Sistemas Lineares I - Last Chance

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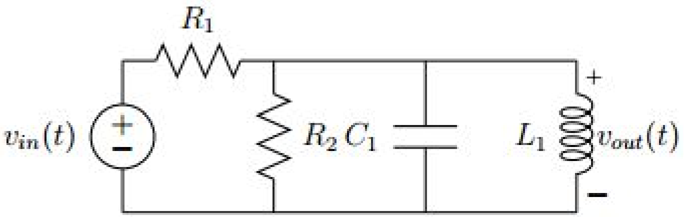
Agenda

- Analise de Circuitos
- Funções de Transferencia
- Polos e Zeros
- Diagrama de Bode
- Resposta do Sistema
- Serie de Fourier
- Diagrama de Blocos
- Resposta em frequencias variantes

Analise de Circuito



$$\frac{V_{in} - V_{out}}{R1} - \frac{V_{out}}{R2} - \frac{C\partial V_{out}}{\partial t} - \frac{1}{L}\int V_{out}\partial t = 0$$

Resistor: $v_R(t) = Ri_R(t) \rightarrow V_R(s) = RI_R(s)$

Capacitor: $v_c(t) = \int_0^1 i_c(\tau)d\tau \rightarrow V_c(s) = \frac{1}{sC}I_c(s) + \frac{v_c(0)}{s}$

Inductor: $v_L(t) = L\frac{di_L(t)}{dt} \rightarrow V_L(s) = sLI_L(s) - Li_L(0)$

- Leis de Kirchhoff
- Analise Nodal
- Equações dos componentes
- Transformada de Laplace para os componentes

Funções de Transferencia

- Linear, Invariante no Tempo e Causal (Definidos pela transformada de Laplace)
- Entrada unica, saida unica
- Razão entre a saida e a entrada

$$X(S)\left(\frac{1}{R_1}\right) = Y(S)\left(S^2C + S\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{1}{L}\right)$$

$$H(S) = \frac{Y(S)}{X(S)} = \frac{SR_2L}{S^2\left(R_1R_2LC\right) + S\left(R_1L + R_2L\right) + R_1R_2}$$

Polos e Zeros

- $R_1 = 10\Omega;$
- $R_2 = 100\Omega;$
- $C = 1F;$
- $L = 1H;$

Apos aplicar os valores comercias em H(S), temos:

$$H(S) = \frac{100S}{1000S^2 + 110S + 110}$$

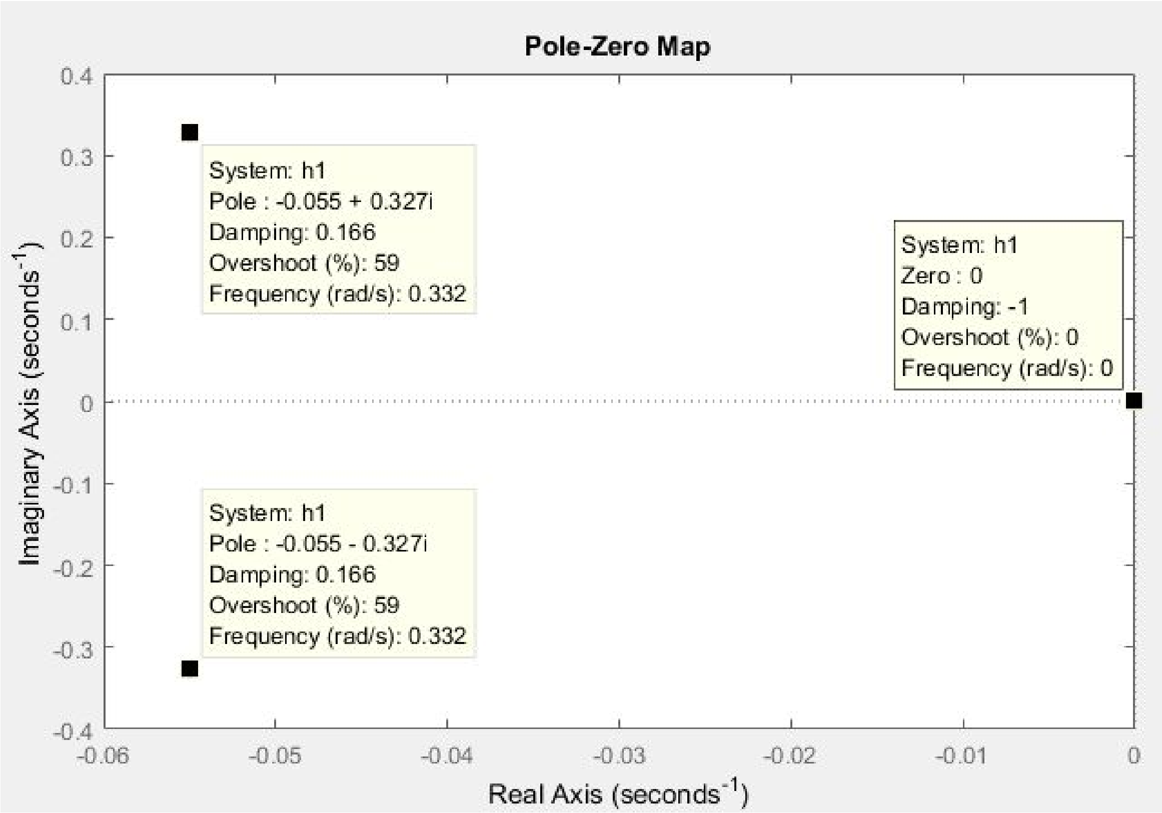
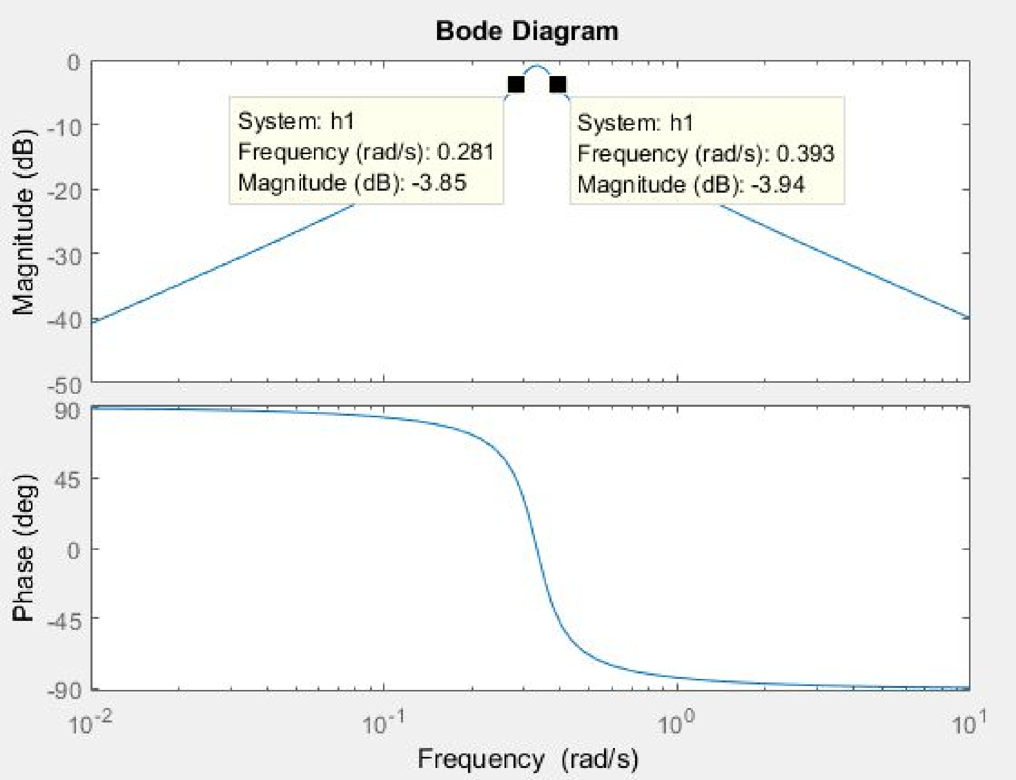


