PSet 4 Notes

Econ 6020: Macro Theory I

Supplemental Notes on Wickens

Notes on Wickens Prob 13.4

Model:

$$x_t = -\beta(R_t - E_t \pi_{t+1} - r) \quad (1)$$

$$\pi_t = E_t \pi_{t+1} + \alpha x_t + e_t \quad (2)$$

$$R_t = \gamma(E_t \pi_{t+1} - \pi^*) \quad (3)$$

(a) Eqn (3) is mis-specified. In the long-run Equil (i.e., the non-stochastic steady state) $\pi_t = E_t \pi_{t+1} = \pi^*$. This in (3) implies that the L.R. Equil value of the nominal fed funds rate is

$$R^* = 0 \quad (4)$$

But if LR real interest rate is r and L.R. inflation rate is π^* then, according to Fisher eqn the L.R. nominal interest rate should be

$$R^* = r + \pi^* \quad (5)$$

Thus, a better specification of the interest rate rule is

$$R_t = r + \pi^* + \gamma\left(E_t \pi_{t+1} - \pi^*\right) \qquad (3')$$

b.) Using (1), (2) and (3') we can find the L.R. equil (Non-stochastic steady state) values of R_t, π_t, and x_t. As already argued R^* is given by (5). This is consistent with (3') when $\pi^* = E_t \pi_{t+1}$.

Use (5) in (1) with $E_t \pi_{t+1} = \pi^*$ to get

$$x^* = 0 \qquad (6)$$

Use (6) in (2) noting that in the Non-stochastic steady state $e_t = 0$ to get

$$\pi^* = \pi^* \qquad (7)$$

c) For "short-run" solution [Rat'l Expectations — equil values of x_t, π_t, R_t] use (3') in (1)

to get

$$x_t = -\beta\left[(r+\pi^*) + \gamma \underset{t}{E}\pi_{t+1} - \gamma\pi^* - \underset{t}{E}\pi_{t+1} - r\right]$$

or

$$x_t = -\beta(\gamma-1)\left[\underset{t}{E}\pi_{t+1} - \pi^*\right] \qquad (8)$$

Use (8) in (2) to get

$$\pi_t = \underset{t}{E}\pi_{t+1} - \alpha\beta(\gamma-1)\underset{t}{E}\pi_{t+1} + \alpha\beta(\gamma-1)\pi^* + e_t$$

or

$$\pi_t = \left[1-\alpha\beta(\gamma-1)\right]\underset{t}{E}\pi_{t+1} + \alpha\beta(\gamma-1)\pi^* + e_t \qquad (9)$$

Re-write as

$$\left[1-\alpha\beta(\gamma-1)\right]\underset{t}{E}\pi_{t+1} - \pi_t = -\alpha\beta(\gamma-1)\pi^* - e_t$$

or

$$\underset{t}{E}\pi_{t+1} - \lambda\pi_t = -\lambda\alpha\beta(\gamma-1)\pi^* - \lambda e_t \qquad (10)$$

where $\lambda \equiv \left[1-\alpha\beta(\gamma-1)\right]^{-1}$

Note that, if $\gamma > 1$, $[1 - \alpha\beta(\gamma - 1)] < 1$ and $\lambda > 1$.

Write (10) as

$$[1 - \lambda L] \underset{t}{E} \pi_{t+1} = -\lambda \alpha\beta(\gamma-1)\pi^* - \lambda e_t$$

and solve λ forward

$$\underset{t}{E} \pi_{t+1} = \left(\frac{1}{1-\lambda L}\right)\left[-\lambda\alpha\beta(\gamma-1)\pi^* - \lambda e_t\right]$$

$$\underset{t}{E} \pi_{t+1} = \left[\frac{-\lambda^{-1}L^{-1}}{1-\lambda^{-1}L^{-1}}\right]\left[-\lambda\alpha\beta(\gamma-1)\pi^* - \lambda e_t\right]$$

or

$$\pi_t = \left[\frac{1}{1-\lambda^{-1}L^{-1}}\right]\alpha\beta(\gamma-1)\pi^* + \left[\frac{1}{1-\lambda^{-1}L^{-1}}\right]e_t \qquad (11)$$

Note that

$$\left[\frac{1}{1-\lambda^{-1}L^{-1}}\right]\alpha\beta(\gamma-1)\pi^* = \left[\frac{1}{1-\lambda^{-1}}\right]\alpha\beta(\gamma-1)\pi^* = \pi^* \qquad (12)$$

and that

$$\left[\frac{1}{1-\lambda^{-1}L^{-1}}\right]e_t = \sum_{s=0}^{\infty} \lambda^{-s} \underset{t}{E}\, e_{t+s} = \sum_{s=0}^{\infty}[1-\alpha\beta(\gamma-1)]^s \underset{t}{E}\, e_{t+j} \qquad (13)$$

Note by comparing RHS(13) to ~~Solution~~ same eqn in Wickens's Soln manual that there is a typo in Wickens's Soln.

Use (12) and (13) in (11) and Note that, since $e_t \sim iid(0, \sigma_e^2)$ $\underset{t}{E}\, e_{t+j} = 0$ for $j = 1, 2, 3, \ldots$ to get

$$\pi_t = \pi^* + e_t \qquad (14)$$

~~For R_t Note that $\underset{t}{E}\, \pi_{t+1} = 0$~~

For R_t, Note that $\pi_{t+1} = \pi^* + e_{t+1}$ So $\underset{t}{E}\, \pi_{t+1} = \pi^*$ and Thus, using (3′)

$$R_t = r + \pi^* \qquad (15)$$

This and $\underset{t}{E}\, \pi_{t+1} = \pi^*$ in (1) give

$$x_t = 0 \qquad (16)$$

Answers to 13.4, d i, e and 13.5 are available at the website listed on page ~~x~~ 14 of the text.

Additional Problem 1:

$$x_t = -(R_t - E_t \pi_{t+1} - r) \quad (1)$$

$$\pi_t = \beta E_t \pi_{t+1} + x_t + e_t \quad (2)$$

$$R_t = r + \gamma \pi_t \quad (3)$$

where $0 < \beta < 1$ and $1 < \gamma$.

A.) Eqn (1) is an "IS curve". It says that an increase in the real interest rate will lower x_t. This would arise from the intertemporal Euler eqn via an intertemporal substitution effect where

$\uparrow (R_t - E_t \pi_{t+1}) \Rightarrow \downarrow C_t$

Eqn (2) is the Calvo AS curve. Firms that are setting their prices in the current period will set them ~~higher~~ higher if expected future π is higher or if x_t is high (so that marginal costs are high). As a result current inflation, π_t, increases if either $E_t \pi_{t+1}$ or x_t increases.

Eqn(3) is a Taylor Rule. The Taylor Rule in general form is

$$R_t = r + \pi^* + \gamma(\pi - \pi^*) \qquad (3')$$

where π^* is the target inflation rate. Comparing (3') to (3) it is clear that $\pi^* = 0$, that is, the target inflation rate is zero. From (3), (or (3')) it is clear that the Fed will increase ~~unless~~ the interest rate in response to an increase in π_t. The Taylor Principle requires that the Fed increase the Fed Funds Rate by an amount that is greater than any increase in π_t. In terms of ~~eqn~~ eqn(3) this requires that $\gamma > 1$ which holds in this specification: The Taylor principle is satisfied here.

B.) Derive the REE values of x_t, π_t, R_t.

① First π_t. Use (3) in (1) to get

$$x_t = -\left[r + \gamma\pi_t - E_t\pi_{t+1} - r\right] \text{ or}$$

$$x_t = E_t\pi_{t+1} - \gamma\pi_t \qquad (4)$$

Use (4) in (2) to get

$$\pi_t = \beta E_t\pi_{t+1} + E_t\pi_{t+1} - \gamma\pi_t + e_t \quad \text{or}$$

$$(1+\gamma)\pi_t = (1+\beta)E_t\pi_{t+1} + e_t \quad \text{or}$$

$$(1+\beta)E_t\pi_{t+1} - (1+\gamma)\pi_t = -e_t \quad \text{or}$$

$$E_t\pi_{t+1} - \left[\frac{1+\gamma}{1+\beta}\right]\pi_t = \left(\frac{-1}{1+\beta}\right)e_t \qquad (5)$$

Let $\left[\frac{1+\gamma}{1+\beta}\right] \equiv \lambda$ and note that, since $0 < \beta < 1 < \gamma$, it follows that $\lambda > 1$.

