# Supplementary material:
## Understanding the impact of disease and vaccine mechanisms on the importance of optimal vaccine allocation

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## 1 Model details

In this section we briefly restate our model and parameter definitions, and we list the default values for each parameter. This is to provide the full model in a single, succinct section. We then provide extra results in Section 2.

We consider the model as defined by Eqs. (2)–(16):

$$\textit{Unvaccinated:} \tag{1}$$

$$\frac{dU_i}{dt} = -\lambda_i(t)U_i(t) \tag{2}$$

$$\frac{dE_{Ui}}{dt} = \lambda_i(t)U_i(t) - \theta E_{Ui}(t) \tag{3}$$

$$\frac{dI_{USi}}{dt} = \omega_i \theta E_{Ui}(t) - \gamma I_{USi}(t) \tag{4}$$

$$\frac{dI_{UAi}}{dt} = (1 - \omega_i)\theta E_{Ui}(t) - \gamma I_{UAi}(t) \tag{5}$$

$$\frac{dR_{USi}}{dt} = (1 - \sigma_i)\gamma I_{USi}(t) \tag{6}$$

$$\frac{dD_{Ui}}{dt} = \sigma_i \gamma I_{USi}(t) \tag{7}$$

$$\frac{dR_{UAi}}{dt} = \gamma I_{SAi}(t) \tag{8}$$

$$\textit{Vaccinated:} \tag{9}$$

$$\frac{dV_i}{dt} = -\alpha_1 \lambda_i(t)V_i(t) \tag{10}$$

$$\frac{dE_{Vi}}{dt} = \alpha_1 \lambda_i(t)V_i(t) - \theta E_{Vi}(t) \tag{11}$$

$$\frac{dI_{VSi}}{dt} = \alpha_2 \omega_i \theta E_{Vi}(t) - \gamma I_{VSi}(t) \tag{12}$$

$$\frac{dI_{VAi}}{dt} = (1 - \alpha_2 \omega_i)\theta E_{Vi}(t) - \gamma I_{VAi}(t) \tag{13}$$

$$\frac{dR_{VSi}}{dt} = (1 - \alpha_3 \sigma_i)\gamma I_{VSi}(t) \tag{14}$$

$$\frac{dD_{Vi}}{dt} = \alpha_3 \sigma_i \gamma I_{VSi}(t) \tag{15}$$

$$\frac{dR_{VAi}}{dt} = \gamma I_{VAi}(t) \tag{16}$$

<div align="center"><em>Force of Infection:</em></div>

$$\lambda_i = \sum_{j=1}^{2} \beta C_{ij} \frac{I_{USj}(t)}{LP_j(t)} + \delta_A \beta C_{ij} \frac{I_{UAj}(t)}{LP_j(t)} + \alpha_4 \left[ \beta C_{ij} \frac{I_{VSj}(t)}{LP_j(t)} + \delta_A \beta C_{ij} \frac{I_{VAj}(t)}{LP_j(t)} \right], \tag{17}$$

where $LP_i(t)$ denotes the living population at time $t$. That is,

$$LP_i(t) = P_i - D_{Ui}(t) - D_{Vi}(t),$$

where $P_i$ is the total population of group $i$ at $t = 0$.

## 1.1 Parameter definitions

We firstly define the compartments $(X_{jki})$ included in our model:

- $X \in \{U, V, E, I, R, D\}$: unvaccinated, vaccinated, exposed, infected, recovered, dead

- $j \in \{U, V\}$: unvaccinated or vaccinated disease progression

- $k \in \{S, A\}$: symptomatic or asymptomatic infection

- $i \in \{1, 2\}$: population group (1: high risk of infection, 2: low risk of infection)

The following parameters are defined to be constant across the population groups:

- $1/\theta = 1$: average time spent exposed

- $1/\gamma = 1$: average time spent infected

- $R_0 = 2$ (unless otherwise stated): reproduction number

- $\delta_A = 0.5$ (unless otherwise stated): reduction in infectivity due to asymptomatic infection

The following parameters are defined as specific to each population group $i$:

- $\omega_i = 0.5$ (unless otherwise stated): proportion of infected people in group $i$ that experience symptomatic disease

- $\sigma_i = 0.5$ (unless otherwise stated): proportion of symptomatic infecteds in group $i$ that die due to disease

- $P_i = 5000$: initial population size for group $i$

- $[C_{ij}] = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ (unless otherwise stated): Contact matrix

- $I_{0i} = 50$: Initial infected people in group $i$ (half symptomatic, half asymptomatic)

Finally, the following parameters characterise vaccination in our model:

- Vaccine coverage = 5000 (unless otherwise stated): Number of vaccines available

- $1 - \alpha_i = 0.25$ (unless otherwise stated): effectiveness of vaccine $i$

- $1 - \alpha_1$: reduction in susceptibility from Vaccine 1

<div align="center">2</div>

- $1 - \alpha_2$: reduction in proportion of infected cases that experience symptomatic infection from Vaccine 2

- $1 - \alpha_3$: reduction in proportion of symptomatic infecteds that die due to infection from Vaccine 3

- $1 - \alpha_4$: reduction in infectivity from Vaccine 4

## 1.2 Implicit Assumptions

In our model formulation, we assume that asymptomatic infection is less infective than symptomatic infection. This reduction in infectivity due to asymptomatic infection is denoted by $1 - \delta_A$. For this work we assume $\delta_A > 0$.

Furthermore, we assume people experiencing asymptomatic infection do not experience severe disease. This is implicitly assumed in our model as asymptomatic infecteds can only transition to a recovered compartment, they cannot die due to disease.

## 2 Results

Here we show how the difference between strategies differs for every combination of vaccine mechanism and allocation objective as we vary specific parameters.

- Varying the contact matrix (2.1)

- Varying $\omega_i$ (2.2)

- Varying $\sigma_i$ (2.3)

- Varying $\delta_A$ (2.4)

- Varying $R_0$ (2.5)

- Varying vaccine coverage (2.6)

- Varying vaccine effectiveness (2.7)

## 2.1 Varying the contact matrix
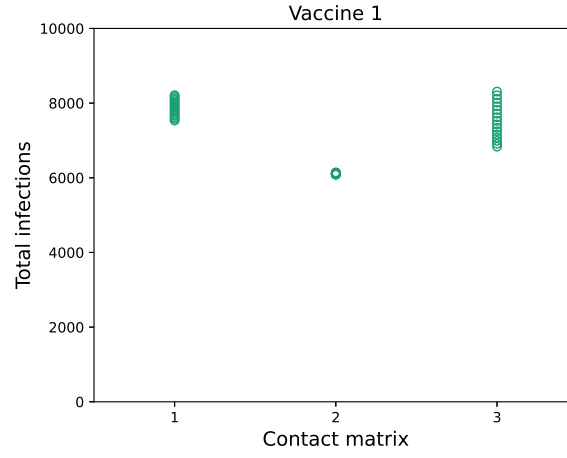


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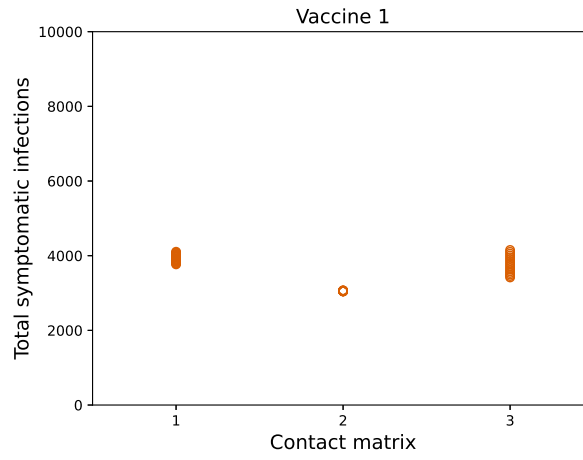


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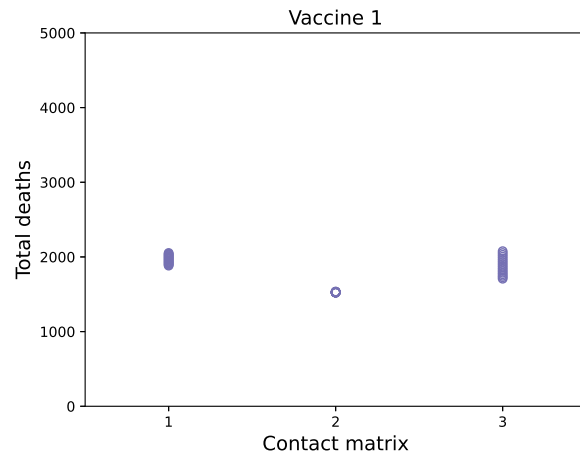


(c)

