

Universidade Federal do Rio de Janeiro

Lista II - Sistemas Lineares I

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Conteúdo

| | | |
|----------|---|----------|
| 1 | Análise de Sistema no Domínio do Tempo | 1 |
| 1.1 | Questão 1 | 1 |
| 1.1.1 | Item a | 1 |
| 1.1.2 | Item b | 1 |
| 1.2 | Questão 2 | 1 |
| 1.2.1 | Item a | 1 |
| 1.2.2 | Item b | 1 |
| 1.3 | Questão 3 | 1 |
| 1.3.1 | Item a | 1 |
| 1.3.2 | Item b | 2 |
| 1.4 | Questão 4 | 2 |
| 1.4.1 | Item a | 2 |
| 1.4.2 | Item b | 2 |
| 1.5 | Questão 5 | 2 |
| 1.6 | Questão 6 | 3 |
| 1.7 | Questão 7 | 3 |
| 1.8 | Questão 8 | 3 |
| 1.9 | Questão 9 | 3 |
| 1.10 | Questão 10 | 3 |
| 1.10.1 | Item a | 3 |
| 1.10.2 | Item b | 4 |
| 1.10.3 | Item c | 4 |
| 1.10.4 | Item d | 4 |
| 1.11 | Questão 11 | 4 |
| 1.11.1 | Item a | 4 |
| 1.11.2 | Item b | 5 |
| 1.12 | Questão 12 | 5 |
| 1.13 | Questão 13 | 6 |
| 1.14 | Questão 14 | 6 |
| 1.15 | Questão 15 | 7 |
| 1.15.1 | Item a | 7 |
| 1.15.2 | Item b | 8 |
| 1.15.3 | Item c | 8 |
| 1.16 | Questão 16 | 8 |
| 2 | Análise de Sistema no Domínio Laplace | 9 |
| 2.1 | Questão 1 | 9 |
| 2.1.1 | Item a | 9 |
| 2.1.2 | Item b | 9 |

| | | |
|-------|---------------------|----|
| 2.1.3 | Item c | 9 |
| 2.1.4 | Item d | 9 |
| 2.1.5 | Item e | 9 |
| 2.1.6 | Item f | 9 |
| 2.1.7 | Item g | 10 |
| 2.2 | Questão 2 | 10 |
| 2.2.1 | Item a | 10 |
| 2.2.2 | Item b | 10 |
| 2.2.3 | Item c | 10 |
| 2.3 | Questão 3 | 10 |
| 2.3.1 | Item a | 10 |
| 2.3.2 | Item b | 10 |
| 2.3.3 | Item c | 11 |
| 2.4 | Questão 4 | 11 |
| 2.4.1 | Item a | 11 |
| 2.4.2 | Item b | 11 |
| 2.5 | Questão 5 | 11 |
| 2.5.1 | Item a | 11 |
| 2.5.2 | Item b | 11 |
| 2.6 | Questão 6 | 11 |
| 2.6.1 | Item a | 11 |
| 2.6.2 | Item b | 11 |
| 2.6.3 | Item c | 11 |
| 2.6.4 | Item d | 11 |
| 2.7 | Questão 7 | 12 |
| 2.7.1 | Item a | 12 |
| 2.7.2 | Item b | 12 |
| 2.8 | Questão 8 | 12 |

Lista de Figuras

| | | |
|---|--|---|
| 1 | Resposta ao Estado Nulo - Questão 11 | 5 |
| 2 | Sinal $\frac{1}{t^2+1}$ | 5 |
| 3 | Sinal Degrau | 5 |
| 4 | Convolução dos sinais | 6 |
| 5 | $\sin(t) * u(t)$ | 7 |

1 Análise de Sistema no Domínio do Tempo

1.1 Questão 1

1.1.1 Item a

$$\lambda^2 + 5\lambda + 6 = 0 \rightarrow \lambda_1 = -2; \lambda_2 = -3 \rightarrow c_1 e^{-2t} + c_2 e^{-3t}$$

1.1.2 Item b

$$\begin{aligned} c_1 + c_2 &= 2 \\ -2c_1 - 3c_2 &= -1 \\ c_1 &= 5 \\ c_2 &= -3 \\ y_0 &= 5e^{-2t} - 3e^{-3t} \end{aligned}$$

1.2 Questão 2

1.2.1 Item a

$$\lambda^2 + \lambda = 0 \rightarrow \lambda_1 = 0; \lambda_2 = -1 \rightarrow c_1^{-2+3j} + c_2 e^{-t}$$

1.2.2 Item b

$$\begin{aligned} c_1 + c_2 &= 1 \\ -c_2 &= 1 \\ c_1 &= 2 \\ c_2 &= -1 \\ y_0 &= 2 - e^{-t} \end{aligned}$$

1.3 Questão 3

1.3.1 Item a

$$\lambda^2 + 4\lambda + 13 = 0 \rightarrow \lambda_1 = -2+3j; \lambda_2 = -2-3j \rightarrow c_1 e^{(-2+3j)t} + c_2 e^{(-2-3j)t} = c e^{-2t} \cos(3t+\phi)$$

1.3.2 Item b

$$\begin{aligned}ccos(\phi) &= 1 \\ -2ccos(\phi) - 3csen(\phi) &= 15.98 \\ c &= 10 \\ \phi &= \frac{-\pi}{3} \\ y_0 &= 10e^{-2t}cos(3t - \frac{\pi}{3})\end{aligned}$$

1.4 Questão 4

1.4.1 Item a

$$(\lambda+1)(\lambda^2+5\lambda+6) = 0 \rightarrow \lambda_1 = -1; \lambda_2 = -2; \lambda_3 = -3 \rightarrow c_1e^{-t} + c_2e^{-2t} + c_3e^{-3t}$$

1.4.2 Item b

$$\begin{aligned}c_1 + c_2 + c_3 &= 2 \\ -c_1 - 2c_2 - 3c_3 &= -1 \\ c_1 + 4c_2 + 9c_3 &= 5 \\ c_1 &= 6 \\ c_2 &= -7 \\ c_3 &= 3 \\ y_0 &= 6e^{-t} - 7e^{-2t} + 3e^{-3t}\end{aligned}$$

1.5 Questão 5

$$\begin{aligned}A_c &= \int_{-\infty}^{\infty} c(t) \\ c(t) &= \int_{-\infty}^{\infty} x(\tau)g(t-\tau)d\tau \\ A_c &= \int_{-\infty}^{\infty} [\int_{-\infty}^{\infty} x(\tau)g(t-\tau)d\tau]dt \rightarrow \int_{-\infty}^{\infty} [\int_{-\infty}^{\infty} x(\tau)d\tau][g(t-\tau)]dt \Rightarrow \\ &\int_{-\infty}^{\infty} A_x[g(t-\tau)]dt = A_x \int_{-\infty}^{\infty} g(t-\tau)dt = A_x A_g\end{aligned}$$

1.6 Questão 6

$$c(t) = x(t) * g(t) \rightarrow \int_{-\infty}^{\infty} x(\tau)g(t - \tau)d\tau$$

$$c(at) = x(at)*g(at) \rightarrow \int_{-\infty}^{\infty} x(a\tau)g(a[t-\tau])d\tau \rightarrow \frac{1}{a} \int_{-\infty}^{\infty} x(w)g(at-w)dw = \frac{c(at)}{a}$$

1.7 Questão 7

$$y(t) = x_{par}(t) * g_{impar}(t) \rightarrow \int_{-\infty}^{\infty} x(\tau)g(t - \tau)d\tau \Rightarrow$$
$$\int_{-\infty}^{\infty} \frac{[x(\tau) + x(-\tau)]}{2} \frac{g(t - \tau) - g(\tau - t)}{2} d\tau$$

Travei

1.8 Questão 8

$$e^{-at}u(t)*e^{-bt}u(t) \rightarrow \int_0^{\infty} e^{-a\tau}e^{-b(t-\tau)}d\tau \rightarrow \int_0^{\infty} e^{-a\tau}e^{-bt}e^{b\tau}d\tau \rightarrow e^{-bt} \int_0^{\infty} e^{-\tau(a-b)}d\tau \Rightarrow$$

$$e^{-bt} \left[\frac{-1}{a-b} \right] = \frac{-e^{-bt}}{a-b} u(t)$$

1.9 Questão 9

$$\int_{-\infty}^{+\infty} \sin(\tau)u(t - \tau)d\tau \rightarrow \int_0^t \sin(\tau)d\tau = (-\cos(t) + 1)u(t)$$

$$\int_{-\infty}^{+\infty} \cos(\tau)u(t - \tau)d\tau \rightarrow \int_0^t \cos(\tau)d\tau = \sin(t)u(t)$$

1.10 Questão 10

1.10.1 Item a

$$\int_{-\infty}^{+\infty} e^{-\tau}u(t)u(t - \tau)d\tau \rightarrow \int_0^t e^{-\tau}d\tau = (-e^{-t} + 1)u(t)$$

1.10.2 Item b

$$\int_{-\infty}^{+\infty} e^{-\tau} u(t) e^{-(t-\tau)} u(t) d\tau \rightarrow \int_0^t e^{-\tau} e^{-(t-\tau)} d\tau = e^{-t} \int_0^t d\tau = t e^{-t}$$

1.10.3 Item c

$$\int_{-\infty}^{+\infty} e^{-\tau} u(t) e^{-2(t-\tau)} u(t) d\tau \rightarrow \int_0^t e^{-\tau} e^{-2(t-\tau)} d\tau \rightarrow e^{-2t} \int_0^t e^{\tau} d\tau = e^{-2t} (e^t - 1)$$

1.10.4 Item d

$$\int_{-\infty}^{+\infty} e^{-\tau} u(t) \sin(3(t-\tau)) u(t) d\tau \rightarrow \int_0^t e^{-\tau} \sin(3t-3\tau) d\tau = \frac{3e^{-t} + \sin(3t) - 3\cos(3t)}{10}$$

1.11 Questão 11

1.11.1 Item a

$$\begin{aligned} \int_{-\infty}^{+\infty} e^{-\tau} u(t) [-\delta(t-\tau) + 2e^{-(t-\tau)}] u(t) d\tau &\rightarrow \int_0^{+\infty} e^{-\tau} - \delta(t-\tau) d\tau + \int_0^t e^{-\tau} 2e^{-(t-\tau)} d\tau \Rightarrow \\ &-e^{-t} + 2te^{-t} \end{aligned}$$