Diagrama de Bode



Resposta a sinais em geral

Exemplo: Encontre a resposta da função de transferência: $H(S) = \frac{S+2}{S^2+5S+4}$ para o sinal $x(t) = 5cos(2t + 30°)$

$$H(j\omega) = \frac{j\omega + 2}{-\omega^2 + 5j\omega + 4}$$

sabemos que:

$$|H(j\omega)| = \frac{\sqrt{(\mathbb{R}\mathrm{e}_1)^2 + (\mathbb{I}\mathrm{m}_1)^2}}{\sqrt{(\mathbb{R}\mathrm{e}_2)^2 + (\mathbb{I}\mathrm{m}_2)^2}} \leftrightarrow \angle = \arctan\left(\frac{\mathbb{I}\mathrm{m}_1}{\mathbb{R}\mathrm{e}_1}\right) - \arctan\left(\frac{\mathbb{I}\mathrm{m}_2}{\mathbb{R}\mathrm{e}_2}\right)$$

e que:

$$y(t) = |H(j\omega)| \cos(\omega t + \phi + \angle H(j\omega))$$

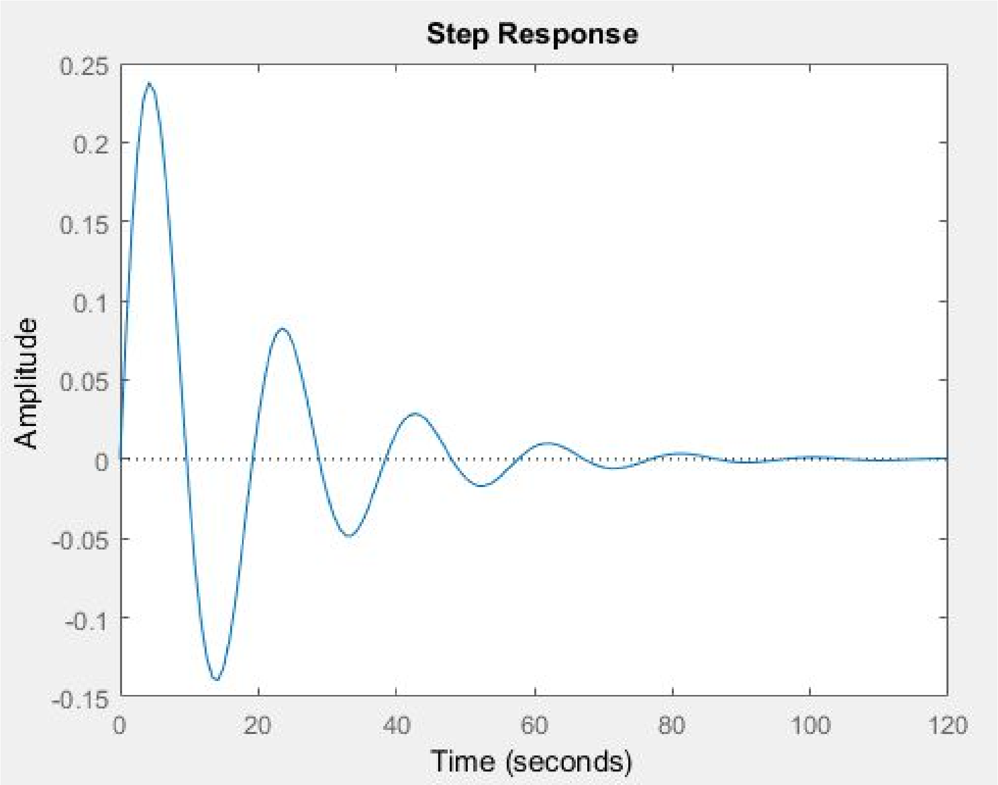
logo, substituindo os valores em:

$$|H(j\omega)| = \frac{\sqrt{\omega^2 + 4}}{\sqrt{(5j\omega)^2 + (4 - \omega^2)}} \leftrightarrow \angle H(j\omega) = \arctan\left(\frac{\omega}{2}\right) - \arctan\left(\frac{5\omega}{4 - \omega^2}\right)$$

