Partial fraction

$$\frac{2x^3}{x^2} = 2x$$

$$\frac{2x^3 - 11x^2 + 13x - 1}{x^2 - 5x + 16}$$

Sol

As the degree of num is greater than denom then we will divide

$$\begin{array}{r|l} & 2x - 1 \\ \hline x^2 - 5x + 6 & 2x^3 - 11x^2 + 13x - 1 \\ & 2x^3 - 10x^2 + 12x \\ & - \quad + \quad - \\ \hline & -x^2 + x - 1 \\ & -x^2 + 5x - 6 \\ & + \quad - \quad + \\ \hline & -4x + 5 \end{array}$$

$$\Rightarrow \frac{2x^3 - 11x^2 + 13x - 1}{x^2 - 5x + 6} = 2x - 1 + \frac{-4x + 5}{(x-2)(x-3)}$$

Case 1

All factors are linear & unequal

let $\frac{-4x+5}{x(x-2)(x-3)} = \frac{A}{x(x-2)} + \frac{B}{x-3} \rightarrow (2)$

$(x-2)(x-2)$

$2x^3 - 10x^2 + 12x$

let $\frac{-4x+5}{(x-2)(x-3)} = \frac{A}{x-2} + \frac{B}{x-3} \rightarrow (2)$

The no. of arbitrary constant must equal to degree of denominator

$(2) \Rightarrow -4x+5 = A(x-3) + B(x-2) \rightarrow (3)$

put $x-2=0 \Rightarrow x=2$ in eq (3)

$(3) \Rightarrow -4(2)+5 = A(2-3)$

$\Rightarrow -3 = -A \Rightarrow \boxed{A=3}$

put $x-3=0 \Rightarrow x=3$ in eq (3)

$(3) \Rightarrow -4(3)+5 = B(3-2)$

$\Rightarrow \boxed{-7 = B}$

$\therefore \frac{-4x+5}{(x-2)(x-3)} = \frac{3}{x-2} + \frac{-7}{x-3} = \frac{3}{x-2} - \frac{7}{x-3}$

$\therefore \frac{2x^3 - 11x^2 + 13x - 1}{x^2 - 5x + 6} = 2x - 1 + \frac{3}{x-2} - \frac{7}{x-3}$

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If all the factor of denominator are linear & some are equal.

Case - 3

If some factor are non linear

Case-2 If some linear factor are equal

Linear some factor

for e.g

$$\frac{4x^2 - 3x + 1}{(x-1)(x-2)^2} = \frac{A}{x-1} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2} \to (1)$$

Arbitrary constant = degree of denominator $(x-2)(x-2)$

$\Rightarrow 4x^2 - 3x + 1 = A(x-2)^2 + B(x-1)(x-2) + C(x-1) \to (2)$

$(Ax^2 - 2Ax + 4) + Bx^2 - 2Bx - Bx - 2B$

put $x - 1 = 0 \Rightarrow x = 1$ in eq (2)

$$4 - 3 + 1 = A(1-2)^2 \Rightarrow \boxed{A = 2}$$

put $x - 2 = 0 \Rightarrow x = 2$ in eq (2)

$$4(2)^2 - 3(2) + 1 = C(2-1) \Rightarrow \boxed{C = 11}$$

(2) $\Rightarrow 4x^2 - 3x + 1 = A(x^2 - 4x + 4) + B(x^2 - 3x + 2) + C(x-1) \to (3)$

$$x^2 \to 4 = A + B \Rightarrow \boxed{B = 2}$$

comparing coefficient of x^2

(2) ⇒

$x^2 \rightarrow 4 = A + B = \boxed{B = 2}$

eq (i) $\frac{4x^2 - 3x + 1}{(x-1)(x-2)^2} = \frac{2}{x-1} + \frac{2}{x-2} + \frac{11}{(x-2)^2}$

e.g $\int \frac{4x^2 - 3x + 1}{(x-1)} dx = \int \left\{ \frac{2}{x-1} + \frac{2}{x-2} + \frac{11}{(x-2)^2} \right\} dx$

$= 2\ln(x-1) + 2\ln(x-2) - \frac{11}{(x-2)} + c$ Ans

Case 3

$$I = \int \frac{4x+9}{(x+1)(x^2+4)}\,dx$$

Consider

$$\frac{4x+9}{(x+1)(x^2+4)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+4} \rightarrow (i)$$

$$\Rightarrow 4x+9 = A(x^2+4) + (Bx+C)(x+1) \rightarrow (2)$$

Ax^2+4 $\quad$ $Bx^2+Bx+Cx+C$

Put $x+1=0 \Rightarrow x=-1$ in eq (2)

