2nd
International
Olympiad
on Astronomy
and Astrophysics

The 2nd International Olympiad on Astronomy and Astrophysics

Bandung, Indonesia

Saturday, 23 August 2008

Theoretical Competition

Please read this carefully:

1. *Every student receives problem sheets in English and/or in native language, an answer book and a scratch book.*
2. *The time available is 5 hours for the theoretical competition. There are fifteen short questions (Theoretical Part 1), and three long questions (Theoretical Part 2).*
3. *Use only Black or dark blue pen*
4. *Use only the front side of **answer sheets**. Write only inside the boxed area.*
5. ***Begin answering each question on a separate sheet.***
6. *Numerical results should be written with as many digits as are appropriate.*
7. *Write on the blank **answer sheets** whatever you consider is required for the solution of each question. Please express your answer primarily in term of equations, numbers, figures, and plots. If necessary provide your answers with concise text. Full credit will be given to correct answer with detailed steps for each question. Underline your final result.*
8. *Fill in the boxes at the top of each sheet of paper with your country code and your student code.*
9. *At the end of the exam place the books inside the envelope and leave everything on your desk.*

Astronomical and Physical Constants

Quantity	Value
Astronomical unit (AU)	149 597 870 691 m
Light year (ly)	9.4605×10^{15} m = 63,240 AU
Parsec (pc)	3.0860×10^{16} m = 206,265 AU
Sidereal year	365.2564 days
Tropical year	365.2422 days
Gregorian year	365.2425 days
Sidereal month	27.3217 days
Synodic month	29.5306 days
Mean sidereal day	$23^h56^m4^s.091$ of mean solar time
Mean solar day	$24^h3^m56^s.555$ of sidereal time
Mean distance, Earth to Moon	384 399 000 m
Earth mass ($M_\oplus$)	5.9736×10^{24} kg
Earth mean radius	6 371 000 m
Earth mean velocity in orbit	29 783 m/s
Moon mass ($M_☽$)	7.3490×10^{22} kg
Moon mean radius	1 738 000 m
Sun mass ($M_\odot$)	1.9891×10^{30} kg
Sun radius ($R_\odot$)	6.96×10^8 m
Sun luminosity ($L_\odot$)	3.96×10^{26} J s^{-1}
Sun effective temperature ($T_{eff\odot}$)	5 800 ^{o}K
Sun apparent magnitude ($m_\odot$)	-26.8
Sun absolute magnitude ($M_\odot$)	4.82
Sun absolute bolometric magnitude ($M_{bol\odot}$)	4.72
Speed of light (c)	2.9979×10^8 m/s
Gravitational constant (G)	6.6726×10^{-11} N m^2 kg^{-2}
Boltzmann constant (k)	1.3807×10^{-23} J K^{-1}
Stefan-Boltzmann constant (σ)	5.6705×10^{-8} J s^{-1} m^{-2} K^{-4}
Planck constant (h)	6.6261×10^{-34} J s
Electron charge (e)	1.602×10^{-19} C = 4.803×10^{-10} esu
Electron mass (m_e)	$5.48579903 \times 10^{-4}$ amu = 9.11×10^{-31} kg
Proton mass (m_p)	1.00727647 amu = 1.67268×10^{-27} kg

Neutron mass (m_n)	1.008664904 amu = 1.67499 × 10^{-27} kg
Deuterium nucleus mass (m_d)	2.013553214 amu = 3.34371 × 10^{-27} kg
Hydrogen mass	1.00794 amu = 1.67379 × 10^{-27} kg
Helium mass	4.002603 amu = 1.646723 × 10^{-27} kg

Conversion table

1 Å	0.1 nm = 10^{-10} m
1 barn	10^{-28} m^2
1 G	10^{-4} T
1 erg	10^{-7} J = 1 dyne cm
1 esu	3.3356 × 10^{-10} C
1 amu (atomic mass unit)	1.6606 × 10^{-27} kg
1 atm (atmosphere)	101,325 Pa = 1.01325 bar
1 dyne	10^{-5} N

THEORETICAL PART 1

(300 points for 15 Theoretical Part-1 questions, 20 points for each question)

Show your method of solution step by step in the answer sheets completely as your final answer. The scratch sheet is to be used for your personal calculation and will not be marked. Partial credits will be given for answers without showing method of solution.

1. Two persons, on the equator of the Earth separated by nearly 180° in longitude, observe the Moon's position with respect to the background star field at the same time. If the declination of the Moon is zero, sketch the situation and calculate the difference in apparent right ascension seen by those two persons.

2. On April 2, 2008 a telescope (10 cm diameter, *f*/10) at the Bosscha Observatory was used to observe the Sun and found an active region 0987 (based on the NOAA number) at 8° South and 40° West from the center of the solar disk. The region was recorded with a CCD SBIG ST-8 Camera (1600 × 1200 pixels, (9 μm × 9 μm)/ pixel) and its size was 5 × 4 pixels. According to the Astronomical Almanac, the solar diameter is 32'. How large is the corrected area of the active region in unit of millionth of solar hemisphere (msh)?

