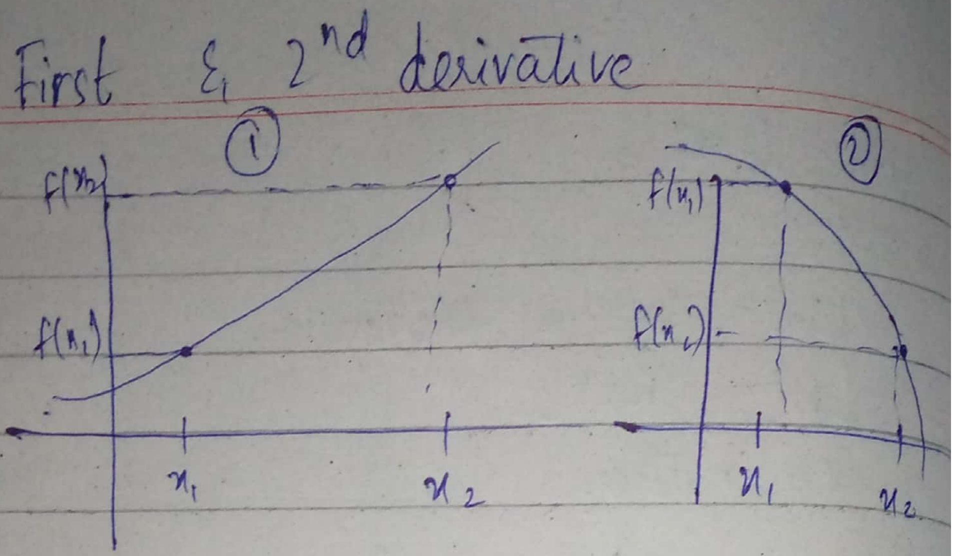
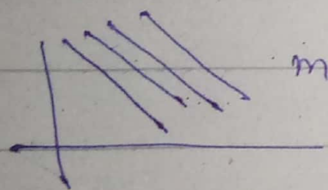
First & 2^{nd} derivative



① $\left.\begin{array}{l} x_1 < x_2 \\ f(x_1) < f(x_2) \end{array}\right\}$ increasing function

② $\left.\begin{array}{l} f(x_1) > f(x_2) \\ x_1 < x_2 \end{array}\right\}$ decreasing function



