta = 20{,}5\%$

(3)



Дано:

1-2 – изохора

2-3 – изобара

3-1 – $p \sim V$

$i = 3$

$\frac{V_3}{V_1} = 2$

Найти: η – ?

Решение:

1) Т.к. $p \sim V$, то $V_1 = V_0$ $V_3 = 2V_0$

и $p_1 = p_0$ $p_2 = p_3 = 2p_0$

2) $A_ц = (2p_0 - p_0)(2V_0 - V_0) \cdot \frac{1}{2} = \frac{1}{2} p_0 V_0$

3) $\eta = \frac{A_ц}{Q_н} = \frac{A_ц}{Q_{12} + Q_{23}}$

4) $Q_{12} = A_{12} + \Delta U_{12}$

$A_{12} = 0$ $\Delta U_{12} = \frac{3}{2} \nu R (T_2 - T_1) = \frac{3}{2}(2p_0 V_0 - p_0 V_0) = \frac{3}{2} p_0 V_0$

$Q_{12} = \frac{3}{2} p_0 V_0$

5) $Q_{23} = A_{23} + \Delta U_{23}$

$A_{23} = 2p_0 (2V_0 - V_0) = 2p_0 V_0$

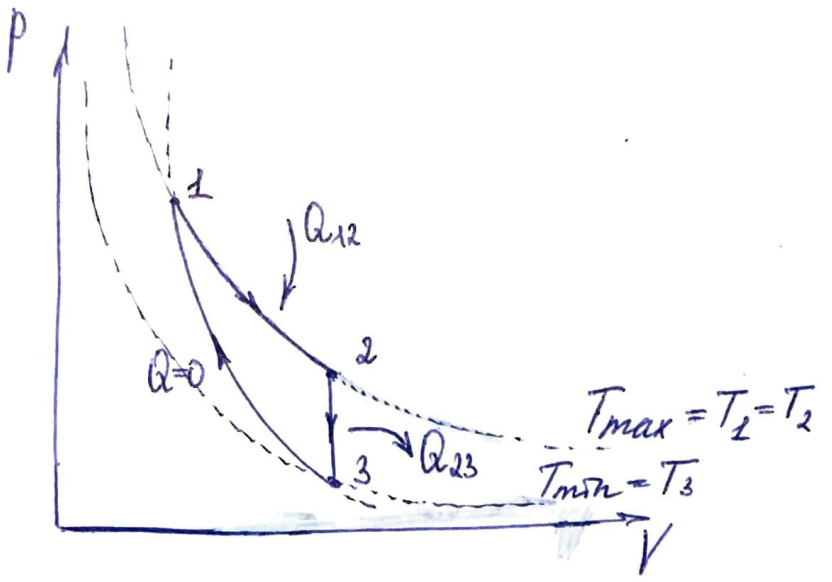
$\Delta U_{23} = \frac{3}{2} \nu R (T_3 - T_2) = \frac{3}{2}(2p_0 2V_0 - 2p_0 V_0) = 3 p_0 V_0.$

$Q_{23} = 5 p_0 V_0$

$\eta = \frac{A_ц}{Q_{12} + Q_{23}} = \frac{\frac{1}{2} p_0 V_0}{\frac{3}{2} p_0 V_0 + 5 p_0 V_0} = \frac{1}{3 + 10} = \frac{1}{13}$

Ответ: $\eta = \frac{1}{13}$

(4)



Дано:

1-2 – изотерма

2-3 – изохора

3-1 – адиабата

КПД цикла: η

$\Delta T = T_{max} - T_{min}$

?

Найти: A_{12} – ?

Решение:

$$\eta = \frac{\sum Q_i}{Q_н} = \frac{Q_{31} + Q_{12} + Q_{23}}{Q_{12}} = \frac{Q_{12} + Q_{23}}{Q_{12}} = 1 + \frac{Q_{23}}{Q_{12}}$$