3. A full moon occurred on June 19, 2008 at $00^h\,30^m$ West Indonesian Time (local civil time for western part of Indonesia with meridian of 105° E). Calculate the minimum and maximum possible values of duration of the Moon above the horizon for observers at Bosscha Observatory (longitude: 107° 35' 00″.0 E, latitude: 6° 49' 00″.0 S, Elevation: 1300.0 m). Time zone = UT $+7^h\,00^m$.

4. Suppose a star has a mass of 20 $\mathfrak{M}_\odot$. If 20% of the star's mass is now in the form of helium, calculate the helium-burning lifetime of this star. Assume that the luminosity of the star is 100 $L_\odot$, in which 30% is contributed by helium burning. The carbon mass, ^{12}C, is 12.000000 amu. Helium burning to Carbon: $3\,^4He \longrightarrow\,^{12}C + \gamma$.

5. The average temperature of the Cosmic Microwave Background (CMB) is currently T = 2.73 K, and it yields the origin of CMB to be at redshift z_{CMB} =1100. The current densities of the Dark Energy, Dark Matter, and Normal Matter components of the Universe as a whole are $\rho_{DE} = 7.1 \times 10^{-30}$ g/cm^3, $\rho_{DM} = 2.4 \times 10^{-30}$ g/cm^3, and

$\rho_{NM} = 0.5 \times 10^{-30}$ g/cm^3, respectively. What is the ratio between the density of Dark Matter to the density of Dark Energy at the time CMB was emitted, if we assume that the dark energy is vacuum energy?

6. Radio wavelength observations of gas cloud swirling around a black hole in the center of our galaxy show that radiation from the hydrogen spin-flip transition (rest frequency = 1420.41 MHz) is detected at a frequency of 1421.23 MHz. If this gas cloud is located at a distance of 0.2 pc from the black hole and is orbiting in a circle, determine the speed of this cloud and whether it's moving toward or away from us and calculate the mass of the black hole.

7. A main sequence star at a distance 20 pc is barely visible through a certain space-based telescope which can record all wavelengths. The star will eventually move up along the giant branch, during which time its temperature drops by a factor of 3 and its radius increases 100 times. What is the new maximum distance at which the star can still be (barely) visible using the same telescope?

8. Gravitational forces of the Sun and the Moon lead to the raising and lowering of sea water surfaces. Let φ be the difference in longitude between points A and B, where both points are at the equator and A is on the sea surface. Derive the horizontal acceleration of sea water at position A due to Moon's gravitational force at the time when the Moon is above point B according to observers on the Earth (express it in φ, the radius $\boldsymbol{R}$ of Earth, and the Earth-Moon distance r).

9. The radiation incoming to the Earth from the Sun must penetrate the Earth's atmosphere before reaching the earth surface. The Earth also releases radiation to its environment and this radiation must penetrate the Earth's atmosphere before going out to the outer space. In general, the transmittance (t_1) of the Sun radiation during its penetration through the Earth's atmosphere is higher than that of the radiation from the Earth (t_2). Let $T_{\text{eff}\odot}$ be the effective temperature of the Sun, $R_\odot$ the radius of the Sun, $r_\oplus$ the radius of the Earth, and x the distance between the Sun and the Earth. Derive the temperature of the Earth's surface as a function of the aforementioned parameters.

10. The coordinates of the components of Visual Binary Star μ Sco on August 22, 2008 are given in the table below

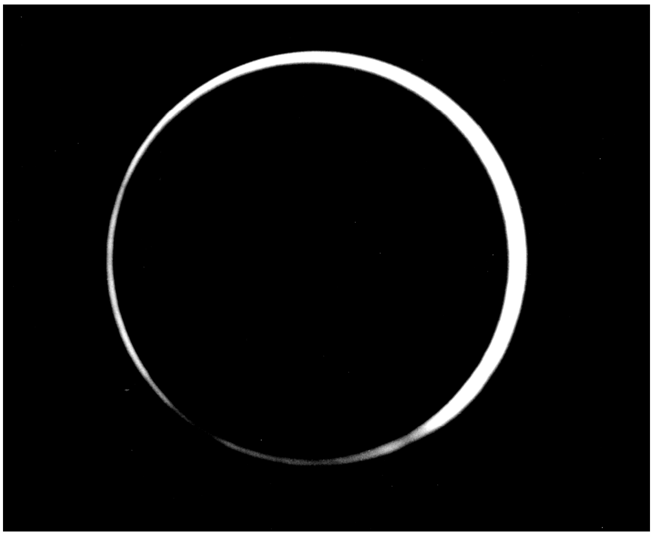
	α(RA)	δ(Dec)
μ Sco 1 (primary)	$20^h\ 17^m\ 38^s.90$	-12° 30' 30"
μ Sco 2 (secondary)	$20^h\ 18^m\ 03^s.30$	-12° 32' 41"

The stars are observed using Zeiss refractor telescope at the Bosscha Observatory with aperture and focal length are 600 mm and 10 780 mm, respectively. The telescope is equipped with 765 × 510 pixels CCD camera. The pixel size of the chip is 9 μm × 9 μm.

a. Can both components of the binary be inside the frame? ("YES" or "NO", show it in your computation!)

b. What is the position angle of the secondary star, with respect to the North?

11. Below is a picture on a 35 mm film of annular solar eclipse in Dumai, Riau, Indonesia on August 22, 1998, taken with a telescope having effective diameter 10 cm and f-ratio 15. The diameter of the Sun's disk in original picture on the film is 13.817 mm and the diameter of the Moon's disk is 13.235 mm. Determine the distances of the Sun and the Moon (expressed in km) from the Earth and the percentage of the solar disk covered by the Moon during the annular solar eclipse.



12. Consider a type Ia supernova, in a distant galaxy, which has a luminosity of $5.8 \times 10^9 L_\odot$ at maximum light. Suppose you observe this supernova using your telescope and find that its brightness is 1.6×10^{-7} times the brightness of Vega. The redshift of its host galaxy is known to be $z = 0.03$. Calculate the distance of this galaxy (in pc) using the data of the supernova and also the Hubble time.

13. In the journey of a space craft, scientists make a close encounter with an object and they would like to investigate the object more carefully using their on-board telescope. For simplicity, we assume this to be a two-dimensional problem and that the position of the space craft is stationary in (0,0). The shape of the object is a disk and the boundary has the equation

$$x^2 + y^2 - 10x - 8y + 40 = 0.$$

Find the exact values of maximum and minimum of tan φ where φ is the angle of the telescope with respect to the x direction during investigation from one edge to the other edge.

14. Consider a Potentially Hazardous Object (PHO) moving in a closed orbit under the influence of the Earth's gravitational force. Let u be the inverse of the distance of the object from the Earth and p be the magnitude of its linear momentum. As the object travels, the graph of u as a function of p passes through points A and B as shown in the following table. Find the mass and the total energy of the object, and express u as a function of p and sketch the shape of u curve from A to B.

	p ($\times 10^9$ kg m s^{-1})	u ($\times 10^{-8}$ m^{-1})
A	0.052	5.15
B	1.94	194.17

15. Galaxy NGC 2639 is morphologically identified as an Sa galaxy with measured maximum rotational velocity $v_{\max}$ of 324 km/s. After corrections for any extinction, its apparent magnitude in B is $m_B = 12.22$. It is customary to measure a radius R_{25} (in units of kpc) at which the galaxy's surface brightness falls to 25 $\mathrm{mag}_B/\mathrm{arcsec}^2$. Spiral galaxies tend to follow a typical relation:

$$\log R_{25} = -0.249 M_B - 4.00,$$

where M_B is the absolute magnitude in B. Apply the B-band Tully-Fisher relation for Sa spirals

$$M_B = -9.95 \log v_{max} + 3.15 \quad (v_{max} \text{ in km/s})$$

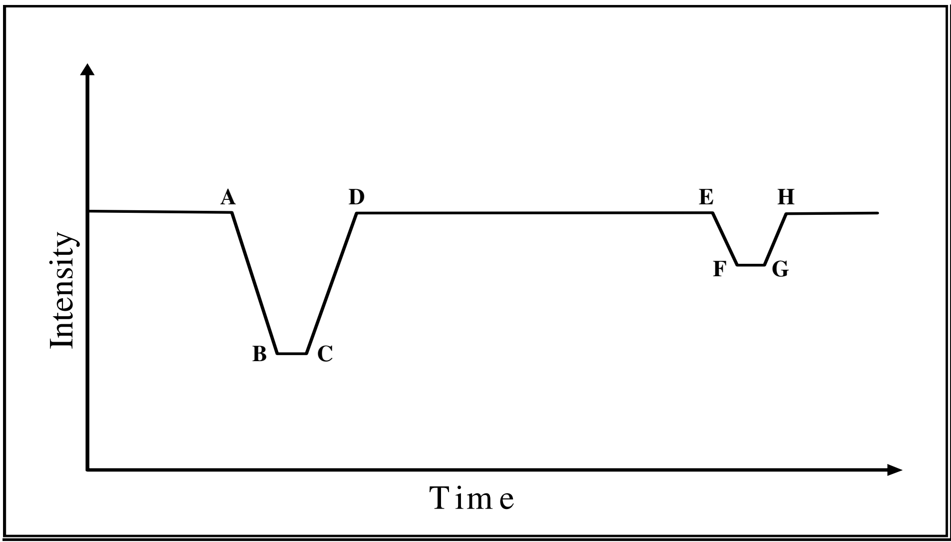
to calculate the mass of NGC 2639 out to R_{25}. If colour index of the sun is $(m_{B_\odot} - m_{V_\odot}) = 0.64$, write the mass (of NGC 2639) in units of solar mass $\mathfrak{M}_\odot$ and its luminosity B-band in unit of $L_\odot$.

