$V = x^2$
$V' = 2x$
$V'' = 2$
$V''' = 0$

Q $f(x) = x^2 \cos 4x$ (2^{nd} derivative)

$f'(x) = 4x^2 \cos 4x \cancel{+ 2x \cos 4x}$

$f'(x) = \cancel{8x}\ 16x^2 \cos 4x$

Product Rule :-

$f'(x) = -x^2 \sin 4x\,(4) + \cos 4x\,(2x)$

$f'(x) = -4x^2 \sin 4x + 2x \cos 4x$

$f''(x) = [-4x^2 \cos 4x\,(4) + (-4 \sin 4x\,(2))]$

$+ [-2x \sin 4x\,(4) + (\cos 4x\ 2)]$

$= -16x^2 \cos 4x - 8 \sin 4x -$
$8 \sin 4x + 2 \cos 4x$

Leibnitz Theorem :-

$f(x)\ y = x^2 \cos 4x$

$U = \cos 4x$, $V = x^2$

$U' = -4 \sin 4x$ $\quad V' = 2x$

$U'' = -16 \cos 4x$ $\quad V'' = 2$

$V''' = 9n_{(2}^{-1)} - 1)$

• V aisa choose kry gy jis ka derivative ky to reduce hojaye.

$(x^2 \cos 4x)'' = U''V + nU'V' + \frac{2(2-1)}{2!} U^0 V''$

$= -16 \cos 4x\, x^2 + 2(-4 \sin 4x)(2x) + \frac{\cancel{2}}{\cancel{2!}}$
$+ \frac{2}{2!} \cos 4x\ 2$ $\quad \cos 4x\ 2$

$= -16x^2 \cos 4x - 16 \sin 4x + 2 \cos 4x$

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Q16 If $y = \cancel{e^{m \sin^{-1} x}}$ $e^{m\sin^{-1} x}$
show that
$$(1-x^2)y^{(n+2)} - (2n+1)xy^{(n+1)} - (n^2+m^2)y^n = 0$$

$$y' = e^{m\sin^{-1}x} \cdot \frac{d}{dx} m\sin^{-1}x$$

$$y' = \frac{m e^{m\sin^{-1}x}}{\sqrt{1-x^2}}$$

$$\sqrt{1-x^2}\, y' = m y$$

S.B.S

$$(1-x^2)[y']^2 = m^2 y^2$$

Diff on b.s

$$2(1-x^2) y' y'' + (-2x)[y']^2 = 2m^2 y \cdot y'$$

$$\cancel{2y'}[(1-x^2)y'' - xy'] = \cancel{2y'} \cdot m^2 y$$

$$I = 1-x^2 y'' - xy' - m^2 y = 0 \quad \text{--- } (1)$$

$$(I)^n = (I_1)^n - (I_2)^n - (I_3)^n \quad \text{--- } (2)$$

$$(I_1)^n = [(1-x^2)y'']^n$$

$U = y''$ $\quad V = 1-x^2$

$U' = y'''$ $\quad V' = -2x$

$U'' = y''''$ or (y^{iv}) $\quad V'' = -2$

$U^{(n-1)} = y^{(n+1)}$ $\quad V''' = 0$

$U^n = y^{n+2}$

+2 karke har derivative

$$[(1-x^2)y'']^n = y^{n+2}(1-x^2) + n y^{(n+1)}(-2x) + \frac{n(n-1)}{2!} y^n (-2)$$

$$= (1-x^2)y^{n+2} - 2nx\, y^{n+1} - \frac{2n(n-1)}{2!} y^n$$

$$(I_1)^n = (1-x^2)y^{(n+2)} - 2nxy^{(n+1)} - n(n-1)y^n \quad \text{— (3)}$$

consider I_2 :—

$(I_2)^n = (xy')^n$

$U = y'$ $\qquad V = x$

$U' = y''$ $\qquad V' = 1$

$U'' = y'''$ $\qquad V = 0$

$U^{n-2} = y^{n-1}$

$U^{n-1} \quad y^n$

$U^n = y^{n+1}$

$x(xy')^n = xy^{n+1} + ny^n(1)$

$(I_2)^n = xy^{n+1} + ny^n$ —— (4)

consider I_3 :—

$(I_3)^n = [m^2 y]^n = m^2 y^n$ —— (5)