($Q_{31} = 0$, т.к. 3-1 – адиабата)

$$Q_{23} = A_{23} + \Delta U_{23} = 0 - \frac{3}{2}\nu R \Delta T = -\frac{3}{2}\nu R \Delta T$$

$$Q_{12} = A_{12} + \Delta U_{12} = A_{12} + 0 = A_{12}$$

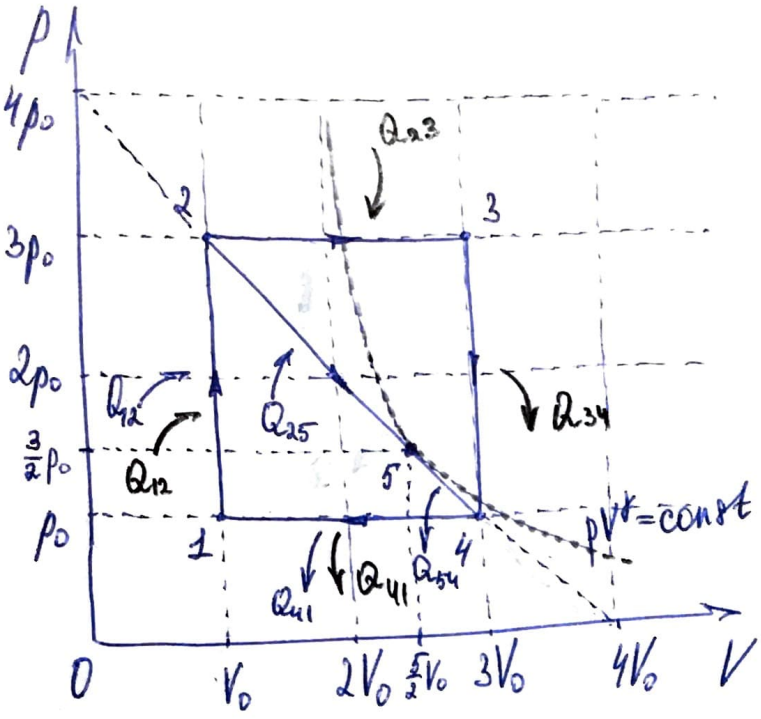
$$\eta = \frac{-\frac{3}{2}\nu R \Delta T}{A_{12}} + 1$$

$$\eta - 1 = -\frac{3}{2} \cdot \frac{\nu R \Delta T}{A_{12}}$$

$$A_{12} = \frac{-\frac{3}{2}\nu R \Delta T}{\eta - 1} = \frac{3\nu R \Delta T}{2(1-\eta)}$$

Ответ: $A_{12} = \frac{3}{2} \cdot \frac{\nu R \Delta T}{1-\eta}$

(5)



Дано:

1-2-3-4-1

и

1-2-4-1 циклы

Найти:

$\eta_{12341} - ?$

$\eta_{1241} - ?$

Решение:

1) $\eta_{12341} = \dfrac{A_y}{Q_н} = \dfrac{A_y}{Q_{12} + Q_{23}}$

$A_y = (3V_0 - V_0)(3p_0 - p_0) = 4p_0V_0$

$Q_{12} = A_{12} + \Delta U_{12} = 0 + \frac{3}{2}\nu R(T_2 - T_1) =$

$= \frac{3}{2}(3p_0V_0 - p_0V_0) = 3p_0V_0$

$Q_{23} = A_{23} + \Delta U_{23} = (3V_0 - V_0)\cdot 3p_0 + \frac{3}{2}(3V_0 \cdot 3p_0 - 3p_0V_0) =$

$= 6p_0V_0 + \frac{3}{2}\cdot 6p_0V_0 = 15p_0V_0$

$\eta_{12341} = \dfrac{4p_0V_0}{3p_0V_0 + 15p_0V_0} = \dfrac{4}{18} = \dfrac{2}{9}$

2) Найдем η_{1241}:

Найдем сначала т.5.:

а) $p_5V_5^{\gamma} = pV^{\gamma}$

б) Из графика 2-4: $p = 4p_0 - \dfrac{p_0}{V_0}\cdot V$

$$\begin{cases} p = p_5V_5^{\gamma}\cdot V^{-\gamma} \\ p = 4p_0 - \dfrac{p_0}{V_0}\cdot V \end{cases} \qquad \begin{cases} p' = p_5V_5^{\gamma}\cdot(-\gamma)\cdot V^{-\gamma-1} \\ p' = -\dfrac{p_0}{V_0} \end{cases}$$

$$\begin{cases} p_5V_5^{\gamma}\cdot V^{-\gamma} = 4p_0 - \dfrac{p_0}{V_0}\cdot V \\ p_5V_5^{\gamma}\cdot V^{-\gamma-1}\cdot(-\gamma) = -\dfrac{p_0}{V_0} \end{cases}$$