THEORETICAL PART 2

(300 points for 3 Theoretical Part-2, 100 points for each question)

Show your method of solution step by step in the answer sheets completely as your final answer. The scratch sheet is to be used for your personal calculation and will not be marked. Partial credits will be given for answers without showing method of solution.

1. An eclipsing binary star system has a period of 30 days. The light curve in the figure below shows that the secondary star eclipses the primary star (from point A to point D) in 8 hours (measured from the time of first contact to final contact), whereas from point B to point C, the total eclipse period is 1 hour and 18 minutes. The spectral analysis yields the maximum radial velocity of the primary star to be 30 km/s and of the secondary star to be 40 km/s. If we assume that the orbits are circular and has an inclination of $i = 90^o$, determine the radii and the masses of both stars in unit of solar radius and solar mass.



2. A *UBV* photometric (*UBV* Johnson's) observation of a star gives U = 8.15, B = 8.50, and V = 8.14. Based on the spectral class, one gets the intrinsic color $(U - B)_o$ = -0.45. If the star is known to have radius of 2.3 $R_\odot$, absolute bolometric magnitude of -0.25, and bolometric correction (BC) of -0.15, determine:
   a. the intrinsic magnitudes *U, B*, and *V* of the star (take, for the typical interstellar matters, the ratio of total to selective extinction (color excess) R_V = 3.2),
   b. the effective temperature of the star,
   c. the distance to the star in pc.

Note: The relation between color excess of U - B and of $B - V$ is $E(U - B) = 0.72\ E(B - V)$.

Let A_v be the interstellar extinction and $R = 3.2$, then $A_v = 3.2\ E(B\text{-}V)$.

3. Measurement of the cosmic microwave background radiation (CMB) shows that its temperature is practically the same at every point in the sky to a very high degree of accuracy. Let us assume that light emitted at the moment of recombination ($T_r \approx 3000$ K, $t_r \approx 300000$ years) is only reaching us now ($T_o \approx 3$ K, $t_o \approx 1.5 \times 10^{10}$ years). Scale factor S is defined as such $S_0 = S(t = t_o) = 1$ and $S_t = S(t < t_o) < 1$. Note that the radiation dominated period was between the time when the inflation stopped ($t = 10^{-32}$ seconds) and the time when the recombination took place, while the matter dominated period started at the recombination time. During the radiation dominated period S is proportional to $t^{1/2}$, while during the matter dominated period S is proportional to $t^{2/3}$.

a. Estimate the horizon distances when recombination took place. Assume that temperature T is proportional to $1/S$, where S is a scale factor of the size of the Universe.

b. Note: Horizon distance in degrees is defined as maximum separation between the two points in CMBR imprint such that the points could "see" each other at the time when the CMBR was emitted.

c. Consider two points in CMBR imprint which are currently observed at a separation angle $\alpha = 5°$. Could the two points communicate with each other using photon? (Answer with "YES" or "NO" and give the reason mathematically)

d. Estimate the size of our Universe at the end of inflation period.

The 2nd International Olympiad on Astronomy and Astrophysics
Bandung, Indonesia

Thursday, August 21, 2008

Practical Competition: Observations

PART I: Naked Eye

Instructions:

1. *All participants will receive a naked-eye observation problem set, a writing board, a pen, a ruler and a flash light at the examination room.*
2. *Part I consists of three steps: distribution of problem sheets in examination room, naked eye observation on the observing ground (7 minutes), and returning to the examination room to answer questions (8 minutes). Bell ring will indicate the beginning and the end of each step.*
3. *All participants will be guided by assistants to go to the observing ground until return to the examination room. Assistants will collect the answer sheets from your table, after the 8 minute time is up.*
4. *Never forget to fill in the boxes at the top of each answer sheet with your country and your student codes. Otherwise, it will be ignored.*

Country Code	Student Code

PROBLEM

Figure 1 shows a part of the southern sky chart for August 21, 2008 at 07.00 p.m. local time. Unfortunately, a number of bright stars in Capricorn and Scorpio constellations are missing. Now, you have to find those missing bright stars in both constellations by looking at the sky directly. To help you remember, the common names of many bright stars are listed in Table 1. Draw small circles on the locations of the missing bright stars in the Capricorn and Scorpio constellations (Point: 60) and identify them by **putting the numbers on the sky chart**, as many as possible based on the Table 1 (Point: 60). Afterwards, draw on the sky chart, the border of Scorpio constellation (Point: 15) and Capricorn constellation (Point: 15).