$m = -ve$

$f'(x) < 0$ decreasing

$f'(x) > 0$ increasing

$f'(x) = 0$ constant function

Q $f(x) = x^3 - 27x - 20$

$f'(x) = 3x^2 - 27$

For $f(x)$ is increasing

$f'(x) > 0$

$3x^2 - 27 > 0$

$x^2 - 9 > 0$

$f(x)$ is decreas

$f'(x) < 0$

$3x^2 - 27 < 0$

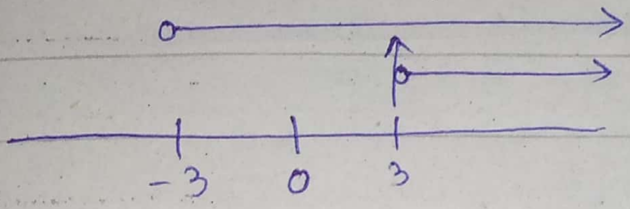
$x^2 - 9 < 0$

$\cancel{x^2}$ $x^2 - 9 > 0 \Rightarrow (x-3)(x+3) > 0$

Case - I

$x - 3 > 0$ $\qquad$ $x + 3 > 0$

$x > 3$ $\qquad$ $x > -3$

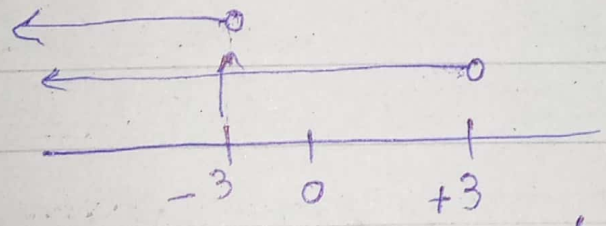


$x \in (3, \infty)$

Case - II

$x - 3 < 0$ $\qquad$ $x + 3 < 0$

$x < 3$ $\qquad$ $x < -3$



$x \in (-\infty, -3)$

$(-\infty, -3) \cup (3, \infty)$
increase function

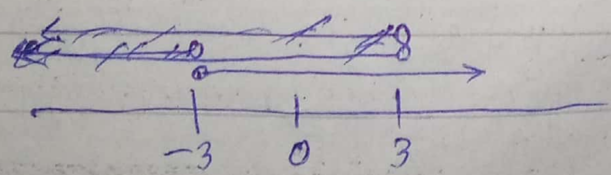
$f(x)$ is decreasing :-

$x^2 - 9 < 0$ $\qquad$ $(x-3)(x+3) < 0$

Case - I

$x - 3 < 0$ $\qquad$ $x + 3 > 0$

$x < 3$ $\qquad$ $x > -3$



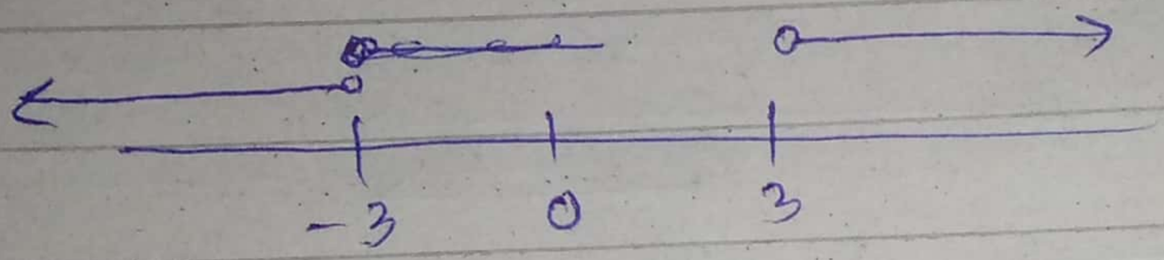
$x \in (-3, 3)$

~~$x \in (-\infty, -3)$~~

Case-II :-

$x - 3 > 0$ $\quad$ $x + 3 < 0$

$x > 3$ $\quad$ $x < -3$



$x \in (-3, 3)$

decreasing function

limit of finite Sums.

Σ = sigma

$$\sum_{k=0}^{5} a_k = a_0 + a_1 + a_2 + a_3 + a_4 + a_5$$

$$\sum_{k=1}^{4} (1+k) = (1+1) + (1+2) + (1+3) + (1+4)$$

(i) $1+2+3+ \dots + n = \frac{n(n+1)}{2}$ (formula of sum of natural no)

$$1+2+3+ \dots +100 = \frac{100(100+1)}{2}$$

$$= 5050$$

(ii) $1+3+5+7+9$ $(1, 3, 5, 7, 9, 11, \dots)$

$T_n = a+(n-1)d$

$= 1+2n-2$

$T_n = 2n-1$

$T_k = (2k-1)$

$S_n = a + ar + ar^2 + \dots + ar^n$ (finite G.S)

$S_\infty = a + ar + ar^2 + \dots$ (Infinite G.S)

$\frac{a}{1-r}$ $r < 1$ (infinite G.S)

$S_n = \frac{a(r^n-1)}{r-1}$, $r > 1$ (finite G.S)

$\frac{r}{1-}$

a) $1+3+9+27+81+ \cdots$

$r = \frac{9}{3} = \frac{3}{1} = 3$

$r > 1$

$S = \frac{a(r^n - \cancel{1})}{r - \cancel{1}}$ $\quad \cancel{\frac{a}{1-r}}$ $\quad \frac{a}{r-1}$

$= 1 \ (3$ $\quad \cancel{\frac{1}{1-3}}$ $\quad \frac{1}{3-1}$

$= -\frac{1}{2}$ $\quad 1/2$

b) $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \cdots$

$r = \frac{1/3}{1} = \frac{1/9}{1/3} = \frac{1}{3}$

$S_\infty = \frac{a}{r-1} = \frac{1}{\frac{1}{3} - 1}$

$= -\frac{3}{2}$

$1 + 1 + 1 + 1 + \cdots\cdots \cancel{+n} = n$

$f = 1$

$2 + 2 + 2 + \cdots\cdots\cdots = 2n$

Rules :-

1) $\sum (a_k + b_k) = \sum a_k + \sum b_k$

2) $\sum (a_k - b_k) = \sum a_k \cancel{b_k} - \sum b_k$

3) $\sum c a_k = c \cdot \sum a_k$

e.g:

$\sum_{k=1}^{5} 2k$

$2 \sum_{k=1}^{5} k$

* $\sum_{k=1}^{n} a = na$

~~e.g:~~

(4) $\sum_{k=1}^{n} c = nc$

e.g: $c + c + c + c \cdots + c = nc$

- series expension
- sum of natural no Formula.

First natural no :

$1+2+3+ \cdots + n = \dfrac{n(n+1)}{2} = \sum_{k=1}^{n} k$

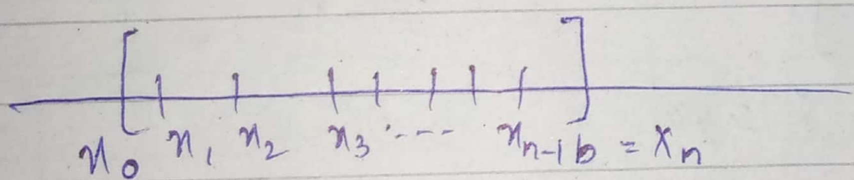
First natural no sq:

$1+2^2+3^2+ \cdots + n = \dfrac{n(n+1)(2n+1)}{6} = \sum_{k=1}^{n} k^2$

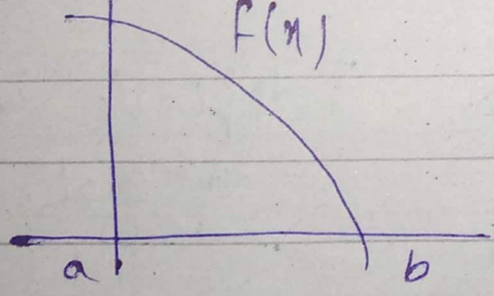
First natural no cube:

$1+2^3+3^3+ \cdots + n = \sum_{k=1}^{n} k^3 = \left(\dfrac{n(n+1)}{2}\right)^2$

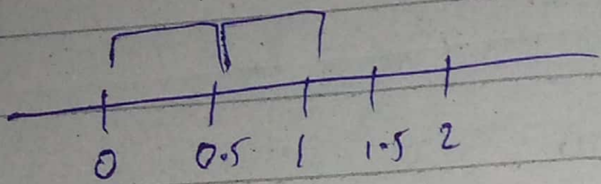
: Interal :



$[x_0, x_1], [x_1, x_2] \cdots [x_{n-1}, x_n]$



Partition of interval :



[0, 0.5]

$\Delta x_1 = 0.5 - 0$

$= 0.5$

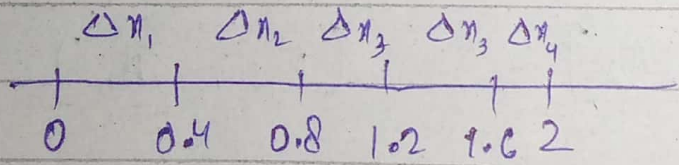
[0.5, 1]

$\Delta x_1 = 1 - 0.5$

$= 0.5$

[0, 2] $\qquad \frac{2-0}{5} = \frac{2}{5}$

5 interval



★ Definite ~~interval~~ or integral :-   Indefinite ~~interval~~ integral.

To find integral b/w interval.

Properties of Definite integral :-

1- $\int_a^b f(x)\,dx = -\int_a^b f(x)\,dx$.

e.g: $\int_a^b f(x)\,dx = F(x)\Big|_a^b$

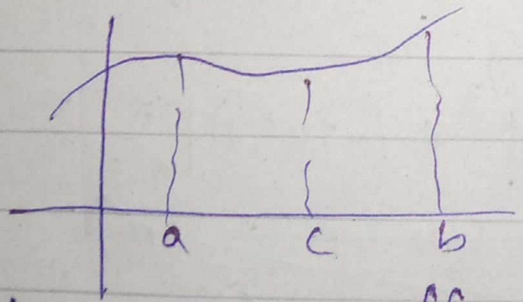
$= F(b) - F(a)$

$= F(a) - F(b)$

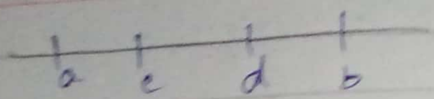
$= \int_b^a f(x)\,dx$

2- $\int_a^a f(x)\,dx = 0$ e.g $\int_2^2 \frac{1}{x}\,dx = 0$

e.g $\int_{-1}^{-1} \sin^2 x \cos x\,dx = 0$



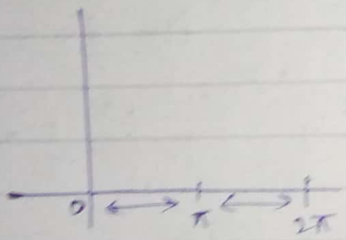
$\int_a^b f(x)\,dx = \int_a^c f(x) + \int_c^b f(x)$



$$\int_a^c + \int_c^d + \int_d^b = \int_a^b$$

Q Integrate $f(x) = \sin x$ in $[0, 2\pi]$ by Partition method. $n = 2$

no. of intervals.



$$\int_0^\pi \sin x\,dx + \int_\pi^{2\pi} \sin x\,dx$$

$$-\cos x\Big|_0^\pi + (-\cos x)\Big|_\pi^{2\pi}$$

$$\cancel{\sin\pi - \sin 0 + \sin 2\pi - \sin\pi}$$

$$\cancel{0 - 0 + 0 - 0}$$

$$\cancel{0}$$

$$-(\cos\pi - \cos 0)$$

$$-(\cos\pi - \cos 0) - (\cos 2\pi - \cos\pi)$$

$$-((-1) - 1) - (1 - (-1))$$

$$= \cancel{1 + 1 - 1 - 1}$$

$$2 - 2$$

$$-(-1-1) - (1+1)$$

$$2 - 2$$

$$0$$

Q Evaluate $\int_{-1}^{4} f(x)\,dx$, If $\int_{-1}^{0} f(x)\,dx = 2$, $\int_0^2 f(x)\,dx = 3$, $\int_2^4 f(x)\,dx = 4$