BAHRIA UNIVERSITY (KARACHI CAMPUS)

Department of Software Engineering.

Assignment 04 (Fall 2022)

Course Title: Calculus and Analytical Geometry
Class: BSE 1B
Course Instructor: Mr. Daniyal ur Rehman
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Shift: Morning
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Max Marks: 05 Points

Assignment No 04

Submitted By:

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Section: 1 B

Question No (1):

1) $\int \sin x \cdot dx$;

$= -\cos x + C$ Ans

2) $\int \cos 5x \cdot dx$;

$= \sin 5x \cdot \left(\frac{1}{5}\right) + C$

$= \frac{\sin 5x}{5} + C$ Ans

3) $\int \cos x \sin^4 x \, dx$

$= \int \sin^4 x \cos x \, dx$

$= \frac{\sin^{4+1} x}{4+1}$

$= \frac{\sin 5x}{5} + C$ Ans

4) $\int \tan x \cdot dx$

$= \int \frac{\sin x}{\cos x} dx$

let $u = \cos x$

$du = -\sin x \cdot dx$

$-du = \sin x \cdot dx$

$= -\int \frac{du}{u}$

$= -\ln(u) + C$

$= -\ln(\cos x) + C$ Ans

5) $\int \cot x \cdot dx$

$= \int \frac{\cos x}{\sin x} \cdot dx$

let $u = \sin x$

$du = \cos x$

$= \int \frac{du}{u}$

$= \ln(u) + C$

$= \ln(\sin x) + C$ Ans

6) $\int \sec x \cdot dx$

$\ln(\tan x + \sec x) + C$ Ans

7): $\int \sin^2 x \, dx$

$$\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{(n-1)}{n} \int \sin^{n-2} x \, dx$$

$$\int \sin^2 x \, dx = -\frac{\sin^{2-1} x \cos x}{2} + \frac{(2-1)}{2} \int \sin^{2-2} x \cdot dx$$

$$\int \sin^2 x \, dx = -\frac{\sin x \cos x}{2} + \frac{1}{2} \int dx$$

$$\int \sin^2 x \, dx = -\frac{\sin x \cos x}{2} + \frac{x}{2} + C \quad \text{Ans}$$

8) $\int \sin^3 x \cdot dx$

$$\int \sin^3 x \, dx = -\frac{\sin^{3-1} x \cos}{3} + \frac{(3-1)}{3} \int \sin^{3-2} x \, dx$$

$$\int \sin^3 x \, dx = -\frac{\sin^2 x \cos x}{3} + \frac{2}{3} \int \sin x \cdot dx$$

$$\int \sin^3 x \cdot dx = -\frac{\sin^2 x \cos x}{3} + \frac{2}{3} (-\cos x) + C$$

$$\int \sin^3 x \cdot dx = -\frac{\sin^2 x \cos x}{3} - \frac{2}{3} \cos x + C \quad \text{Ans}$$

9) $\int \sin 4x \, dx$

$$\int \sin 4x \cdot dx = -\frac{\sin^{4-1} \cos x}{4} + \frac{(4-1)}{4} \int \sin^{4-2} x \, dx$$

$$\int \sin^4 x\,dx = -\frac{\sin^3 x\cos x}{4} + \frac{3}{4}\left(-\frac{\sin x\cos x}{2} + \frac{x}{2}\right) + C$$

$$\int \sin^4 x\,dx = -\frac{\sin^3 x\cos x}{4} - \frac{3\sin x\cos x}{8} + \frac{3x}{8} + C$$ Ans

10): $\int \sin^5 x\,dx$

$$\int \sin^5 x\,dx = -\frac{\sin^{5-1} x\cos x}{5} + \frac{(5-1)}{5}\int \sin^{5-2} x\,dx$$

$$\int \sin^5 x\,dx = -\frac{\sin^4 x\cos x}{5} + \frac{4}{5}\int \sin^3 x\,dx$$

$$\int \sin^5 x\,dx = -\frac{\sin^4 x\cos x}{5} + \frac{4}{5}\left(-\frac{\sin^2 x\cos x}{3} - \frac{2\cos x}{3}\right) + C$$

$$\int \sin^5 x\,dx = -\frac{\sin^4 x\cos x}{5} - \frac{4\sin^2 x\cos x}{15} - \frac{8\cos x}{15} + C$$ Ans

:- Question No 2:-

1) $\int \cos 3x\,dx.$

$$\sin 3x \cdot \left(\frac{1}{3}\right) + C$$

$$\frac{\sin 3x}{3} + C$$ Ans

2) $\int \sec\theta \tan\theta \, d\theta$.

$\int \sec\theta \tan\theta \, d\theta = \sec\theta + c$ Ans.

3) $\int \sec^2\theta \, d\theta$

$\int \sec^2\theta \, d\theta = \tan\theta + c$ Ans.