**Figure 1:** *Comparing total infections, symptomatic infections and deaths for Vaccine 1 as we vary contact matrices. The three contact matrices considered are (1)* $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$, *(2)* $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$, *and (3)* $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$.
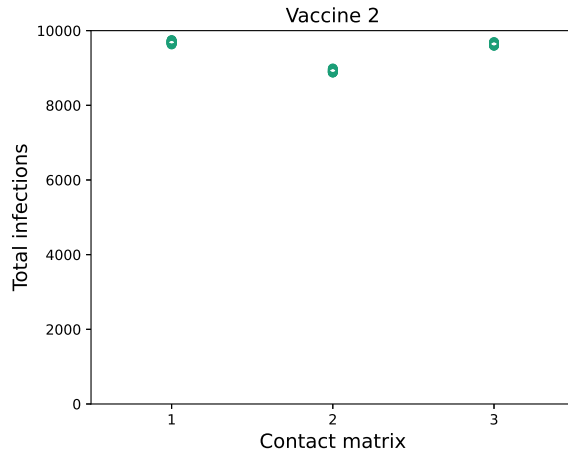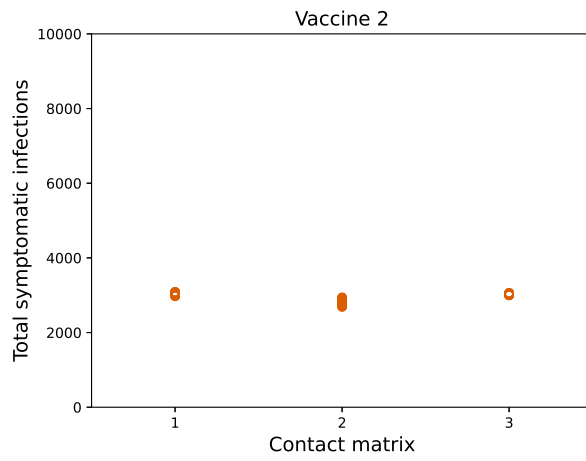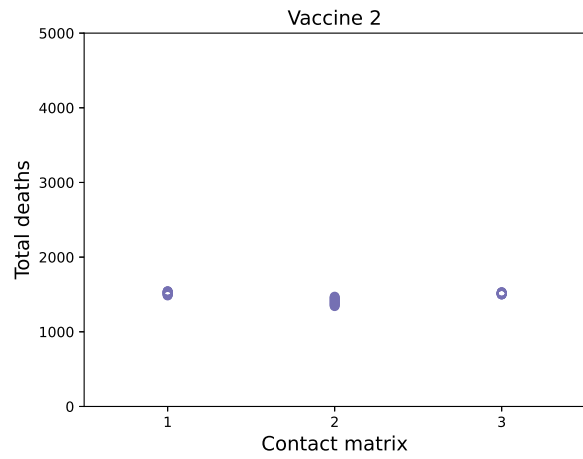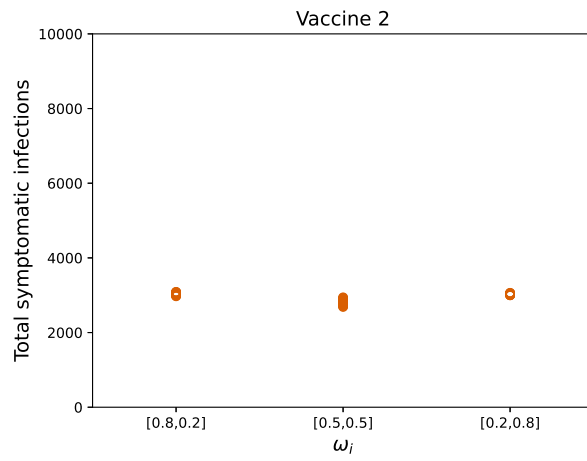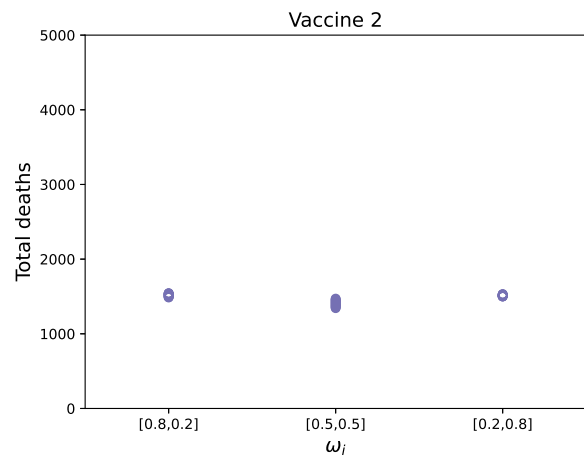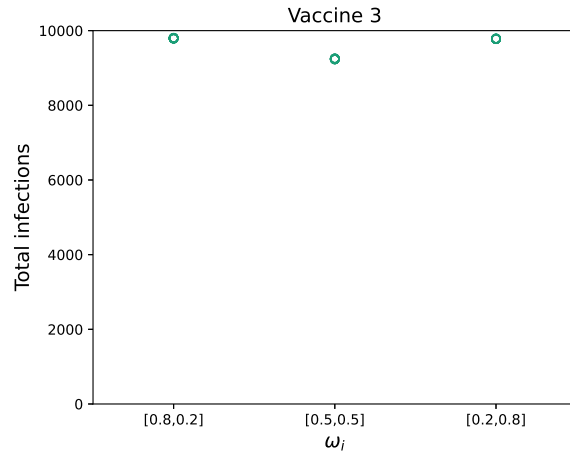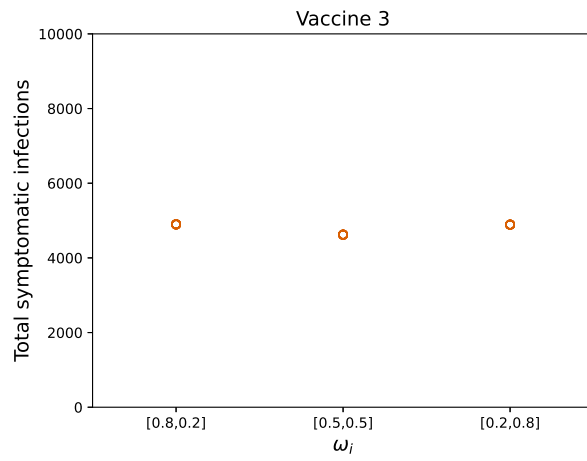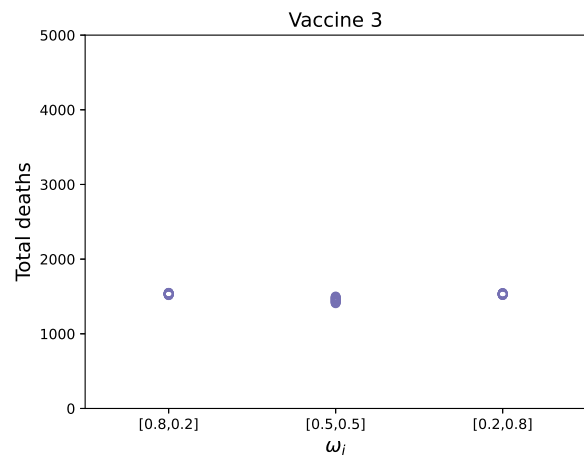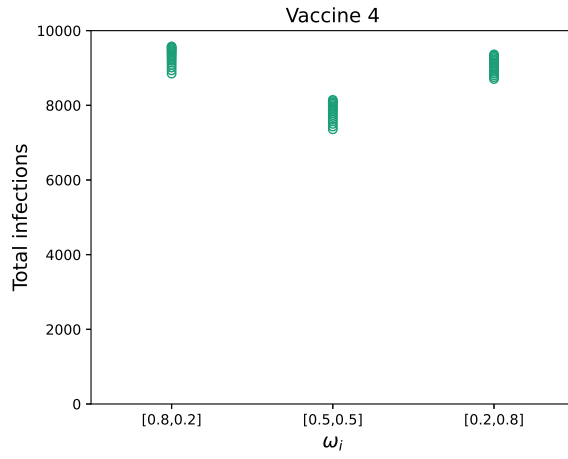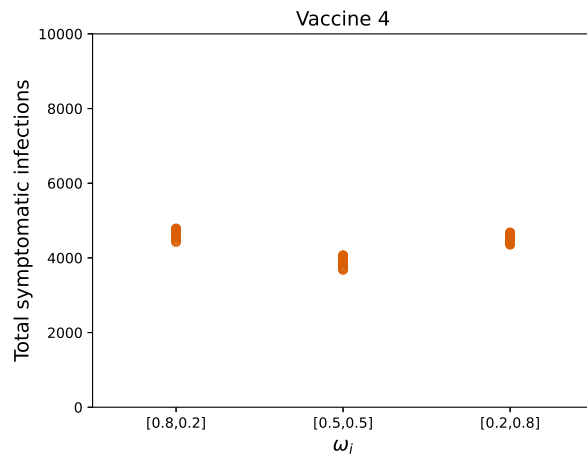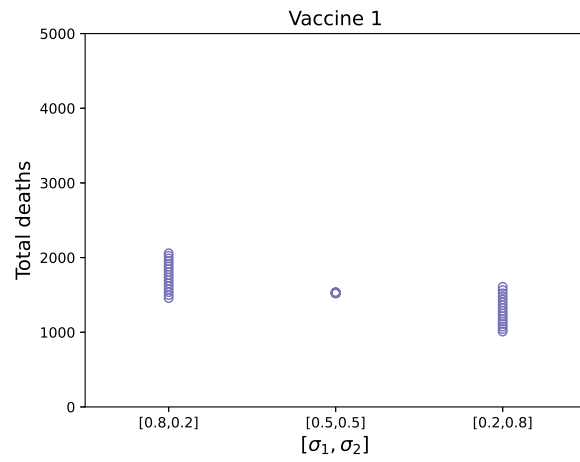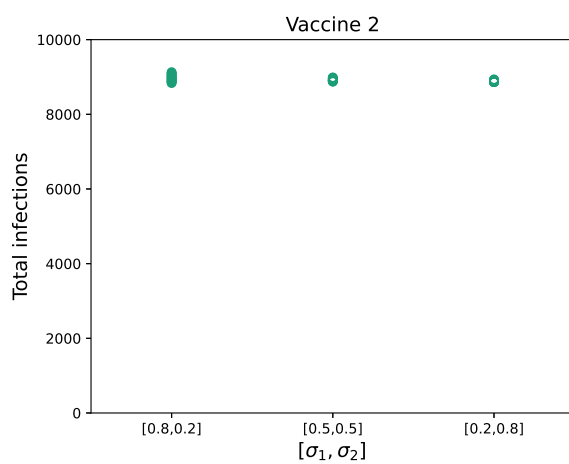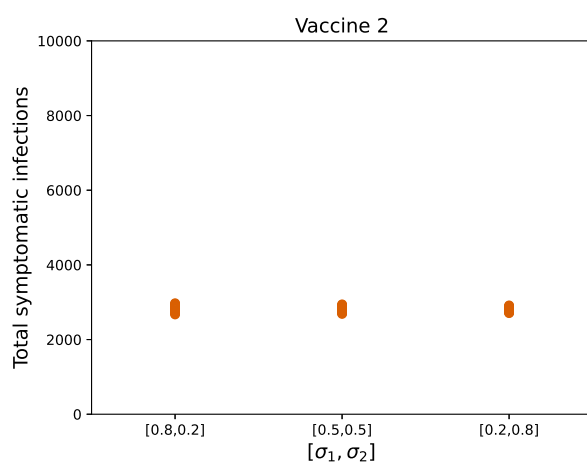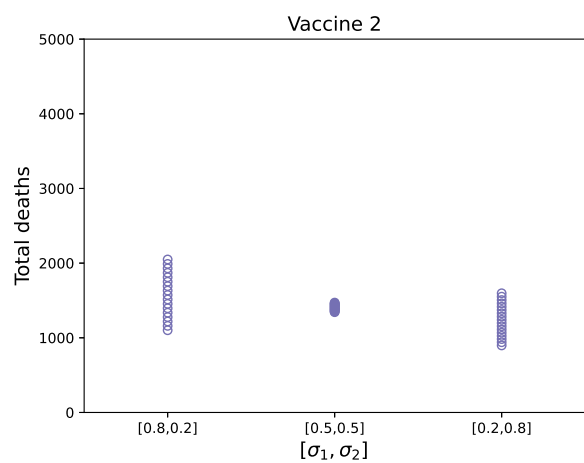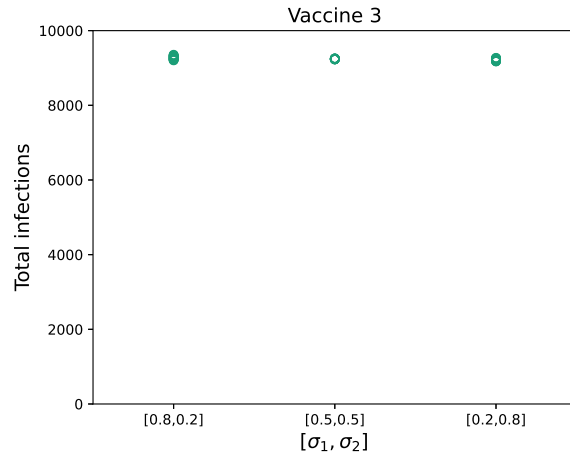
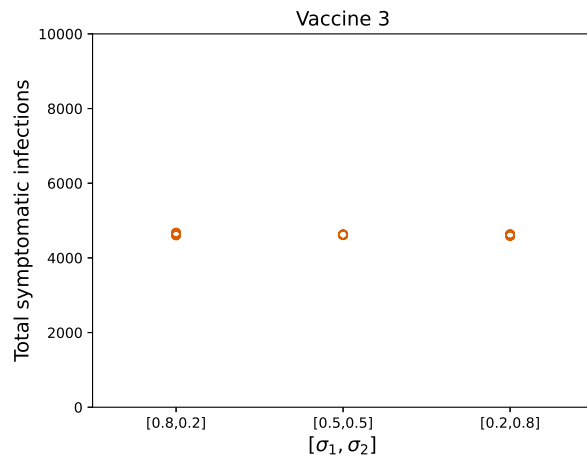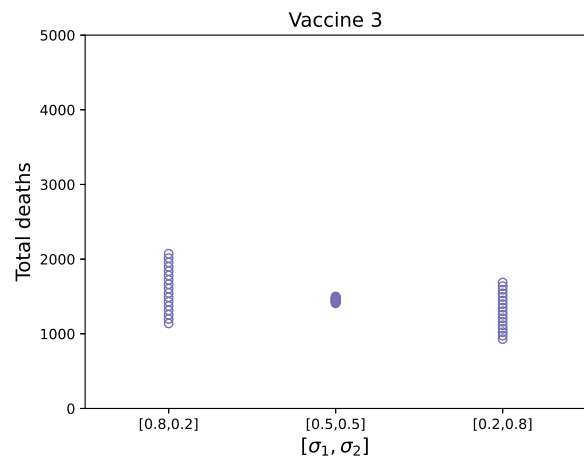**Figure 2:** *Comparing total infections, symptomatic infections and deaths for Vaccine 2 as we vary contact matrices. The three contact matrices considered are (1) $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$, (2) $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$, and (3) $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$.*
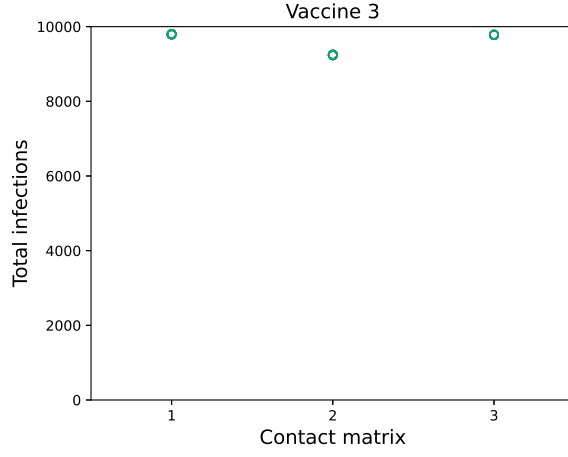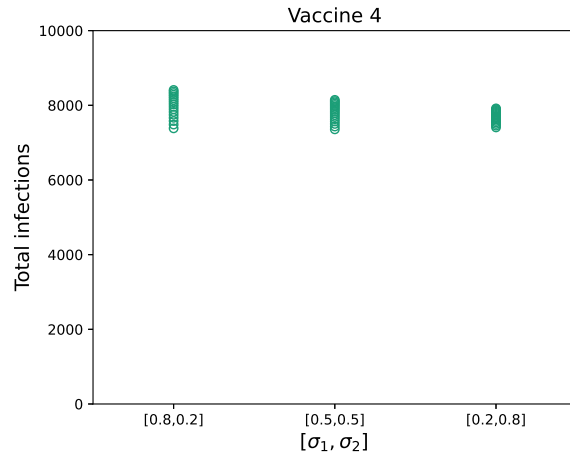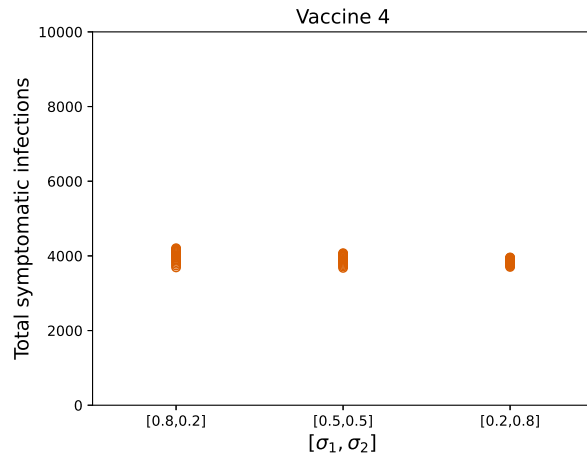
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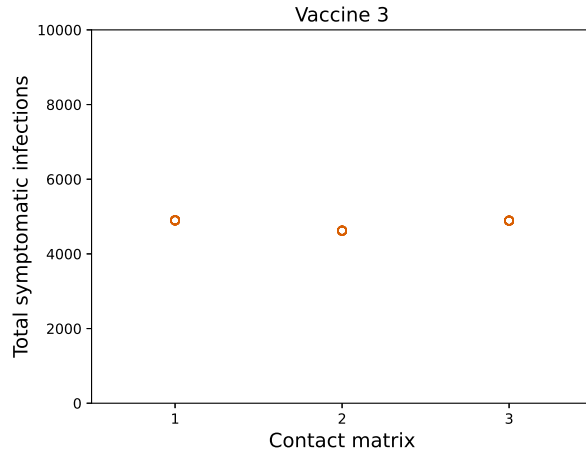


