The 3rd International Olympiad on Astronomy and Astrophysics

Tehran, Iran

Problems and solutions

- Theoretical Problems
- Practical Problems
- Observasional Problems
- Students Answer sheets

Theoretical Competition

Please read these instructions carefully:

1. Each student will receive problem sheets in English and/or in his/her native language.
2. The available time for answering theoretical problems is 5 hours. You will have 15 short problems (Theoretical Part 1, Problem 1 to 15), and 2 long problems (Theoretical Part 2, Problem 16 and 17).
3. Use only the pen that is provided on your desk.
4. Do **Not** use the back side of your writing sheets. Write only inside the boxed area.
5. Yellow scratch papers are not considered in marking.
6. Begin answering each problem in separate sheet.
7. Fill in the boxes at the top of each sheet of your paper with your "country name", your "student code", "problem number", and total number of pages which is used to answer to that problem.
8. **Write the final answer for each problem in the box, labeled "Answer Sheet".**
9. Starting and the end of the exam will be announced by ringing a bell.
10. The final answer in each question must be accompanied by units, which should be in SI or appropriate units as specified in the problem. 20% of the marks available for that part will be deducted for a correct answer without units.
11. The required numerical accuracy for the final answer depends on the number of significant figures given in the data values in the problem. 20% of the marks available for the final answer in each question part will be deducted for answers without required accuracy as given in the problem. Use the constant values exactly as given in the table of constants.
12. At the end of the exam put all papers, including scratch papers, inside the envelope and leave everything on your desk.

Table of Constants

(All constants are in SI)

Parameter	Symbol	Value
Gravitational constant	G	$6.67 \times 10^{-11} N\ m^2\ kg^{-2}$
Plank constant	h	$6.63 \times 10^{-34} \mathrm{J\ s}$
	$\hbar$	$1.05 \times 10^{-34}\ J\ s$
Speed of light	c	$3.00 \times 10^{8}\ m\ s^{-1}$
Solar Mass	$M_\odot$	$1.99 \times 10^{30}\ kg$
Solar radius	$R_\odot$	$6.96 \times 10^{8}\ m$
Solar luminosity	$L_\odot$	$3.83 \times 10^{26}\ W$
Apparent solar magnitude (V)	$m_\odot$	-26.8
Solar constant	$b_\odot$	$1.37 \times 10^{3}\ W\ m^{-2}$
Mass of the Earth	$M_\oplus$	$5.98 \times 10^{24}\ kg$
Radius of the Earth	$R_\oplus$	$6.38 \times 10^{6}\ m$
Mean density of the Earth	$\rho_\oplus$	$5 \times 10^{3}\ kg\ m^{-3}$
Gravitational acceleration at sea level	g	$9.81\ m\ s^{-2}$
Tropical year		365.24 days
Sidereal year		365.26 days
Sidereal day		86164 s
Inclination of the equator with respect to the ecliptic	ε	23.5°
Parsec	pc	$3.09 \times 10^{16}\ m$
Light year	ly	$9.46 \times 10^{15}\ m$
Astronomical Unit	AU	$1.50 \times 10^{11}\ m$
Solar distance from the center of the Galaxy	R	$8 \times 10^{3} pc$
Hubble constant	H	$75\ kms^{-1}\ M\ pc^{-1}$
Mass of electron	m_e	$9.11 \times 10^{-31}\ kg$
Mass of proton	m_p	$1.67 \times 10^{-27}\ kg$
Central wavelength of V-band	λ	$550\ n\ m$
Refraction of star light at horizon		$34'$
	π	3.1416

Useful mathematical formula:

$$\ln(1+x) \sim x \qquad for\ \ x \to 0$$

Short Problems: (10 points each)

Problem 1: Calculate the mean mass density for a super massive black hole with total mass of $1\times10^8\ M_\odot$ inside the Schwarzschild radius.

Problem 2: Estimate the number of photons per second that arrive on our eye at $\lambda = 550\ nm$ (V-band) from a G2 main sequence star with apparent magnitude of $m = 6$ (the threshold of naked eye visibility). Assume the eye pupil diameter is $6\ mm$ and all the radition from this star is in $\lambda = 550\ nm$.

Problem 3: Estimate the radius of a planet that a man can escape its gravitation by jumping vertically. Assume density of the planet and the Earth are the same.

Problem 4: In a typical Persian architecture, on top of south side windows there is a structure called "Tabeshband" (shader), which controls sunlight in summer and winter. In summer when the Sun is high, Tabeshband prevents sunlight to enter rooms and keeps inside cooler. In the modern architecture it is verified that the Tabeshband saves about 20% of energy cost. Figure (1) shows a vertical section of this design at latitude of $36°.0$ N with window and Tabeshband.

Using the parameters given in the figure, calculate the maximum width of the Tabeshband, "x", and maximum height of the window , "h" in such a way that:

i) No direct sunlight can enter to the room in the summer solstice at noon.
ii) The direct sunlight reaches the end of the room (indicated by the point **A** in the figure) in the winter solstice at noon.

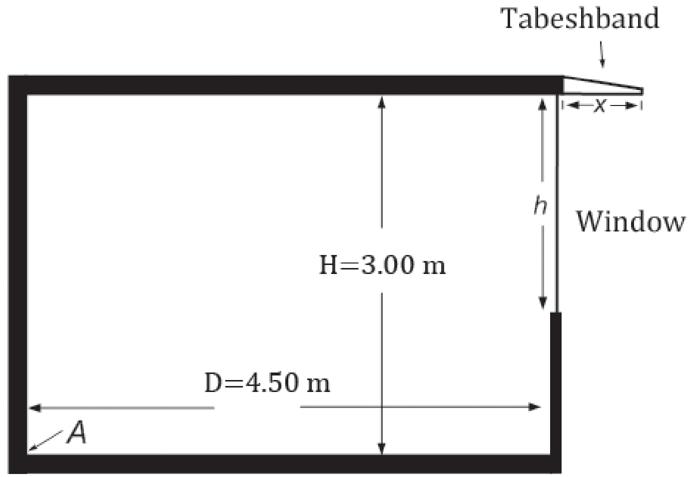


Figure (1)

Problem 5: The Damavand Mountain is located at the North part of Iran, in south coast of Caspian Sea. Consider an observer standing on the Damavand mountain top (latitude = $35°57'$N; longitude = $52°6'$E; altitude $5.6 \times 10^3\ m$ from the mean sea level) and looking at the sky over the Caspian Sea. What is the minimum declination for a star, to be seen marginally circumpolar for this observer. Geodetic radius of the Earth at this latitude is $6370.8\ km$. Surface level of the Caspian Sea is approximately equal to the mean sea level.

Problem 6: Derive a relation for the escape velocity of an object, launched from the center of a proto-star cloud. The cloud has uniform density with the mass of M and radius R. Ignore collisions between the particles of the cloud and the launched object. If the object were allowed to fall freely from the surface, it would reach the center with a velocity equal to $\sqrt{\frac{GM}{R}}$.

Problem 7: A student tries to measure field of view (FOV) of the eyepiece of his/her telescope, using rotation of the Earth. To do this job, the observer points the telescope towards Vega (alpha Lyr., RA: 18.5^h, Dec: $+39°$), turns off its "clock drive" and measures trace out time, t=5.3 minutes, that Vega crosses the full diameter of the FOV. What is the FOV of this telescope in arc-minutes?

Problem 8: Estimate the mass of a globular cluster with the radius of $r = 20\,pc$ and root mean square velocity of stars equal to $v_{rms} = 3\,kms^{-1}$.

Problem 9: The Galactic longitude of a star is $l = 15°$. Its radial velocity with respect to the Sun is $V_r = 100\,kms^{-1}$. Assume stars in the disk of the Galaxy are orbiting the center with a constant velocity of $V_0 = 250\,kms^{-1}$ in circular orbits in the same sense in the galactic plane. Calculate distance of the star from the center of the Galaxy.

Problem 10: A main sequence star with the radius and mass of $R = 4R_{\odot}, M = 6M_{\odot}$ has an average magnetic field of 1×10^{-4} T. Calculate the strength of the magnetic field of the star when it evolves to a neutron star with the radius of $20\,km$.

Problem 11: Assume the mass of neutrinos is $m_{\nu} = 10^{-5} m_e$. Calculate the number density of neutrinos (n_{ν}) needed to compensate the dark matter of the universe. Assume the universe is flat and 25 % of its mass is dark matter. Hint: Take the classical total energy equal to zero

Problem 12: Calculate how much the radius of the Earth's orbit increases as a result of the Sun losing mass due to the thermo-nuclear reactions in its center in 100 years. Assume the Earth's orbit remains circular during this period.

Problem 13: Assume that you are living in the time of Copernicus and do not know anything about Kepler's laws. You might calculate Mars-Sun distance in the same way as he did. After accepting the revolutionary belief that all the planets are orbiting around the Sun, not around the Earth, you measure that the orbital period of Mars is 687 days, then you observe that 106 days after opposition of Mars, the planet appears in quadrature. Calculate Mars-Sun distance in astronomical unit (AU).

Problem 14: A satellite is orbiting around the Earth in a circular orbit in the plane of the equator. An observer in Tehran at the latitude of $\varphi = 35.6^{\circ}$ N observes that the satellite has a zenith angle of $z = 46.0°$, when it transits the local meridian. Calculate the distance of the satellite from the center of the Earth (in the Earth radius unit).

Problem 15: An eclipsing close binary system consists of two giant stars with the same sizes. As a result of mutual gravitational force, stars are deformed from perfect sphere to the prolate spheroid with $a = 2b$, where a and b are semi-major and semi-minor axes (the major axes are always co-linear). The inclination of the orbital plane to the plane of sky is $90°$. Calculate the amplitude of light variation in magnitude (Δm) as a result of the orbital motion of two stars. Ignore temperature variation due to tidal deformation and limb darkening on the surface of the stars. Hint: A prolate spheroid is a geometrical shape made by rotating of an ellipse around its major axis, like rugby ball or melon.

Long Problems:

Problem 16: High Altitude Projectile (45 points)

A projectile which initially is put on the surface of the Earth at the sea level is launched with the initial speed of $v_{\circ} = \sqrt{(GM/R)}$ and with the projecting angle (with respect to the local horizon) of $\theta = \frac{\pi}{6}$. M and R are the mass and radius of the Earth respectively. Ignore the air resistance and rotation of the Earth.

a) Show that the orbit of the projectile is an ellipse with a semi-major axis of $a = R$.
b) Calculate the highest altitude of the projectile with respect to the Earth surface (in unit of Earth radius).
c) What is the range of the projectile (distance between launching point and falling point)?
d) What is eccentricity (e) of the ellipse?
e) Find the flying time for the projectile.

Problem 17: Apparent number density of stars in the Galaxy (45 points)

Let us model the number density of stars in the disk of Milky Way Galaxy with a simple exponential function of $n = n_0 \exp\left(-\frac{r-R_0}{R_d}\right)$, where r represents the distance from the center of the Galaxy, R_0 is the distance of the Sun from the center of the Galaxy, R_d is the typical size of disk and n_0 is the stellar density of disk at the position of the Sun. An astronomer observes the center of the Galaxy within a small field of view. We take a particular type of Red giant stars (red clump) as the standard candles for the observation with approximately constant absolute magnitude of $M = -0.2$,

a) Considering a limiting magnitude of $m = 18$ for a telescope, calculate the maximum distance that telescope can detect the red clump stars. For simplicity we ignore the presence of interstellar medium so there is no extinction.

b) Assume an extinction of $0.70\ mag/kpc$ for the interstellar medium. Repeat the calculation as done in the part (a) and obtain a rough number for the maximum distance these red giant stars can be observed.

c) Give an expression for the number of these red giant stars per magnitude within a solid angle of Ω that we can observe with apparent magnitude in the range of m and $m + \Delta m$, (i.e. $\frac{\Delta N}{\Delta m}$). Red giant stars contribute f of overall stars. In this part assume no extinction in the interstellar medium as part (a).

Hint : the Tylor expansion of $y = \log_{10} x$ is :

$$y = y_0 + \frac{1}{ln10}\frac{x - x_0}{x}$$

Solutions

Solution 1:

Schwarzschild radius of a black hole with mass M is

$$R = \frac{2GM}{c^2}$$ (4 points)

Then the mass density can be estimated as

$$\rho = \frac{M}{\frac{4}{3}\pi\frac{8G^3M^3}{c^6}} = \frac{3c^6}{32\pi}\frac{1}{G^3M^2}$$ (2 points)

$$= \frac{3\times(3.00\times10^8)^6}{32\times3.1416}\frac{1}{(6.67\times10^{-11})^3(10^8\times1.99\times10^{30})^2}$$

$$= 1.85\times10^3\ kg\ m^{-3}$$ (4 points)

Solution 2:

To calculate flux of a $m = 6$ star we use the Sun as standard candle

$$m_1 - m_2 = -2.5\log\frac{f_1}{f_2}$$ (1 point)

$$6 - (-26.8) = -2.5\log\frac{f_1}{1.37\times10^3}$$

$$f_1 = 1.04\times10^{-10}(w/m^2)$$ (3 points)

We need to know how much energy a visual photon has (at 550 nm)

$$E_p = h\nu = \frac{hc}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{550 \times 10^{-9}}$$

(3 points)

$$= 3.62 \times 10^{-19}\, J$$

Then number of photon which arrive to our eye per second is

(3 points)

$$N = \frac{f_1 \pi r_e^2}{E_p} = \frac{1.04 \times 10^{-10}}{3.62 \times 10^{-19}} \times 3.1416 \times 0.003^2 = 8 \times 10^3 \; s^{-1}$$

Solution 3:

We must compare the jumping speed of a normal human with escape velocity of the planet .
A normal human can jump up to 50 cm then his initial velocity is

$$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.5}$$
$$v = 3.13\, ms^{-1}$$

(3 points)

Comparing this velocity with escape velocity of the planet

$$v = \sqrt{\frac{2GM}{R}}$$

(3 points)

$$v^2 = \frac{2GM}{R} = \frac{2G}{R}\frac{4\pi}{3}\rho R^3$$

$$= \frac{8\pi G}{3}\rho R^2$$ (2 points)

$$R^2 = \frac{3v^2}{8\pi G\rho}$$

$$R = 2 \times 10^3\ m$$ (2 points)

Solution 4:

The zenith angle of the sun at summer solstice will be

$$z_s = \phi - 23.5 = 12.5$$ (1.5 points)

And in the winter solstice

$$z_w = \phi + 23.5 = 59.5$$ (1.5 points)

Figures shows that in summer solstice we have

$$\tan(z_s) = \frac{x}{h} = 0.22$$ (1.5 points)

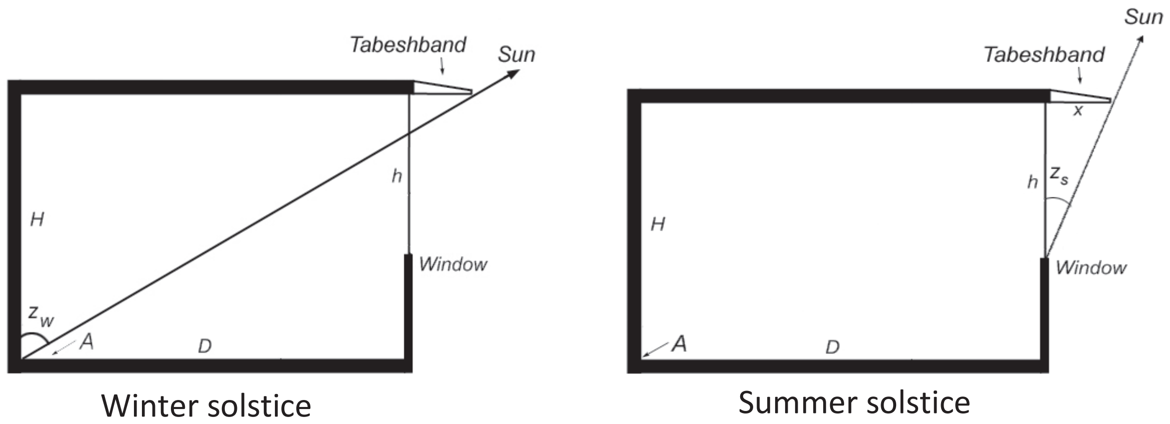
And in the winter solstice

Then

$$\tan(z_w) = \frac{D+x}{H} = 1.70$$ (1.5 points)

$$x = 1.70H - D = 0.60\ m$$ (2 points)

$$h = 2.73\ m$$ (2 points)



Solution 5:

To calculate accurate value for minimum declination for circumpolar stars two major effect must be considered.

1. Refraction in earth atmosphere, which is $34'$ at horizon. (3 points)
2. Horizon depression which is

$$\mathrm{Cos}\,\theta = \frac{R}{R+h} = \frac{6370.8}{6370.8+5.6} \Longrightarrow \theta = 2^\circ 24^{'}$$ (3 points)

Then

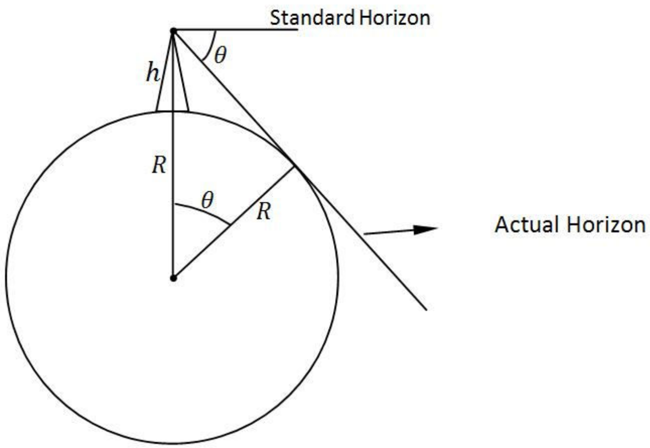
$$\delta_{min} = 90 - Latitude - Refraction - Horizon\ depression$$

(2 points)

$$= 90 - 35^\circ 57^{'} - 34' - 2^\circ 24'$$

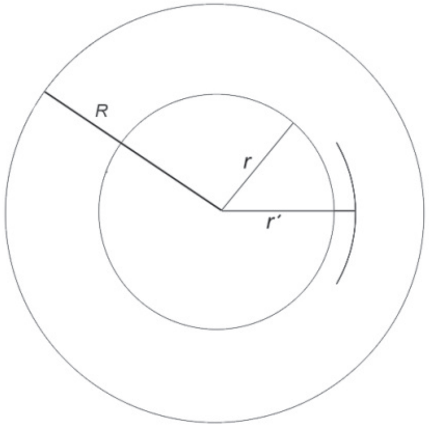
(2 points)

$$\Longrightarrow \delta_{min} = 51^\circ 5'$$



Solution 6:

To solve this problem we must calculate gravitational potential at the center of the cloud, letting $\phi(\infty) = 0$. For a uniform density and spherical mass distribution we have



$$\frac{1}{2}\, m\, v^2(r=R) - \frac{GMm}{R} = E = \frac{1}{2} m v^2(r=0) + \varphi(0)$$ (4 points)

$$v(r=R) = 0 \qquad \& \qquad v(r=0) = \sqrt{\frac{GM}{R}}$$ (1 point)

$$\varphi(0) = -\frac{1}{2} m v_0^2 - \frac{GmM}{R}$$ (1 point)

$$\varphi(0) = \frac{-1}{2} m \left(\sqrt{\frac{GM}{R}}\right)^2 - \frac{GmM}{R}$$ (2 points)

$$\varphi(0) = \frac{-3}{2} \times \frac{GMm}{R}$$

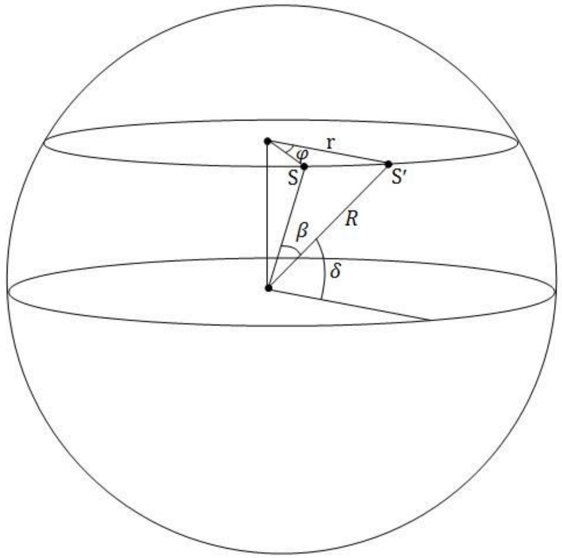
To escape from the cloud, the particle should have total energy equal to zero

$$E = \frac{1}{2}mv_e{}^2 + \phi(r=0) = \frac{1}{2}mv_e{}^2 - \frac{3}{2}\left(\frac{GMm}{R}\right) = 0$$

$$v_e{}^2 = \frac{3GM}{R} \quad \rightarrow \quad v_e = \sqrt{\frac{3GM}{R}}$$ (2 points)

Solution 7:

Figure shows that if FOV of telescope is β then we have :



$$\beta = \phi cos\delta$$ (4 points)

As the earth rotate, Vega moves through the FOV with constant angular velocity of the earth

$$\omega = \frac{2\pi}{86164} = 7.29 \times 10^{-5}(rad/s)$$ (2 points)

$$\phi = \omega t = 7.29 \times 10^{-5} \times 5.3 \times 60 = 0.023\ (\text{rad})$$

$$FOV = \beta = \phi cos\delta = 0.023cos39^{\circ} = 0.018\ (\text{rad}) \simeq 62\ min$$ (4 points)

Solution 8:

$$\frac{1}{2}Mv_{esc}^2 = \frac{1}{2}M\frac{2GM}{R}$$ (4 points)

The velocity must be smaller than the escape velocity ($v_{esc} \approx \sqrt{2}v_{rms}$) and since the problem is an estimation, any velocities smaller than escape velocity is accepted and therefore the escape velocity can be replaced by v_{rms}.

$$Mv_{rms}^2 = \frac{GM^2}{R}$$ (2 points)

$$M = \frac{Rv_{rms}^2}{G} = \frac{20 \times 3.09 \times 10^{16} \times 9 \times 10^6}{6.67 \times 10^{-11}} = 8.3 \times 10^{34} Kg$$

(4 points)

$$M = 4.2 \times 10^4 M_\odot$$

Solution 9:

In the figure, S is the Sun and R_0 and V_0 are Sun distance and velocity. The distance and velocity of star P is denoted by R and $V = V_0$. The radial velocity of star P respect to the Sun is

$$V_r = V\cos\alpha - V_0 \sin l = V_0(\cos\alpha - \sin l)$$ (4 points)

In SCP triangle we have

$$\frac{\sin l}{R} = \frac{\cos\alpha}{R_0} \Rightarrow \cos\alpha = \frac{R_0}{R}\sin l$$ (1 point)

So

$$V_r = V_0\left(\frac{R_0}{R} - 1\right)\sin l$$ (2 points)

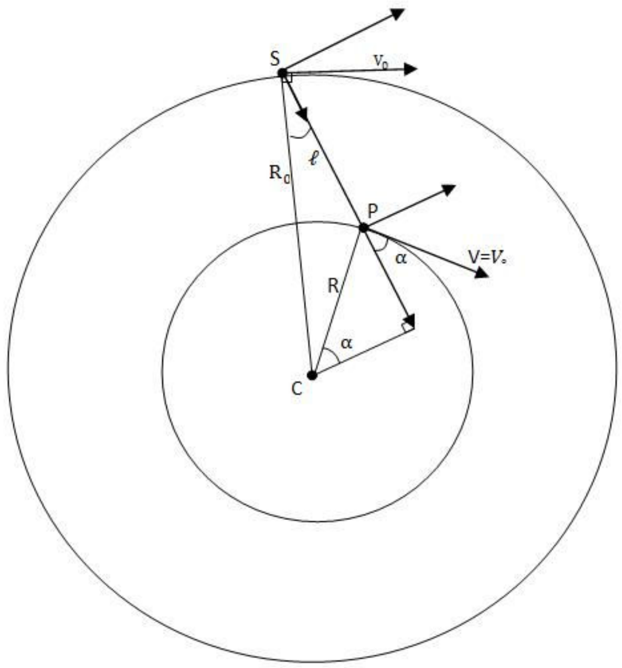
$$\frac{R_0}{R} - 1 = \frac{V_r}{V_0 sinl}$$

$$\frac{R_0}{R} = \frac{V_r + V_0 sinl}{V_0 \sin l}$$

$$R = R_0 \frac{V_0 \sin l}{V_r + V_0 sinl}$$ (2 points)

$$R = 8\times 10^3 \frac{250 sin15^\circ}{100 + 250 sin15^\circ}$$

$$R = 3\times 10^3\, pc$$ (1 point)



Solution 10:

Because of high conductivity of plasma inside the star, flux of magnetic field will be conserved through contraction then:

$$4\pi R^2 B = 4\pi R_n^2 B_n$$ (6 points)

Where R_n and B_n are radius and magnetic field of neutron star, thus:

$$B_n = \left(\frac{R}{R_n}\right)^2 B = \left(\frac{4 \times 6.96 \times 10^5}{20}\right)^2$$ (2 points)

$$B_n = 1.93 \times 10^{10}\ Gauss$$

$$= 1.93 \times 10^6\ T$$ (2 points)

Solution 11:

In a flat universe

$$\rho = \rho_c = \frac{3H^2}{8\pi G}$$ (4 points)

$$H = 75\ kms^{-1}Mpc^{-1} = 2.4 \times 10^{-18}s^{-1}$$ (2 points)

$$\rho_c = 1.1 \times 10^{-26}\ kgm^{-3}$$ (2 points)

$$n_v = \frac{0.25\ \rho_c}{10^{-5}\ m_e} = 3 \times 10^8\ m^{-3}$$ (2 points)

Solution 12:

The rate of change of solar mass could be estimated from solar luminosity:

(2 points)

$$L_\odot = -\frac{\Delta E}{\Delta t} = -\frac{\Delta M c^2}{\Delta t} = -\dot{M}c^2$$

$$\dot{M} = -\frac{L_\odot}{c^2} = -\frac{3.83 \times 10^{26}}{(3.00 \times 10^8)^2} = -4.26 \times 10^9 (kgs^{-1})$$

Newton second law will give us:

$$\frac{v^2}{r} = \frac{GM}{r^2} \implies v^2 = \frac{GM}{r}$$ (2 points)

where v and r are orbital velocity and orbital radius of the Earth.
From conservation of angular momentum:

$$l = rmv \implies v = \frac{l}{mr}$$ (2 points)

Where m and I are the Earth mass and angular momentum which are constant:

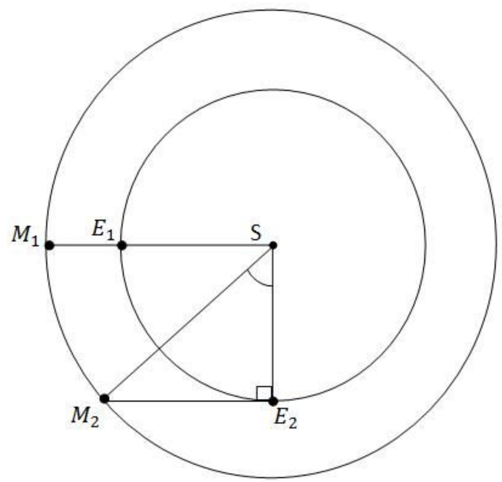
$$\left.\begin{aligned}
&\frac{l^2}{m^2 r^2} = \frac{GM}{r} \Rightarrow r = \frac{l^2}{GMm^2} \Rightarrow \dot{r} = -\frac{l^2}{Gm^2}\frac{\dot{M}}{M^2} = -\frac{r^2 m^2 v^2}{GM^2 m^2}\dot{M} \\
&\Rightarrow \dot{r} = -\frac{(GM/r)r^2}{G}\frac{\dot{M}}{M^2} = -r\frac{\dot{M}}{M} \Rightarrow \frac{\dot{r}}{r} = -\frac{\dot{M}}{M} \Rightarrow \dot{r} = -r\frac{\dot{M}}{M} \\
&\Rightarrow \dot{r} = -\frac{1.50 \times 10^{11} \times 4.26 \times 10^9}{1.99 \times 10^{30}} = 3.21 \times 10^{-10} (m/s)
\end{aligned}\right\}$$ (2 points)

$$\Delta r = 3.19 \times 10^{-10} \times 100 \times 86400 \times 365.24 = 1.01m \quad (for\ 100\ years)$$ (2 points)

Solution 13:

We know the orbital period of Mars then the angle $\angle M_1SM_2$ can be determined simply

$$\angle M_1SM_2 = \frac{106}{687} \times 360 = 55.5°$$ (2 points)



By the same way we determine $\angle E_1SE_2$:

$$\angle E_1SE_2 = \frac{106}{365} \times 360 = 104.5°$$ (2 points)

Then angle $\angle M_2SE_2$ is:

$$\angle M_2SE_2 = 104.5 - 55.5 = 49.0°$$ (2 points)

$$\cos(\angle M_2SE_2) = \frac{SE_2}{SM_2}$$ (2 points)

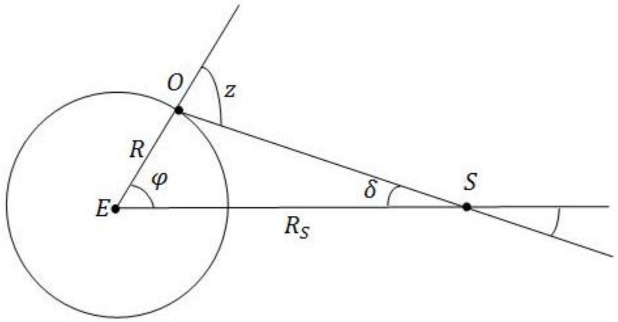
$$r_{mars} = \frac{SM_2}{SE_2} = \frac{1}{\cos(\angle M_2SE_2)} = \frac{1}{\cos(49°)} = 1.52\ AU$$ (2 points)

Solution 14:

In the figure, the observer is at *O* and the satellite is in *S* the angle $\angle EOS$ will be

$$\angle EOS = 180 - z = 180 - 46.0 = 134° \qquad \text{(2 points)}$$

$$\delta = 180 - \varphi - \angle EOS = 10.4° \qquad \text{(3 points)}$$



In *EOS* triangle we have:

$$\frac{R_S}{\sin(\angle EOS)} = \frac{R}{\sin\delta} \qquad \text{(3 points)}$$

$$\frac{R_S}{R} = \frac{\sin(\angle EOS)}{\sin\delta} = 3.98 \qquad \text{(2 points)}$$

Solution 15:

Ignoring temperature variation on the stars surface, the brightness of star system will be proportional to projected surface of both stars on plane of the sky. Maximum brightness will occur when two stars are seen like figure 1 and minimum light will happen when one of the stars is in total eclipse and projected surface of the other is a circle with radius *b* (Figure 2). In maximum brightness

$$I_{max} \propto 2\pi ab \qquad \text{(3 points)}$$

In minimum brightness

$$I_{min} \propto \pi b^2 \qquad \text{(3 points)}$$

So

$$\Delta m = -2.5 \log \frac{I_{max}}{I_{min}} = -2.5 \log \frac{2\pi ab}{\pi b^2} = -2.5 \log 4$$ (2 points)

$$\Delta m = -1.5$$ (2 points)

Solution 16:

a) Total energy of the projectile is

$$E = \frac{1}{2} m v_\circ^2 - \frac{GMm}{R} = -\frac{GMm}{2R} < 0$$

$E < 0$ means that orbit might be ellipse or circle. As $\theta > 0$, the orbit is an ellipse.
Total energy for an ellipse is

$$E = -\frac{GMm}{2a}$$

Then

$$a = R$$ (7 points)

b) In figure (1) we have

$$OA + O'A = 2a$$

$$O'A = a$$

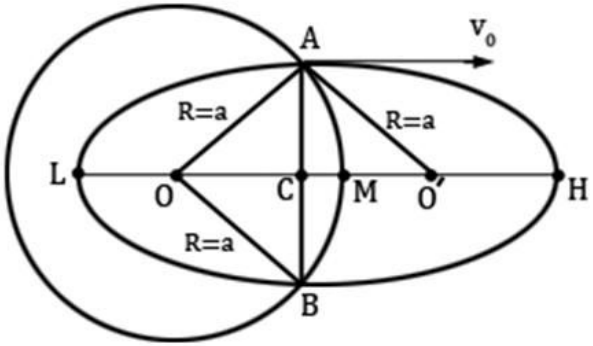


Figure (1)

In OAO' triangle it is obvious that

$$OC = CO'$$

Then C must be the center of the ellipse with the initial velocity vector $v_\circ$ parallel to the ellipse major-axis (LH).

In figure (2)

$$HM = CH - CM = a - (R - R\sin\theta) = R - R + R\sin\theta = R\sin\theta = \frac{R}{2}$$

(15 points)

c) Range of the projectile is $\widehat{AB}$

$$\widehat{AB} = 2\left(\frac{\pi}{2} - \theta\right)R = (\pi - 2\theta)R = \frac{2\pi}{3}R$$

(6 points)

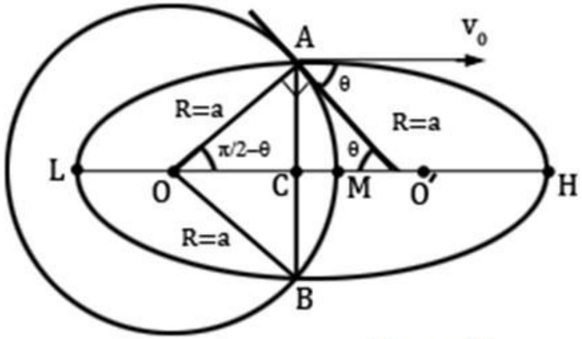


Figure (2)

d) Start with ellipse equation in polar coordinates

$$r = \frac{a(1-e^2)}{1+ecos\varphi}$$

For point A

$$R = \frac{R(1-e^2)}{1-ecos(\frac{\pi}{2}+\theta)}$$

$$e = sin\theta = \frac{1}{2}$$

(5 points)

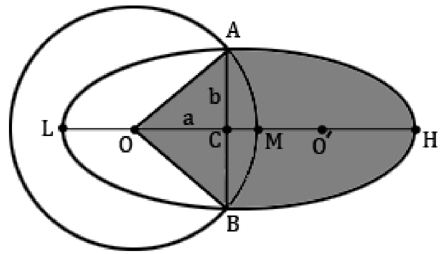
e) Using Kepler's second law

$$\frac{\Delta S}{S_0} = \frac{\Delta T}{T}$$

$$\Delta S = S_{AOBH} = S_{\Delta AOB} + \frac{S_0}{2}$$

$$= 2 \times \frac{\mathrm{bae}}{2} + \frac{\pi \mathrm{ab}}{2} = \mathrm{bae} + \frac{\pi \mathrm{ab}}{2}$$

$$\frac{\Delta S}{S} = \frac{\mathrm{bae} + \frac{\pi \mathrm{ab}}{2}}{\pi ab} = \frac{e + \frac{\pi}{2}}{\pi} = \frac{0.5 + \frac{\pi}{2}}{\pi}$$



Kepler’s third law

$$T = \sqrt{\frac{4\pi^2 R^3}{GM}} = 84.5\, min$$

$$\Delta T = T \times \frac{0.5 + \frac{\pi}{2}}{\pi} = 55.7\, min$$

(12 points)

Solution 17:

a) Relation between the apparent and absolute magnitude is given by

$$m = M + 5log\left(\frac{d}{10}\right)$$ (3 points)

where d is in terms of parsec. Substituting $m = 18$ and $M = -0.2$, results in

(5 points)

$$d = 4.37 \times 10^4\ pc$$

b) Adding the term for the extinction, changes the magnitude distance relation as follows

$$m = M + 0.7x + 5\log\,(100x)$$

where x is given in terms of kilo parsec. To have a rough value for x, after substituting m and M, this equation reduces to

$$8.2 = 0.7x + 5\log\,(x)$$ (6 points)

To solve this equation, we examine

$$x = 5\,, 5.5\,, 6\,, 6.5$$

where the best value is obtained roughly $x \cong 6.1\ kpc$. (8 points)

c) For a solid angle Ω, the number of observed red clump stars at the distance in the range of x and $x + \Delta x$ is given by

$$\Delta N = \Omega\, x^2\, n(x) f\ \Delta x$$

So the number of stars observed in Δx is given by

$$\frac{\Delta N}{\Delta x} = \Omega\, x^2\, n(x) f$$

(6 points)

From the relation between the distance and apparent magnitude we have

$$m_1 = M + 5log\left(\frac{x}{10}\right)$$

$$m_2 = M + 5log\left(\frac{x + \Delta x}{10}\right)$$

$$\Delta m = 5log\left(\frac{x + \Delta x}{x}\right)$$

$$\Delta m = 5log\left(1 + \frac{\Delta x}{x}\right)$$

$$\Delta m = \frac{5}{ln10} ln\left(1 + \frac{\Delta x}{x}\right)$$

$$\Delta m = \frac{5}{ln10}\left(\frac{\Delta x}{x}\right)$$

Replacing Δx with Δm, results in

$$\frac{\Delta N}{\Delta m} = \frac{\Delta N}{\Delta x} \times \frac{\Delta x}{\Delta m}$$

So the number of stars for a given magnitude is obtained by (5 points)

$$\frac{\Delta N}{\Delta m} = \frac{\Omega ln10}{5} n(x) x^3 f$$

Finally we substituting x in terms of apparent magnitude using $\mathrm{x} = 10^{\frac{\mathrm{m}+5.2}{5}}$.