1.11.2 Item b

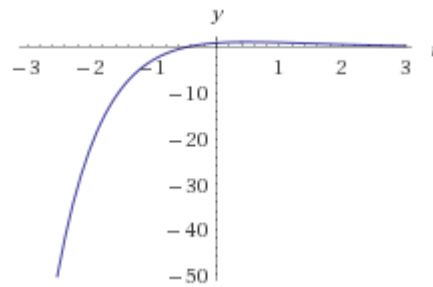


Figura 1: Resposta ao Estado Nulo - Questão 11

1.12 Questão 12

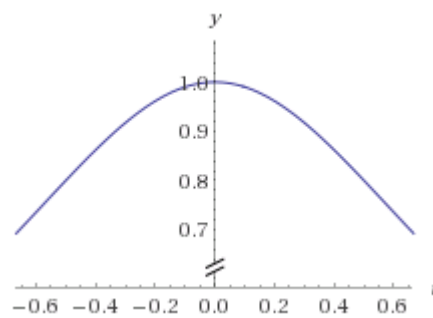


Figura 2: Sinal $\frac{1}{t^2+1}$

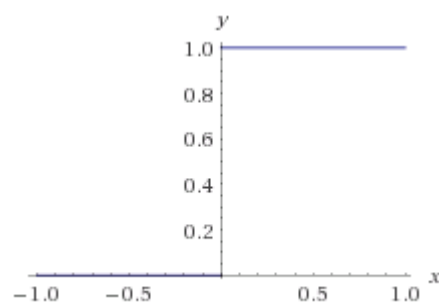


Figura 3: Sinal Degrau

$$\int_{-\infty}^{+\infty} \frac{1}{\tau^2 + 1} u(t) d\tau \rightarrow \int_0^{+\infty} \frac{1}{\tau^2 + 1} = \arctan(t) + \frac{\pi}{2}$$

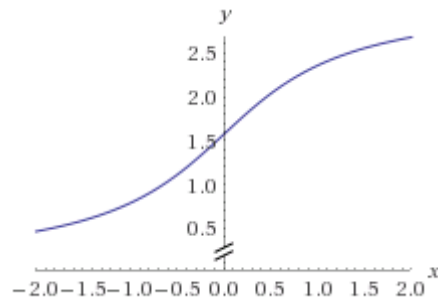
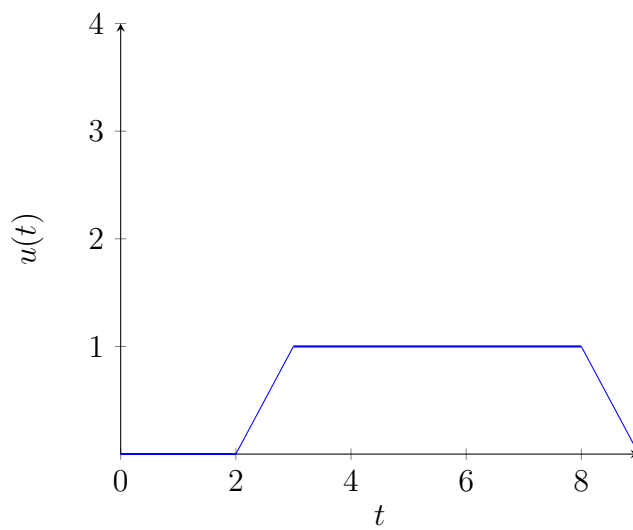


Figura 4: Convolução dos sinais

1.13 Questão 13



1.14 Questão 14

$$\int_{-\infty}^{+\infty} \sin(\tau) u(t - \tau) d\tau \rightarrow \int_0^t \sin(\tau) d\tau = 1 - \cos(t)$$

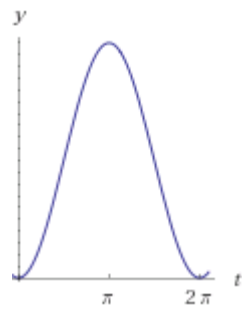
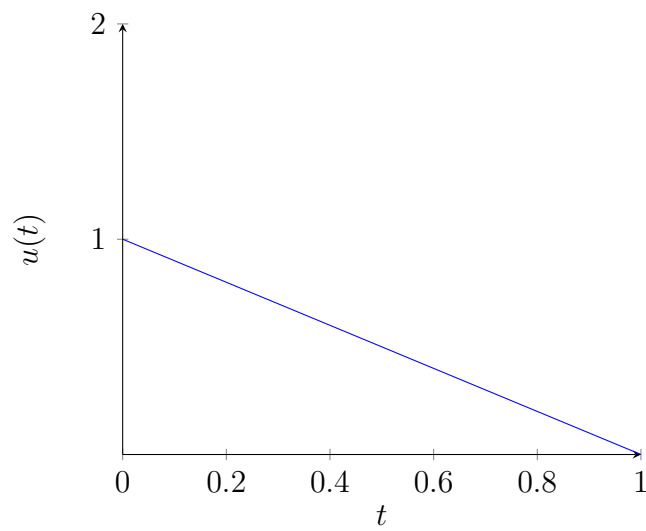
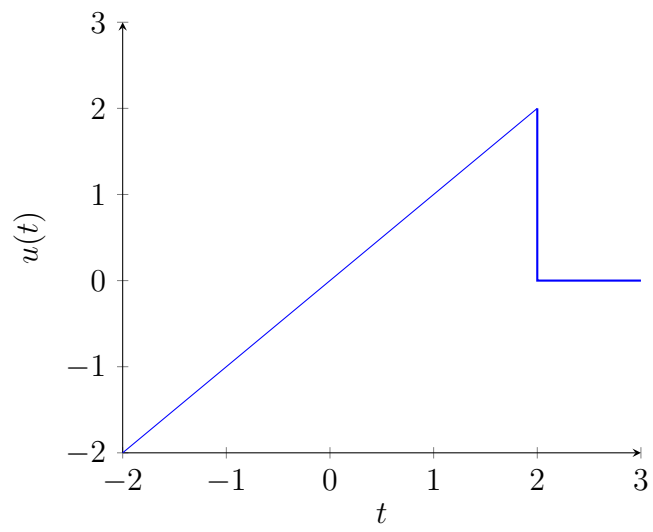


Figura 5: $\sin(t) * u(t)$

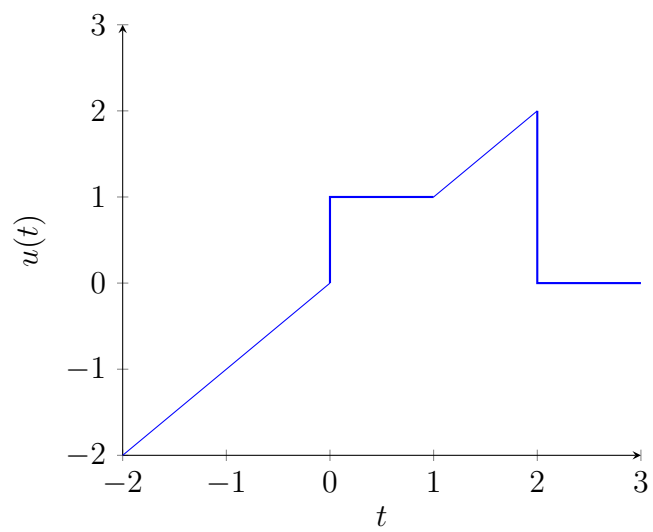
1.15 Questão 15

1.15.1 Item a





1.15.2 Item b



1.15.3 Item c

Não consegui fazer

1.16 Questão 16

Não consegui fazer

2 Análise de Sistema no Domínio Laplace

2.1 Questão 1

2.1.1 Item a

$$\int_0^1 e^{-st} dt = \frac{(1 - e^{-s})}{s}$$

2.1.2 Item b

$$\int_0^\infty te^{-t}e^{-st}dt = \int_0^\infty te^{-t(s+1)}dt = \frac{1}{(s+1)^2}$$

2.1.3 Item c

$$\begin{aligned}\int_0^\infty t\cos(w_0t)e^{-st}dt &= \frac{1}{2}\int_0^\infty (e^{jw_0t} + e^{-jw_0t})te^{-st}dt = \frac{1}{(s+1)^2} \Rightarrow \\ \frac{1}{2}\int_0^\infty (te^{t(s-jw_0)} + te^{t(s+jw_0)})dt &= \frac{1}{2(s-jw_0)^2} + \frac{1}{2(s+jw_0)^2}\end{aligned}$$

2.1.4 Item d

$$\int_0^\infty (e^{2t} - 2e^{-t})e^{-st}dt = \int_0^\infty e^{-t(s-2)}dt - 2\int_0^\infty e^{-t(s+1)}dt = \frac{1}{s-2} - \frac{2}{s+1}$$

