Data Analytics Assignment - II Report by Abhishek Kumar (Sr. no. 15648)

1. Problem Statement:

Use the Mars opposition data and ignore the latitudes. Assuming that the projection of Mars orbit on the ecliptic plane is a circle and that this centre, the average Sun, and the Sun are collinear. Find the following:
(i) For each fixed value of X (offset of the average Sun) and Y (angle of the line with respect to the reference Aries), write down an expression for the projection of Mars position on the ecliptic plane and the distance of this projection to the centre, in terms of a paired observation.
(ii) Then using the loss function log (arithmetic mean) - log (geometric mean), find the best fit value of x and y.

Dataset: Mars Opposition data (Taken from Kepler's data set on Mars)

2. Solution

2.1) **Data Pre-Processing**
The given data file contains Mars opposition data observed by kepler. It has 12 (rows) data points where each row contains the value of angles calculated by kepler using some constellation as reference point. These angles have been calculated w.r.t Sun and Avg. Sun. To calculate the value of X and Y, We need following information :

Angle made by mars when seen from Sun and avg. Sun w.r.t the reference point,i.e., Longitude which can be calculated using the following columns from the dataset: Zodiac index, Degree, Minute, Second
Given, Longitude = ZodiacIndex$*$30 + Degree + Minute/60 + Second/3600 $*\pi/180$ (Radians)

2.2) **Solution Approach**
I have used the following algorithm to Calculate the value of X (offset of the average Sun) and Y (angle of the line with respect to the reference Aries)

Result: Parameter X, Y and Loss
Initialization : Intialize X and Y with some initial values;
Calculate the values of longitude $l1$ and $l2$ for Sun and Avg. Sun respectively using the given formula of longitude.;
while *loss is not minimized* **do**
    Find Coordinates of Mars using the longitude and coordinates of sun and avg.sun;
    Find Distance of mars from origin for each of the 12 points. ;
    Loss : log(Arithmetic mean of distances) - log(geometric mean of the distances.);
    Calculate new value of (X and Y) using some method such that the loss is minimized.;
    update X and Y;
end

Algorithm 1: Find the value of X (offset of the average Sun) and Y (angle of the line with respect to the reference Aries)

To minimize the loss, I have used **scipy.optimize.minimize** function which takes the loss function, X and Y as input and produces the updated value of the parameter and total loss using these parameters.

Method to calculate positions (coordinates on the elliptical plane) of Mars
1) Calculate the polar coordinates of Sun and Avg. Sun using X and Y i.e., $(Cos(Y), Sin(Y))$ and $(-XCos(Y), -XSin(Y))$ respectively.

2) Find the equation of lines passing through Sun and Mars (line 1) and another line passing through Avg. Sun and Mars (line 2).

Let's $m_1 = \tan(\alpha)$ where α is the angle made by sun and mars with the reference line.
$m_2 = \tan(\beta)$ where β is the angle made by avg. sun and mars with the reference line.

Equation of line$_1$ is : $y = m_1x + sinY$ - m_1cosY

Equation of line$_2$ is : $y = m_2x + m_2X\ cosY$ - $XsinY$

3) Find the intersection point of l_1 and l_2 :

x_coord =(cosY($\mathrm{m}_1 + m_2X$) - $sinY(1+X)$)/($\mathrm{m}_1 - m_2$)

y_coord = $(m_1m_2cosY(X+1) - sinY(m_1X+m_2))$ / $(m_1 - m_2)$

Hence, we get the position of mars using the above formula for calculating x_coordinate and y_coordinate.

3. Result

I have used initial value of X and Y as 5 and 3 respectievely. Hence, The final value of X and Y after running the algorithm is as follows:

$X = 0.96792145$ and $Y = 2.5983632$

Also, the loss in this case is : 0.0040977282210210575

Plots:

The plot of 12 position of mars is as below: