Lecture 3

Quadratic inequalities:

$\sqrt{a^2} = |x|$

$x^2 = 2$

$x = \sqrt{2}$

a) $x^2 < 2$

$|x| < \sqrt{2}$

$-\sqrt{2} < x < \sqrt{2}$

$I = (-\sqrt{2}, \sqrt{2})$

(b) $4 \le x^2$

$\pm 2 \le x$

$\boxed{2 \le x}$, $\boxed{-2 \le x}$

$I = (-\infty, -2] \cup [2, \infty)$

$2 \le x$

$x \geq 2$

$x > 2$

$(2, \infty)$

$-2 \le x$

$\boxed{x \le -2}$

$[2, \infty)$

$R = (-\infty, \infty)$

c) $\frac{1}{9} < x^2 < \frac{1}{4}$

$\frac{1}{3} < \pm x < \frac{1}{2}$

$\frac{1}{3} < x < \frac{1}{2}$

$\frac{1}{3} < -x < \frac{1}{2}$

$-\frac{1}{3} < x < -\frac{1}{2}$

$\boxed{-\frac{1}{2} < x < -\frac{1}{3}}$

$I = (\frac{1}{3}, \frac{1}{2}) \cup (-\frac{1}{2}, -\frac{1}{3})$

Quadratic Inequalities

Solve the inequalities in Exercises 35–42. Express the solution sets as intervals or unions of intervals and show them on the real line. Use the result $\sqrt{a^2} = |a|$ as appropriate.

35. $x^2 < 2$

36. $4 \le x^2$

37. $4 < x^2 < 9$

38. $\frac{1}{9} < x^2 < \frac{1}{4}$

39. $(x - 1)^2 < 4$

40. $(x + 3)^2 < 2$

41. $x^2 - x < 0$

42. $x^2 - x - 2 \ge 0$

EXAMPLE 6 Solve the inequality and show the solution set on the real line:

(a) $|2x - 3| \leq 1$ (b) $|2x - 3| \geq 1$

Solution

(a)

$$\begin{aligned} |2x - 3| &\leq 1 \\ -1 \leq 2x - 3 &\leq 1 && \text{Property 8} \\ 2 \leq 2x &\leq 4 && \text{Add 3.} \\ 1 \leq x &\leq 2 && \text{Divide by 2.} \end{aligned}$$

The solution set is the closed interval [1, 2] (Figure 1.4a).

(b)

$$|2x - 3| \geq 1$$

$$\begin{aligned} 2x - 3 \geq 1 \quad &\text{or} \quad 2x - 3 \leq -1 && \text{Property 9} \\ x - \frac{3}{2} \geq \frac{1}{2} \quad &\text{or} \quad x - \frac{3}{2} \leq -\frac{1}{2} && \text{Divide by 2.} \\ x \geq 2 \quad &\text{or} \quad x \leq 1 && \text{Add } \tfrac{3}{2}. \end{aligned}$$

The solution set is $(-\infty, 1] \cup [2, \infty)$ (Figure 1.4b).

Relation and Functions

1. Domain and Range of a Relation

In many naturally occurring phenomena, two variables may be linked by some type of relationship. For instance, an archeologist finds the bones of a woman at an excavation site. One of the bones is a femur. The femur is the large bone in the thigh attached to the knee and hip. Table 4-1 shows a correspondence between the length of a woman's femur and her height.

Table 4-1

Length of Femur (cm) x	Height (in.) y		Ordered Pair
45.5	65.5	→	(45.5, 65.5)
48.2	68.0	→	(48.2, 68.0)
41.8	62.2	→	(41.8, 62.2)
46.0	66.0	→	(46.0, 66.0)
50.4	70.0	→	(50.4, 70.0)

Each data point from Table 4-1 may be represented as an ordered pair. In this case, the first value represents the length of a woman's femur and the second, the woman's height. The set of ordered pairs {(45.5, 65.5), (48.2, 68.0), (41.8, 62.2), (46.0, 66.0), (50.4, 70.0)} defines a relation between femur length and height.

Definition of a Relation in x and y

Any set of ordered pairs (x, y) is called a **relation in x and y**. Furthermore,

- The set of first components in the ordered pairs is called the **domain of the relation**.
- The set of second components in the ordered pairs is called the **range of the relation**.

Section 4.2 Introduction to Functions

Key Concepts

Given a relation in x and y, we say "**y is a function of x**" if for every element x in the domain, there corresponds exactly one element y in the range.

The Vertical Line Test for Functions

Consider a relation defined by a set of points (x, y) in a rectangular coordinate system. Then the graph defines y as a function of x if no vertical line intersects the graph in more than one point.

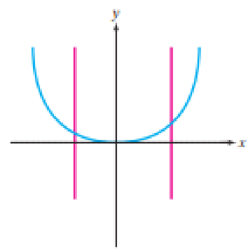
Examples

Example 1

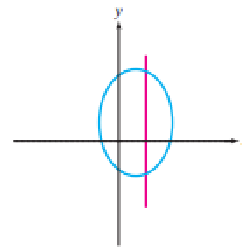
Function $\{(1, 3), (2, 5), (6, 3)\}$

Nonfunction $\{(1, 3), (2, 5), (1, 4)\}$

Example 2



Function



Not a Function

Function Notation

$f(x)$ is the value of the function f at x.

Example 3

Given $f(x) = -3x^2 + 5x$, find $f(-2)$.

$$f(-2) = -3(-2)^2 + 5(-2)$$
$$= -12 - 10$$
$$= -22$$

The domain of a function defined by $y = f(x)$ is the set of x-values that when substituted into the function produces a real number. In particular,

- Exclude values of x that make the denominator of a fraction zero.
- Exclude values of x that make a negative value within a square root.

Example 4

Find the domain.

1. $f(x) = \dfrac{x + 4}{x - 5}$; $(-\infty, 5) \cup (5, \infty)$
2. $f(x) = \sqrt{x - 3}$; $[3, \infty)$
3. $f(x) = 3x^2 - 5$; $(-\infty, \infty)$

EXAMPLE 1 Identifying Domain and Range

Verify the domains and ranges of these functions.

Function	Domain (x)	Range (y)
$y = x^2$	$(-\infty, \infty)$	$[0, \infty)$
$y = 1/x$	$(-\infty, 0) \cup (0, \infty)$	$(-\infty, 0) \cup (0, \infty)$
$y = \sqrt{x}$	$[0, \infty)$	$[0, \infty)$
$y = \sqrt{4 - x}$	$(-\infty, 4]$	$[0, \infty)$
$y = \sqrt{1 - x^2}$	$[-1, 1]$	$[0, 1]$

Solution The formula $y = x^2$ gives a real y-value for any real number x, so the domain is $(-\infty, \infty)$. The range of $y = x^2$ is $[0, \infty)$ because the square of any real number is nonnegative and every nonnegative number y is the square of its own square root, $y = \left(\sqrt{y}\right)^2$ for $y \geq 0$.

The formula $y = 1/x$ gives a real y-value for every x except $x = 0$. *We cannot divide any number by zero*. The range of $y = 1/x$, the set of reciprocals of all nonzero real numbers, is the set of all nonzero real numbers, since $y = 1/(1/y)$.

The formula $y = \sqrt{x}$ gives a real y-value only if $x \geq 0$. The range of $y = \sqrt{x}$ is $[0, \infty)$ because every nonnegative number is some number's square root (namely, it is the square root of its own square).