Expand eq (2)

$$(2) \Rightarrow 4(-1)+9 = A\{(1)^2+4\} \Rightarrow 5 = 5A \Rightarrow A = 1$$

$$(2) \Rightarrow 4x+9 = A(x^2+4) + B(x^2+x) + C(x+1) \rightarrow (3)$$

$A = -B$

$x^2 \rightarrow 0 = A+B \Rightarrow \boxed{B=-1}$

$x - 4 = B+C \Rightarrow \boxed{C=5}$

$4 = B+C$

$$\frac{4x+9}{(x+1)(x^2+4)} = \frac{1}{x+1} + \frac{-x+5}{x^2+4}$$

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Magic Substitution :-

The integration of any rational trigonom function may be difficult, so we use magic substitution

$$\boxed{z = \tan\frac{x}{2}}$$

$$\frac{dz}{dx} = \sec^2\frac{x}{2}\cdot\left(\frac{1}{2}\right)$$

$$\Rightarrow dx = \frac{2dz}{\sec^2 x/2}$$

$$= \frac{2dz}{1+\tan^2 x/2}$$

$$\boxed{\Rightarrow dx = \frac{2dz}{1+z^2}}$$

Consider $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$\times$ing & $(\div)$ing by $\cos\frac{x}{2}$

$$= \frac{2\sin x/2}{\cos x/2}\cdot\cos^2 x/2$$

$$= \frac{2\tan x/2}{\sec^2 x/2}$$

$$\Rightarrow \boxed{\sin x = \frac{2z}{1+z^2}}$$

consider

$$\cos x = 2\cos^2\frac{x}{2} - 1 = \frac{2}{\sec^2\frac{x}{2}} - 1$$

$$= \frac{2}{1+z^2} - 1$$

$$\Rightarrow \boxed{\cos x = \frac{1-z^2}{1+z^2}}$$

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$$I = \int \frac{2 - \cos x}{2 + \cos x}\, dx$$

put $z = \tan \frac{x}{2}$, $dx = \frac{2dz}{1+z^2}$

$$\cos x = \frac{1 - z^2}{1+z^2}$$

$$I = \int \frac{2 - \frac{1-z^2}{1+z^2}}{2 + \frac{1-z^2}{1+z^2}} \cdot \frac{2dz}{1+z^2}$$

$$I = 2\int \frac{3z^2 + 1}{(z^2+3)(z^2+1)}\, dz$$

Consider

$$\frac{3z^2+1}{(z^2+3)(z^2+1)} = \frac{3u+1}{(u+3)(u+1)}$$

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$$\frac{3U+1}{(U+3)(U+1)} = \frac{A}{U+3} + \frac{B}{U+1} = \frac{3U+1}{(U+3)(U+1)}$$

put $U+3=0 \Rightarrow U=-3$ in eq (1)

$$(1) \Rightarrow \cancel{3(-3)+1} \quad \frac{A}{\cancel{-3+3}} + \frac{B}{-3+1}$$

$$\Rightarrow A = 0 +$$

$$3U+1 = A(U+1) + B(U+3) \rightarrow (1)$$

put $U+3=0 \Rightarrow U-3$ in eq (1)

$$3U+1 = A(-3+1) + B(\cancel{-3+3})$$

$$3(-3)+1 = A(-2)$$

$$-9+1 = A(-2)$$

$$-8 = A(-2)$$

$$\frac{-8}{-2} = A$$

$$\boxed{A = 2}$$

$$\frac{3U+1}{(U+3)(U+1)} = \frac{A}{U+3} + \frac{B}{U+1} = \frac{4}{U+3} + \frac{-1}{U+1}$$

$$\therefore I = 2\int\left\{\frac{-1}{z^2+1} + \frac{4}{z^2+3}\right\}dz$$

$$= -2\tan^{-1}z + 4\left\{\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{z}{3}\right)\right\} + c$$

$$= -2\tan^{-1}\left(\tan\frac{x}{2}\right) + \frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right) + c$$

$$= -x + \frac{4}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right) + c \text{ A}$$

$$I = \int \frac{d\theta}{\cot 4\theta + \operatorname{cosec} 4\theta}$$

$$= \int \frac{d\theta}{\frac{\cos 4\theta}{\sin 4\theta} + \frac{1}{\sin 4\theta}}$$

$$= -\frac{1}{4} \int \frac{-4\sin 4\theta}{\cos 4\theta + 1} \, d\theta$$

$$= -\frac{1}{4} \ln(\cos 4\theta + 1) + C \text{ Ans!}$$