$(-1)^2 = 1$
$(2)^2 = 4$

Chapter 01: Preliminaries

- Real Numbers: $Q \cup Q'$

Union of Rational and irrational numbers

Square is positive number $(-\infty, \infty)$, $\mathbb{R}$

$W = \{0, 1, 2, 3, \cdots\}$
$N = \{1, 2, 3, \cdots\}$
$Z = \{\cdots -1, 0, 1, \cdots\}$
$Z = \{0, \pm 1, \pm 2, \cdots\}$
$Q = \{p/q ;\ p, q \in Z,\ q \neq 0\}$
$\frac{2}{3}, \frac{1}{1},$
$Q' \neq p/q$
$\sqrt{25}$ $\sqrt{2}, \sqrt{3}$ π
$\sqrt{5^2} = 5$

- Real Lines:

Real numbers can be representing in lines is called Real lines

$-\infty$ -3 -2 -1 0 1 2 ∞

- Interval:

A subset of the real lines is called interval

It contains at least two numbers:

$[\,[c]\ (b)\ (a)\,]$ R

$a \in R$
$b \in R$
$c \in R$

[1, 5]

$f(x) = x^2$

$f(1)$
$f(2)$
$f(3)$
$f(4)$ $f(5)$

[a, b]

Types

If its contains both of its points
closed

(1, 5)

(a, b)

$f(2)$
$f(3)$
$f(4)$

if it contains neither end points
open

Example: 01

Solve the following inequalities and show their solution set on the real lines:

a) $2x - 1 < x + 3$

$2x - x < 3 + 1$
$x < 4$

$I = (-\infty, 4) \rightarrow$ open

or $x \le 4$

$I = (-\infty, 4]$ half open / closed

$-\infty$ ∞ 6 4

$(-\infty, \infty)$

b) $-\frac{x}{3} < 2x + 1$

$-x < (2x+1)3$
$-x < 6x + 3$
$-3 < 6x + x$
$-3 < 7x$
$-3/7$

c) $\frac{6}{x-1} \geq 5$

$6 \geq 5(x-1)$

$6 \geq 5x-5$

$6+5 \geq 5x$

$\boxed{11/5 \geq x}$

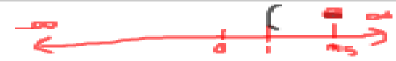
Sol set: $I = (1, 11/5]$

$x-1=0$

$x=1$

at $x=1$

$\frac{6}{0x} \to \infty$



$\bar{I} = (-\infty, 1) \cup (1, 11/5]$

- Absolute values:

The absolute value of a number denoted by $|x|$ is defined by

$$|x| \overset{\text{or}}{=} \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases} \quad ; \quad \sqrt{(x)^2} = |x|$$

Exp: 3

$|3| = \sqrt{(3)^2} = \sqrt{9} = 3$

0

$|0| = \sqrt{0^2} = 0$

-5

$|-5| = \sqrt{(-5)^2} = \sqrt{25} = 5$

Exp:- Solving Eqs with absolute values.

(a) $|2x-3| = 7$

Sol:

$2x-3 = \pm 7$

$2x-3 = 7$

$2x = 7+3$

$x = 10/2$

$\boxed{x=5}$

$2x-3 = -7$

$2x = -7+3$

$x = -4/2$

$\boxed{x = -2}$

$SS = \{x_1, x_2\} = \{5, -2\}$

(b) $\left|5 - \frac{2}{x}\right| < 1$

$-1 < 5 - \frac{2}{x} < 1$

$-6 < -2/x < -4$

$3 < \frac{1}{x} < 2$

$\frac{1}{3} < x < \frac{1}{2}$

Rules for Inequalities

If a, b, and c are real numbers, then:

1. $a < b \Rightarrow a + c < b + c$
2. $a < b \Rightarrow a - c < b - c$
3. $a < b$ and $c > 0 \Rightarrow ac < bc$
4. $a < b$ and $c < 0 \Rightarrow bc < ac$
   Special case: $a < b \Rightarrow -b < -a$
5. $a > 0 \Rightarrow \frac{1}{a} > 0$
6. If a and b are both positive or both negative, then $a < b \Rightarrow \frac{1}{b} < \frac{1}{a}$

TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	
	$[a, b]$	$\{x \mid a \le x \le b\}$	Closed	
	$[a, b)$	$\{x \mid a \le x < b\}$	Half-open	
	$(a, b]$	$\{x \mid a < x \le b\}$	Half-open	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	
	$[a, \infty)$	$\{x \mid x \ge a\}$	Closed	
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	
	$(-\infty, b]$	$\{x \mid x \le b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open and closed	