**(b)**
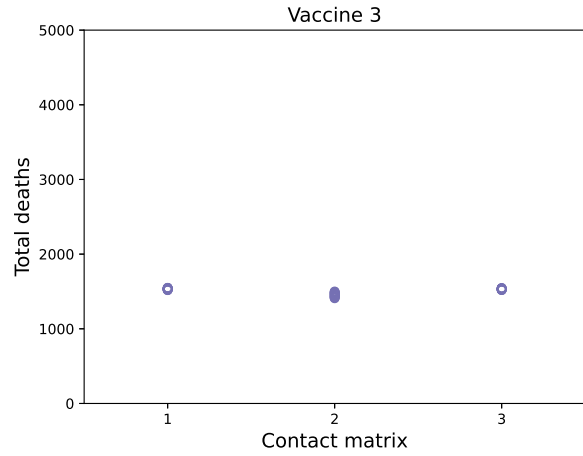


**(c)**

**Figure 3:** *Comparing total infections, symptomatic infections and deaths for Vaccine 3 as we vary contact matrices. The three contact matrices considered are (1)* $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$, *(2)* $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$, *and (3)* $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$.
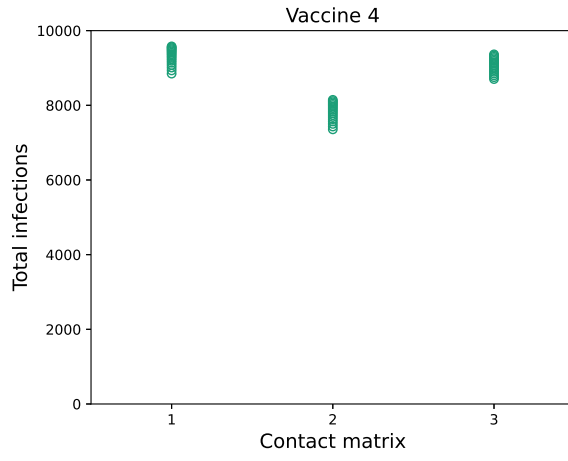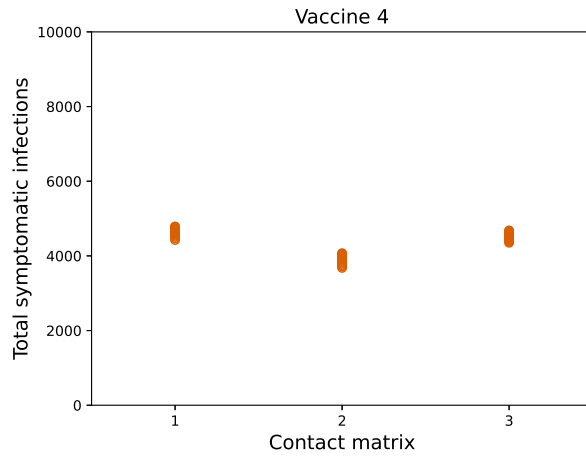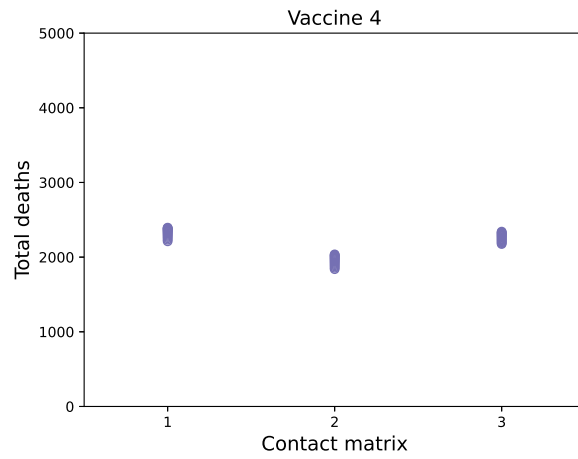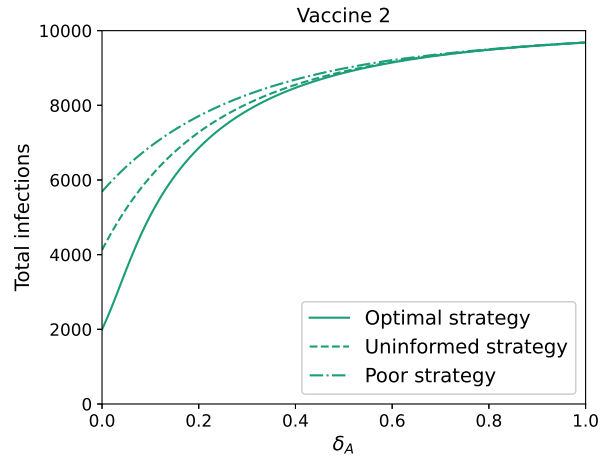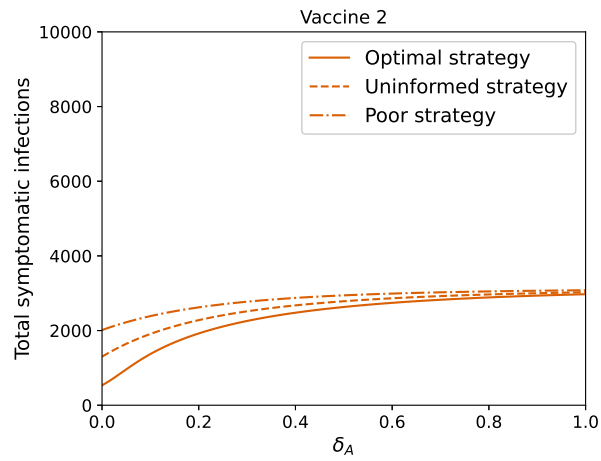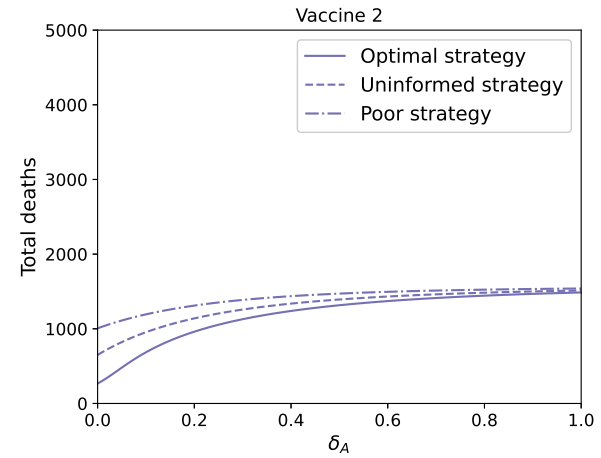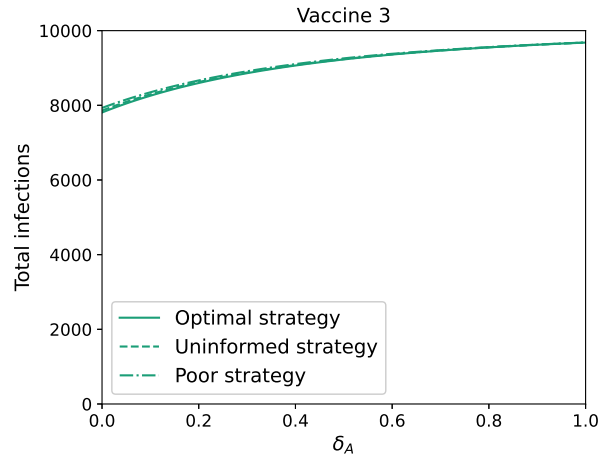
**Figure 4:** *Comparing total infections, symptomatic infections and deaths for Vaccine 4 as we vary contact matrices. The three contact matrices considered are (1)* $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$*, (2)* $\begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$*, and (3)* $\begin{bmatrix} 2 & 4 \\ 1 & 2 \end{bmatrix}$*.*
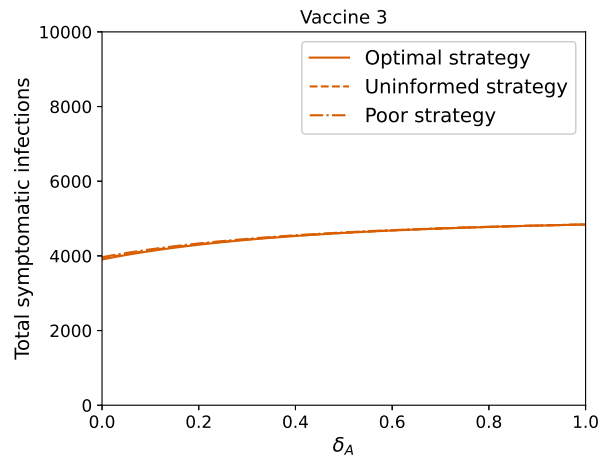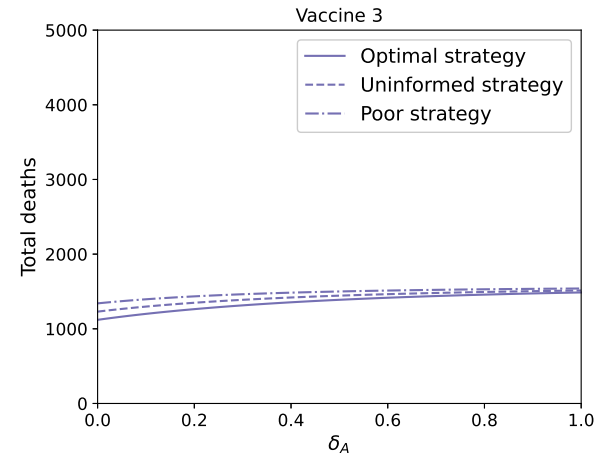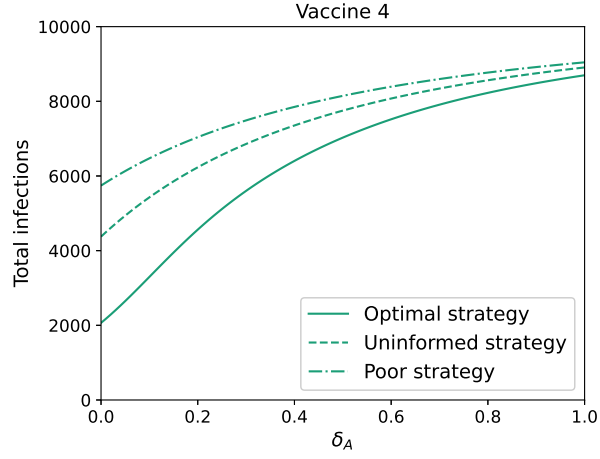
## 2.2   Varying $\omega_i$
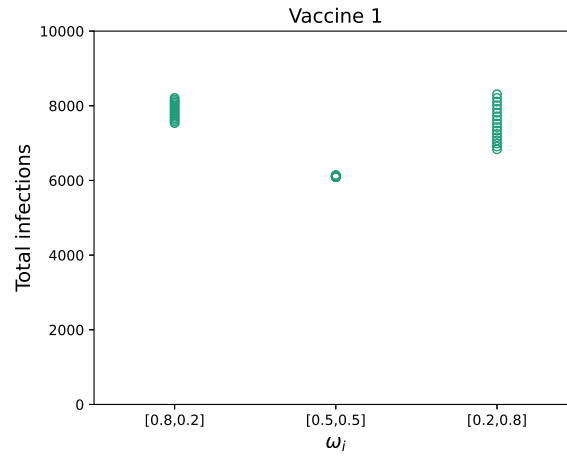


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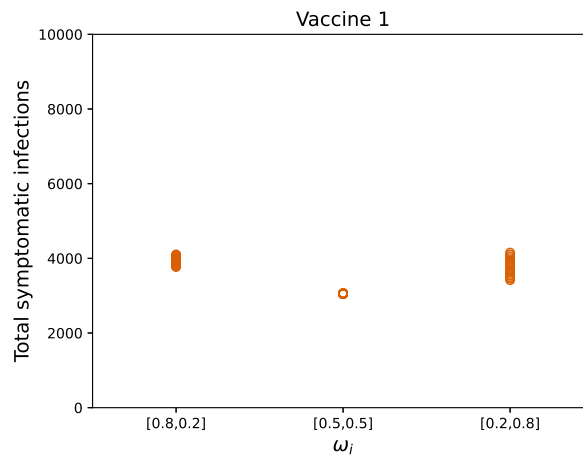


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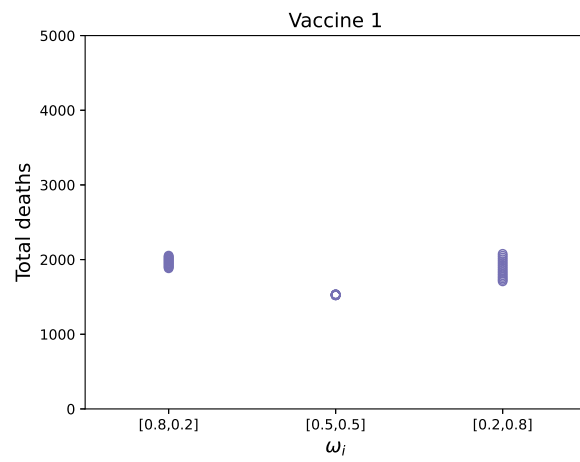


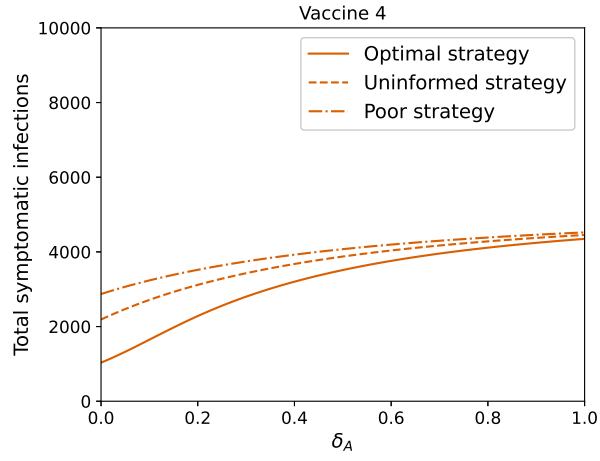**(c)**

**Figure 5:** *Comparing total infections, symptomatic infections and deaths for Vaccine 1 as we vary $\omega_1$ and $\omega_2$.*
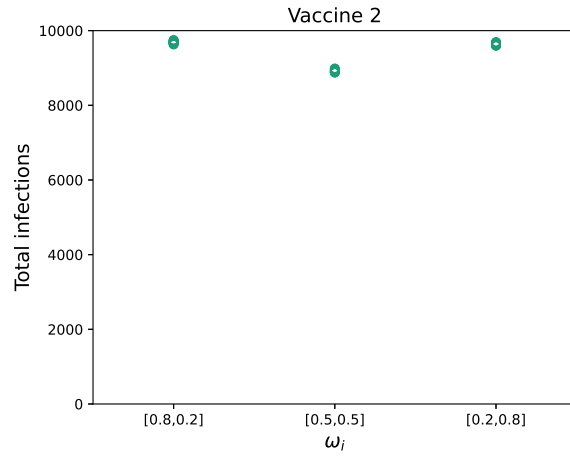
**(a)**



**(b)**



**(c)**

**Figure 6:** *Comparing total infections, symptomatic infections and deaths for Vaccine 2 as we vary $\omega_1$ and $\omega_2$.*

**(a)**



**(b)**



**(c)**

**Figure 7:** *Comparing total infections, symptomatic infections and deaths for Vaccine 3 as we vary* $\omega_1$ *and* $\omega_2$.