Table 1: The number and common name of bright stars

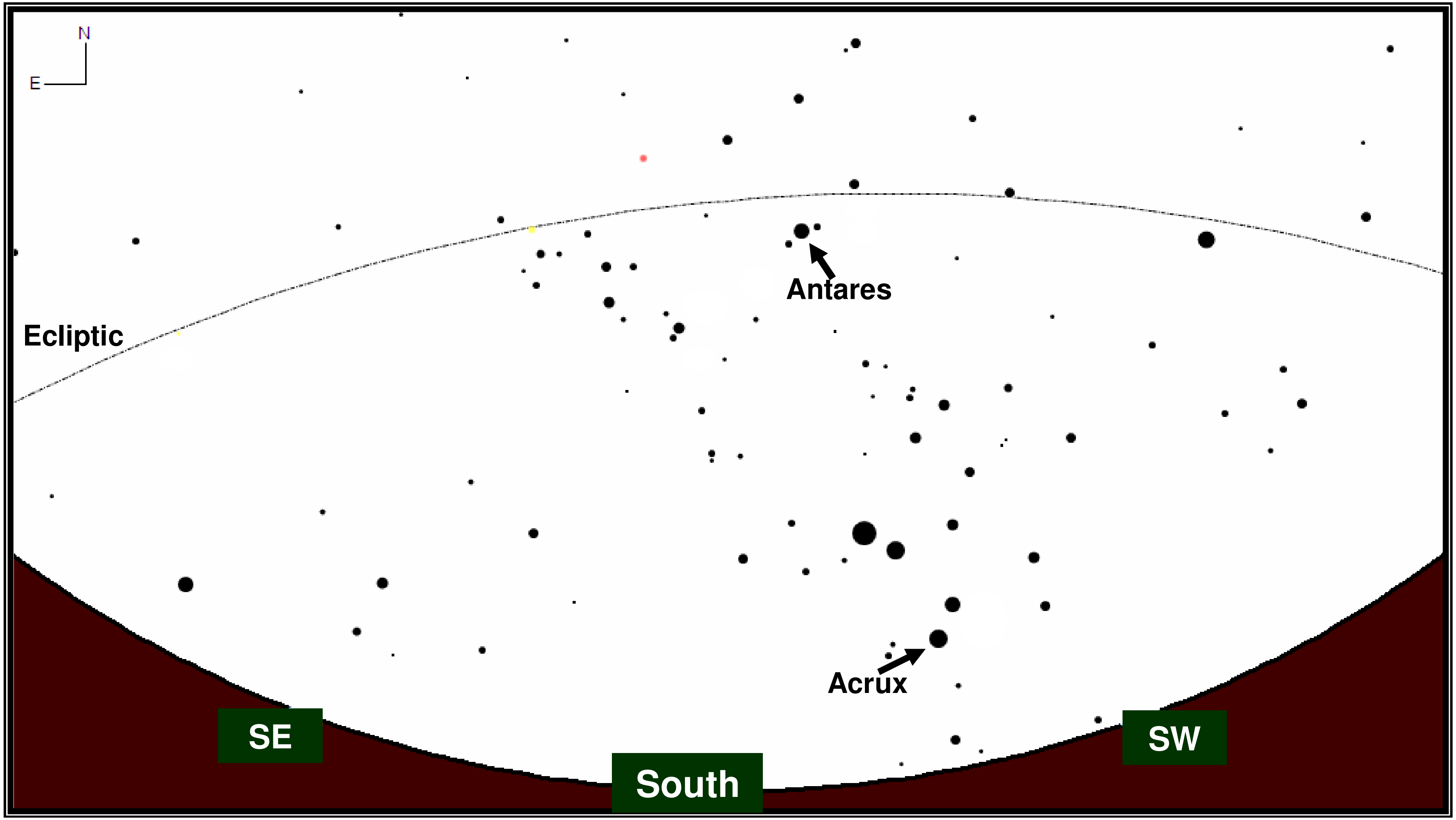
Number	Common Names	Number	Common Names
1	Rukbat (α Sgr)	18	Albali (ε Aqr)
2	Graffias (β Sco)	19	Altair (α Aql)
3	Nunki (σ Sgr)	20	Shaula (λ Sco)
4	Deneb (α Cyg)	21	Vrischika (π Sco)
5	Zaniah (η Vir)	22	Arich (γ Vir)
6	Tarazed (γ Aql)	23	Deneb Algedi (δ Cap)
7	Dabih (β Cap)	24	Heze (ζ Vir)
8	Girtab (κ Sco)	25	Nusakan (β CrB)
9	Spica (α Vir)	26	Wei (ε Sco)
10	Sabik (η Oph)	27	Syrma (ι Vir)
11	Dschubba (δ Sco)	28	Nashira (γ Cap)
12	Kaus Australis (ε Sgr)	29	Lesath (υ Sco)
13	Algiedi (α Cap)	30	Zavijava (β Vir)
14	Sadr (γ Cyg)	31	Arcturus (α Boo)
15	Vindemiatrix (ε Vir)	32	Megrez (δ UMa)
16	Antares (α Sco)	33	Chara (β CVn)
17	Yen (ζ Cap)	34	Sargas (θ Sco)

PART I: Naked Eye

Sky chart at 19.00 on August 21, 2008

Country and Student Code: ....