In the case of no extinction, we are able to observe the Galaxy beyond the center. So $\frac{dN}{dm}$ has two terms in

$x < R_0$ and $x > R_0$. The relation between x and r for these two cases are

$$x = R_0 - r \qquad x < R_0$$

(6 points)

and

$$x = R_0 + r \qquad x > R_0$$

So in general we can write $\frac{\Delta N}{\Delta m}$ as

(6 points)

$$\frac{\Delta N}{\Delta m}=\frac{\Omega ln10}{5}n_0\exp\left(\frac{10^{\frac{\mathrm{m}-5.2}{5}}}{R_d}\right)\times 10^{\frac{3(m-5.2)}{5}}f \qquad x<R_0$$

$$\frac{\Delta N}{\Delta m}=\frac{\Omega ln10}{5}\mathrm{n}_0\exp\left(\frac{2\mathrm{R}_0}{R_d}\right)\exp\left(-\frac{10^{\frac{\mathrm{m}-5.2}{5}}}{R_d}\right)\times 10^{\frac{3(m-5.2)}{5}}f\ \Theta(x_0-x\ )\ \ x>R_0$$

where $\Theta(x)$ is the step function and x_0 is the maximum observable distance.

Practical completion

Problem 1: CCD Image Processing (60 points)

As an exercise of image processing, this problem involves use of a simple calculator and tabular data. Table 1.1 contains the pixel values of an image during the given exposure time. (table 1.1 is given in the accompanying CD). This image, which is a part of a larger CCD image, was taken by a small CCD camera, installed on an amateur telescope and using a *V* band filter. Figure 1.1 shows this 50×50 pixels image that contains 5 stars.

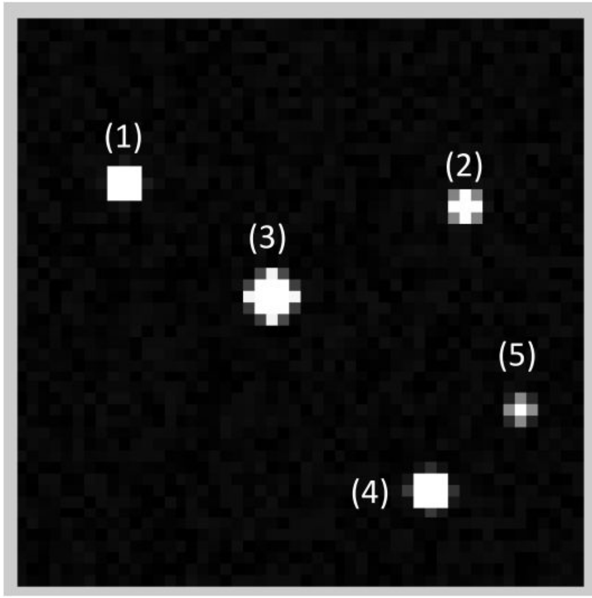


Figure (1.1)

In table 1.1 the first row and column indicates the pixels' x and y coordinates. Table 1.2 gives the telescope and the image specifications.

Table 1.2

Telescope focal length	$1.20\ m$
CCD pixel size	$25 \times 25\ \mu m$
Exposure time	$450\ s$
Telescope zenith angle	25°
Average extinction coefficient in V band	0.3 mag/airmass

The observer identified stars 1, 3 and 4 by comparing this image with standard star catalogues. Table 1.3 shows stars true magnitudes (m_t) as given in the catalogue.

Table 1.3

Star	m_t
1	9.03
3	6.22
4	8.02

a. Using the available data, determine the instrumental magnitudes of the stars in the image. Assume the dark current is negligible and the image is flat fielded. For simplicity you can use a square aperture.
Hint: The instrumental magnitude is calculated using the difference between the measured flux from the star in the aperture and the flux from an equivalent area of dark sky.

b. The instrumental magnitude of a star in a CCD image is related to true magnitude as

$$m_I = m_t + KX - Zmag$$

where K is extinction, X is airmass, m_I and m_t are respectively instrumental and true magnitude of star and $Zmag$ is zero point constant. Calculate the zero point constant ($Zmag$) for identified stars. Calculate average zero point constant ($Zmag$). **Hint**: Zero point constant is the constant reducing extinction-free magnitudes to the true magnitude.

c. Calculate true magnitudes of stars 2 and 5.
d. Calculate CCD pixel scale for the CCD camera in units of arcsec.
e. Calculate average brightness of dark sky in magnitude per square arcsec (m_{sky}).
f. Use a suitable plot to estimate astronomical seeing in arcsec.

Problem 2: Venus(60 point)

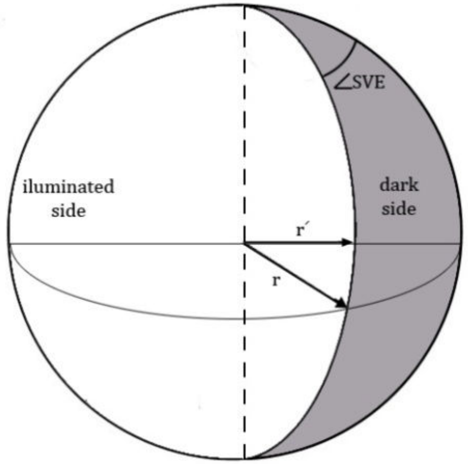
An observer in Deh-Namak (you will be taking the observational part of the exam in this region tonight) has observed Venus for seven months, started from September 2008 and continued until March 2009. During the observation, a research CCD camera and an image processing software were used to take high resolution images and to extract high precision data. Table 2.1 shows the collected data during the observation.

Table 2.1 description:

Column 1	Date of observation.
Column 2	Earth-Sun distance in astronomical unit (AU) for observation date and time. This value is taken from high precision tables.
Column 3	Phase of Venus, Percent of Venus disk illuminated by the Sun as observed from the Earth.
Column 4	Elongation of Venus, the angular distance between center of the Sun and center of Venus in degrees as observed from the Earth.

a) Using given data in table 2.1, calculate the Sun-Venus-Earth angle($\angle SVE$) . This is angular separation between the Sun and the Earth as seen from Venus. Write $\angle SVE$ angle in column 2 of Table 2.2 in your answer sheet for the all observing dates.

Hint: Remember that the line between light and shadow, in the phases, is an ellipse arc.



b) Calculate Sun – Venus distance in AU and write it down in column 3 of table 2.2 for all observation dates.

c) Plot Sun – Venus distance versus observing date.

d) Find perihelion ($r_{v,min}$) and aphelion ($r_{v,max}$) distances of Venus from the Sun.

e) Calculate semi-major axis (a) of the Venus orbit.

f) Calculate eccentricity (e) of Venus orbit.

Table 2.1

Column 1	Column 2	Column 3	Column 4
Date	Earth - Sun Distance (AU)	Phase (%)	Elongation (SEV; degree)
20/9/2008	1.0043	88.4	27.56
10/10/2008	0.9986	84.0	32.29
20/10/2008	0.9957	81.6	34.53
30/10/2008	0.9931	79.0	36.69
9/11/2008	0.9905	76.3	38.71
19/11/2008	0.9883	73.4	40.62
29/11/2008	0.9864	70.2	42.38
19/12/2008	0.9839	63.1	45.29
29/12/2008	0.9834	59.0	46.32
18/1/2009	0.9838	49.5	47.09
7/2/2009	0.9863	37.2	44.79
17/2/2009	0.9881	29.6	41.59
27/2/2009	0.9904	20.9	36.16
19/3/2009	0.9956	3.8	16.08

Solutions

Solution 1: CCD Image Processing

a) To measure instrumental magnitude we should choose an aperture. Careful investigation of the image, shows that a 5×5 pixel aperture is enough to measure m_I for all stars. m_I can be calculated using:

$$m_I = -2.5\log\left(\frac{\sum_{i=1}^{N} I_{i(star)} - N\bar{I}_{Sky}}{Exp}\right)$$

where $I_{i(star)}$ is the pixel value for each pixel inside the aperture, N is number of pixels inside the aperture, $\bar{I}_{Sky}$ is the average of sky value per pixel taken from dark part of image and Exp is the exposure time. Table (1.4) lists values for m_I and $Zmag$ calculated for all three identified stars.

$$\bar{I}_{Sky} = 4.42$$

$$N = 25$$

$$Exp = 450$$

Table (1.4)

Star	m_I	m_t	$Zmag$
1	-3.02	9.03	12.38
3	-5.85	6.22	12.40
4	-4.04	8.02	12.39

b) Average $Zmag$ = 12.4

c) Following part (a) for stars 2 and 5, we can calculate true magnitudes (m_t) for these stars

Table (1.5)