**(a)**



**(b)**
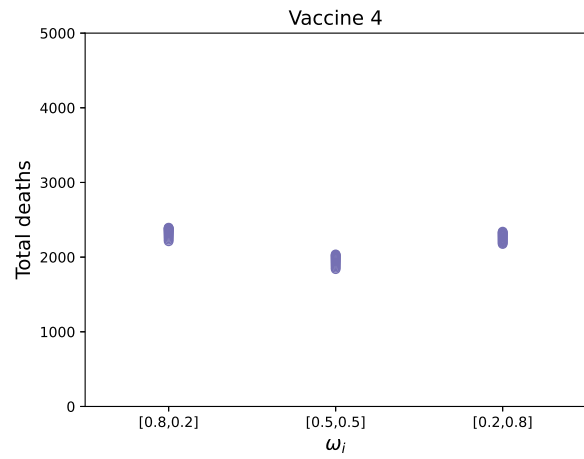


**(c)**

**Figure 8:** *Comparing total infections, symptomatic infections and deaths for Vaccine 4 as we vary $\omega_1$ and $\omega_2$.*

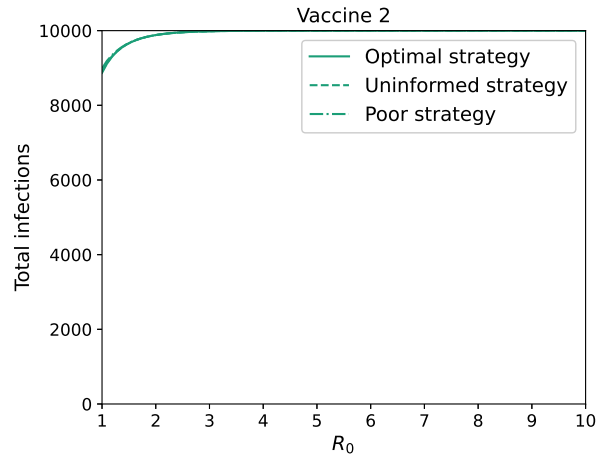## 2.3 Varying $\sigma_i$
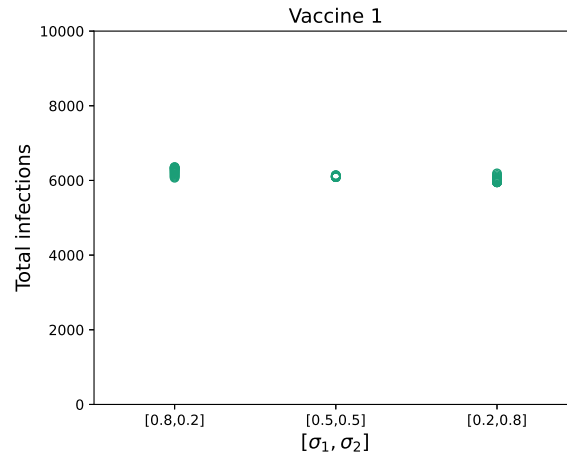


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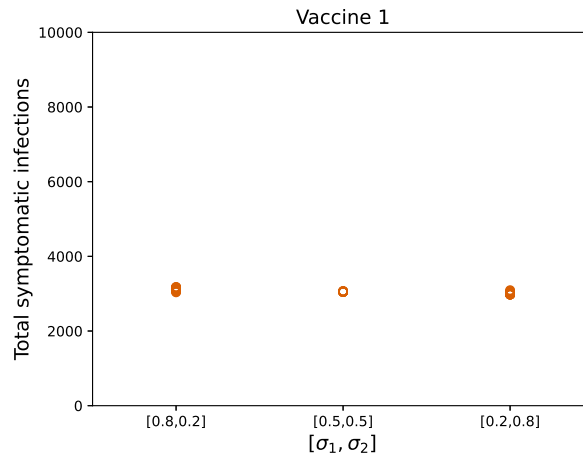


**(b)**



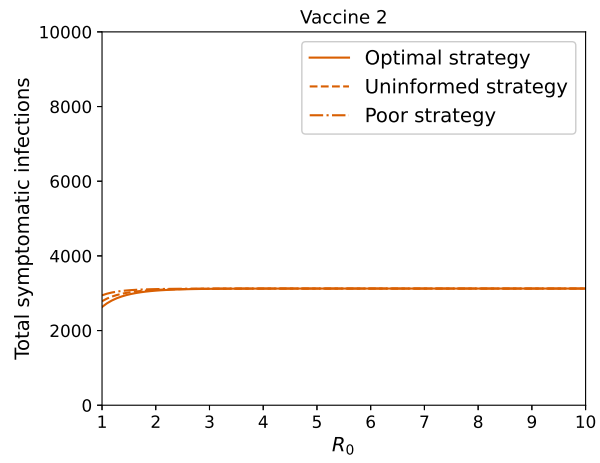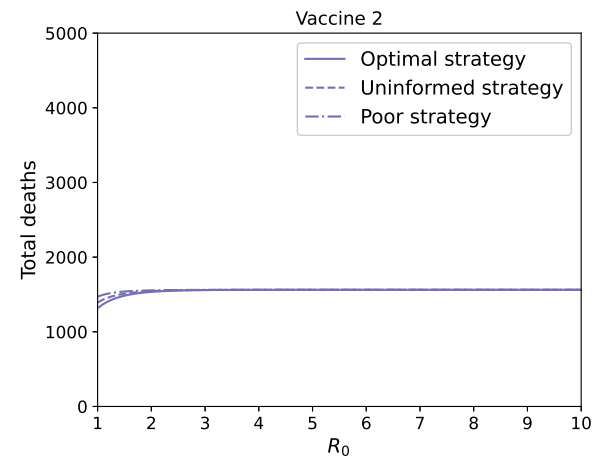**(c)**

**Figure 9:** *Comparing total infections, symptomatic infections and deaths for Vaccine 1 as we vary $\sigma_1$ and $\sigma_2$.*

**Figure 10:** *Comparing total infections, symptomatic infections and deaths for Vaccine 2 as we vary $\sigma_1$ and $\sigma_2$.*

**(a)**



**(b)**



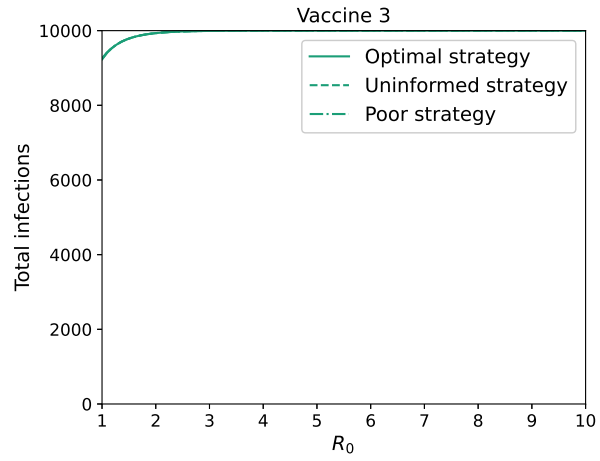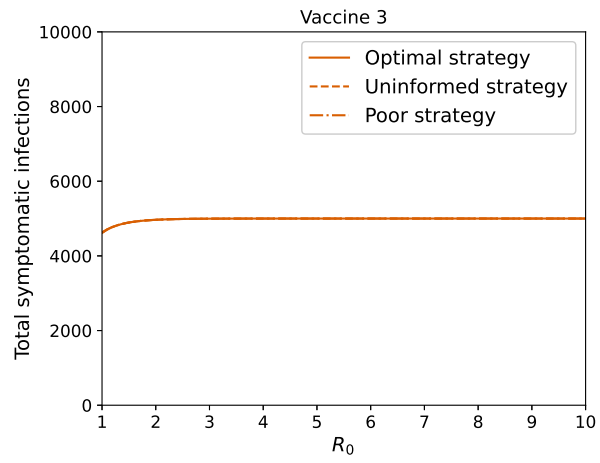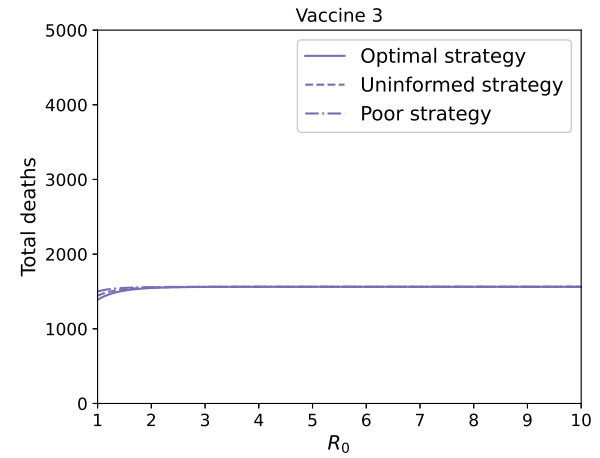**(c)**

**Figure 11:** *Comparing total infections, symptomatic infections and deaths for Vaccine 3 as we vary $\sigma_1$ and $\sigma_2$.*
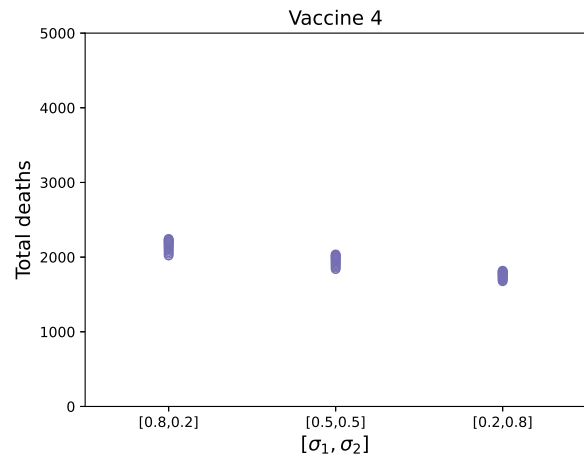
**Figure 12:** *Comparing total infections, symptomatic infections and deaths for Vaccine 4 as we vary $\sigma_1$ and $\sigma_2$.*
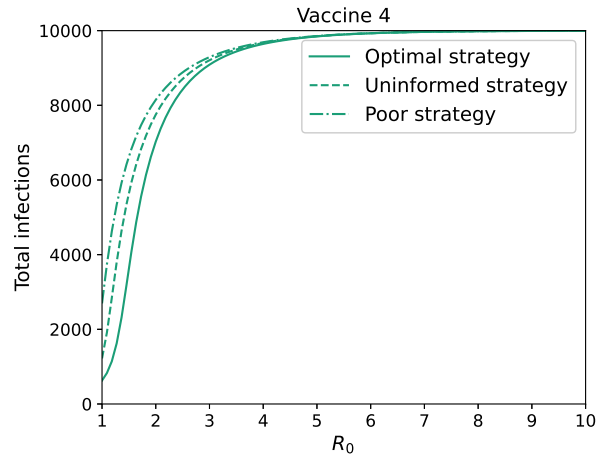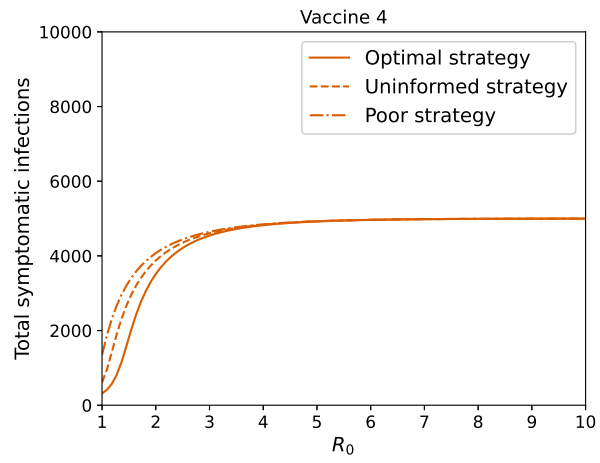
## 2.4   Varying $\delta_A$
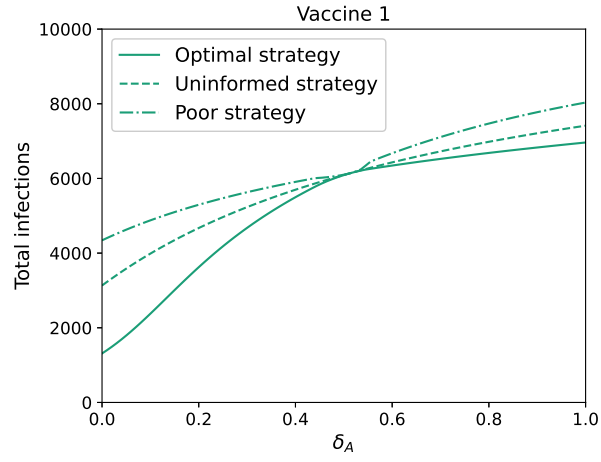