2.1.5 Item e

$$\begin{aligned}\int_0^\infty \cos(w_1t)\cos(w_2t)e^{-st}dt &= \frac{1}{4}\int_0^\infty (e^{jw_1t} + e^{-jw_1t})(e^{jw_2t} + e^{-jw_2t})e^{-st}dt \Rightarrow \\ \frac{1}{2}\left[\frac{s}{s^2 + (w_1 + w_2)^2} + \frac{s}{s^2 + (w_1 - w_2)^2}\right]\end{aligned}$$

2.1.6 Item f

Cosseno Hiperbólico é tenso!

2.1.7 Item g

Seno Hiperbólico é tenso!

2.2 Questão 2

2.2.1 Item a

$$\int_0^1 te^{-st} dt = \frac{1 - e^{-s}(s+1)}{s^2}$$

2.2.2 Item b

$$\int_0^\pi \sin(t)e^{-st} dt = \frac{1 + e^{-\pi s}}{s^2 + 1}$$

2.2.3 Item c

$$\int_0^1 \frac{t}{e} e^{-st} dt + \int_1^\infty e^{-t(s+1)} dt = \frac{e^{-s+1}}{s+1} + \frac{1 - e^{-s}(s+1)}{es^2}$$

2.3 Questão 3

2.3.1 Item a

$$\begin{aligned} s^2 Y(s) - sy(0^-) - y'(0^-) + 3sY(s) - 3y(0^-) + 2y(t) &= sX(s) - x(0^-) \Rightarrow \\ s^2 Y(s) - 0 - 0 + 3sY(s) - 0 + 4Y(s) &= \frac{s}{s} \rightarrow Y(s) = \frac{1}{s^2 + 3s + 2} = \frac{1}{s+1} + \frac{1}{s+2} \\ y(t) &= e^{-t} + e^{-2t} \end{aligned}$$

2.3.2 Item b

$$\begin{aligned} s^2 Y(s) - sy(0^-) - y'(0^-) + 4sY(s) - 4y(0^-) + 4y(t) &= sX(s) - x(0^-) + X(s) \Rightarrow \\ s^2 Y(s) - 2s - 1 + 4sY(s) - 8 + 4Y(s) &= 1 \rightarrow (s^2 + 4s + 2)Y(s) = 2s + 10 \\ Y(s) &= \frac{2}{s+2} + \frac{6}{(s+2)^2} \Rightarrow (2+6t)e^{-2t} \end{aligned}$$

2.3.3 Item c

$$\begin{aligned}s^2Y(s) - sy(0^-) - y'(0^-) + 6sY(s) - 6y(0^-) &= sX(s) - x(0^-) + 2X(s) \Rightarrow \\s^2Y(s) - s - 1 + 6sY(s) - 6 + 25Y(s) &= 25 + \frac{50}{s} \rightarrow (s^2 + 6s + 25)Y(s) = \frac{s^2 + 32s + 50}{s} \\Y(s) &= \frac{s^2 + 32s + 50}{s(s^2 + 6s + 25)}\end{aligned}$$

2.4 Questão 4

2.4.1 Item a

2.4.2 Item b

2.5 Questão 5

2.5.1 Item a

2.5.2 Item b

2.6 Questão 6

2.6.1 Item a

$$H(s) = \frac{5s + 3}{s^2 + 11s + 24}$$

2.6.2 Item b

$$H(s) = \frac{3s^2 + 7s + 5}{s^3 + 6s^2 - 11s + 6}$$

2.6.3 Item c

$$H(s) = \frac{3s + 2}{s^4 + 4s}$$

2.6.4 Item d

$$H(s) = \frac{s - 1}{s^2 - 1}$$

2.7 Questão 7

2.7.1 Item a

2.7.2 Item b

2.8 Questão 8