Star	m_I	m_t
2	-2.13	9.93
5	-0.66	11.4

d) Pixel scale for this CCD is calculated as

$$p = \frac{pixel\ size}{focal\ length} \times \frac{180 \times 3600}{\pi}$$

$$= 4.30''$$

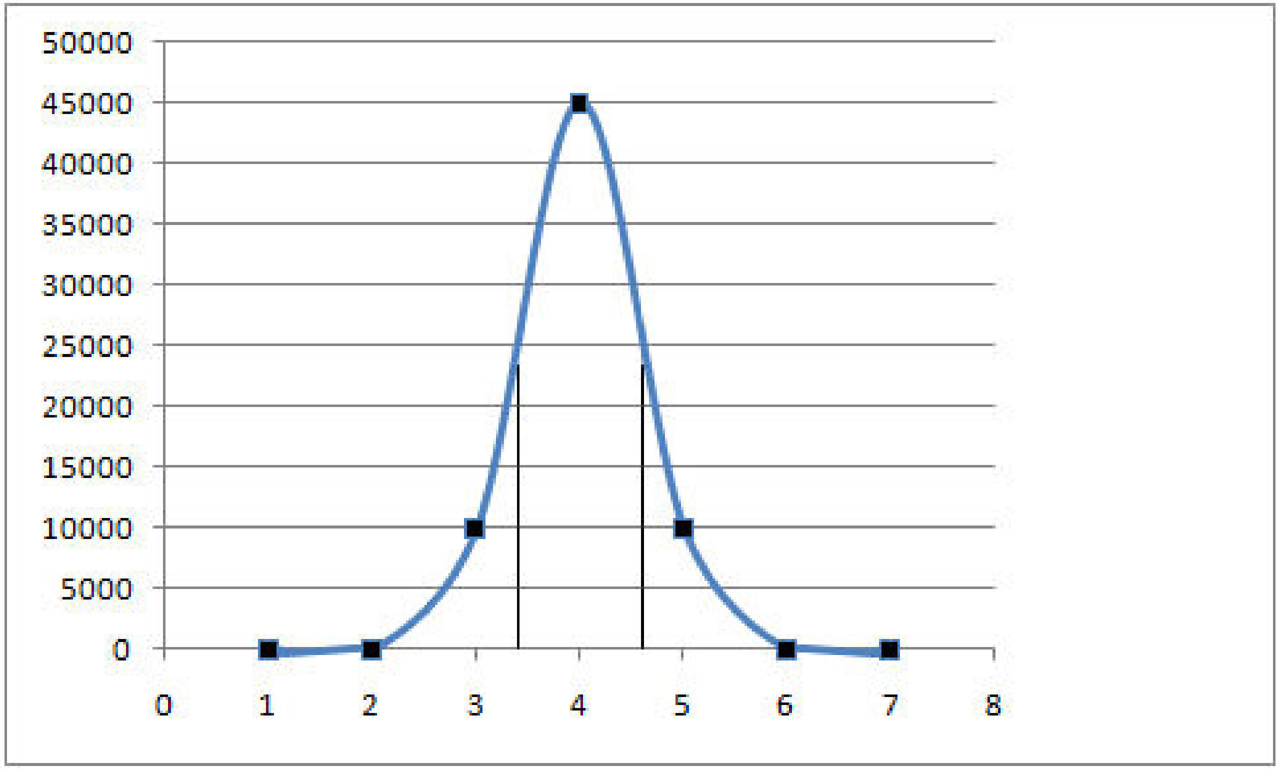
e) Average sky brightness:

$$m_{sky} = -2.5 \log \frac{\bar{I}_{Sky}}{(Exp)(p)^2} + Zmag$$

$$= 20.6$$

f) To estimate astronomical seeing, first we plot pixel values in x or y direction for one of the bright stars in the image. As plot (1) shows, the FWHM of pixel values which is plotted for star 3, is 1 pixel , hence astronomical seeing is equal to

$$seeing \cong 4''$$



Plot (1)

CCD Image Problem Marking Scheme

Part	Tot. Pts.	Details	Max.	Explanation
a	10	Relation	2	Each value :+2 $\bar{I}_{sky}$ (within calculation) : +2 m_I relation (in calculation) +2
		m_I	6	
		$\bar{I}_{sky}$	2	
b	10	Z_{mag}	10	$3Z_{mag}$ and average , for each less: - 2
c	10	m_t	10	For each one:+ 5, for each numerical mistake: -2
d	10	P (pixel Scale)	10	
e	10	Relation of m_{sky}	5	
		Value of m_{sky}	5	
f	10	Seeing	10	Seeing: +4, Gaussian profile: +3, FWHM: +3

Solution 2: Venus

a) The $\angle SVE$ angle should be calculated from the phase of Venus. Figure 2.1 shows that projected area of Venus disk which is illuminated by the Sun is

$$\frac{\pi r^2}{2} + \frac{\pi r r'}{2}$$

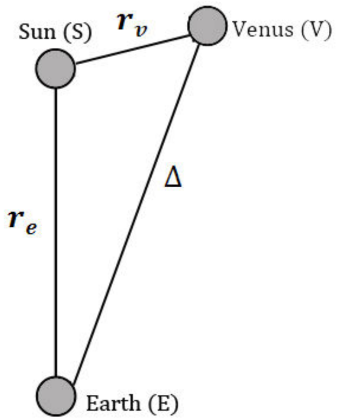
where

$$r^{'} = rcos(\angle SVE)$$

Then,

$$Phase = (\frac{\frac{\pi r^2}{2} + \frac{\pi r^2 \cos(\angle SVE)}{2}}{\pi r^2}) \times 100 = \frac{100}{2}(1 + \cos(\angle SVE)) = 100cos^2(\frac{\angle SVE}{2})$$

The angle $\angle SVE$ is calculated and written in table 2.2, column 2.



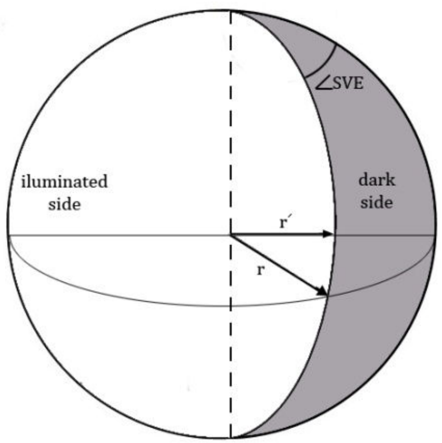


Figure 2.1

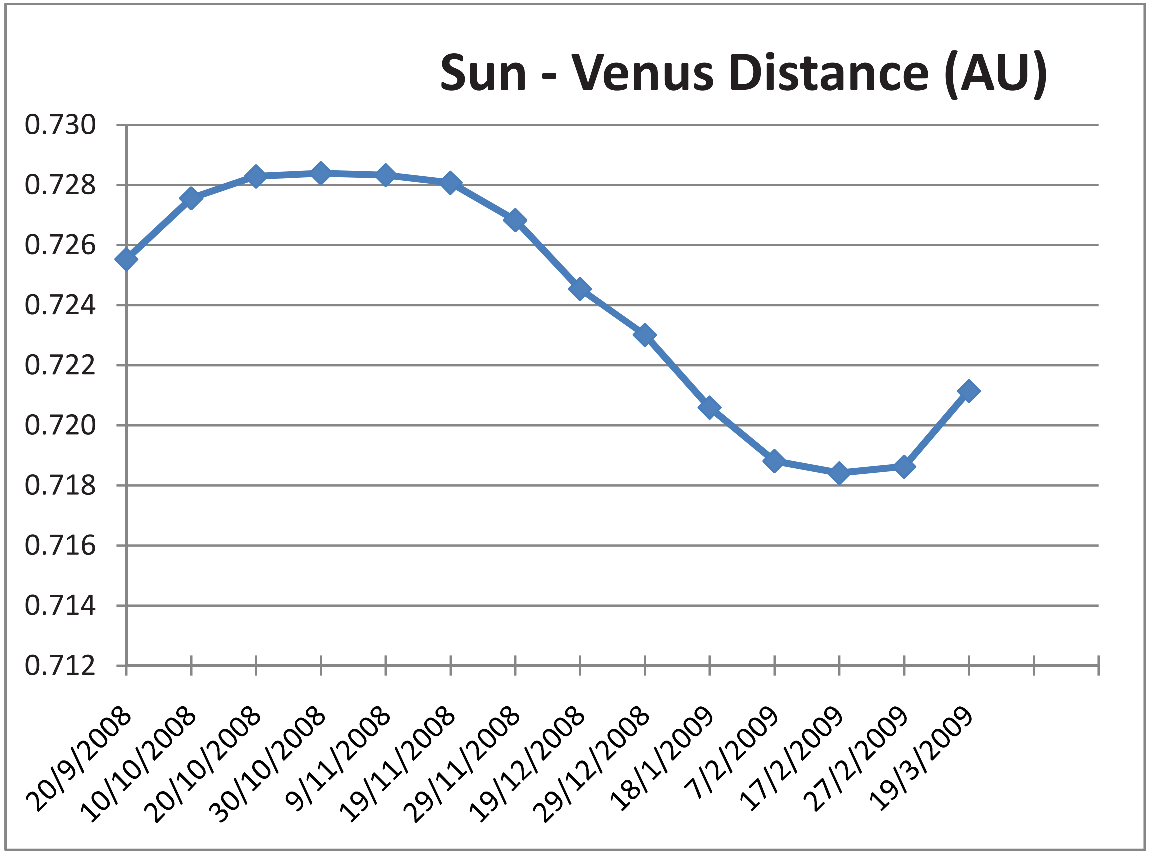
b) As in figure 2.1, in SEV triangle we have,

$$\frac{r_e}{\sin(\angle SVE)} = \frac{r_v}{\sin(\angle SEV)}$$

$$r_v = r_e \frac{\sin(\angle SEV)}{\sin(\angle SVE)}$$

where r_e and $\angle SEV$ (elongation) is given in table 2.1 then, r_v for all observing dates is calculated and written in table 2.2 column 3.

c)



Sun - Venus Distance (AU)
0.730
0.728
0.726
0.724
0.722
0.720
0.718
0.716
0.714
0.712
20/9/2008
10/10/2008
20/10/2008
30/10/2008
9/11/2008
19/11/2008
29/11/2008
19/12/2008
29/12/2008
18/1/2009
7/2/2009
17/2/2009
27/2/2009
19/3/2009

d) According to the obtained values written in table 2.2 column 3,

$$r_v^{max} = 0.728\ AU$$
$$r_v^{min} = 0.718\ AU$$

e) Semi-major axis is

$$a = \frac{(r_v^{max} + r_v^{min})}{2} = 0.723\ AU$$

f) Eccentricity could be calculated from both of aphelion and perihelion distances as

$$e = \frac{r_v^{max} - r_v^{min}}{2a} = 6.92 \times 10^{-3}$$

Table 2.2

Column 1	Column 2	Column 3
Date	SVE (°)	Sun - Venus Distance (AU)
2008-Sep-20	39.83	0.726
2008-Oct-10	47.16	0.728
2008-Oct-20	50.80	0.728
2008-Oct-30	54.55	0.728
2008-Nov-09	58.26	0.728
2008-Nov-19	62.10	0.728
2008-Nov-29	66.17	0.727
2008-Dec-19	74.81	0.725
2008-Dec-29	79.63	0.723
2009-Jan-18	90.57	0.721
2009-Feb-07	104.83	0.719
2009-Feb-17	114.08	0.718
2009-Feb-27	125.59	0.719
2009-Mar-19	157.52	0.721

Venus Problem Marking Scheme

part	Tot. Pts	Details	Max
a	16	Angle derivation	6
		Calculation of $\angle SVE$	10
b	14	Relation	4
		Sun-Venus distance	10
c	6	Plotting Sun-Venus distance	6
d	8	Perihelion	4
		Aphelion	4
e	8	a (relation)	4
		a (value)	4
f	8	e (relation)	4
		e (value)	4

Note: reported numbers in table 2 are not acceptable if they are out of 0.75 and 1.25 times of designer answer.