**(a)**
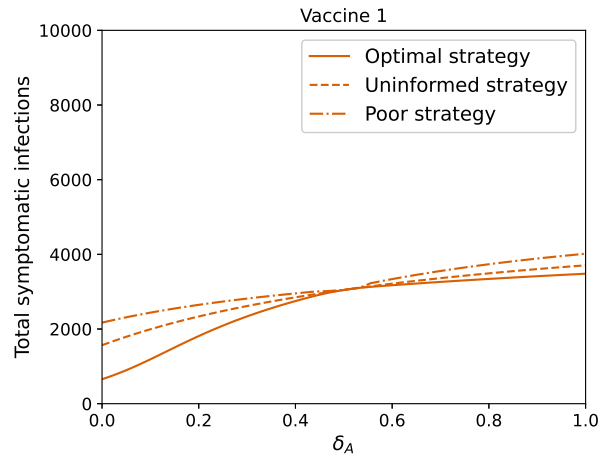


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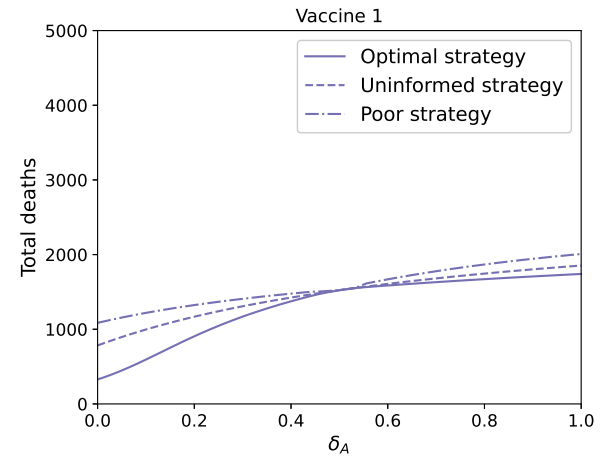


**(c)**

**Figure 13:** *Comparing total infections, symptomatic infections and deaths for Vaccine 1 as we vary $\delta_A$.*

**(a)**



**(b)**



**(c)**

**Figure 14:** *Comparing total infections, symptomatic infections and deaths for Vaccine 2 as we vary* $\delta_A$.

**(a)**



**(b)**



**(c)**

**Figure 15:** *Comparing total infections, symptomatic infections and deaths for Vaccine 3 as we vary* $\delta_A$.

**(a)**



**(b)**
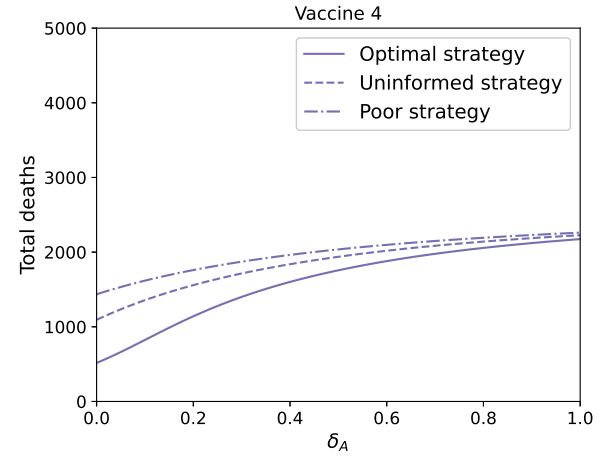


**(c)**

**Figure 16:** *Comparing total infections, symptomatic infections and deaths for Vaccine 4 as we vary* $\delta_A$.

## 2.5  Varying $R_0$
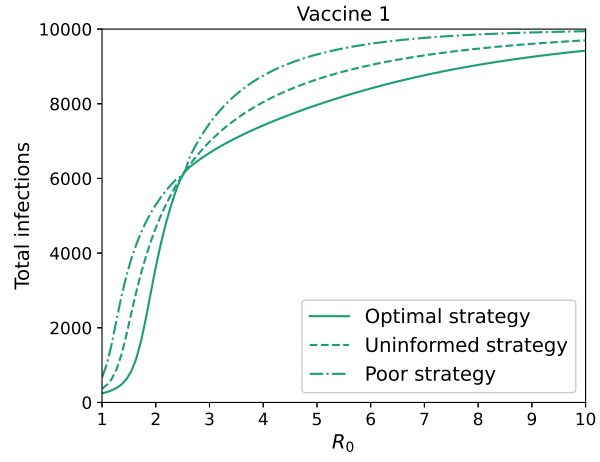


**(a)**
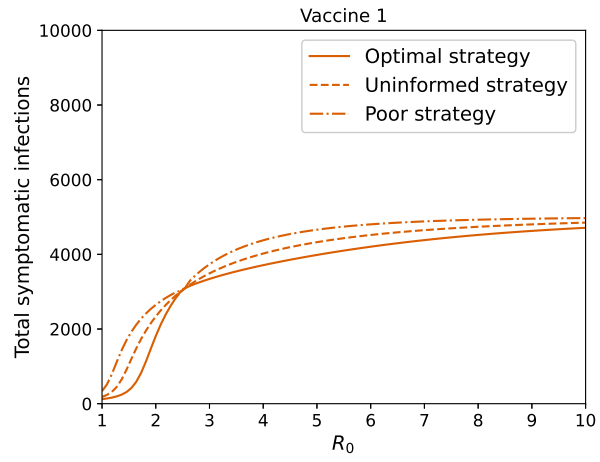


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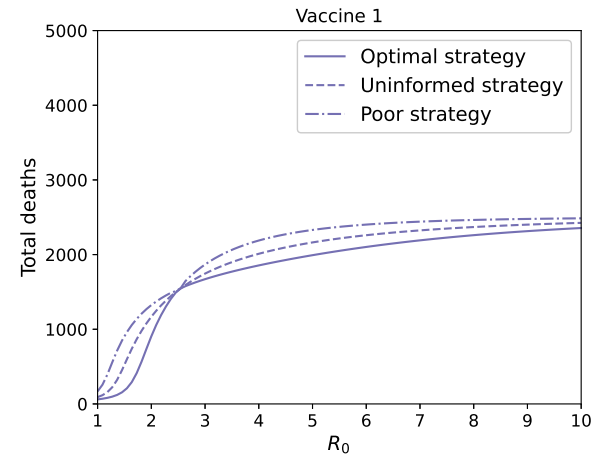


**(c)**

**Figure 17:** *Comparing total infections, symptomatic infections and deaths for Vaccine 1 as we vary $R_0$.*

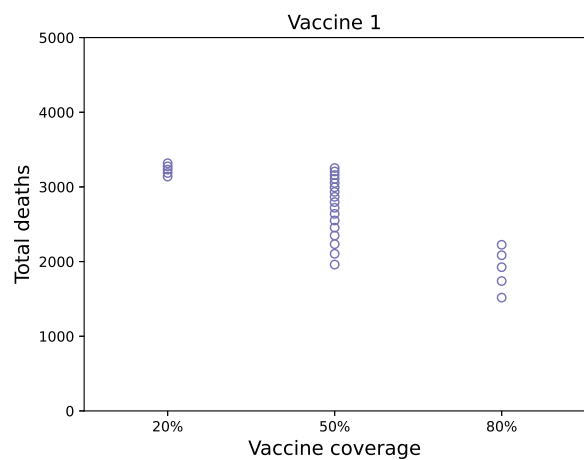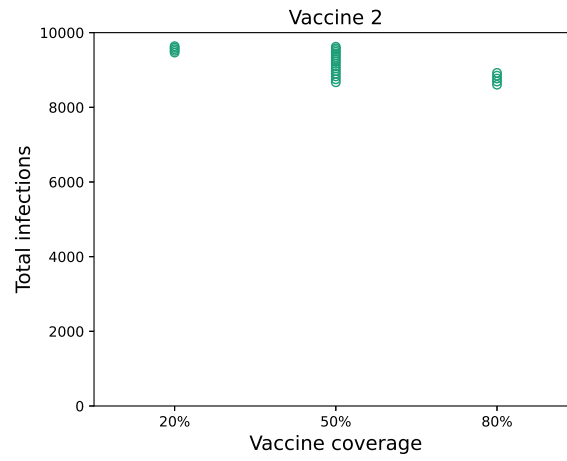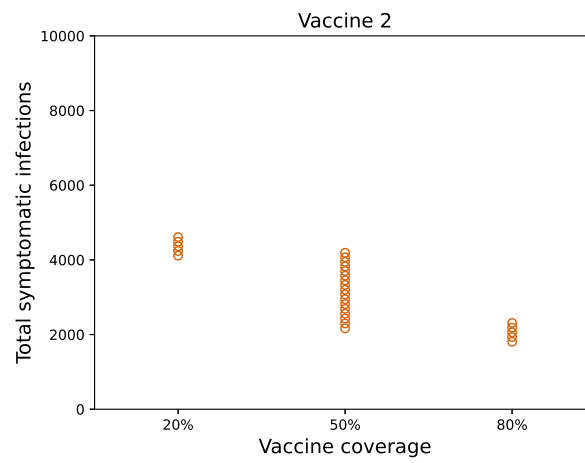**Figure 18:** *Comparing total infections, symptomatic infections and deaths for Vaccine 2 as we vary $R_0$.*