$-n(n-1)-n$
$-n^2+\cancel{n}-\cancel{n}-m^2$
$-n^2-m^2$

Put 3, 4, 5 in ②

$(I)^n = (1-x^2)y^{n+2} - 2nxy^{(n+1)} - n(n-1)y^n$
$- xy^{n+1} - ny^n - m^2y^n$

Add 3, 4, 5

$(1-x^2)y^{n+2} - 2nxy^{n+1} - n(n-1)y^n$
$- xy^{n+1} - ny^n$
$- m^2y^n$

$+$

$(1-x^2)y^{n+2} - (2nx+x)y^{n+1} - (n^2+m^2)y^n$

$(1-x^2)y^{n+2} - (2n+1)xy^{n+1} - (n^2+m^2)y^n = 0$

$I = (1-x^2)y^{n+2} - (2n+1)xy^{n+1} - (n^2+m^2)y^n = 0$

$\therefore I = 0$

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ROLLE'S THEOREM :

Let $f(x)$ be

i) continuous function on $[a,b]$

(ii) Differentiable function on (a,b)

iii) $f(a) = f(b)$

then $\exists$ a point (there exist), $c \in (a,b)$ such that $f'(c) = 0$.

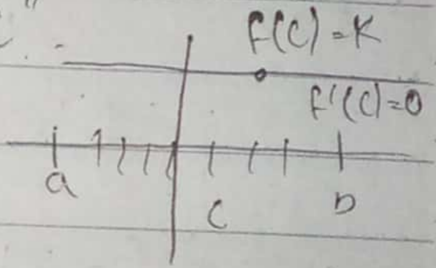
Proof :–

Case - I :

$f(c) = k$ (constant)

in the interval (a,b)

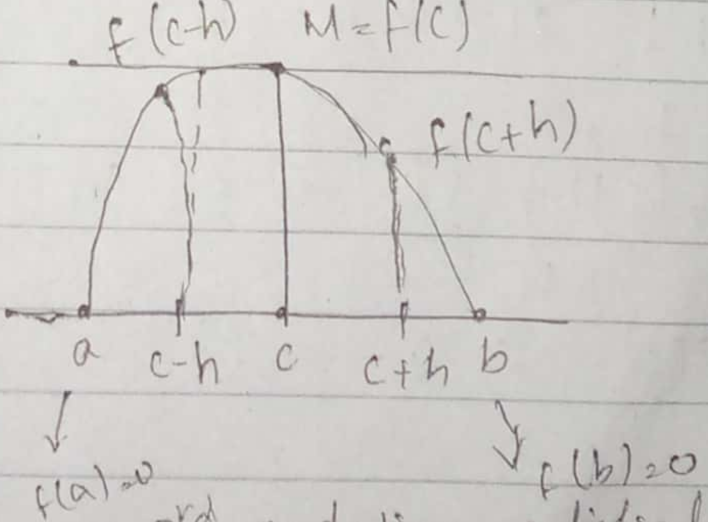
then for every point "c" in (a,b) $\boxed{f'c = 0}$



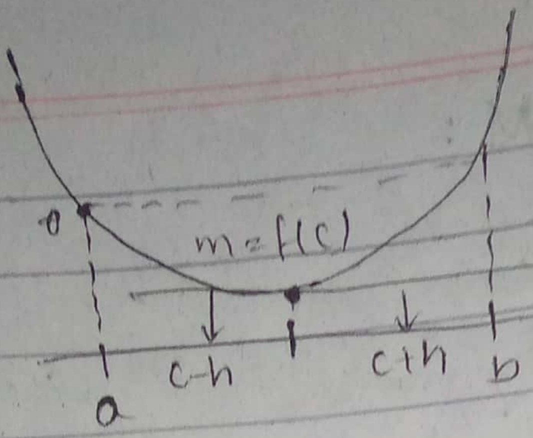
Case - II :

$f(c) \neq$ constant

either $f(c) = M$ OR $f(c) = m$



3rd condition satisfied.



let $f(c) = M$

$f(c-h) \le M$

$f(c-h) \le f(c)$

$f(c-h) - f(c) \le 0$

$$\frac{f(c-h) - f(c)}{-h} \ge \frac{0}{-h}$$

$$\lim_{h \to 0} \frac{f(c-h) - f(c)}{-h} \ge 0$$

$f'c \ge 0$ — (1)

$f(c+h) \le M$

$f(c+h) \le f(c)$

$f(c+h) - f(c) \le 0$

$$\frac{f(c+h) - f(c)}{h} \le \frac{0}{h}$$

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h} \le 0$$

$f'(c) \le 0$ — (2)

Trichotomy Property

$a \le b$
$b \le a$
$\boxed{a = b}$

From (1) & (2)

$f'(c) \ge 0$ and $f'(c) \le 0$

$\Rightarrow f'(c) = 0$

★ First principle.

Left hand $f'(x) = \lim_{h \to 0} \dfrac{f(x-h) - f(x)}{-h}$

Right hand $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

Q Discuss the validity of:
Rolle's Theorem if $f(x) = x(x+3)e^{-x/2}$
on the interval $[-3, 0]$.

Sol :-

$a = -3$, $b = 0$

$f(a) = 0$

$f(b) = 0$

$f(a) = f(b)$ Condition satisfied

$f(x) = (x^2+3x)e^{-x/2}$

$f'(x) = e^{-x/2} \cdot \frac{d}{dx}(x^2+3x) + (x^2+3x) \cdot \frac{d}{dx} e^{-x/2}$

$= e^{-x/2}(2x+3) + (x^2+3x)\, e^{-x/2} \cdot -\frac{1}{2}$

$= e^{-x/2}(2x+3) + (x^2+3x)\, e^{-x/2} \cdot \frac{-1}{2}$

$= e^{-x/2}(2x+3) - \frac{1}{2}(x^2+3x)\, e^{-x/2}$

$= \frac{e^{-x/2}}{2}\left[4x+6 - x^2 - 3x\right]$

$f'(x) = \frac{e^{-x/2}}{2}(-x^2+x+6)$

$f'(x) = \frac{-e^{-x/2}}{2}(x^2-x-6)$

$f'(c) = \frac{-e^{-c/2}}{2}(c^2-c-6)$

$0 = \frac{-e^{-c/2}}{2}(c^2-c-6)$

$\frac{-e^{-c/2}}{2} \neq 0$ OR $c^2-c-6=0$

$c = -2, 3$

~~3 is out~~

$c = -2$

e ki power koi bhi real no ho to vo h zero k equal nhi hoga.

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3 is out of the interval [-3, 0]

Q $f(x) = \frac{x^3}{3} - 3x$ on $[-3, 3]$

$f'(x) = \frac{\cancel{3}x^2 \cdot \cancel{3}}{\cancel{3}} - 3$

$f'(x) = x^2 - 3$

~~$x - 3$~~

$f'(c) = c^2 - 3$

$0 = c^2 - 3$

~~$0 = (c+3)(c-3)$~~

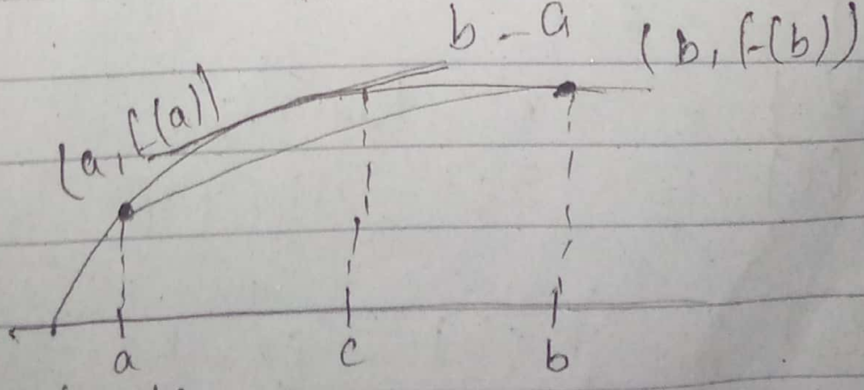
$c^2 = 3$

$c = \pm\sqrt{3}$

~~$c = \pm 1.73$~~

Mean Value Theorem :-

(1) $f'(c) = \frac{f(b) - f(a)}{b - a}$



$c \in (a, b)$

(1) let $f(x)$ be a continuous function on a ~~closed~~ $[a,b]$ & differentiable on (a,b), then there exist atleast one point c in (a,b) at which ~~$f'(c)$~~

$$f'(c) = \frac{f(b)-f(a)}{b-a} \Rightarrow (f'(c) = m)$$

★ ~~Mean value~~ theorem ~~special case hai. f~~

★ Rolle's theorem special case hai mean value theorem ka.

Q $f(x) = x^3 - 5x^2 + 4x - 2$ on $[1,3]$

sol

$a = 1$; $b = 3$

$f(1) = -2$

$f(3) = -8$

$f'(x) = 3x^2 - 10x + 4$

$f'(c) = 3c^2 - 10c + 4$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$3c^2 - 10c + 4 = \frac{-8 - (2)}{3 - 1}$$

$\cancel{3c^2 - 20}$

$3c^2 - 10c \cancel{+4} + 7 = 0$

$c = \frac{7}{3}, 1$

~~Session~~ Section 4.2 Pg # 255

Exercise 4.2 Q1 – 4

$$f'(c) = \frac{f(b) - f(a)}{b - a} \quad \text{(Mean value)}$$

Taylor and Maclaurin :-

$$f(x) = \sum_{n=0}^{\infty} a_n (x-a)^n$$

$$f(x) = a_0 + a_1(x-a) + a_2(x-a)^2 + \cdots + a_n(x-$$
$$f'(x) = 0 + a_1 + 2a_2(x-a) + 3a_3(x-a)^2 + \cdots$$
$$f''(x) = 2!\,a_2 + 6a_3(x-a) + \cdots$$
$$f'''(x) = 3!\,a_3 + \cdots$$

$$f^{(n)}x = n!\,a_n$$

$$a_n = \frac{f^{(n)}(x)}{n!}$$

It is used for series expansion.

$$\sum_{k=0}^{\infty} \frac{f^{k}(a)}{k!}(x-a)^k = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)$$

$x=0$ (maclaurin series)

$x=a$ (Taylor series) (a will be any value)

a) $\sum_{k=0}^{\infty} \frac{f^{(k)} a}{k!} (x-a)^k$ at ~~$x=0$~~

$f(x) = \sin x$ at $x=0$

$f'(x) = \cos x$

$f'(0) = \cos(0) = 1$

$f''(x) = -\sin x = 0$

$f'''(x) = -\cos x = 1$

$$f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!} + \dots$$

$$= 0 + 1 \cdot x + \frac{0x^2}{2} - \frac{x^3}{3!} + 0 + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

Q $f(x) = \cos x$ Expand M.S up to 4 terms

$f'(x) = -\sin x$

$f'(0) = -\sin(0)$

$f'(0) = 0$

$f''(x) = -\cos x$

$f''(0) = -1$

$f'''(x) = \sin x$

$f'''(0) = 0$

$f''''(x) = \cos x$

$f''''(0) = 1$

$\therefore f(x) = f(0) + f'(0)x + \frac{f''(0)x^2}{2!} + \frac{f'''(0)x^3}{3!}$

$+ \frac{f''''(0)x^4}{4!}$

$= 1 + (0)x + \frac{(-1)x^2}{2!} + \frac{(0)x^3}{3!} + \frac{1x^4}{4!}$

$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!}$

Taylor series :-

$f(x) = \frac{1}{x}$ at $a = 2$

* $f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)(x-a)^2}{2!} + \cdots$

$+ \frac{f^{(n)}(a)(x-a)^n}{n!} + \cdots$

$f'(x) = \frac{-1}{x^2}$

$f'(2) = \frac{-1}{4}$

$f'(x) = -x^{-2}$

Q find the taylor series for
$f(x) = \frac{1}{x}$ at $a = 2$

$\Rightarrow \sum_{k=0}^{\infty} \frac{f^{(k)}(a)(x-a)^k}{n!} = f(a) + \frac{f'(a)(x-a)}{1!} + \frac{f''(a)(x-a)^2}{2!}$

$+ \ldots + \frac{f^{(n)}(a)(x-a)^n}{n!} + \ldots$

$f'(x) = \frac{1}{x} = x^{-1} = -1x^{-2}$ | $f''' =$

$f'(2) = \frac{-1}{2^2}$

$f''(x) = (-1)(-2)x^{-3}$

$f''(2) = \frac{2}{2^3} = \frac{1}{2^2}$

~~$f'''(2) = \frac{1}{}$~~

① $\Rightarrow = f(2) + f(2)(x-2) + \frac{f''(2)(x-2)^2}{2!} + \ldots +$

$\frac{f^{(n)}(2)(x-2)^n}{n!} + \ldots$

$= \frac{1}{2} - \frac{1}{2}(x-2) + \frac{1}{2^3}(x-2)^2 - \frac{1}{2^4}(x-2)^3 +$

$\ldots + \frac{(-1)^n (x-2)^n}{2^{n+1}}$

$* \quad \frac{(-1)^n}{2^{n+1}} (x-2)^n$

$f^0(x) = x^{-1}$

$f'(x) = -1x^{-2}$

$f''(x) = (-1)(-2)x^{-3}$

$\vdots \qquad \vdots \qquad \vdots$

$f^n(x) = (-1)^n \, n! \, x^{-(n+1)}$

$\frac{f^n(x)}{n!} = (-1)^n x^{-(n+1)}$

$\frac{f^n(x)}{n!} = \frac{(-1)^n}{x^{n+1}}$

$\frac{f^n(2)}{n!} = \frac{(-1)^n}{2^{n+1}}$

② ⇒

$f(x) = \frac{1}{2} - \frac{1}{2^2}(x-2) + \frac{1}{2^3}(x-2)^2 - \frac{1}{2^4}(x$

$+ \cdots + \frac{(-1)^n}{2^{n+1}}(x-2)^n + \cdots$

$r \neq \frac{-1/2\,(x-2)}{1/2} = \frac{-(x-2)}{2} < 1$

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$$S_n = \frac{a}{1-r}$$

$$= \frac{1/2}{1 - \left(-\frac{(x-2)}{2}\right)}$$

$$= \frac{1/\cancel{2}}{\frac{2 + x - 2}{\cancel{2}}}$$

$$= \frac{1}{x}$$

Q function ki taylor or Machr series expand up to Sum terms or n^{th} term.

Roll's theorem, Mean value, Taylor, Macla

Reduction formula :-

— high order integration.

$I = \int \sin^n x \, dx$ $\quad n \geq 2$

$\int \sin^{n-1} x \sin x \, dx$

$V = \sin x \quad U = \sin^{n-1} x$

$\therefore \int (Uv)\, dv = U \int v \, dv - \int \left(U' \int v \, dv \right) dv$

$= \cancel{\sin^{(n-1)} x \cos x} - \int \left((n-1) \sin^{n-2} x \cos x \right. \left(-\cos x\right) dx$

$I = \sin^{n-1} x \cos x - \int (n-1) \sin^{n-2} \cos x \, (-\cos x)\, dx$

$I = \sin^{n-1} x \cos x + \int (n-1) \sin^{n-2} x \cos^2 x \, dx$

$I = -\sin^{(n-1)} x \cos x + (n-1) \int \sin^{n-2} x \, (1 - \sin^2 x)$

$I = -\sin^{(n-1)} x \cos x + (n-1) \int \sin^{n-2} x - \sin^{n-2}$

$$= -\sin^{(n-1)} x \cos x + (n-1)\int \sin^{n-2} dx \cancel{- n-1} \int$$

$$- (n-1)\int \sin^{n} x \, dx$$

$$I = -\sin^{(n-1)} x \cos x + (n-1)\int \sin^{n-2} x \, dx -$$

$$(n-1) I$$

$$I + (n-1) I = -\sin^{(n-1)} x \cos x + (n-1)\int \sin^{n-2} x \, dx$$

$$\cancel{I} + nI - \cancel{I} = -\sin^{(n-1)} x \cos x + (n-1)\int \sin^{n-2} x \, dx$$

$$\therefore \quad I = \frac{-\sin^{(n-1)} x \cos x}{n} + \frac{(n-1)}{n}\int \sin^{n-2} x \, dx$$

Q1 $I = \int \cos^{n} x \, dx$

Q2 $$\int \sin^{3} x \, dx = \frac{-\sin^{(3-1)} \cos x}{3} + \frac{(3-1)}{3}\int \sin^{(3-2)} x \, dx$$

$$\int \sin^{3-1} x \sin x = \frac{-\sin^{2} x \cos x}{3} + \frac{2}{3}\int \sin x \, dx$$

$$= \frac{-\sin^{2} x \cos x}{3} \mp \frac{2}{3} \cdot \cos x$$

$F(x,y)$ = function ~~of~~ in 2 variable

Partial diffrentiation

If $z = f(x,y)$ is a function of two independent variables "x" and "y" then partial derivatives of function $z = f(x,y)$ is given by

~~$\frac{dz}{dx}$ and $\frac{dz}{dy}$~~ $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$

E.g:-

$z = f(x,y) = x^2y + 2xy + x$

$\frac{\partial z}{\partial x} = 2x + 2y + 1$ ~~y~~

* $\frac{\partial z}{\partial x}$ ly gy to y ko as a constant treat krygy.

• Agr product form mai hai y to usit is y ajaye ga or agr + horha hai to 0 likhygy.

$\frac{\partial z}{\partial y} = x^2 + 2x$

* $\frac{\partial z}{\partial y}$ ly gy to x ko as a constant treat krygy.

Q $f(x,y) = x + 2x^2y^3 + \cos(xy)$

$$\frac{\partial z}{\partial x} = 1 + 4xy^3 - \sin(xy) \cdot \frac{d}{dx}(xy)$$

$$= 1 + 4xy^3 - y \sin xy$$

$$\frac{\partial z}{\partial y} = 0 + 6x^2y^2 - x\sin(xy)$$

$$= 6x^2y^2 - x\sin(xy)$$

Q $f(x,y) = xy\cos(xy)$

calculate $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$

$$\frac{\partial z}{\partial x} = y\left\{\frac{\partial}{\partial x}(x\cos(xy))\right\}$$

$$\frac{\partial z}{\partial x} = y\left\{1\cos(xy) - xy\sin(xy)\right\}$$

$$= y\cos(xy) - xy^2\sin(xy)$$

$$\frac{\partial z}{\partial y} = x\left\{1\cos(xy) - xy\sin(xy)\right.$$

$$= x\cos(xy) - x^2y\sin(xy)$$

Q $z = f(x,y) = e^{x^3+y^2}$

$$\frac{\partial z}{\partial x} = e^{x^3+y^2} \cdot \frac{\partial}{\partial x}(x^3+y^2)$$