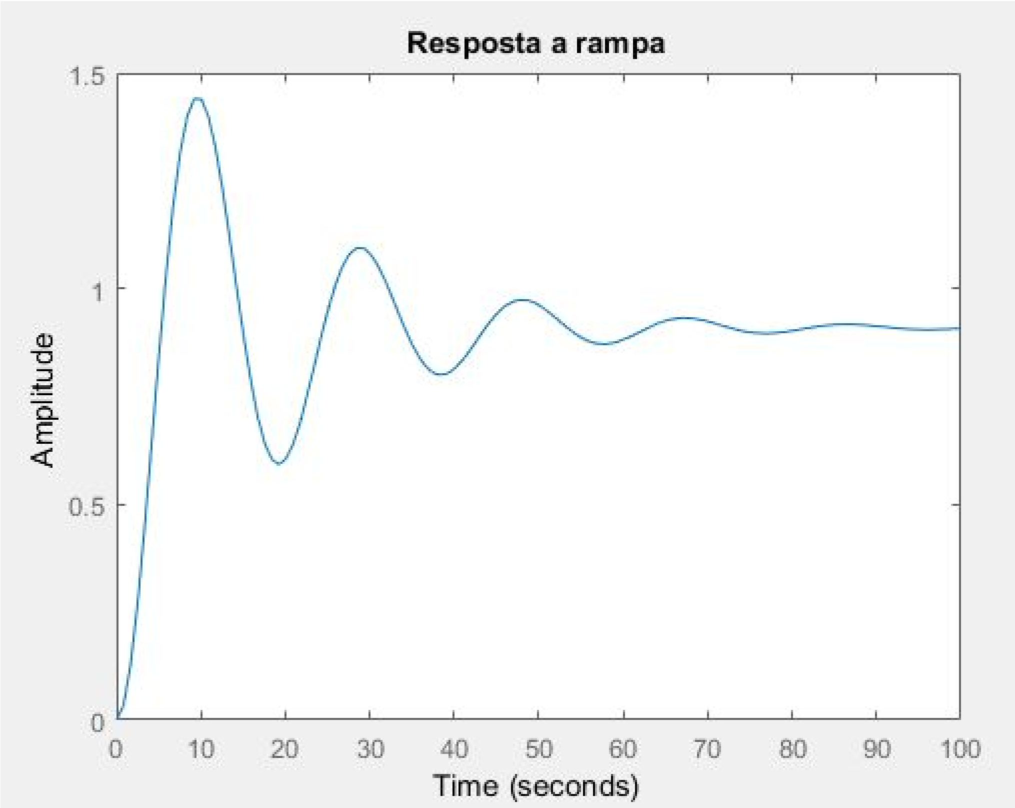
temos:

$$y(t) = \sqrt{2}\cos(2t - 15°)$$

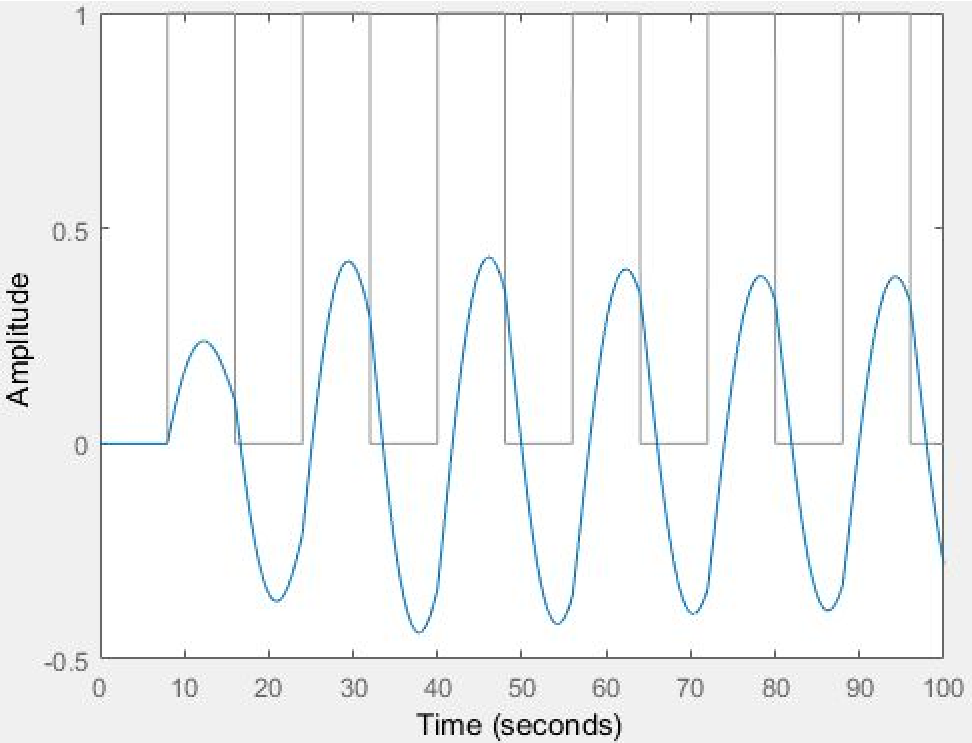
Resposta ao Degrau



Resposta a Rampa



Resposta a onda quadrada



Serie de fourier

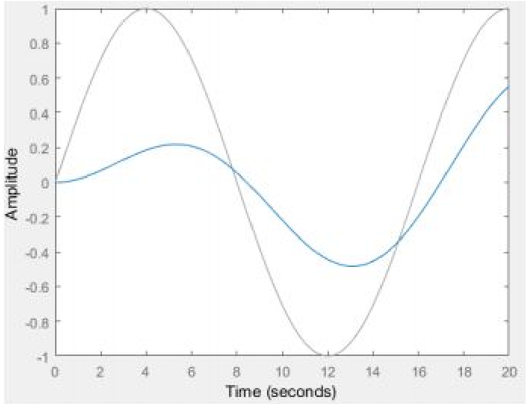


Figura 7: Circuito 1 - Resposta ao primeiro harmônico da série de Fourier de um onda quadrada com $\omega = \frac{1}{8}\pi$

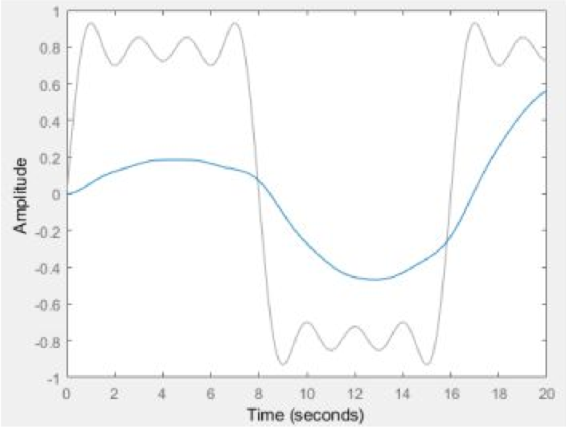
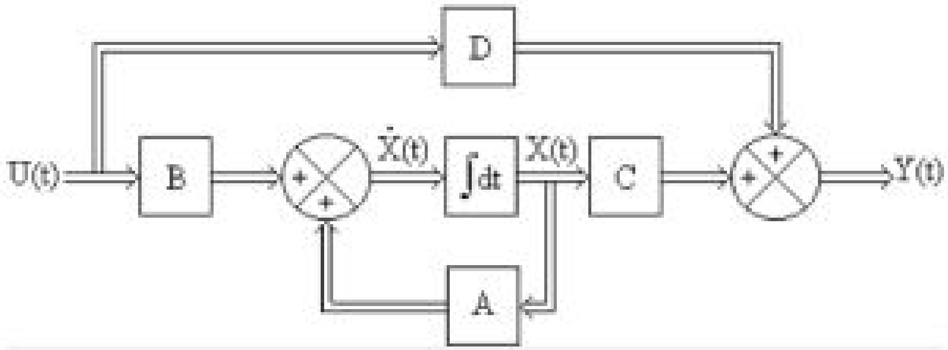


Figura 10: Circuito 1 - Resposta ao sétimo harmônico da série de Fourier de um onda quadrada com $\omega = \frac{1}{8}\pi$

Diagrama de Blocos



- a = -22;
- b = 7;
- c = 3;
- d = 4;