Observational Competition

Please read these instructions carefully:

1. All participants will receive a question set, a writing board, a pen, a ruler and a headlight by the organizers.
2. This competition consists of two parts:
   a) Two questions on “Naked Eye observation”. You have 12 minutes to answer these two questions.
   b) One question on “Using a telescope”. Each part of this question has a specific time, which is mentioned in your question sheet.
3. All participants will be guided by assistants to the observing site until returning to the waiting hall. Assistants will collect the answer and problem sheets.
4. **Do not forget** to fill out the boxes at the top of each answer sheet with your country name and your student code.
5. You have 2 minutes to familiarize yourself with observing ground and darkness of your environment, just before starting the exam time in observing ground.
6. Examiner’s alarm will indicate the beginning and the end of each part of your exam.
7. Each problem has a specific guideline which helps you during the exam.

Naked Eye Observations

You Have 12 minutes to answer the questions of the Naked Eye Observations (Question 1 and Question 2)

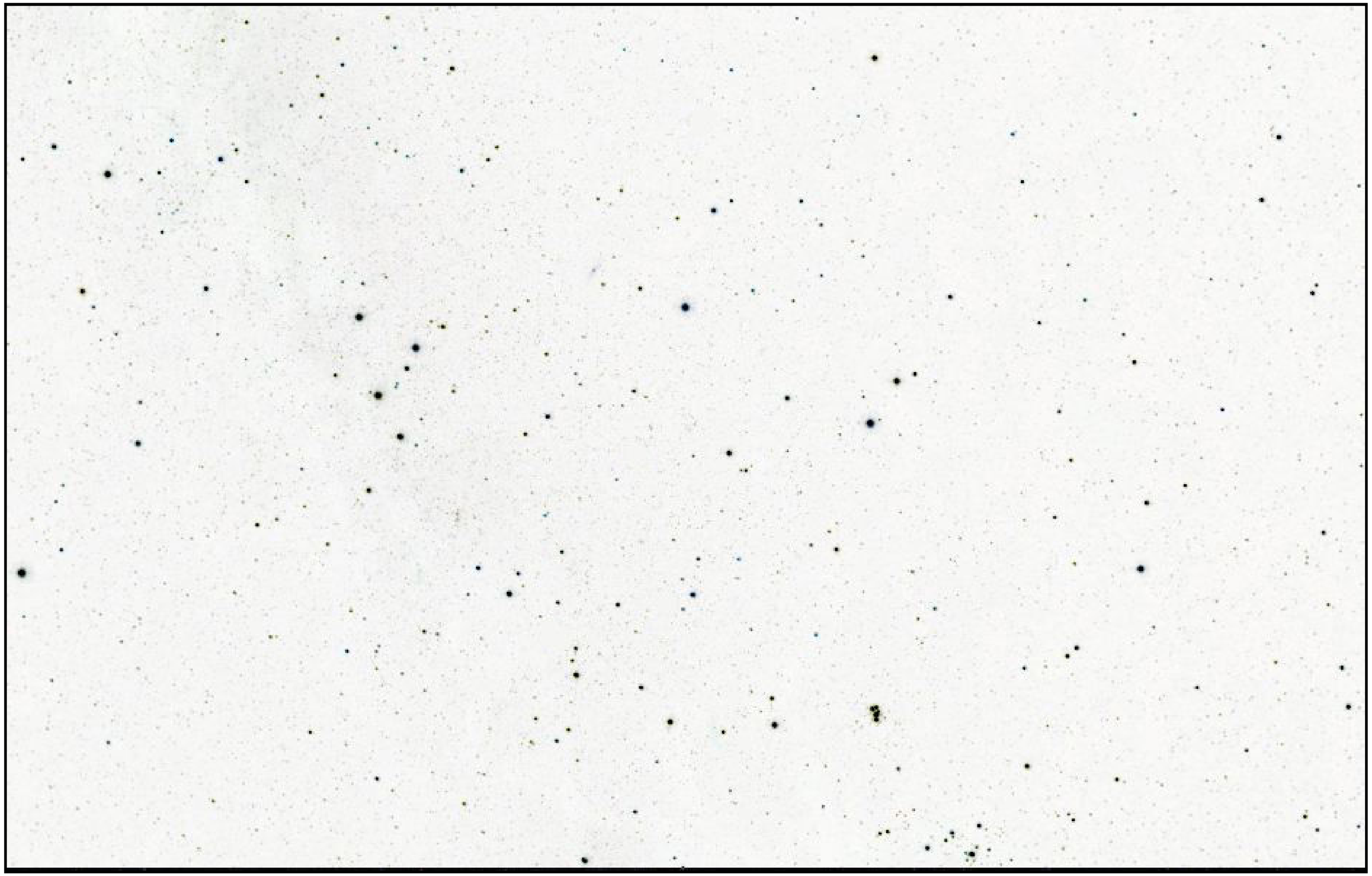
Question 1: (40 Points)

1.1: Figure 1 (frame size $\cong 100^{\circ} \times 70^{\circ}$) shows a part of the sky, for 22 October 2009 at 21:00 local time. Four bright stars in Perseus and Andromeda constellations are missing in this chart. Find these missing stars by looking at the sky. Then, draw a cross on the location of each missing bright star in these two constellations on the chart (i.e. figure 1). Use numbers in table 1 to indicate these crosses.

Table 1

Number	Common Name	Bayer Names
1	Mirfak	Alpha Persei
2	Alpheratz	Alpha Andromeda
3	-	Epsilon Persei
4	Menkib	Xi Persei
5	-	Gamma Persei
6	Algol	Beta Persei
7	Almach	Gamma Andromeda
8	-	Delta Andromeda
9	-	51 Andromeda
10	Mirach	Beta Andromeda
11	Atik	Zeta Persei

Question 1 - Figure 1



Question 2:

2.1: Figure 2 shows a part of the sky which contains **Cepheus constellation**, for 22 October 2009 at 22:00 local time. Five bright stars in Cepheus constellation are identified by numbers (1, 2, … , 5) and common names. Estimate the angular distances (in units of degrees) between two pairs of stars shown in table 2 and complete this table with your answers. **(40 Points)**

Table 2

Angular Distance	
Pairs of stars	**Angular Distance (degrees)**
1 (Errai) and 2 (Alfirk)	
1 (Errai) and 3 (Alderamin)	

2.2: Use table 3 and figure 2, then estimate the “apparent visual magnitude” of stars 2 (Alfirak) and 3 (Alderamin) and complete table 4. **(40 Points)**

Table 3

Star Name	Apparent Visual Magnitude
Polaris	1.95
Altais	3.05
Segin	3.34
All of these stars, are marked in the figure 2	

Table 4

Magnitude Estimation		
Star Number	**Star Name**	**Apparent Visual Magnitude**
2	Alfirk	
3	Alderamin	

