BAHRIA UNIVERSITY (KARACHI CAMPUS)

Department of Software Engineering.

Assignment 03 (Fall 2022)

Course Title: Applied Physics	**Course Code: GSC-114**
Class: BSE 1B	**Shift: Morning**
Course Instructor: Engr. Rizwan Fazal	**Date: 04 Jan 2023**
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Assignment No 03

Submitted By:

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Section: 1 B

Question #(1): find if the following vectors are collinear?

a): $\vec{A} = (3,4,5)$ and $\vec{B} = (6,8,10)$

for two collinear vectors: $\vec{A} \times \vec{B} = 0$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 6 & 8 & 10 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 5 \\ 8 & 10 \end{vmatrix} \hat{i} - \begin{vmatrix} 3 & 5 \\ 6 & 10 \end{vmatrix} \hat{j} + \begin{vmatrix} 3 & 4 \\ 6 & 8 \end{vmatrix} \hat{k}$$

$$= (40-40)\hat{i} - (30-30)\hat{j} + (24-24)\hat{k}$$

$$= 0 - 0 + 0$$

$$\boxed{\vec{A} \times \vec{B} = 0}$$

Hence, $\vec{A}$ and $\vec{B}$ are two collinear vectors

b): $\vec{A} = (3,4,0)$, and $\vec{B} = (2,2,1)$

for two collinear vectors $\vec{A} \times \vec{B} = 0$.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 0 \\ 2 & 2 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 4 & 0 \\ 2 & 1 \end{vmatrix} \hat{i} - \begin{vmatrix} 3 & 0 \\ 2 & 1 \end{vmatrix} \hat{j} + \begin{vmatrix} 3 & 4 \\ 2 & 2 \end{vmatrix} \hat{k}$$

$$= (4-0)\hat{i} - (3-0)\hat{j} + (6-8)\hat{k}$$

$$\boxed{\vec{A} \times \vec{B} = 4\hat{i} - 3\hat{j} - 2\hat{k}}$$

since, $\vec{A} \times \vec{B} \neq 0$, hence $\vec{A}$ and $\vec{B}$ are not collinear.

Question #(2): Find if following vectors are Coplanar?

a): $\vec{A} = (1,2,3)$, $\vec{B} = (2,4,6)$ and $\vec{C} = (3,4,5)$

for three coplanar vectors $(\vec{A} \times \vec{B}) \cdot \vec{C} = 0$.

for $$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 \\ 4 & 6 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 2 \\ 2 & 4 \end{vmatrix} \hat{k}$$

$$= (12-12)\hat{i} - (6-6)\hat{j} + (4-4)\hat{k}$$

$$= 0 - 0 + 0$$

$$= 0$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (0) \cdot (3\hat{i} + 4\hat{j} + 5\hat{k})$$

$$\boxed{(\vec{A} \times \vec{B}) \cdot \vec{C} = 0}$$

Hence, $\vec{A}$, $\vec{B}$ and $\vec{C}$ are coplanar vectors

(b) $\vec{A} = (5, -1, 1)$, $\vec{B} = (-2, 3, 4)$ and $\vec{C} = (3, 4, 5)$

for three coplanar vectors $(\vec{A} \times \vec{B}) \cdot \vec{C} = 0$

for $\vec{A} \times \vec{B}$:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 1 \\ -2 & 3 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 1 \\ 3 & 4 \end{vmatrix} \hat{i} - \begin{vmatrix} 5 & 1 \\ -2 & 4 \end{vmatrix} \hat{j} + \begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix} \hat{k}$$

$= (-4-3)\hat{i} - (20+)\hat{j} + (15-2)\hat{k}$

$= -7\hat{i} - 22\hat{j} + 13\hat{k}$

$(\vec{A} \times \vec{B}) \cdot \vec{C} = (-7\hat{i} - 22\hat{j} + 13\hat{k}) \cdot (3\hat{i} + 4\hat{j} + 5\hat{k})$

$= -21 - 88 + 65$

$\boxed{(\vec{A} \times \vec{B}) \cdot \vec{C} = -44}$

Hence, $(\vec{A} \times \vec{B}) \cdot \vec{C} \neq 0$. $\vec{A}$, $\vec{B}$ and $\vec{C}$ are not coplanar

Question # (3):- find the scalar Tripple product of given vectors.

a): $\vec{A} = (1,2,3)$, $\vec{B} = (4,5,6)$ and $\vec{C} = (2,6,5)$

= formula

$(\vec{A} \times \vec{B}) \cdot \vec{C}$ = scalar Tripple product

for $\vec{A} \times \vec{B}$:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 3 \\ 4 & 0 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} \hat{k}$$

$$= (12 - 15)\hat{i} - (6 - 12)\hat{j} + (5 - 8)\hat{k}$$

$$\vec{A} \times \vec{B} = -3\hat{i} + 6\hat{j} - 3\hat{k}$$

Now $(\vec{A} \times \vec{B}) \cdot \vec{C}$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (-3\hat{i} + 6\hat{j} - 3\hat{k}) \cdot (2\hat{i} + 5\hat{j} + 6\hat{k})$$

$$= -6 + 30 - 18$$

$$\boxed{(\vec{A} \times \vec{B}) \cdot \vec{C} = 6}$$ Ans

(b) $\vec{A} = (5, -1, 1)$, $\vec{B} = (-2, 0, 4)$ and $\vec{C} = (3, 4, 5)$

scalar Triple product $= (\vec{A} \times \vec{B}) \cdot \vec{C}$

for $\vec{A} \times \vec{B}$:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 1 \\ -2 & 0 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 1 \\ 0 & 4 \end{vmatrix} \hat{i} - \begin{vmatrix} 5 & 1 \\ -2 & 0 \end{vmatrix} \hat{j} + \begin{vmatrix} 5 & -1 \\ -2 & 0 \end{vmatrix} \hat{k}$$