(5) Урав-ние:

$$\begin{cases} p_5 V_5^{\gamma} \cdot V^{-\gamma} = 4p_0 - \frac{p_0}{V_0} \cdot V \\ p_5 V_5^{\gamma} \cdot V^{-\gamma-1}(-\gamma) = -\frac{p_0}{V_0} \end{cases}$$

$$\begin{cases} (p_5 V_5^{\gamma} \cdot V^{-\gamma}) = 4p_0 - \frac{p_0}{V_0} \cdot V \\ (p_5 \cdot V_5^{\gamma} \cdot V^{-\gamma}) \cdot V^{-1}(+\gamma) = -\frac{p_0}{V_0} \end{cases}$$

$$-\gamma \cdot \frac{\left(4p_0 - \frac{p_0}{V_0} \cdot V\right)}{V} = -\frac{p_0}{V_0}$$

$$-\gamma \cdot \frac{4 - \frac{V}{V_0}}{V} = -\frac{1}{V_0}$$

$$-\gamma\left(4 - \frac{V}{V_0}\right) = -\frac{V}{V_0}$$

$$\gamma = \frac{C_p}{C_v} = \frac{i+2}{2} \cdot \frac{2}{i} \cdot \frac{R}{R} = \frac{3+2}{3} = \frac{5}{3}$$

$$\frac{5}{3}\left(\frac{V}{V_0} - 4\right) = -\frac{V}{V_0}$$

$$V\frac{5}{3V_0} - \frac{20}{3} = -\frac{V}{V_0}$$

$$V\left(\frac{5}{3} \cdot \frac{1}{V_0} + \frac{1}{V_0}\right) = \frac{20}{3}$$

$$V = \frac{20}{\frac{1}{V_0} 3\left(\frac{5}{3} + 1\right)} = \frac{20V_0}{5+3} = \frac{20V_0}{8} = \frac{5}{2}V_0$$

$$p = 4p_0 - \frac{p_0}{V_0} \cdot \frac{5}{2}V_0 = \frac{3}{2}p_0$$

$$\boxed{V = \frac{5}{2}V_0 \; ; \; p = \frac{3}{2}p_0} - \text{т. 5}$$

Найдем η_{1241}:

$$\eta_{1241} = \frac{A_ц}{Q_н} = \frac{A_{цикла}}{Q_{12} + Q_{25}}$$

$$A_{цикла} = (3V_0 - V_0)(3p_0 - p_0) \cdot \frac{1}{2} = 2p_0V_0$$

$$Q_{12} = A_{12} + \Delta U_{12} = 0 + \frac{3}{2}\nu R(T_2 - T_1) = \frac{3}{2}(3p_0V_0 - p_0V_0) =$$

$$= 3p_0V_0$$

(5) прод-ние:

$$Q_{25} = A_{25} + \Delta U_{25} = \frac{1}{2}\left(3p_0 + \frac{3}{2}p_0\right)\cdot\left(\frac{5}{2}V_0 - V_0\right) + \frac{3}{2}\nu R(T_5 - T_2) =$$

$$= \frac{1}{2}\cdot\frac{9}{2}\cdot\frac{3}{2}p_0V_0 + \frac{3}{2}\left(\frac{3}{2}p_0\frac{5}{2}V_0 - 3p_0V_0\right) =$$

$$= \frac{27}{8}p_0V_0 + \frac{9}{8}p_0V_0 = \frac{9}{2}p_0V_0.$$

$$\eta_{1241} = \frac{A_{\text{цикла}}}{Q_{12} + Q_{25}} = \frac{2p_0V_0}{3p_0V_0 + \frac{9}{2}p_0V_0} = \frac{4}{6+9} = \frac{4}{15}$$

$\eta_{1241} > \eta_{12341}$ и $\eta_{1241} - \eta_{12341} = \frac{2}{45}$

Ответ: $\eta_{1241} = \frac{4}{15}$

$\eta_{12341} = \frac{2}{9}$