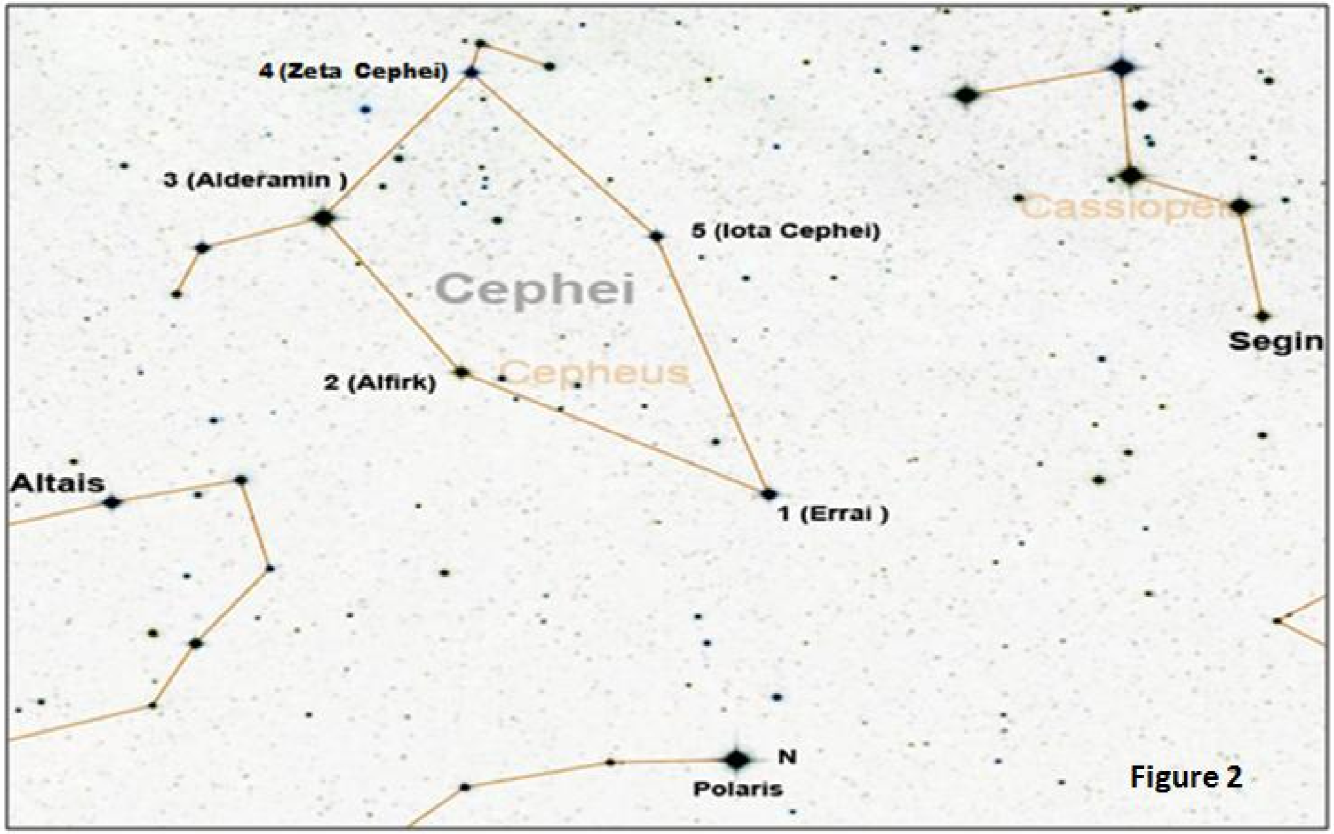


Figure 2

Telescopic Observations

Note: You have only 13 Minutes to answer all parts of this Question

Question 3

Before starting this part, please note:

The telescope is pointed by the examiner towards Caph (α Cas). Please note the readings on the grade circles before moving the telescope (to be used in 3.2).

3.1: Choose one of the 4 recommended stars listed below; write down the name of the selected star in table 5 and point the telescope to that star. Then, notify the examiner to check it. **(6 minutes; 40 Points)**

1- Deneb (Alpha Cygni)
2- Alfirk (Beta Cephei)
3- Algol (Beta Persei)
4- Capella (Alpha Aurigae)

Table 5

Name of selected star

3.2**:** The Telescope is pointed to Caph in Cassiopeia constellation (RA: 0h:9.7m ; Dec: 59°:12’). Using the clock beside the telescope write down the local time (with the format of HH:MM:SS) in the appropriate field in table 6. Then, by using the graded circle on the telescope mount, estimate “declination” and “hour angle” of the target measured from South, which you chose in part one of this question. Then, complete Table 6. **(7 minutes; 40 Points)**

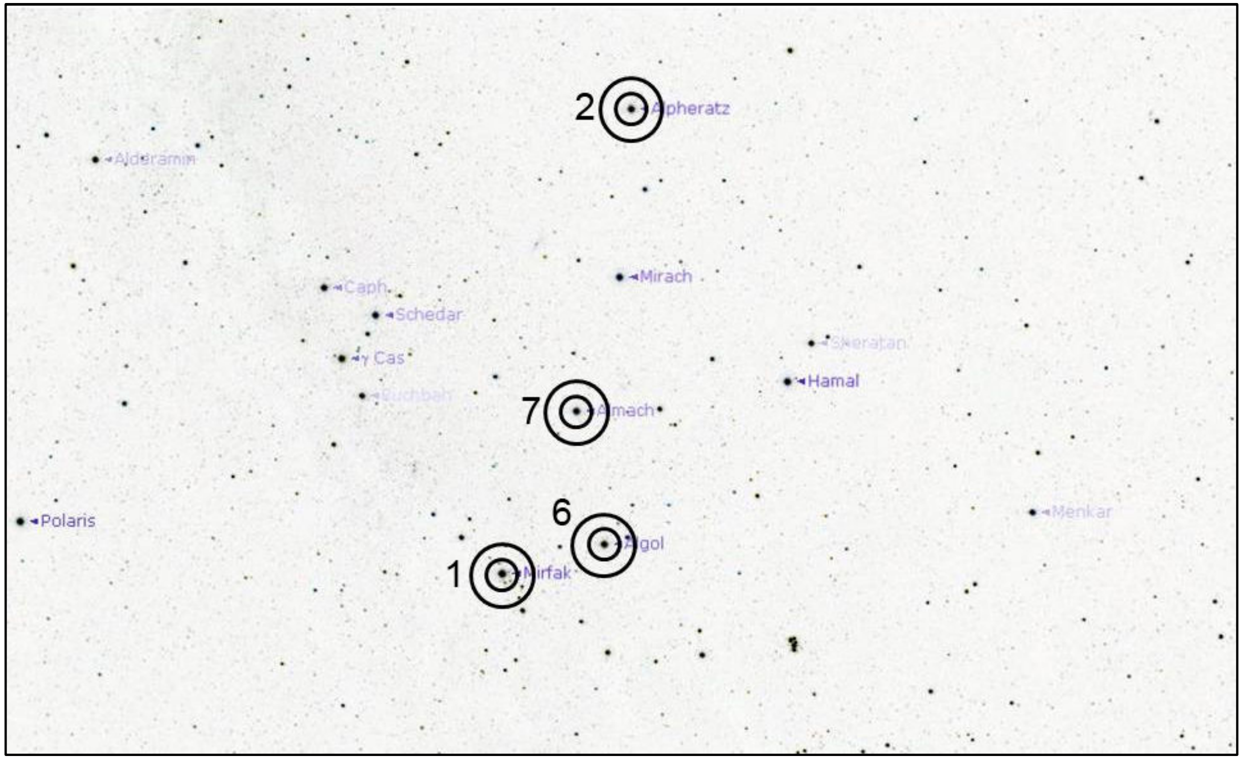
Table 6

<table>
<tr><th colspan="3">Name and Coordinates of the Selected Star</th><th rowspan="3">Local Time :</th></tr>
<tr><th>Name of Selected Star</th><th>Hour Angle (hh:mm)</th><th>Declination (°:’)</th></tr>
<tr><td></td><td></td><td></td></tr>
</table>

Solutions and Marking Scheme

Solution 1:

Figure 2 – Solution of Problem 1



Marking Scheme:

Part 1: Location of each bright star:

A) Small Circle + Correct Number: +10 points.
B) Large Circle+ Correct Number: +5 Points.
C) Small Circle without Identifier Number : +5 Points
D) Large Circle without Identifier Number: 0 Point.
E) Small or Large Circle+ Incorrect Identifier Number: 0 Point.

Solution 2:

Part 1:

Table 2

Angular Distance	
Stars Name	**Angular Distance (Degree)**
1 (Errai) and 2 (Alfirk)	$11^{\circ}:09':10'' \sim 11^{\circ}$
1 (Errai) and 3 (Alderamin)	$18^{\circ}:36':50'' \sim 19^{\circ}$

Marking Scheme:

Part 1: $\Delta =$ Error in estimation of angular distance.

$\Delta \leq 2^{\circ}$: 20 points
$2^{\circ} < \Delta \leq 4^{\circ}$: 10 Points
$\Delta > 4^{\circ}$: 0 Point

Part 2:

Table 4

Magnitude Estimation		
Number	**Star Name**	**Visible Magnitude**
2	Alfirk	3.2
3	Alderamin	2.4

For Each Star:

$\Delta m \leq 0.2$: 20 points
$0.2 < \Delta m \leq 0.5$: 15 Points
$0.5 < \Delta m \leq 0.8$: 10 Points
$0.8 < \Delta m \leq 1.0$: 5 Points
$1.0 < \Delta m \leq 1.2$: 2 Points
$\Delta m > 1.2$: 0 Point

Solution 3:

Coordinate of Selected Star				Local Time :
Star	**Hour Angle**	**R.A.**	**Dec.**	
Deneb (Alpha Cygni)	ST-RA	20h 41m	+45°	
Alfirk (Beta Cephei)	ST-RA	21h 28m	+71°	**Sidereal Time :**
Algol (Beta Persei)	RA-ST	03h 08m	+41°	
Capella (Alpha Aurigae)	RA-ST	05h 18m	+46°	

Marking Scheme:

Part 1: Point the Telescope to the coordinates of the selected stars.

If the examiner confirms the star in the 32 mm eyepiece: 40 Points

If the examiner confirms the star in the finder scope: 20 Points

If the examiner doesn't see the star in Finder : No Point

Part2: Estimate Hour Angle and Declination. ($\Delta =$ Error)

HA:	$\Delta \leq 30\ min$	: 20 points
	$30\ min < \Delta \leq 45\ min$	: 15 Points
	$45\ min < \Delta \leq hour$	: 10 Points
	$1\ hour < \Delta \leq 1.5\ hour$	: 5 Points
	$\Delta > 1.5\ hour$	: 0 Point

If Participant estimates R.A. Instead of H.A.: 20 Point

Dec:	$\Delta \leq 2^{\circ}$	: 20 points
	$2^{\circ} < \Delta \leq 4^{\circ}$	: 15 Points
	$4^{\circ} < \Delta \leq 8^{\circ}$	: 10 Points
	$8^{\circ} < \Delta \leq 10^{\circ}$	: 5 Points
	$\Delta > 10^{\circ}$	: 0 Point