$$= e^{x^3+y^2}\, 3x^2$$

$$= 3x^2 e^{x^3+y^2}$$

$$\frac{\partial z}{\partial y} = e^{x^3+y^2} \cdot \frac{\partial}{\partial y}(x^3+y^2)$$

$$= e^{x^3+y^2}\, 2y$$

Q $f(x,y) = \frac{x-y}{x+y}$

$$\frac{\partial z}{\partial x} = \frac{x+y\frac{\partial}{\partial x}(x-y) - (x-y)\frac{\partial}{\partial x}(x+y)}{(x+y)^2}$$

$$= \frac{x+y(1) - (x-y)(1)}{(x+y)^2}$$

$$= \cancel{x^2+xy - x^2+xy} \frac{\cancel{x}+y-\cancel{x}+y}{(x+y)^2}$$

$$= \frac{2y}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \frac{(x+y)\frac{\partial}{\partial y}(x-y) - (x-y)\frac{\partial}{\partial y}(x+y)}{(x+y)^2}$$

$$= \frac{(x+y)(-1) - (x-y)(1)}{(x+y)^2}$$

$$= \frac{-2x}{(x+y)^2}$$

Q $f(x,y) = (3x-y)^2$

$$\frac{\partial z}{\partial x} = 2(3x-y) \cdot \frac{d}{dx}(3x-y)$$

$$= 2(3x-y)\,3$$

$$= 6(3x-y)$$

$$\frac{\partial z}{\partial y} = 2(3x-y) \cdot \frac{\partial}{\partial y} \cdot (3x-y)$$

$$= 2(3x-y)(-1)$$

$$= -2(3x+y)$$

Q $f(x,y) = e^{(2x^2+y^3)}$

calculate $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$, $\frac{\partial^2 z}{\partial x \partial y}$ & $\frac{\partial^2 z}{\partial y \partial x}$

$\cancel{\frac{\partial z}{\partial x}}$

$\downarrow$ z_{xx}, $\downarrow$ z_{yy}, $\downarrow$ z_{yx}, $\downarrow$ z_{xy}

$$\frac{\partial z}{\partial x} = e^{2x^2+y^3}(4x)$$

$$\frac{\partial^2 z}{\partial x^2} = 4\left[4x^2 e^{2x^2+y^3} + e^{2x^2+y^3}\right]$$

$$= 16x^2 e^{(2x^2+y^3)} + 4e^{(2x^2+y^3)}$$

$$= 4e^{(2x^2+y^3)}(1+4x^2)$$

$$\frac{\partial z}{\partial y} = e^{2x^2+y^3}\,3y^2$$

$$\frac{\partial^2 z}{\partial y^2} = 3\left[\cancel{2y}\, e^{2x^2+y^3}(2y) + 3y^2 e^{2x^2+y^3}(3y^2)\right]$$

$$= 3\left[2y\, e^{2x^2+y^3} + 3y^4 e^{2x^2+y^3}\right]$$

$$= 6y\, e^{2x^2+y^3} + 9y^4 e^{2x^2+y^3}$$

$$\frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\, 4x\, e^{(2x^2+y^3)}$$

$$= 4x\, e^{2x^2+y^3} \cdot \frac{\partial}{\partial y}(2x^2+y^3)$$

$$= 4x\, e^{2x^2+y^3}\, 3y^2$$

$$= 12xy^2\, e^{2x^2+y^3}$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}\left(3y^2 e^{(2x^2+y^3)}\right)$$

$$= 3y^2\, e^{(2x^2+y^3)}\,(4x)$$

$$= 12xy^2\, e^{2x^2+y^3}$$

($Z_{xy} = Z_{yx}$
If this condition is satisfied then the function is continuous.)

Q If z is implicit function of x and y.

$$x \sin z - z^2 y = 1$$

w.r.t "x" :-

$$1 \cdot \sin z + x \cos z \cdot \frac{\partial z}{\partial x} - 2zy \frac{\partial z}{\partial y} = 0$$

$$\sin z + x \cos z \frac{\partial z}{\partial x} - 2zy \frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial x}(x\cos z - 2zy) = -\sin z$$

$$\frac{\partial z}{\partial x} = \frac{-\sin z}{(x \cos z - 2zy)}$$

w.r.t "y"

$$x \cdot \cos z \frac{\partial z}{\partial y} - z^2 - 2yz \frac{\partial z}{\partial y} = 0$$

$$\frac{\partial z}{\partial y}(x\cos z - 2yz) = z^2$$

$$\frac{\partial z}{\partial y} = \frac{z^2}{x\cos z - 2yz}$$

$$\int_a^b f(x)\,dx$$ (definate integral)

$$\iint_{y_0\ x_0}^{y_1\ x_1} f(x,y)\,dx\,dy$$

jo phly given hoga derivative limit usi ki phly hogi.

$$\iint_{x_0\ y_0}^{x_1\ y_1} f(x,y)\,dx\,dy$$

* Agr phly limit x ki di hai to integrate phly x ki respect sy hoga.