**Figure 19:** *Comparing total infections, symptomatic infections and deaths for Vaccine 3 as we vary $R_0$.*

**(a)**



**(b)**
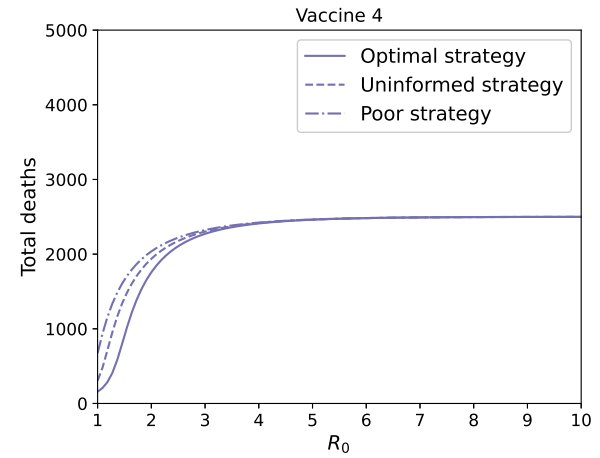


**(c)**

**Figure 20:** *Comparing total infections, symptomatic infections and deaths for Vaccine 4 as we vary $R_0$.*

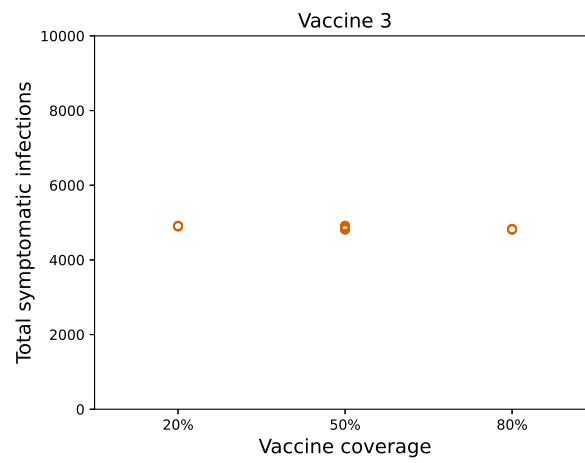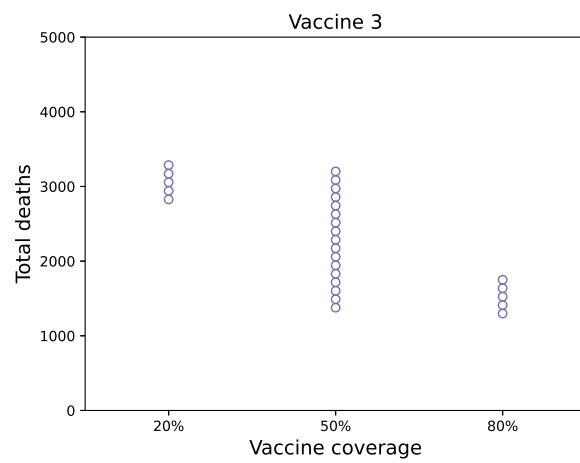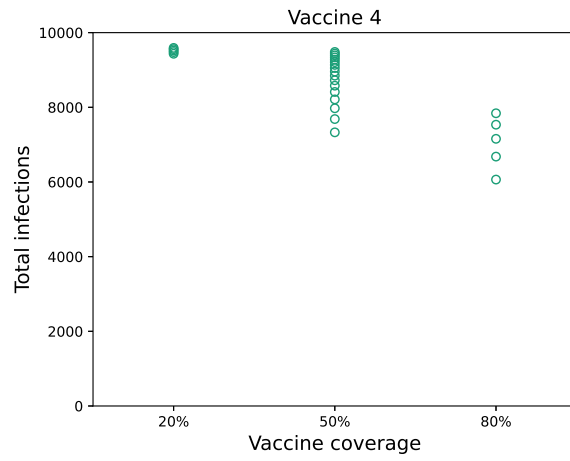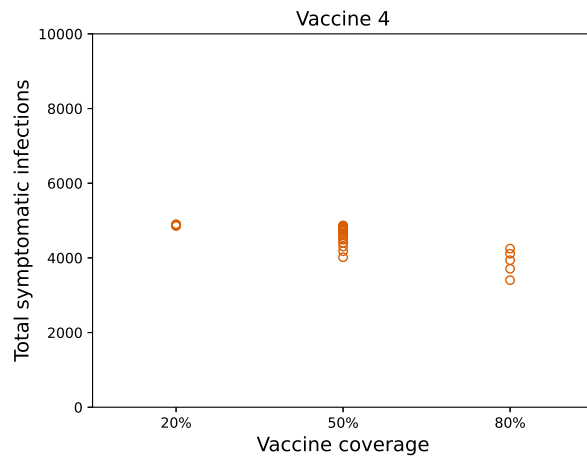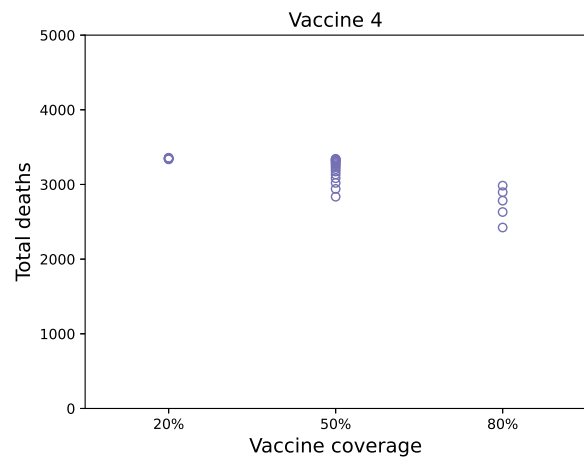## 2.6   Varying vaccine coverage
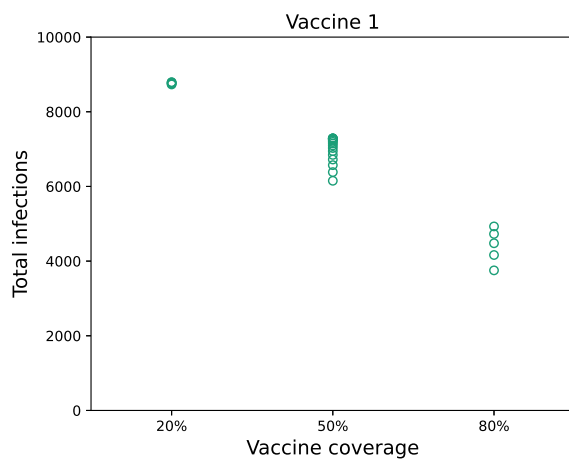


**(a)**
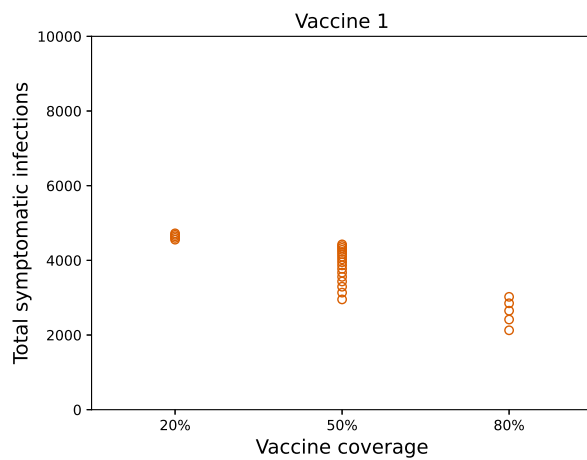


**(b)**



**(c)**

**Figure 21:** *Comparing total infections, symptomatic infections and deaths for Vaccine 1 as we vary vaccine coverage.*
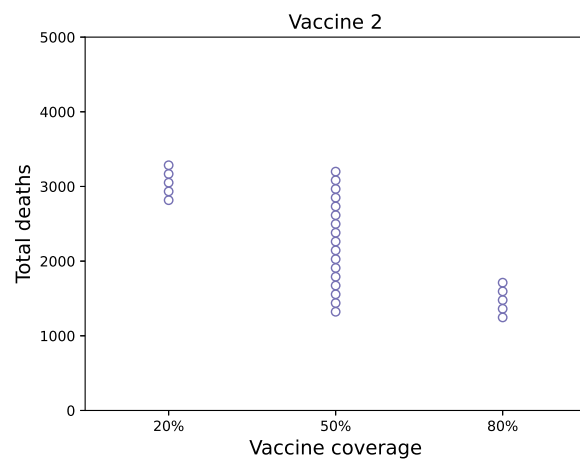
**Figure 22:** *Comparing total infections, symptomatic infections and deaths for Vaccine 2 as we vary vaccine coverage.*
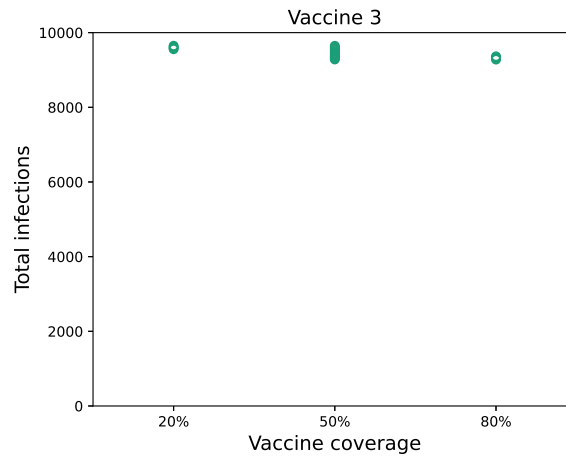
**(a)**



**(b)**



**(c)**

**Figure 23:** *Comparing total infections, symptomatic infections and deaths for Vaccine 3 as we vary vaccine coverage.*
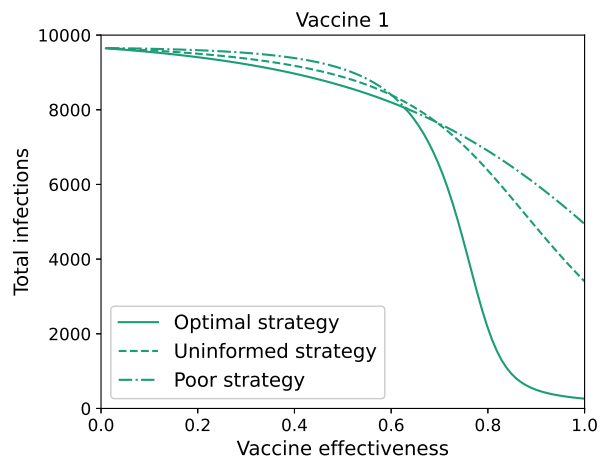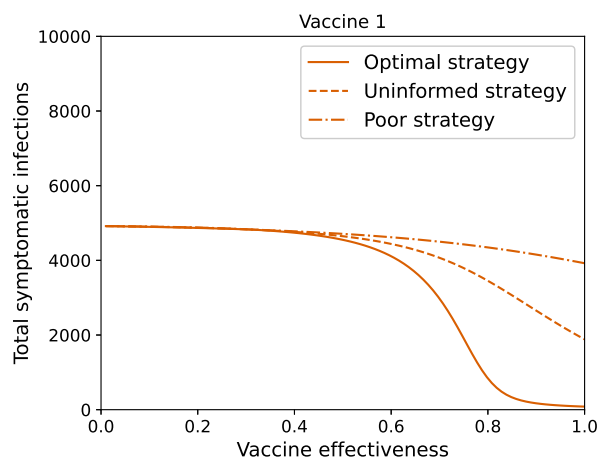
**(a)**



