Data Analysis 1: Scaling Relations (75 points)

Please read the general instructions in the separate envelope before you start this problem.

Spiral galaxies are disk-like rotating structures, whose dynamical state is fairly grasped by the so-called rotation curves, quantifying the mean rotational velocity of the disk at different distances from the center (see Figure 1, curve B). An interesting feature is the flat region of the curve, which is attributed to the presence of dark matter. Without it, rotation velocities would drop steadily at large radii from the center, as depicted in curve A.

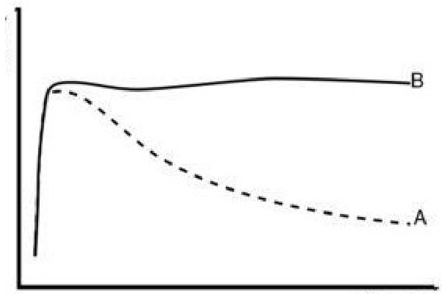

Figure 1: Rotation curves. Circular velocity (Y-axis) vs Radius (X-axis)

In disk galaxies a strong correlation has been observed between the intrinsic luminosity of the whole galaxy and the asymptotic rotational velocity (as given by the rotation curve for the outer edge of the galaxy i.e. R_{max}), a result that is known as the Tully-Fisher relation. This relation also holds if you use the luminosity in a specific band. This is shown on Figure 2 for a number of galaxies in a galaxy cluster. Every dot is a galaxy, and the solid line is the best-fit linear relation between absolute magnitude in K band and $log_{10}(V_{\text{max}})$ for the whole sample.

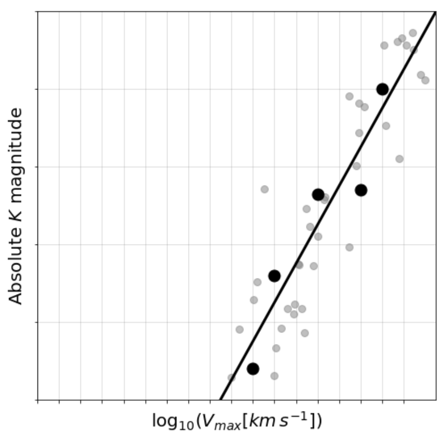

Figure 2: Absolute magnitude in K band vs $\log_{10}(V_{\text{max}}[km s^{-1}])$. Tully-Fisher relation for several galaxies. Every dot represents a galaxy. The dark points are five selected galaxies, for which we will provide some numbers in part 1.2.

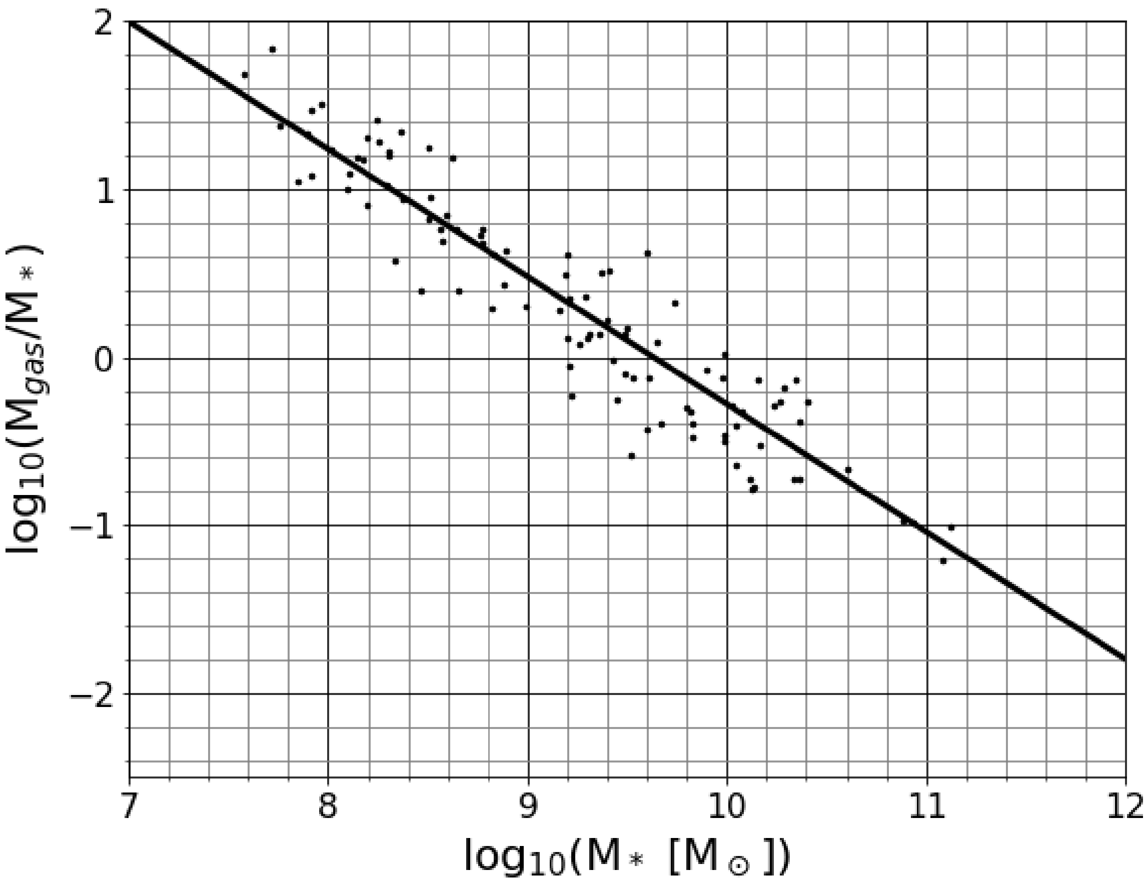

Figure 3: Gas fraction vs stellar mass.

Another interesting trend is shown in Figure 3: disks with larger stellar masses (M_*) tend to have smaller gas fractions (M_{gas}/M_*).

In the following questions you will be asked to extract physical information about the galaxies using the scaling relations just introduced. Consider the following guidelines:

- Assume that V_{max} was measured at the same radius for all galaxies (R_{max}), in the flat part of the rotation curves and well beyond the end of the stellar disk.
- Use M_{dm} for the dark matter mass up to R_{max} and M_{tot} for the sum of all components.(gas, stars and dark matter)
- Assume that all galaxies have identical stellar populations[1], and assume that the gaseous component does not interact with the stellar light. .
- The galaxy cluster is far away. Its distance is much larger than the cluster size.
- In spherically-symmetric mass distributions, to infer the gravitational effect on a particle at distance r from the center, it suffices to consider the total mass enclosed up to that radius $M(\leq r)$ as if it were placed at the very center of the distribution.

[1]The term stellar population refers to the type of stars that are present in a galaxy, and the relative amount of each type with respect to the total number of stars.

Part 1 (20 points).

1.1 From an analysis of Figure 3, find the appropriate constants in the following relation: $M_{gas} = a \times M_*^b$ — 5.0pt

$$a = ?$$

$$b = ?$$

1.2 In the plot of the Tully-Fisher relation there are 5 highlighted points. Data for these 5 galaxies is given in the following table. Use this dataset to find the appropriate constants for TF relation presented below the table, by means of a linear fit using the method of least squares. — 15.0pt

Note: Treat $log_{10}(V_{\text{max}})$ as the x variable and K as the y variable in the linear fit.

$V_{\text{max}}[km/s]$	$K[mag]$
79.4	−16.8
100.1	−19.2
158.5	−21.3
251.2	−21.4
316.2	−24.0

$$K = c \times log_{10}(V_{max}) + d$$

$$c = ?$$

$$d = ?$$

Part 2 (16 points).

For two galaxies, G1 and G2, in the cluster, the recorded *apparent* magnitudes are:

$$k_1 = 19.2 \qquad ; \qquad k_2 = 25.2$$

Using this information and the relations calibrated in Part 1 find the correct exponents in the following equations:

2.1 — 6.0pt

$$\frac{M_{*1}}{M_{*2}} = 10^e \qquad ; \qquad e = ?$$

2.2 4.0pt

$$\frac{M_{gas1}}{M_{gas2}} = 10^f \qquad ; \qquad f = ?$$

2.3 6.0pt

$$\frac{M_{tot1}}{M_{tot2}} = 10^g \qquad ; \qquad g = ?$$

Part 3 (15 points).

3.1 15.0pt

Galaxy	Apparent magnitude k	$M_{gas}[M_\odot]$	$M_*\ [M_\odot]$	$M_{dm}[M_\odot]$	$M_{tot}\ [M_\odot]$
G_1	19.2				4.39×10^{11}

Fill in the missing values in the table using the fact that for galaxy G_1, the dark-to-baryonic mass ratio up to $R_{\max}$ is 6.82.

Part 4 (24 points).

4.1 Consider a systematic uncertainty of $\sigma_{sys} = \pm 0.2$ in each apparent magnitude due to CCD calibration errors. Then k_1 must be read as $k_1 = 19.2 \pm 0.2$, i.e., the only thing we know is that k_1 most likely lies in the interval $[19.0, 19.4]$. The same goes for k_2.
Recalculate the exponent in the scaling relation $\frac{M_{*1}}{M_{*2}} = 10^e$ (found in 2.1), expressing e as an interval estimated by considering the extreme possible variations in $k_1$and k_2 . 4.0pt

$$e \in [?, ?]$$

4.2 Now we consider that there is always a natural spread of the data around any relation. For instance, for a given value of the K magnitude the TF relation gives a single value of $log_{10}(V_{\text{max}})$, but it would be more realistic to report an interval of plausible values, derived from the natural spread of the data around the mean TF relation. We call this the statistical uncertainty, σ_{stat}. **10.0pt**

Estimate the statistical uncertainty if $log_{10}(V_{\text{max}})$ is inferred from K using the TF relation from question 1.2. For this, consider for each point the difference between the value of $log_{10}(V_{\text{max}})$ estimated from K using your linear fit and the actual measurement of $log_{10}(V_{\text{max}})$, and take σ_{stat} as two times the root mean square (RMS) of these differences†.

$$\sigma_{stat} = ?$$

†The RMS of a set of values is the square root of the arithmetic mean of the squares of those values.

4.3 Recalculate the exponent in the scaling relation $\frac{M_{tot1}}{M_{tot2}} = 10^g$, expressing g as an interval estimated by considering the extreme possible variations arising from both the systematic and statistical uncertainties: **10.0pt**

$$g \in [?, ?]$$