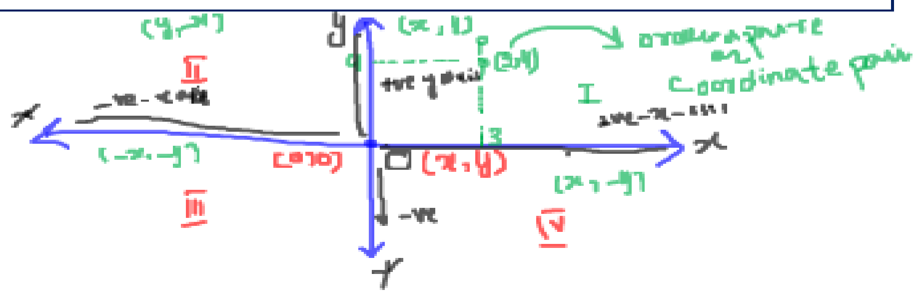
Coordinate axes

Two Perpendicular Coordinate lines that interest at the O- point these lines are called coordinate axes in the plane.



Origin

The coordinate system is the point in the plane where x and y are both zero

Coordinate pair

The ordered pair (a,b) is assigned to the point P

Quadrant

Coordinate axes divide into four regions is called quadrant

Increment and straight lines

When particles move from one point to another point, the net changes is called increment

$A = (4, -3)$ $x_1 = 4$ $y_1 = -3$ | $B = (2, 5)$ $x_2 = 2$ y_2

Example: 01

In going from the point A(4,-3) to the point B(2,5) the increments in the x and y coordinate are?

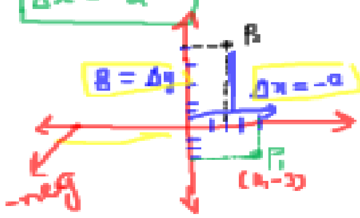
x-coordinate

$x_1 = 4$
$x_2 = 2$

$\Delta x = x_2 - x_1$

$\Delta x = 2 - 4 = -2$

$\boxed{\Delta x = -2}$



y-coordinate

$y_1 = -3$
$y_2 = 5$

$\Delta y = y_2 - y_1$

$\Delta y = 5 - (-3)$

$\Delta y = 5 + 3 = 8$

$\boxed{\Delta y = 8}$

$P_1 = (4, -3)$
$P_2 = (2, 5)$

$|\Delta y$

Δx

Example: 02

The coordinate increments from C (5,6) to D (5,1) ?

$x_1 = 5$ $y_1 = 6$
$x_2 = 5$ $y_2 = 1$

$\Delta x = x_2 - x_1$
$5 - 5$

$\boxed{\Delta x = 0}$

$\Delta y = y_2 - y_1$

$\Delta y = 1 - 6$

$\boxed{\Delta y = -5}$

Absolute Value

The **absolute value** of a number x, denoted by $|x|$, is defined by the formula

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0. \end{cases}$$

EXAMPLE 2 Finding Absolute Values

$$|3| = 3, \quad |0| = 0, \quad |-5| = -(-5) = 5, \quad |-|a|| = |a|$$

Geometrically, the absolute value of x is the distance from x to 0 on the real number line. Since distances are always positive or 0, we see that $|x| \geq 0$ for every real number x, and $|x| = 0$ if and only if $x = 0$. Also,

$$|x - y| = \text{the distance between } x \text{ and } y$$

on the real line (Figure 1.2).

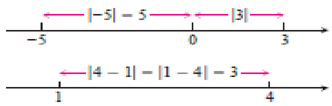


FIGURE 1.2 Absolute values give distances between points on the number line.

Since the symbol $\sqrt{a}$ always denotes the *nonnegative* square root of a, an alternate definition of $|x|$ is

$$|x| = \sqrt{x^2}.$$

It is important to remember that $\sqrt{a^2} = |a|$. Do not write $\sqrt{a^2} = a$ unless you already know that $a \geq 0$.

The absolute value has the following properties. (You are asked to prove these properties in the exercises.)

Absolute Value Properties

1.	$\lvert -a \rvert = \lvert a \rvert$	A number and its additive inverse or negative have the same absolute value.
2.	$\lvert ab \rvert = \lvert a \rvert \lvert b \rvert$	The absolute value of a product is the product of the absolute values.
3.	$\left\lvert \frac{a}{b} \right\rvert = \frac{\lvert a \rvert}{\lvert b \rvert}$	The absolute value of a quotient is the quotient of the absolute values.
4.	$\lvert a + b \rvert \leq \lvert a \rvert + \lvert b \rvert$	The **triangle inequality**. The absolute value of the sum of two numbers is less than or equal to the sum of their absolute values.

Increments and Straight Lines

When a particle moves from one point in the plane to another, the net changes in its coordinates are called *increments*. They are calculated by subtracting the coordinates of the starting point from the coordinates of the ending point. If x changes from x_1 to x_2, the **increment** in x is

$$\Delta x = x_2 - x_1.$$

EXAMPLE 1 In going from the point $A(4, -3)$ to the point $B(2, 5)$ the increments in the x- and y-coordinates are

$$\Delta x = 2 - 4 = -2, \qquad \Delta y = 5 - (-3) = 8.$$

From $C(5, 6)$ to $D(5, 1)$ the coordinate increments are

$$\Delta x = 5 - 5 = 0, \qquad \Delta y = 1 - 6 = -5.$$

See Figure 1.7.

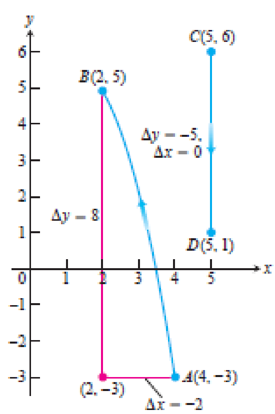


FIGURE 1.7 Coordinate increments may be positive, negative, or zero (Example 1).

Given two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ in the plane, we call the increments $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$ the run and the rise, respectively, between P_1 and P_2. Two such points always determine a unique straight line (usually called simply a line) passing through them both. We call the line P_1P_2.

Any nonvertical line in the plane has the property that the ratio

$$m = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

has the same value for every choice of the two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the line (Figure 1.8). This is because the ratios of corresponding sides for similar triangles are equal.

HISTORICAL BIOGRAPHY*

René Descartes
(1596–1650)