Write (5) as $\underset{t}{E}\,\pi_{t+1} - \lambda \pi_t = \left(\frac{-1}{1+\beta}\right) e_t$ or

$(1-\lambda L)\underset{t}{E}\,\pi_{t+1} = \left(\frac{-1}{1-\beta}\right) e_t$ or, Solving λ Forward,

$$\underset{t}{E}\,\pi_{t+1} = \left(\frac{1}{1-\lambda L}\right)\left(\frac{-1}{1+\beta}\right) e_t = \left(\frac{-\lambda^{-1}L^{-1}}{1-\lambda^{-1}L^{-1}}\right)\left(\frac{-1}{1+\beta}\right) e_t$$

or, multiplying through by L

$$\pi_t = \left(\frac{1}{\lambda}\right)\left(\frac{1}{1-\lambda^{-1}L^{-1}}\right)\left(\frac{1}{1+\beta}\right) e_t \quad \text{or}$$

$$\pi_t = \left(\frac{1+\beta}{1+\gamma}\right)\left(\frac{1}{1+\beta}\right)\sum_{j=0}^{\infty} \lambda^{-j}\,\underset{t}{E}\,e_{t+j} \quad \text{or}$$

$$\boxed{\pi_t = \left(\frac{1}{1+\gamma}\right)\sum_{j=0}^{\infty} \lambda^{-j}\,\underset{t}{E}\,e_{t+j} \qquad (6)}$$

Note $\underset{t}{E}\,e_{t+j} = e_t$ for $j=0$

But $\underset{t}{E}\,e_{t+j} = 0$ for $j = 1, 2, 3, \ldots$

Thus, (6) gives

$$\pi_t = \left(\frac{1}{1+\gamma}\right)\left[e_t + \lambda^{-1}\cdot 0 + \lambda^{-2}\cdot 0 + \cdots\right] \quad \text{or}$$

$$\boxed{\pi_t = \left(\frac{1}{1+\gamma}\right) e_t \qquad (7)}$$

② REE R_t use (7) in (3) to get

$$\boxed{R_t = r + \left(\frac{\gamma}{1+\gamma}\right) e_t \qquad (8)}$$

③ REE x_t. From (7) it follows that $\pi_{t+1} = \left(\frac{1}{1+\gamma}\right) e_{t+1}$.

Thus $\underset{t}{E}\, \pi_{t+1} = \left(\frac{1}{1+\gamma}\right) \underset{t}{E}\, e_{t+1} = 0$. This and (7) in (4)

gives
$$\boxed{x_t = \left(\frac{-\gamma}{1+\gamma}\right) e_t \qquad (9)}$$

C. From (7) it follows that

$$VAR(\pi_t) = \left(\frac{1}{1+\gamma}\right)^2 \sigma^2 \qquad (10)$$

So the policy that minimizes $VAR(\pi_t)$ is $\gamma \to +\infty$.

From (9) it follows that

$$VAR(x_t) = \left(\frac{\gamma}{1+\gamma}\right)^2 \sigma^2 \qquad (11)$$

Note ~~regard~~ regarding the coefficient on σ^2 on the RHS of (11) that

$$\frac{d\left(\frac{\gamma}{1+\gamma}\right)^2}{d\gamma} = 2\left(\frac{\gamma}{1+\gamma}\right)\left[\frac{1\cdot(1+\gamma)-1\cdot\gamma}{(1+\gamma)^2}\right] = 2\left(\frac{\gamma}{1+\gamma}\right)\left[\frac{1}{(1+\gamma)^2}\right]$$

Since $\gamma > 0$ it follows that

$\frac{d\left(\frac{\gamma}{1+\gamma}\right)^2}{d\gamma} > 0$. So to <u>minimize</u> the variance of (x_t) the Central Bank needs to minimize $\left(\frac{\gamma}{1+\gamma}\right)^2$. As $\left(\frac{\gamma}{1+\gamma}\right)^2$ is monotonically increasing in γ the Central Bank needs to set γ as small as possible. Since $\gamma > 1$ this means the Central Bank needs to <u>set γ as close as possible to 1 while maintaining $\gamma > 1$.</u>

Summarize And compare The Two Policies

Min $Var(\pi_t)$: $\gamma \to +\infty$ Set γ As large As Possible. The Central Bank must Raise R_t Aggressively in Response to inflationary Shocks.

Min $Var(x_t)$: $\gamma \downarrow 1$ but $\gamma > 1$. Set γ As small As Possible consistent with The Taylor Principle. The Central Bank must Raise R_t greater Than any increase in π_t to satisfy Taylor Principle – but just greater. The increase in R_t should be as small As Possible without violating $\gamma > 1$.

Note That The goals Min $Var(\pi_t)$ and Min $Var(x_t)$ are in Conflict; They Require opposite Policies. This is Because e_t is A (price) Supply Shock. In The Face of supply shocks The Central Bank's Policy objectives of stabilizing Both π_t and x_t are in conflict.