$$= (-4-0)\hat{i} - (20+2)\hat{j} + (0-2)\hat{k}$$

$$\vec{A}\times\vec{B} = -4\hat{i} - 22\hat{j} - 2\hat{k}$$

Now for $(\vec{A}\times\vec{B})\cdot\vec{C}$

$$(\vec{A}\times\vec{B})\cdot\vec{C} = (-4\hat{i} - 22\hat{j} - 2\hat{k})(3\hat{i} + 2\hat{j} + \hat{k})$$

$$= -12 - 44 - 2$$

$$\boxed{\vec{A}\times\vec{B}\cdot\vec{C} = -56}$$ Ans

Question (4):– Find the Area of Triangle determined by two vectors

a) $\vec{A} = (1,2,3)$, $\vec{B} = (4,5,6)$

∴ formula,

$$\text{Area of triangle} = \frac{1}{2}|\vec{A}\times\vec{B}|$$

$$\text{Area of triangle} = \frac{1}{2}\left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{matrix} \right\|$$

$$= \frac{1}{2}\left\| \left[\begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix}\hat{i} - \hat{j}\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} + \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}\hat{k} \right] \right\|$$

$$= \frac{1}{2}\left|\left[(12-15)\hat{i} - (6-12)\hat{j} + (5-8)\hat{k}\right]\right|$$

$$= \frac{1}{2}\left| -3\hat{i} + 6\hat{j} - 3\hat{k}\right|$$

$$= \frac{1}{2}\sqrt{(-3)^2 + (6)^2 + (-3)^2}$$

$$= \frac{1}{2}\sqrt{9 + 36 + 9}$$

$$\boxed{\Delta = \frac{1}{2}\sqrt{54}} \text{ Unit}^2$$

(b): $\vec{A} = (5, -1, 1)$, $\vec{B} = (-2, 3, 4)$

- formula

$$\Delta = \frac{1}{2}\left|\vec{A} \times \vec{B}\right|$$

for $\vec{A} \times \vec{B}$:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 1 \\ -2 & 3 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 1 \\ 3 & 4 \end{vmatrix} \hat{i} - \begin{vmatrix} 5 & 1 \\ -2 & 4 \end{vmatrix} \hat{j} + \begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix}$$

$$= (-4 - 3)\hat{i} - (20 + 2)\hat{j} + (15 - 2)\hat{k}$$

$$\vec{A} \times \vec{B} = -7\hat{i} - 22\hat{j} + 13\hat{k}$$

$$|\vec{A} \times \vec{B}| = \sqrt{(-7)^2 + (-22)^2 + (13)^2}$$

$$= \sqrt{49 + 484 + 169}$$

$$|\vec{A} \times \vec{B}| = \sqrt{702}$$

$$\boxed{\Delta = \frac{1}{2}\sqrt{702} \text{ unit}^2}$$

Question # (5):- Find the volume of parallelepiped determined by following vector.

a) $\vec{A} = (1, 2, -3)$, $\vec{B} = (3, 0, 6)$ and $\vec{C} = (7, 1, 9)$

: formula

volume of parallelepiped $= V = |\vec{C} \cdot (\vec{A} \times \vec{B})|$

for $\vec{A} \times \vec{B}$:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 3 & 0 & 6 \end{vmatrix}$$

$$= \begin{vmatrix} 2 & 3 \\ 0 & 6 \end{vmatrix} \hat{i} - \begin{vmatrix} 1 & 3 \\ 3 & 6 \end{vmatrix} \hat{j} + \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} \hat{k}$$

$$= (12 - 0)\hat{i} - (6 - 9)\hat{j} + (0 - 6)\hat{k}$$

$$\vec{A} \times \vec{B} = 12\hat{i} + 3\hat{j} - 6\hat{k}$$

Now:

$$V = |\vec{C} \cdot (\vec{A} \times \vec{B})|$$

$$V = |(7\hat{i} + \hat{j} + 9\hat{k}) \cdot (12\hat{i} + 3\hat{j} - 6\hat{k})|$$

$$= |84 + 3 - 54|$$

$$\boxed{V = 33} \text{ unit}^3$$

(b) $\vec{A} = (5, -1, 1)$, $B = (-2, 3, 4)$ and $C = (3, 4, 5)$

$$\therefore V = |\vec{C} \cdot (\vec{A} \times \vec{B})|$$

Now $\vec{A} \times \vec{B}$:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & -1 & 1 \\ -2 & 3 & 4 \end{vmatrix}$$

$$= \begin{vmatrix} -1 & 1 \\ 3 & 4 \end{vmatrix} \hat{i} - \begin{vmatrix} 5 & 1 \\ -2 & 4 \end{vmatrix} \hat{j} + \begin{vmatrix} 5 & -1 \\ -2 & 3 \end{vmatrix} \hat{k}$$

$$= (-4-3)\hat{i} - (20+2)\hat{j} + (15-2)\hat{k}$$

$$\vec{A} \times \vec{B} = -7\hat{i} - 22\hat{j} + 13\hat{k}$$

$$\therefore V = |\vec{C} \cdot (\vec{A} \times \vec{B})|$$

$$V = |(3\hat{i} + 4\hat{j} + 5\hat{k}) \cdot (-7\hat{i} - 22\hat{j} + 13\hat{k})|$$

$$= |-21 - 88 + 65|$$

$$= |-44|$$

$$\boxed{V = 44 \text{ unit}^3}$$