**(b)**



**(c)**

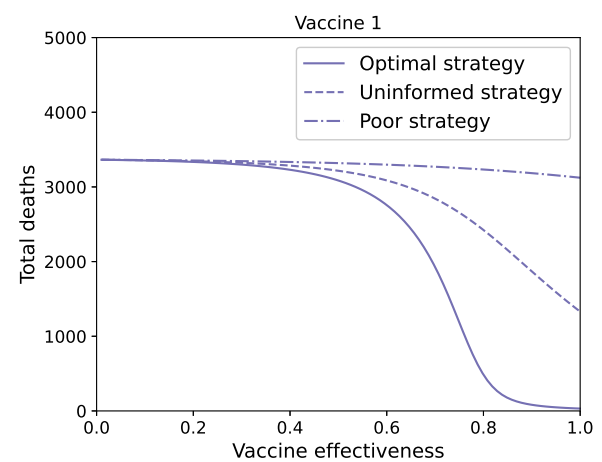**Figure 24:** *Comparing total infections, symptomatic infections and deaths for Vaccine 4 as we vary vaccine coverage.*
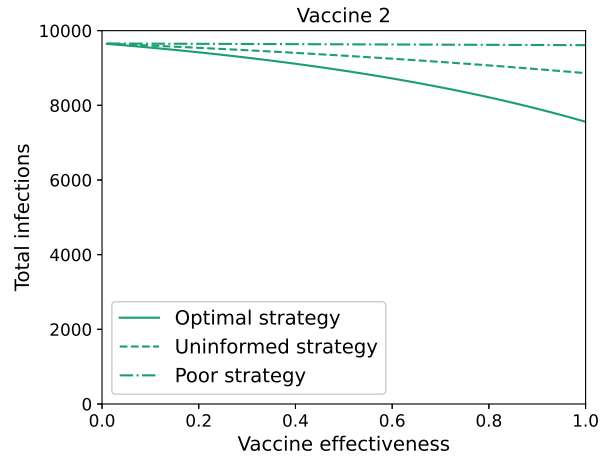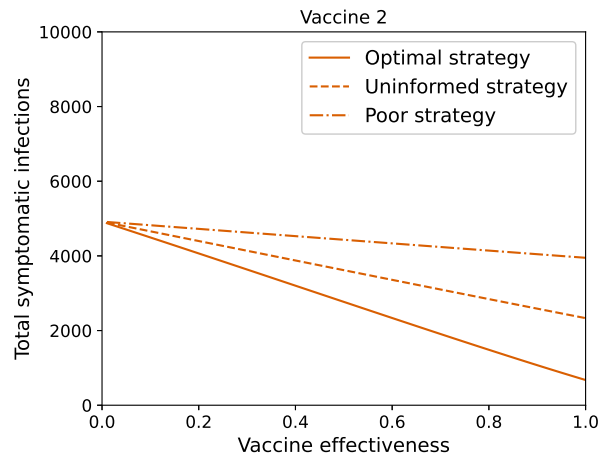
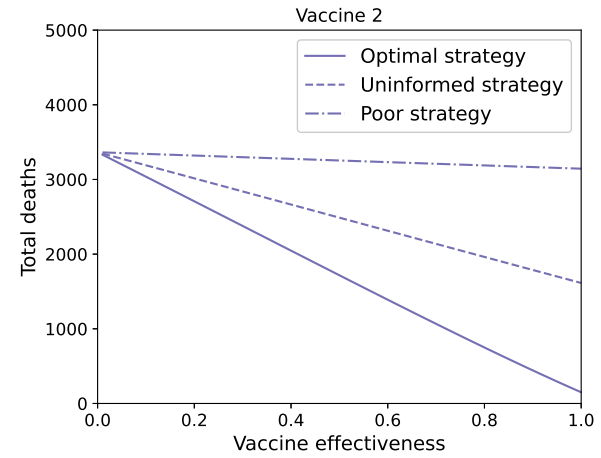## 2.7 Varying vaccine effectiveness



**(a)**



**(b)**



**(c)**

**Figure 25:** *Comparing total infections, symptomatic infections and deaths for Vaccine 1 as we vary vaccine effectiveness.*
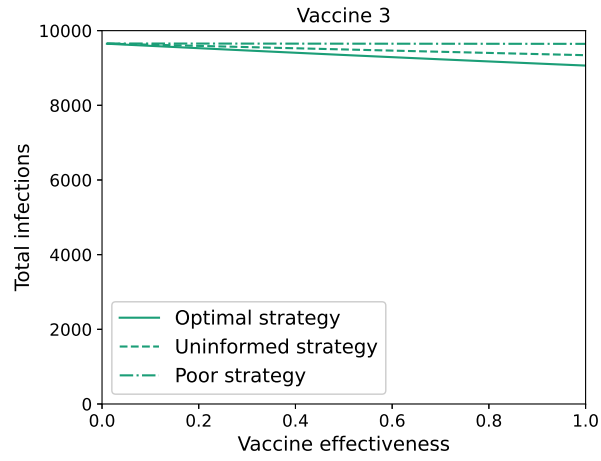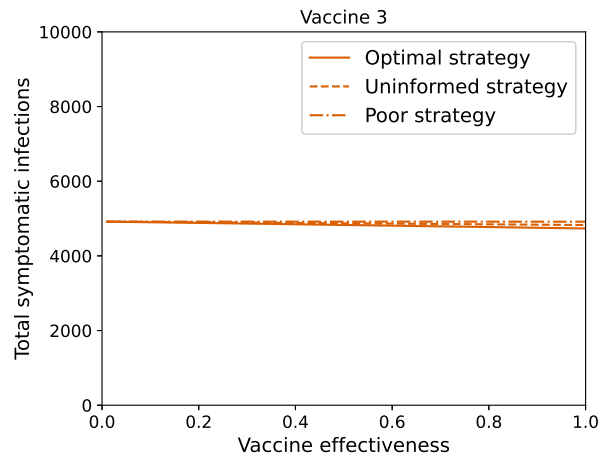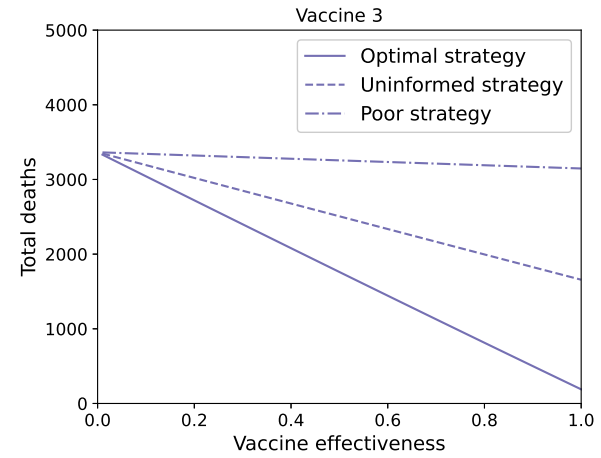
**(a)**



**(b)**



**(c)**

**Figure 26:** *Comparing total infections, symptomatic infections and deaths for Vaccine 2 as we vary vaccine effectiveness.*

**(a)**



**(b)**



**(c)**

**Figure 27:** *Comparing total infections, symptomatic infections and deaths for Vaccine 3 as we vary vaccine effectiveness.*
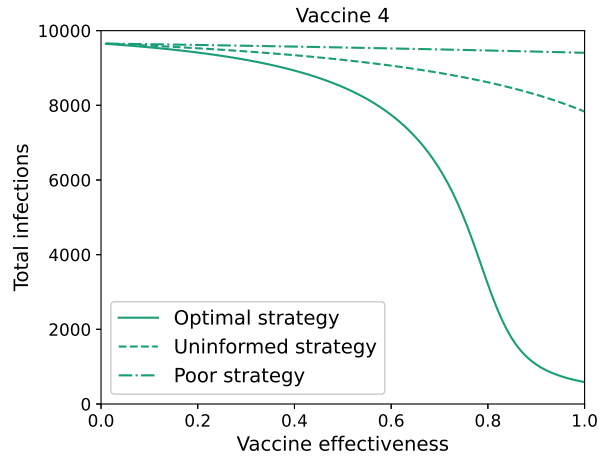
**(a)**
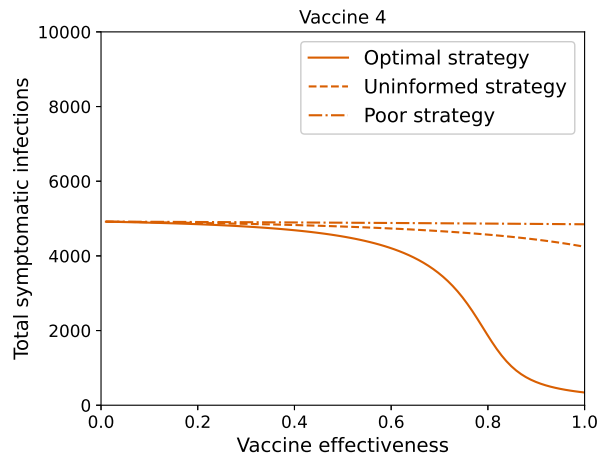


**(b)**
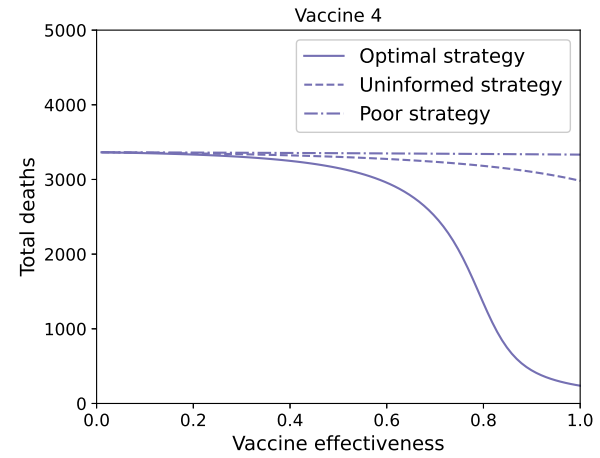


**(c)**

**Figure 28:** *Comparing total infections, symptomatic infections and deaths for Vaccine 4 as we vary vaccine effectiveness.*