4) $\int \cos^2 x \, dx$

$\int \cos^n x \, dx = \cos^{n-1} x \sin x + \frac{(n-1)}{n} \int \cos^{n-2} x \, dx.$

$\int \cos^2 x \, dx = \frac{\cos x^{2-1} \sin x}{2} + \frac{(2-1)}{2} \int \cos^{2-2} x \cdot dx$

$\int \cos^2 x \, dx = \frac{\cos x \sin x}{2} + \frac{1}{2} \int dx$

$\int \cos^2 x \, dx = \frac{\cos x \sin x}{2} + \frac{1}{2} x + c$ Ans.

5) $\int \cos^3 x \, dx$.

$\int \cos^3 x \, dx = \frac{\cos^{3-1} x \sin x}{3} + \frac{(3-1)}{3} \int \cos^{3-2} x \, dx$

$$\int \cos^3 x \cdot dx = \frac{\cos^2 x \sin x}{3} + \frac{2}{3}(\sin x) + C$$

$$\int \cos^3 x \, dx = \frac{\cos^2 x \sin x}{3} + \frac{2}{3}\sin x + C \quad \text{Ans.}$$

6): $\int \cos^4 x \cdot dx$

$$\int \cos^4 x \cdot dx = \frac{\cos^{4-1} x \sin x}{4} + \frac{(4-1)}{4}\int \cos^{4-2} x \cdot dx$$

$$\int \cos^4 x \cdot dx = \frac{\cos^3 x \sin x}{4} + \frac{3}{4}\int \cos^2 x \cdot dx$$

$$\int \cos^4 x \cdot dx = \frac{\cos^3 x \sin x}{4} + \frac{3}{4}\left(\frac{\cos x \sin x}{2} + \frac{1}{2}x\right) + C$$

$$\int \cos^4 x \cdot dx = \frac{\cos^3 x \sin x}{4} + \frac{3\cos x \sin x}{8} + \frac{3}{8}x + C \quad \text{Ans.}$$

7): $\int \cos^5 x \cdot dx$.

$$\int \cos^5 x \, dx = \frac{\cos^{5-1} x \sin}{5} + \frac{(5-1)}{5}\int \cos^{5-2} x \, dx$$

$$\int \cos^5 x \cdot dx = \frac{\cos^4 x \sin x}{5} + \frac{4}{5}\int \cos^3 x \cdot dx$$

$$\int \cos^5 x \, dx = \frac{\cos^4 x \sin x}{5} + \frac{4}{5}\left(\frac{\cos^2 x \sin x}{3} + \frac{2}{3}\sin x\right) + C$$

$$\int \cos^5 x \cdot dx = \frac{\cos^4 x \sin x}{5} + \frac{4\cos^2 5\sin x}{15} + \frac{8\sin x}{15} + C$$

8) $\int \sin x \cos^5 x \cdot dx$

$$= -\int \cos^5 x (-\sin x) \cdot dx$$

$$= -\frac{\cos^{5+1} x}{5+1} + C$$

$$= -\frac{\cos^6 x}{6} + C \quad \text{Ans}$$

9) $\int \tan^2 x \cdot dx$

$$\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} + \int \tan^{n-2} x \cdot dx$$

$$\int \tan^2 x \, dx = \frac{\tan^{2-1} x}{2-1} + \int \tan^{2-2} x \cdot dx$$

$$\int \tan^2 x \cdot dx = \tan x + \int dx$$

$$\int \tan^2 x \cdot dx = \tan x + x + C \quad \text{Ans}$$

10) $\int \sin 2a \cos 8a \, da.$

$$\sin u + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right) \quad \text{(1)}$$

$$\frac{u+v}{2} = 2a \qquad \frac{u-v}{2} = 8a$$

$$u+v = 4a \quad \text{(i)} \qquad u-v = 16a \quad \text{(ii)}$$

Adding (i) and (ii)

$$\begin{aligned} u+v &= 4a \\ u-v &= 16a \\ \hline 2u &= 20a \\ u &= 10a \end{aligned}$$

$$u+v = 4a$$
$$v = 4a - 10a$$
$$v = -6a$$

Now putting value of u and v in (1)

$$\sin 10a + \sin(-6a) = 2\sin 2a \cos 8a$$
$$\sin 10a - \sin 6a = 2\sin 2a \cos 8a$$
$$\sin 2a \cos 8a = \frac{1}{2}\left[\sin 10a - \sin 6a\right]$$

Now

$$\int \sin 2a \cos 8a \, da = \int \frac{1}{2}\left[\sin 10a - \sin 6a\right] da$$

$$= \frac{1}{2}\int \sin 10a\,da - \frac{1}{2}\int \sin 6a\,da$$

$$= \frac{1}{2}\left[\left(-\frac{\cos 10a}{10}\right) - \left(\frac{-\cos 6a}{6}\right)\right] + C$$

$$= \frac{1}{2}\left[\frac{-\cos 10a}{10} + \frac{\cos 6a}{6}\right] + C$$

$$= \frac{-\cos 10a}{20} + \frac{\cos 6a}{12} + C \quad \text{Ans}$$