Figure 1



The 2nd International Olympiad on Astronomy and Astrophysics

Bandung, Indonesia

Thursday, August 21, 2008

Practical Competition: Observations

PART II: Using Telescope and CCD

Instructions:

1. *Part II consists of four steps: distributions of problem sheets in examination room, using telescope and CCD observations on the observing ground (20 minutes), print images and return to the examination room to answer questions (10 minutes). Bell ring will indicate the beginning and the end of each step. Works completed after the allocated time will not be considered.*
2. *All participants will be guided by assistants to go to the observing ground until return to the examination room. Assistants will collect the answer sheets from your table.*
3. *Never forget to fill in the boxes at the top of each answer sheet with your country and your student codes. Otherwise, it will be ignored.*

Country Code	Student Code

PROBLEM

Participants have to identify as many stars as possible in the field of a celestial photograph using the telescope and CCD provided by the committee. Participants have to choose one out of the five recommended regions in the sky listed below. Then, point the telescope to the direction of the selected sky region. Take three photographs with different exposure times and record the images of the sky by the CCD camera. Save the observational data. Transfer the data to the printing facilities to print out the result. Ask the technical assistant for a help. Choose the best prints-out and use the image to identify the stars in the field of observations. The following procedures are,

1. Choose only one out of the following five recommended regions to the directions of (marked by the following bright clusters):

   i. M7 ($\alpha=17^h53.^m3$, $\delta= -34°46.'2$)
   ii. M8 ($\alpha=18^h04.^m2$, $\delta= -24°22.'0$)
   iii. M20 ($\alpha=18^h02.^m4$, $\delta= -22°58.'7$)
   iv. M21 ($\alpha=18^h04.^m2$, $\delta= -22°29.'5$)
   v. M23 ($\alpha=17^h57.^m0$, $\delta= -18°59.'4$)

   You may not change your choice.
2. Type in your country code, your student code, and your choice of region into the answer page provided in the computer.
3. Point the telescope to the chosen cluster by using telescope controller. If necessary you may move the telescope slightly to get the best position in the frame of the CCD by checking the display of CCDops software.
4. Display the region in The Sky map software provided in the computer, to confirm that the telescope is pointed to the selected object in the sky. You may change field of view of the sky chart.
5. You may invert the background images into white color, as in the chart mode. Copy and paste the sky chart from The Sky into the answer page. Use “Ctrl c” and “Ctrl v” buttons from the keyboard, respectively.

6. Type the equatorial coordinates of the centre of that object in the answer page as indicated in The Sky.
7. Take three photographs of the chosen object by using the attached CCD camera and CCDops software, with various exposure times.

   Choose exposure time in the range between 1 and 120 seconds. Image is automatically subtracted with dark frame with the same exposure time.
8. You must invert the background images into white color. Copy and paste images from CCDops into the answer page. Use “Ctrl c” and “Ctrl v” buttons from the keyboard, respectively.
9. Save your answer page into hard disk in a Microsoft Word file format.
10. Print your answer page which consists of the photographs and the corresponding sky chart.
11. Go to the identification room and bring with you the prints-out of the sky chart and the photographs.

    Ask some help from technical assistant, if necessary.
12. Choose the best out of the three printed images and identify as many objects as possible on it.
13. Use the assigned computer and The Sky software to identify objects. Type your identification to your answer page.
14. Type on the answer page the names (or catalogue number), RA, Dec, and magnitude of each identified star and put the sequential number on the photograph.

    Make sure that you list the stars on the answer page following the same order and number as on the photograph.
15. Estimate the limiting magnitude of the photograph you choose, empirically.

The 2nd International Olympiad on Astronomy and Astrophysics

Bandung, Indonesia

Thursday, 21 August 2008

Practical Competition: Data Analysis

Please read this carefully:

1. *Every student receives problem sheets in English and/or in his/her native language, answer sheets, millimeter block papers, and scratch sheets.*
2. *The time available is five hours for the data analysis and observation competitions. There are three data analysis problems, and two observation problems.*
3. *Use only the materials provided.*
4. *Fill in the boxes at the top of each sheet of paper with your country code and your student code.*
5. *Use only the front side of **answer sheets**. Write only inside the boxed area.*
6. *Begin answering each question on a separate sheet.*
7. *Numerical results should be written with as many digits as are appropriate.*
8. *Write on **the answer sheets and the millimeter block papers** whatever you consider is required for the solution of each question. Please express your answer primarily in term of equations, numbers, figures, and plots. If necessary provide your answers with concise text. Full credit will be given to correct answer with detailed steps for each question. Underline your final result.*
9. *At the end of the exam place the answer sheets and the millimeter block papers inside the envelope and leave everything on your desk.*