Q $\int_0^1 \int_0^2 y^2 x\,dy\,dx$

$$\int_0^1 \int_0^2 y^2 dy\ x\,dx$$

$$\int_0^1 \left(\frac{y^3}{3} x \Big|_0^2 \right) dx$$

$$\int_0^1 \frac{8}{3} x\,dx$$

$$\frac{8x^2}{3\times 2}\Big|_0^1 = \frac{4}{3}$$

$$\int_2^3 \int_1^2 \frac{1}{xy}\,dy\,dx$$

$$\int_2^3 \frac{1}{x}\left(\int_1^2 \frac{1}{y}\,dy\right) dx$$

$$\int_2^3 \frac{1}{x}\left(\log y \Big|_1^2\right) dx$$

$$\int_2^3 \frac{1}{x}(\log 2 - \log 1)\,dx$$

$$\int_2^3 \left(\frac{1}{x}\log 2\right) dx$$

$$\log 2 \left[\log x\right]_2^3$$

$$\log 2\,[\log 3 - \log 2]$$

$$\log 2\left[\log\left(\frac{3}{2}\right)\right]$$

$$\cancel{\int_2^3\int_1^2 \frac{1}{xy}\,dy\,dx}$$

Q $$\int_1^2\int_2^3 \frac{1}{xy}\,dx\,dy$$

$$\int_1^2 \frac{1}{y}\left(\int_2^3 \frac{1}{x}\,dx\right)dy$$

$$\int_1^2 \frac{1}{y}\left(\ln x\Big|_2^3\right)dy$$

$$\int_1^2 \frac{1}{y}(\ln 3-\ln 2)\,dy$$

$$\int_1^2 \frac{1}{y}(\ln 3-\ln 2)\,dy$$

$$(\ln 3-\ln 2)\ \ln y\Big|_1^2$$

$$(\ln 3-\ln 2)\ \ln 2-\ln 1$$

$$(\ln 3-\ln 2)\ \ln 2$$

$$\left(\ln\frac{3}{2}\right)\ln 2$$

(r, θ, ϕ)

Q $$\int_0^{\pi}\int_0^{\pi/2}\int_0^1 r^2\sin 2\theta\,dr\,d\theta\,d\phi$$

$$\int_0^{\pi}\int_0^{\pi/2}\left(\frac{r^3}{3}\sin 2\theta\Big|_0^1\right)d\theta\,d\phi$$

$$\int_0^{\pi}\int_0^{\frac{\pi}{2}} \frac{1^3}{3}\sin 2\theta\,d\theta\,d\phi$$

$$\int_0^{\pi}\frac{1}{3}\left(\int_0^{\pi/2}\sin 2\theta\,d\theta\right)d\theta$$

$$\int_0^{\pi}\frac{1}{3}\ \frac{-\cos 2\theta}{2}\Big|_0^{\pi/2}\,d\theta$$

$$\cancel{\int_0^{\pi/6}\frac{1}{3}\cdot\frac{-1}{6}\left[\cos 2\left(\frac{\pi}{2}\right)\right.}$$

$$\int_0^{\pi}\frac{1}{6}\ -\cos 2\theta\Big|_0^{\pi/2}\,d\theta$$

$$\int_0^{\pi}-\frac{1}{6}\ \cos 2\left(\frac{\pi}{2}\right)-\cos 2(0)\,d\phi$$

$$\int_0^{\pi}\frac{-1}{6}\ \cancel{\cos\pi}-1\,d\phi$$

$$\int_0^{\pi} \frac{-1}{6} (-1) - 1$$

$$\frac{1}{6} - 1$$

$$\frac{1-6}{6}$$

$$\frac{-5}{6}$$

$$\int_0^{\pi} \frac{-5}{6} \, d\theta$$

$$\frac{-5}{6} \phi \Big|_0^{\pi}$$

$$\frac{-5}{6} \pi$$

$$\therefore \int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\, dy\, dz}{\sqrt{1-x^2-y^2-z^2}}$$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{(\sqrt{1-x^2-y})^2 - z^2}} \, dz\, dy\, dz$$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \sin^{-1}\left(\frac{z}{\sqrt{1-x^2-y^2}}\right) \Bigg|_0^{\sqrt{1-x^2-y^2}} dy\, dx$$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \left[\sin^{-1}(1) - \sin^{-1}(0)\right] dy\, dx$$