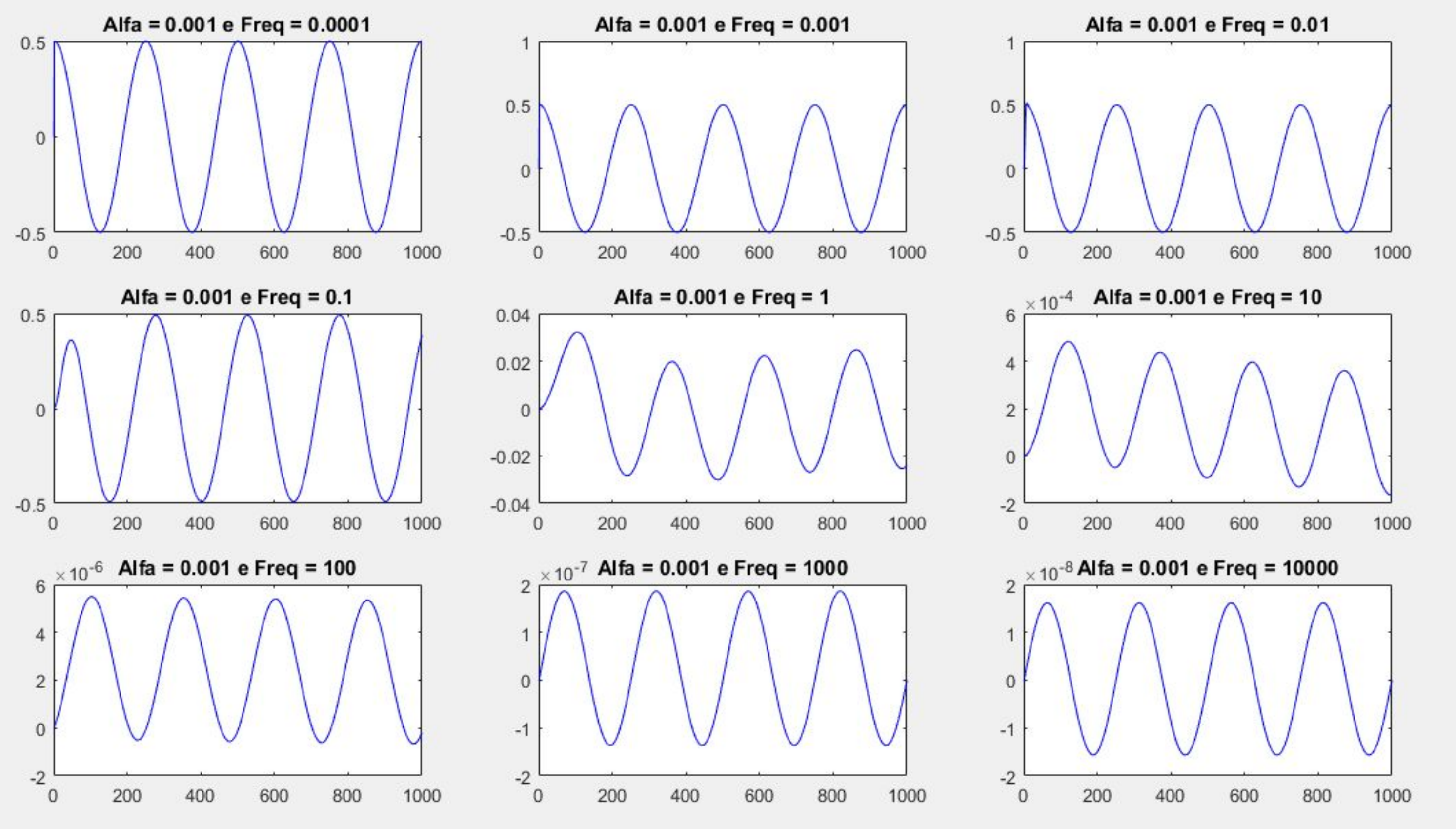
- y(t) = 4u(t) + 3x(t) *(Item (e) da questão 2)*;
- B = 7u(t);
- C = 3x(t);
- D = 4u(t);
- $x'(t) = 7u(t) - 22x(t)$ *(Item (d) da questão 2)*;

Sabendo que $x(t) = \frac{y(t)-4u(t)}{3}$ e $x' = \frac{y'(t)-4u'(t)}{3}$, obtemos a seguinte E.D.O:

$$\frac{\partial y(t)}{\partial t} + 22y(t) = 4\frac{\partial y(t)}{\partial t} + 109u(t)$$

Resposta em frequencias variantes

$$H(S) = \frac{1 + \alpha S}{S^2 + 2S + 2}$$



$$\frac{1+\mathrm{Sin}x}{n} =$$

$$= \frac{1+\mathrm{Si}\cancel{n}x}{\cancel{n}} =$$

$$= 1+\mathrm{Six} =$$

$$= 7$$

Conclusão

Referencias

[1] https://en.wikipedia.org/wiki/Buffer_amplifier;

[2] https://en.wikipedia.org/wiki/Electronic_filter;

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[5] https://en.wikipedia.org/wiki/Butterworth_filter;

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Questions?