Astronomical and Physical Constants

Quantity	Value
Astronomical unit (AU)	149,597,870.691 km
Light year (ly)	9.4605×10^{17} cm = 63,240 AU
Parsec (pc)	3.0860×10^{18} cm = 206,265 AU
Sidereal year	365.2564 days
Tropical year	365.2422 days
Gregorian year	365.2425 days
Sidereal month	27.3217 days
Synodic month	29.5306 days
Mean sidereal day	$23^h56^m4^s.091$ of mean solar time
Mean solar day	$24^h3^m56^s.555$ of sidereal time
Mean distance, Earth to Moon	384,399 km
Earth mass ($\mathfrak{M}_\oplus$)	5.9736×10^{27} g
Earth's mean radius	6,371.0 km
Earth's mean velocity in orbit	29.783 km/s
Moon's mass ($\mathfrak{M}_☽$)	7.3490×10^{25} g
Moon's mean radius	1,738 km
Sun mass ($\mathfrak{M}_\odot$)	1.9891×10^{33} g
Mean Earth radius	6.3710×10^{6} cm
Sun radius ($R_\odot$)	6.96×10^{10} cm
Sun luminosity ($L_\odot$)	3.96×10^{33} erg s^{-1}
Sun effective temperature ($T_{eff\odot}$)	5 800 ^{o}K
Sun apparent magnitude ($m_\odot$)	-26.8
Sun bolometric magnitude ($m_{bol\odot}$)	-26.79
Sun absolute magnitude ($M_\odot$)	4.82
Sun absolute bolometric magnitude ($M_{bol\odot}$)	4.72
Speed of light (c)	2.9979×10^{10} cm/s
Gravitational constant (G)	6.6726×10^{-8} dyne cm^2 g^{-2}
Boltzmann constant (k)	1.3807×10^{-16} erg. K^{-1}
Stefan-Boltzmann constant (σ)	5.6705×10^{-5} erg cm^{-2} K^{-4} s^{-1}
Planck constant (h)	6.6261×10^{-27} erg s
Electron charge (e)	1.602×10^{-19} C = 4.803×10^{-10} esu
Electron mass (m_e)	$5.48579903 \times 10^{-4}$ amu

Proton mass (m_p)	1.007276470 amu
Neutron mass (m_n)	1.008664904 amu
Deuterium nucleus mass (m_d)	2.013553214 amu
Hydrogen mass	1.00794 amu
Helium mass	4.002603 amu
Carbon mass	12.01070 amu

Conversion table	
1 Å	0.1 nm
1 barn	10^{-28} m^2
1 G	10^{-4} T
1 erg	10^{-7} J = 1 dyne cm
1 esu	3.3356×10^{-10} C
1 amu (atomic mass unit)	1.6606×10^{-24} g
1 atm (atmosphere)	101,325 Pa = 1.01325 bar
1 dyne	10^{-5} N

300 points for 3 problems, 100 points for each problem

I. Virgo Cluster

The Virgo cluster of galaxies is the nearest large cluster which extends over nearly 10 degrees across the sky and contains a number of bright galaxies. It is very interesting to find the distance to Virgo and to deduce certain cosmological information from it. The table below provides the distance estimates using various distance indicators (listed in the left column). The right column lists the mean distance $d_i \pm$ the standard deviation s_i.

i	**Distance Indicator**	**Virgo Distance (Mpc**
1	Cepheids	14.9 ± 1.2
2	Novae	21.1 ± 3.9
3	Planetary Nebulae	15.2 ± 1.1
4	Globular Cluster	18.8 ± 3.8
5	Surface Brightness Fluctuation	15.9 ± 0.9
6	Tully-Fisher relation	15.8 ± 1.5
7	Faber-Jackson relation	16.8 ± 2.4
8	Type Ia Supernovae	19.4 ± 5.0

1. By applying a weighted mean, compute the average distance (which can be taken as an estimate to the distance to Virgo)

$$d_{avg} = \frac{\sum_i \frac{d_i}{s_i^2}}{\sum_i \frac{1}{s_i^2}}$$

   where the sum runs over the eight distance indicator used.

2. What is the uncertainty (rms) (in unit of Mpc) in that estimate?
3. Spectra of the galaxies in Virgo indicate an average recession velocity of 1136 km/sec for the cluster. Can you estimate the Hubble constant H_0 and its uncertainty (rms)?
4. What is the Hubble Time (age of the universe) using the value of Hubble constant you found and the uncertainty (rms)?

II. Determination of stellar masses in a visual binary system

The star α-Centauri (Rigel Kentaurus) is a triple star which consists of two main-sequence stars α-Centauri A and α-Centauri B representing visual binary system, and the third star, called Proxima Centauri, which is smaller and fainter than the other two stars. The angular distance between α-Centauri A and α-Centauri B is 17.59". The binary system has an orbital period of 79.24 years. The visual magnitudes of α-Centauri A and α-Centauri B are -0.01 and 1.34 respectively. Their color indices are 0.65 and 0.85 respectively. Use the data below to answer the following questions.

Data for main-sequence stars

$(B-V)_0$	T_{eff}	BC
-0.25	24500	2.30
-0.23	21000	2.15
-0.20	17700	1.80
-0.15	14000	1.20
-0.10	11800	0.61
-0.05	10500	0.33
0.00	9480	0.15
0.10	8530	0.04
0.20	7910	0
0.30	7450	0
0.40	6800	0
0.50	6310	0.03
0.60	5910	0.07
0.70	5540	0.12
0.80	5330	0.19
0.90	5090	0.28
1.00	4840	0.40
1.20	4350	0.75

BC=Bolometric Correction, $(B-V)_0$=Intrinsic Color

Questions:

1. Plot the curve BC versus $(B-V)_0$.

2. Determine the apparent bolometric magnitudes of α-Centauri A and α-Centauri B using the corresponding curve.
3. Calculate the mass of each star.

Notes:

1. **Bolometric correction** (BC) is a correction that must be made to the apparent magnitude of an object in order to convert an object's visible magnitude to its bolometric magnitude:

$$BC = m_v - m_{bol} \text{ or } BC = M_v - M_{bol}$$

2. **Luminosity mass relation** : $M_{bol} = -10.2\log\left(\frac{M}{M_\odot}\right) + 4.9$

III. The Age of Meteorite

The basic equation of radioactive decay can be expressed as:

$$N(t) = N_0 \exp(-\lambda t)$$

where $N(t)$ and N_0 are the number of remaining atoms of the radioactive isotope (or parent isotope) at time t and its initial number at $t = 0$, respectively, while λ is the decay constant. The decay of the parent produces daughter nuclides $D(t)$, or radiogenics, which is defined as

$$D(t) = N_0 - N(t) \;.$$

Based on those ideas, a group of astronomers investigates a number of meteorite samples to determine their ages. They have two kinds of samples: allende chondrite (A) and basaltic achondrite (B). From the samples, they measure the abundance of ^{87}Rb and ^{87}Sr, where it is assumed that ^{87}Sr is entirely produced by the decay of ^{87}Rb. The value of λ is 1.42×10^{-11} per year for this isotopic decay. In addition, non-radiogenic element ^{86}Sr is also measured. Results of measurement are given in the table below, expressed in ppm (part per million).

Sample No	Meteorite type	^{86}Sr (ppm)	^{87}Rb (ppm)	^{87}Sr (ppm)
1	A	29.6	0.3	20.7
2	B	58.7	68.5	44.7
3	B	74.2	14.4	52.9
4	A	40.2	7.0	28.6
5	A	19.7	0.4	13.8
6	B	37.9	31.6	28.4
7	A	33.4	4.0	23.6
8	B	29.8	105.0	26.4
9	A	9.8	0.8	6.9
10	B	18.5	44.0	15.4

<u>Questions:</u>

1. Express the time t in term of $\dfrac{D(t)}{N(t)}$
2. Determine the half-life $t_{1/2}$, i.e., the time needed to obtain a half number of parents after decay.
3. Knowledge on the ratio between two isotopes is more valuable than just the absolute abundance of each isotope. It is quite likely that there was some initial strontium present. By taking $\left(\dfrac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right)$ as independent variable and $\left(\dfrac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)$ as dependent variable, estimate the simple linear regression model to represent the data.
4. Plot $\left(\dfrac{^{87}\mathrm{Rb}}{^{86}\mathrm{Sr}}\right)$ versus $\left(\dfrac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)$ and also the regression line (isochrone) for each type of the meteorites. (Please use minimum 7 decimal digits for intermediate calculations)
5. Subsequently, help this astronomer to determine the age of each type of the meteorites and its error. Which type is older?
6. Determine the initial value of $\left(\dfrac{^{87}\mathrm{Sr}}{^{86}\mathrm{Sr}}\right)_0$ for each type of the meteorites and its error.

Glossary:

A simple linear regression line $y=a+bx$ can be fitted to a set of data (X_i,Y_i), $i=1,\ldots,n$, in which

$$b = \frac{SS_{xy}}{SS_{xx}}$$

$$a = \bar{y} - b\bar{x}$$

where

SS_{xx} : sum of square for $X = \sum_{i=1}^{n} X_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} X_i\right)^2$

SS_{yy} : sum of square for $Y = \sum_{i=1}^{n} Y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} Y_i\right)^2$

SS_{xy} : sum of square for both X and $Y = \sum_{i=1}^{n} X_i Y_i - \frac{1}{n}\sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i$

Standard deviation of each parameter, a and b can be calculated by

$$S_a = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}} \times \sum_{i=1}^{n} X_i^2}$$

$$S_b = \sqrt{\frac{SS_{YY} - \frac{(SS_{XY})^2}{SS_{XX}}}{(n-2)SS_{XX}}}$$