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \frac{\pi}{2} (1) \, dy\, dx$$

$$\int_0^1 \frac{\pi}{2} y \Big|_0^{\sqrt{1-x^2}} dx$$

$$\int_0^1 \frac{\pi}{2} \sqrt{1-x^2} \, dx$$

$$\int \sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + c$$

$$\frac{\pi}{2}\int_0^1 \sqrt{1 - x^2}\, dx$$

$$\frac{\pi}{2}\left[\frac{x}{2}\sqrt{1 - x^2} + \frac{1}{2}\sin^{-1}(x)\right]_0^1$$

$$\frac{\pi}{2}\left[\frac{\pi}{4} - 0\right]$$

$$\frac{\pi^2}{8}$$

Multiple Integral

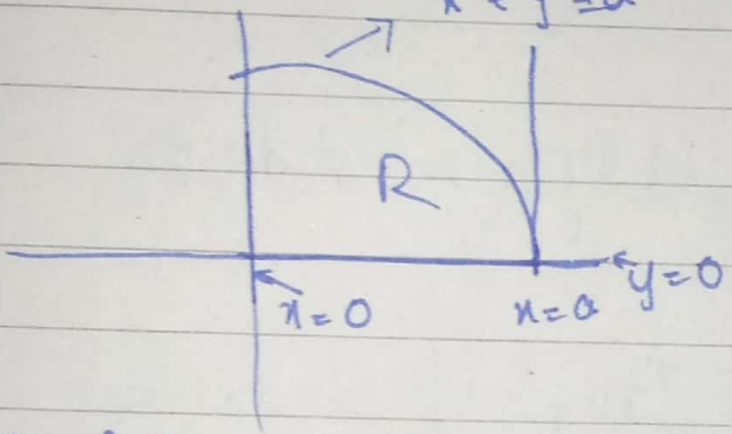
Scratch:-

jis region k ~~around~~ along integration karni hoti hai

$$\int_0^a \int_0^{\sqrt{a^2-x^2}} f(x,y)\,dy\,dx$$

$y = 0$, $y^2 = a^2 - x^2$ | $x = 0$, $x = a$

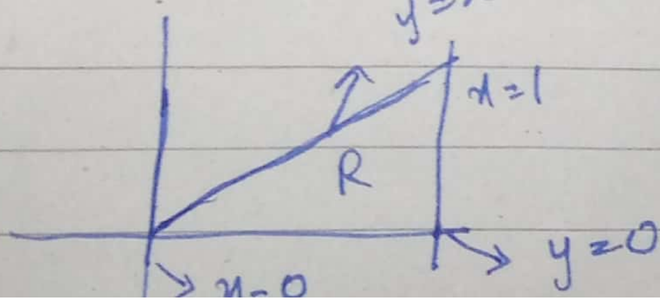
$x^2 + y^2 = a^2$



$$\int_0^1 \int_0^x f(x,y)\,dy\,dx$$

$y = 0$, $y = x$

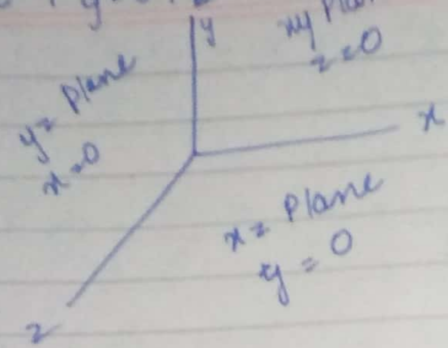
$x = 0$, $x = 1$



$$\iiint_V dx\,dy\,dz$$

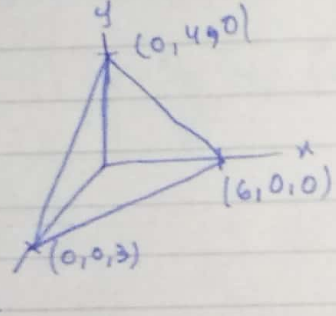
$2x + 3y + 4z = 12$

$x = 0,\ y = 0,\ z = 0$



Traces:-

x ka trace nikaly gy to y, z 0 hoga and vice versa.



$2x + 3y + 4z = 12$

$2x + 0 + 0 = 12$

$x = 6$

$2x + 3y + 4z = 12$

$y = 4$

$z = 3$

$$\int_0^3\int_0^4\int_0^6 dx\,dy\,dz$$

$$\int_0^3\int_0^4 x\Big|_0^6\,dy\,dz$$

$$\int_0^3\int_0^4 6\,dy\,dz$$

$$\int_0^3 6\left(y\Big|_0^4\right)dz$$

$$\int_0^3 6(4)\,dz$$

$$24\ x\Big|_0^3$$

$$24(3)$$

$$72$$

Ans